Algebra and Number Theory Seminar

Title: Complex Hadamard matrices, minimal polynomials and morphisms

Speaker: Padraig O'Cathain (Worcester Polytechnic Institute)

Date: Thu 12th March 2020 at 2:00PM

Location: Seminar Room SCN 1.25

Abstract: Let M be a matrix with complex entries of unit norm. A well-known theorem of Hadamard bounds the magnitude of the determinant of M as a function of its dimension, and M is a (complex) Hadamard matrix if M meets Hadamard's bound with equality.

The Hadamard conjecture concerns the existence of real Hadamard matrices (that is, with entries in $\{\pm 1\}$. While existence of real Hadamard matrices has been studied for over 100 years, (complex) Hadamard matrices with entries in some extension field of $\mathbb{Q}$ have not received the same attention. In fact, apart from matrices with entries $\{\pm 1, \pm i\}$, existence and non-existence of complex Hadamard matrices is poorly understood.

In this talk, I will discuss relations between the minimal polynomial of a complex Hadamard matrix and tensor-product-like constructions for Hadamard matrices with entries in a smaller field. These results generalise theorems of Turyn and Compton-Craigen-de Launey which give real Hadamard matrices from certain complex Hadamard matrices. Applying these techniques to certain biquadratic extensions of $\mathbb{Q}$ recov-

ers a construction of Mukhopadhyay-Seberry for skew-Hadamard matrices, and time permitting, I will report on techniques for controlling the minimal polynomial of a Hadamard matrix. I will finish the talk with some open questions and directions for future research.

Joint work with Ronan Egan, Phillip Heikoop, Guillermo Nunez Ponasso, Jack Pugmire and Eric Swartz.

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