Applied and Computational Mathematics Seminar

Title: Integrable Elliptic Billiards Ballyards

Speaker: Peter Lynch (University College Dublin)

Date: Mon 9th September 2019 at 1:00PM

Location: Seminar Room SCN 1.25

Abstract: The billiard problem concerns a point particle moving freely in a region of the horizontal plane bounded by a closed curve Γ, and reflected at each impact with Γ. The region is called a 'billiard', and the reflections are specular: the angle of reflection equals the angle of incidence. We review the dynamics in the case of an elliptical billiard. In addition to conservation of energy, the quantity $L_1 L_2$ is an integral of the motion, where L_1 and L_2 are the angular momenta about the two foci.

We can regularize the billiard problem by approximating the flat-bedded, hard-edged surface by a smooth function. We then obtain solutions that are everywhere continuous and differentiable. We call such a regularized potential a 'ballyard'. A class of ballyard potentials will be defined that yield systems that are completely integrable. We find a new integral of the motion that corresponds, in the billiards limit $N \to \infty$, to $L_1 L_2$.

Just as for the billiard problem, there is a separation of the orbits into boxes and loops. The discriminant that determines the character of the solution is the sign of

L_1L_2 on the major axis.