Analysis Seminar

Title: Lipschitz-free spaces and integral representation II

Speaker: R. Smith

Date: Tue 7th March 2023 at 3:00PM

Location: Seminar Room SCN 1.25

Abstract: Given a complete metric space (M, d) with base point 0, let $\mathcal{F}(M)$ denote the Lipschitz-free space over M, which is a natural isometric predual of the space $\mathrm{Lip}_0(M)$ of real-valued Lipschitz functions on M that vanish at 0. Let $\widetilde{M}$ denote the set of ordered pairs of M having different coordinates, and let $\Phi : \mathrm{Lip}_0(M) \to C(\beta\widetilde{M})$ denote the de Leeuw representation map, given by $\Phi f(x, y) = (f(x) - f(y))/d(x, y)$, $(x, y) \in \widetilde{M}$, and extended continuously to the Stone-Čech compactification $\beta\widetilde{M}$. Then the dual map $\Phi^* : C(\beta\widetilde{M})^* \to \mathrm{Lip}_0(M)^*$ is a non-expansive surjection.

In this second talk, we introduce the 'Lipschitz realcompactification' $M^{\mathcal{R}}$ of M and a natural lower semicontinuous extension $\bar{d}$ of the metric d to $M^{\mathcal{R}}$. Using $(M^{\mathcal{R}}, \bar{d})$, we analyse those elements $\psi \in \mathrm{Lip}_0(M)^* \equiv \mathcal{F}(M)^{**}$ that 'avoid infinity' (these include all elements of $\mathcal{F}(M)$). Our main result is that if $\psi \in \mathrm{Lip}_0(M)^*$ avoids infinity and the measure $\lambda \in C(\beta\widetilde{M})^*$ is an optimal representation of ψ, in the sense that $\lambda \geq 0$, $\Phi^*\lambda = \psi$ and $\|\lambda\| = \|\psi\|$, then λ is concentrated on a set $H \subseteq \beta\widetilde{M}$ whose natural image in $M^{\mathcal{R}} \times M^{\mathcal{R}}$ is $\bar{d}$-cyclically monotonic. Cyclical monotonicity is a key property in optimal transport theory enjoyed by optimal transport plans. In addition, we present the best possible partial converse.

This is joint work with R. J. Aliaga (Valencia Polytechnic University) and E. Pernecká (Czech Technical University, Prague).

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