Applied and Computational Mathematics Seminar

Title: Exact discrete resonances in the Fermi-Pasta-Ulam-Tsingou system

Speaker: Miguel Bustamante (University College Dublin)

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Abstract: In systems of N coupled anharmonic oscillators, exact resonant interactions play an important role in the energy exchange between normal modes. In the weakly nonlinear regime, those interactions may facilitate energy equipartition in Fourier space. We consider analytically resonant wave-wave interactions for the celebrated Fermi-Pasta-Ulam-Tsingou (FPUT) system. Using a number-theoretical approach based on cyclotomic polynomials, we show that the problem of finding exact resonances for a system of N particles is equivalent to a Diophantine equation whose solutions depend sensitively on the set of divisors of N. We provide an algorithm to construct all possible resonances, based on two methods: pairing-off and cyclotomic, which we introduce to build up explicit solutions to the 4-, 5- and 6-wave resonant conditions. Our results shed some light in the understanding of the long-standing FPUT paradox, regarding the sensitivity of the resonant manifolds with respect to the number of particles N and the corresponding time scale of the interactions leading to thermalisation. In this light we demonstrate that 6-wave resonances always exist for any N, while 5-wave resonances exist if N is divisible by 3 and N ¿ 6. It is known (for finite N) that 4-wave resonances do not mix energy across the spectrum, so we investigate whether 5-wave resonances can produce energy mixing across a significant region of the Fourier spectrum by analysing the interconnected network of

Fourier modes that can interact nonlinearly via resonances. The answer depends on the set of odd divisors of N that are not divisible by 3: the size of this set determines the number of dynamically independent components, corresponding to independent constants of motion (energies). We show that 6-wave resonances connect all these independent components, providing in principle a restoring mechanism for full-scale thermalisation. This work was done in collaboration with Kevin Hutchinson (UCD), Y. V. Lvov and M. Onorato.