Analysis Seminar

Title: More Spectral Disjointness

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Location: Seminar Room SCN 1.25

Abstract: Spectral disjointness confers a certain "independence" upon linear operators. If G is a ring with identity I then an idempotent $Q = Q^2 \in G$ gives the ring G a block structure

$$G \cong \begin{pmatrix} A & M \\ N & B \end{pmatrix}$$

where for example $A = QGQ$; then

$$T = \begin{pmatrix} a & m \\ n & b \end{pmatrix} \in G$$

commutes with Q iff it is a "block diagonal":

$$TQ = QT \iff T = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} \ .$$

Specialising to complex Banach algebras, for block diagonals there is two way implication

$$\sigma_A(a) \cap \sigma_B(b) = \emptyset \iff Q \in \mathrm{Holo}(T) \ \ :$$

$Q = f(T)$ with $f : U \to G$ holomorphic on an open neighbourhood of $\sigma_G(T)$. Weaker spectral disjointness gives a little less:

$$\sigma_A^{left}(a) \cap \sigma_B^{right}(b) = \emptyset = \sigma_A^{right}(a) \cap \sigma_B^{left}(b) \implies Q \in \mathrm{comm}^2(T) \ \ :$$

the block structure idempotent Q "double commutes" with $T \in G$. Specializing to $G = B(X)$, the bounded operators on a Banach space, closed complemented subspaces $Y \subseteq X$ give us again the block structure, and operators $T \in G$ for which Y is "invariant" become "block triangles":

$$T(Y) \subseteq Y \Longleftrightarrow T = \begin{pmatrix} a & m \\ 0 & b \end{pmatrix} \ .$$

When $Y \subseteq X$ is not complemented then the block structure is missing and we must resort to the restriction and the quotient:

$$a = T_Y \in A = B(Y) \ ; \ b = T_{/Y} \in B(X/Y) \ .$$

Now spectral disjointness

$$\sigma_A(a) \cap \sigma_B(b) = \emptyset$$

ensures that the subspace $Y \subseteq X$ is both hyperinvariant and reducing, in particular complemented.