Analysis Seminar

Title: Lipschitz-free spaces and integral representation I

Speaker: R. Smith

Date: Tue 28th February 2023 at 3:00PM

Location: Seminar Room SCN 1.25

Abstract: Given a complete metric space (M, d) with base point 0, let $\mathrm{Lip}_0(M)$ denote the real Banach space of Lipschitz functions that vanish at 0. This space has an isometric predual,denoted $\mathcal{F}(M)$, which is the norm-closure in the dual $\mathrm{Lip}_0(M)^*$ of the set of elementary molecules $m_{x,y} : \mathrm{Lip}_0(M) \to R$, $x \neq y \in M$, defined by $m_{x,y}(f) = (f(x) - f(y))/d(x, y)$. This predual is sometimes called the Lipschitz-free space of M; these spaces have close links with optimal transport theory.

Given $m \in \mathcal{F}(M)$ and $\varepsilon > 0$, we can always express m as a sum of the form $m = \sum_{i=1}^{\infty} a_n m_{x_i,y_i}$, where $\sum_{i=1}^{\infty} |a_n| \leq \|m\| + \varepsilon$. Distinguished among the elements of $\mathcal{F}(M)$ are those for which we can set $\varepsilon = 0$ in the sum above; equivalently, these elements can be viewed as Bochner integrals in $\mathcal{F}(M)$ with respect to discrete measures concentrated on the set of elementary molecules.

In the first of two talks, we analyse the properties of elements that can be represented as Bochner integrals with respect to more general measures and derive some consequences, e.g. applications to the extremal structure of $\mathcal{F}(M)$. This analysis will be done indirectly, using the integral representation of elements of $\mathrm{Lip}_0(M)^*$

established by de Leeuw.

This is joint work with R. J. Aliaga (Valencia Polytechnic University) and E. Pernecká (Czech Technical University, Prague).

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