



General Interest Seminar

Title: Why is the 4-dimensional smooth Poincare Conjecture still open?

Speaker: Brendan Guilfoyle (IT Tralee)

Date: Mon 9th December 2019 at 3:30PM

Location: Seminar Room SCN 1.25

Abstract: The Poincare conjectures roughly state that any closed n -manifold that looks like the n -sphere is the n -sphere. There are various versions of the conjecture in different categories: if a manifold is homotopy equivalent to the n -sphere, is it homeomorphic to the n -sphere? If it is homeomorphic to the n -sphere, is it diffeomorphic to the n -sphere? The former is referred to as the topological Poincare Conjecture, the latter as the smooth Poincare Conjecture. It is claimed that these conjectures have been resolved in all cases except for the 4-dimensional smooth Poincare Conjecture, which remains shrouded in mystery.

In this talk, we will explore the reasons for this anomaly, pointing the finger at our incomplete understanding of Freedman's claimed resolution of the 4-dimensional topological case. The talk will centre on a series of open MathOverflow questions on the topic which have been widely discussed:

<https://mathoverflow.net/questions/87674/independent-evidence-for-the-classification-of-topological-4-manifolds> <https://mathoverflow.net/questions/108631/fake-versus-exotic>
<https://mathoverflow.net/questions/252563/the-freedman-dichotomies>

We will describe the background and motivation of these questions and explain why Freedman's Disk Theorem is so problematic. In addition, we will outline a number of possible approaches to disproving Freedman's result and the implications of such a disproof.

There will be tea and coffee after the seminar. Everyone is very welcome.