Analysis Seminar

Title: The continuity of betweenness

Speaker: R. Smith

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Abstract: Given a set X, we can use a suitable ternary relation $[\cdot,\cdot,\cdot] \subseteq X^3$ to express the notion of 'betweenness' on X: x is between a and b if and only if $[a,x,b]$ holds. We assume that this relation is "basic": $[a,a,b]$ and $[a,b,b]$ always hold, $[a,x,b]$ implies $[b,x,a]$, and $[a,x,a]$ implies $x = a$. Many natural examples of betweenness arise when X is endowed with some additional order-theoretic or topological structure. Given $a,b \in X$, we can define the "interval" $[a,b] = \{x \in X : [a,x,b]\}$ $(= [b,a])$. If X has additional topological structure, it is reasonable to ask whether the assignment $\{a,b\} \mapsto [a,b]$ has good continuity properties, given a suitable hyperspace topology. We examine this question in the context of "Menger betweenness" on metric spaces (X,d) ($[a,x,b]$ holds if and only if $d(a,b) = d(a,x)+d(x,b)$), and the "K-interpretation of betweenness" on topological continua ($[a,x,b]$ holds if and only if x is an element of every subcontinuum that includes a and b).