

WRF 4D-Var

**The Weather Research and Forecasting model based
4-Dimensional Variational data assimilation system**

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The WRF 4D-Var Team

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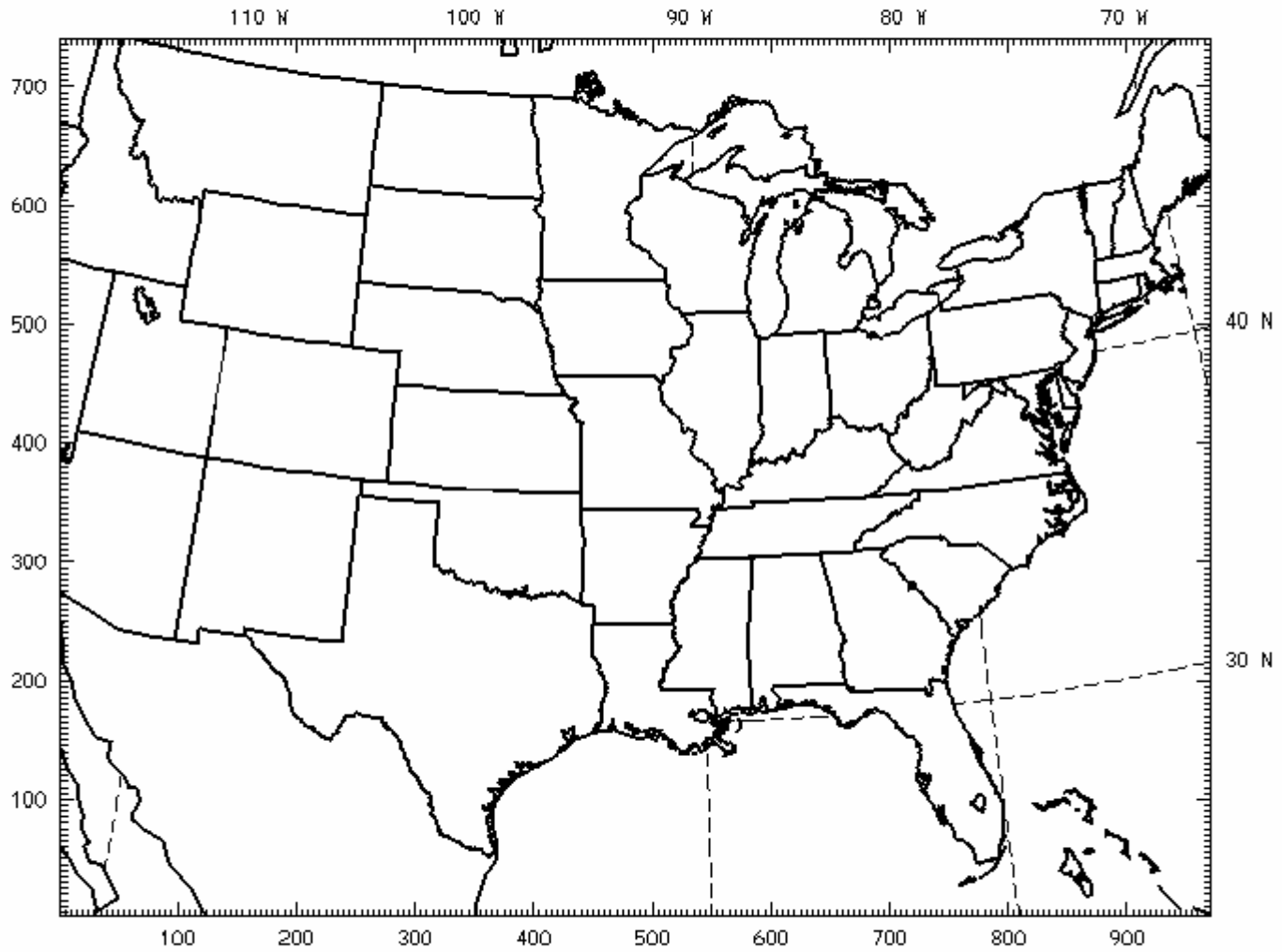
Acknowledgments. The WRF 4D-Var development has been primarily supported by the Air Force Weather Agency.

Outline

1. WRF
2. 4D-Var
3. Current status of WRF 4D-Var
4. Single ob experiments
5. Noise control
6. Typhoon (Haitang) forecasts
7. Work plan
8. Summary

WRF overview

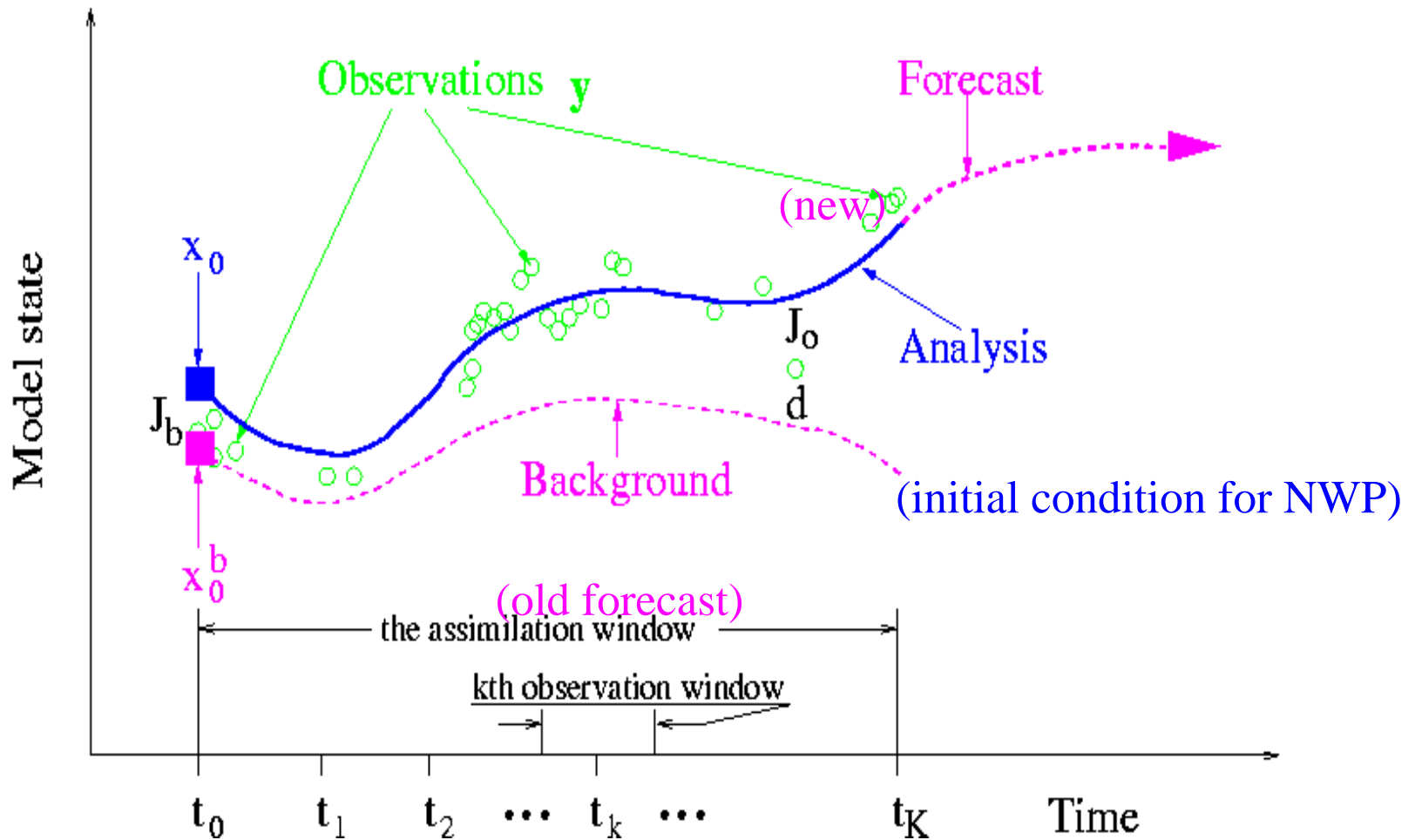
- Eight-year, multi-agency collaboration to develop advanced community mesoscale model and data assimilation system with direct path to operations
- Current release WRFV2.1 (Next release 2.2 November 2006)
 - Two dynamical cores, numerous physics, chemistry
 - Variational Data Assimilation (3D-Var released) and Ensemble Kalman Filter (in development)
- Rapid community growth
 - More than 3,000 registered users
 - June 2005 Users Workshop: 219 participants, 117 inst., 65 countries
 - Scientific papers: real-time NWP, atmos. chemistry, data assimilation, climate, wildfires, mesoscale processes
- Operational capabilities implemented or planned
 - Air Force Weather Agency
 - National Centers for Environmental Prediction
 - BMB (Beijing), KMA (Korea), IMD (India), CWB (Taiwan), IAF (Israel), WSI (U.S.)



Observations are not enough for initializing NWP models:

- Observations have errors.
- Observations are not evenly distributed in time and/or in space.
- Many observations are indirect, e.g. radiance. (not “model variables”, e.g. p , T , u , v , q).
- ...

Variational methods: 3D-Var and 4D-Var



4D-Var

$$J = J_b + J_o$$

$$J_b(\mathbf{x}_0) = \frac{1}{2} \left[(\mathbf{x}_0 - \mathbf{x}_b)^T \mathbf{B}^{-1} (\mathbf{x}_0 - \mathbf{x}_b) \right]$$

$$J_o(\mathbf{x}_0) = \frac{1}{2} \sum_{k=1}^K \left[(H_k \mathbf{x}_k - \mathbf{y}_k)^T \mathbf{R}_k^{-1} (H_k \mathbf{x}_k - \mathbf{y}_k) \right]$$

$$\mathbf{x}_k = M(\mathbf{x}_0)$$

WRF 4D-Var

Black – WRF-3DVar [\mathbf{B} , \mathbf{R} , $\mathbf{U}=\mathbf{B}^{1/2}$, $\mathbf{v}^n=\mathbf{U}^{-1}(\mathbf{x}^n-\mathbf{x}^{n-1})$]

Green – modification required

Blue – existing (for 4DVar)

Red – new development

$$J'_{\mathbf{v}^n} = \mathbf{v}^n + \sum_{i=1}^{n-1} \mathbf{v}^i + \mathbf{U}^T \mathbf{S}_{\mathbf{v}-\mathbf{w}}^T \sum_{k=1}^K \mathbf{M}_k^T \mathbf{S}_{\mathbf{w}-\mathbf{v}}^T \mathbf{H}_k^T \mathbf{R}^{-1} [\mathbf{H}_k \mathbf{S}_{\mathbf{w}-\mathbf{v}} \mathbf{M}_k \mathbf{S}_{\mathbf{v}-\mathbf{w}} \mathbf{U}^{-1} \mathbf{v}^n + H_k(\mathbf{M}_k(\mathbf{x}^{n-1})) - \mathbf{y}_k]$$

(Huang, et.al. 2006: Preliminary results of WRF 4D-Var.
WRF users' workshop, Boulder, Colorado.)

Necessary components of 4D-Var

- H observation operator, including the tangent linear operator \mathbf{H} and the adjoint operator \mathbf{H}^T .
- M forecast model, including the tangent linear model \mathbf{M} and adjoint model \mathbf{M}^T .
- \mathbf{B} background error covariance ($N*N$ matrix).
- \mathbf{R} observation error covariance which includes the representative error ($K*K$ matrix).

Why 4D-Var?

- Use observations over a time interval, which suits most asynoptic data.
- Use a forecast model as a constraint, which ensures the dynamic balance of the analysis.
- Implicitly use flow-dependent background errors, which ensures the analysis quality for fast developing weather systems.

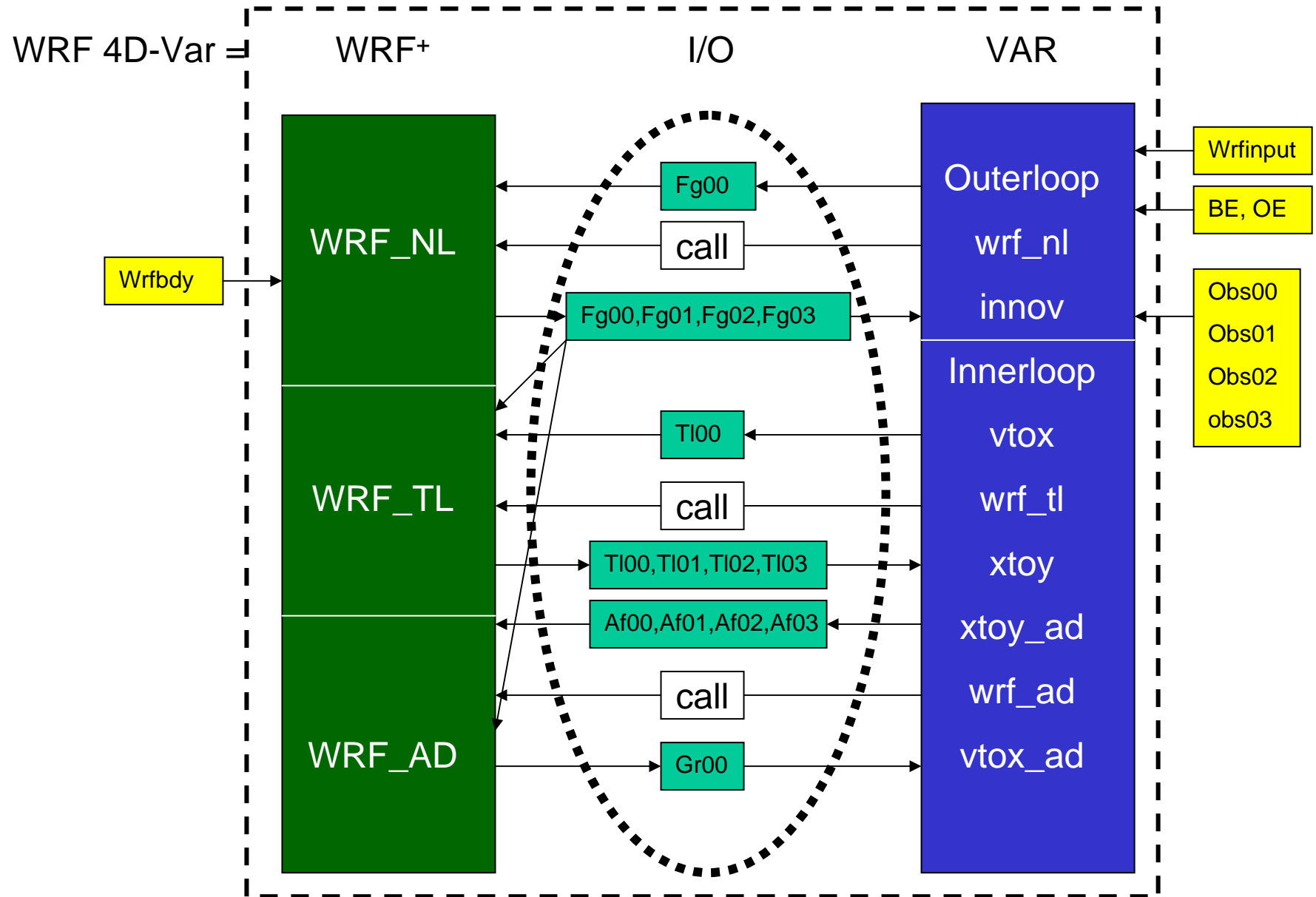
A short 4D-Var review

- The idea: Le Dimet and Talagrand (1986); Lewis and Derber (1985)
- Implementation examples:
 - Courtier and Talagrand (1990); a shallow water model
 - Thepaut and Courtier (1991); a multi-level primitive equation model
 - Navon, et al. (1992); the NMC global model
 - Zupanski M (1993); the Eta model
 - Zou, et al. (1995); the MM5 model
 - Sun and Crook (1998); a cloud model
 - Rabier, et al. (2000); the ECMWF model
 - Huang, et al. (2002); the HIRLAM model
 - Zupanski M, et al. (2005); the RAMS model
 - Ishikawa, et al. (2005); the JMA mesoscale model
 - [Huang, et al. \(2005\); the WRF model](#)
- Operation: ECMWF, Meteo France, JMA, UKMO, MSC.
- Pre-operation: HIRLAM

Current status of WRF 4D-Var

- Necessary modifications to WRF 3D-Var have been completed.
- WRF tangent-linear and adjoint models have been developed.
- WRF 4D-Var framework has been developed.
- The prototype has been put together and can run. An implementation of it has been made at AFWA in Jan 2006.

The prototype: Use separate executables, communicate through I/O



Single observation experiment

The idea behind single ob tests:

The solution of 3D-Var should be

$$\mathbf{x}^a = \mathbf{x}^b + \mathbf{B}\mathbf{H}^T \left[\mathbf{H}\mathbf{B}\mathbf{H}^T + \mathbf{R} \right]^{-1} \left[\mathbf{y} - \mathbf{H}\mathbf{x}^b \right]$$

Single observation

$$\underline{\mathbf{x}^a - \mathbf{x}^b} = \mathbf{B}_i \left[\sigma_b^2 + \sigma_o^2 \right]^{-1} \left[\mathbf{y}_i - x_i \right]$$

3D-Var \rightarrow 4D-Var: $\mathbf{H} \rightarrow \mathbf{H}\mathbf{M}$; $\mathbf{H}^T \rightarrow \mathbf{M}^T\mathbf{H}^T$

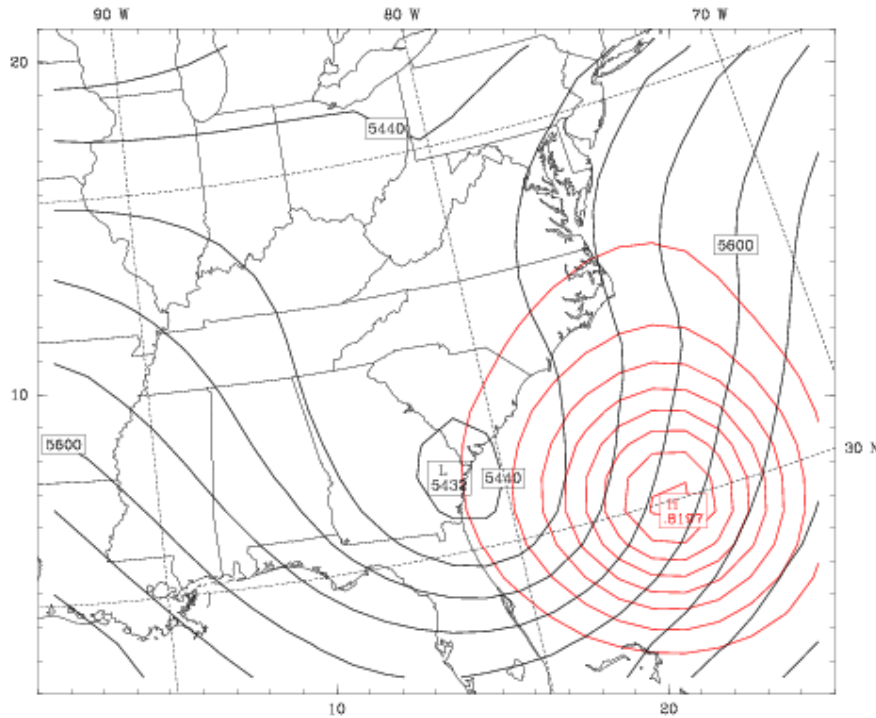
The solution of 4D-Var should be

$$\mathbf{x}^a = \mathbf{x}^b + \mathbf{B}\mathbf{M}^T\mathbf{H}^T \left[\mathbf{H}(\mathbf{M}\mathbf{B}\mathbf{M}^T)\mathbf{H}^T + \mathbf{R} \right]^{-1} \left[\mathbf{y} - \mathbf{H}\mathbf{M}\mathbf{x}^b \right]$$

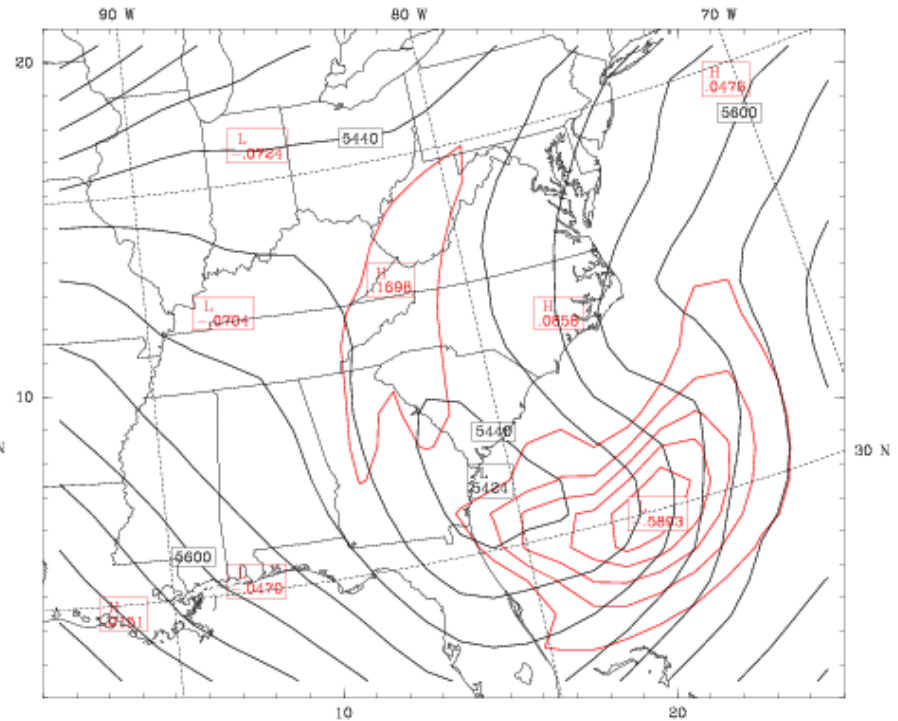
Single observation, solution at observation time

$$\underline{\mathbf{M}(\mathbf{x}^a - \mathbf{x}^b)} = \left(\mathbf{M}\mathbf{B}\mathbf{M}^T \right)_i \left[\sigma_b^2 + \sigma_o^2 \right]^{-1} \left[\mathbf{y}_i - x_i \right]$$

500mb θ increments from 3D-Var at 00h and from 4D-Var at 06h due to a 500mb T observation at 06h



FGAT(3D-Var)



4D-Var

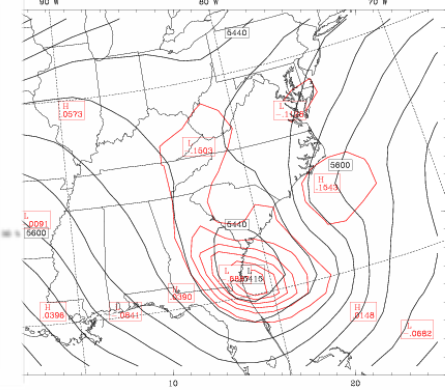
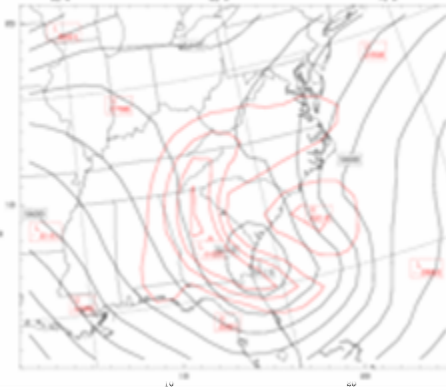
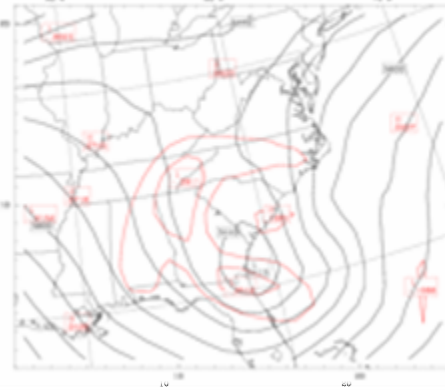
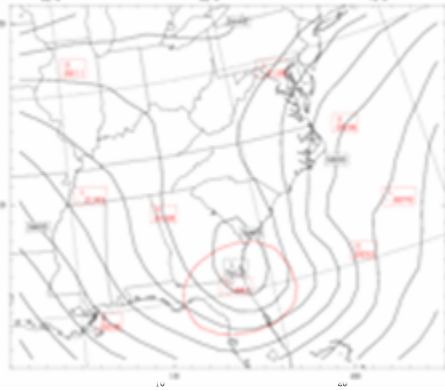
500mb θ increments at 00,01,02,03,04,05,06h to a 500mb T ob at 06h

00h

01h

02h

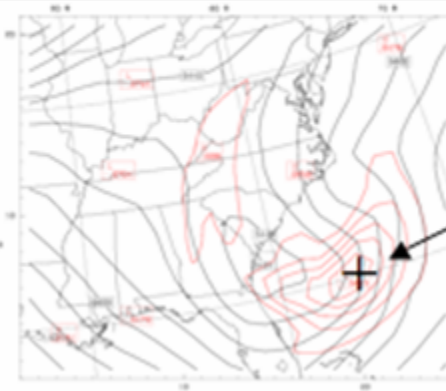
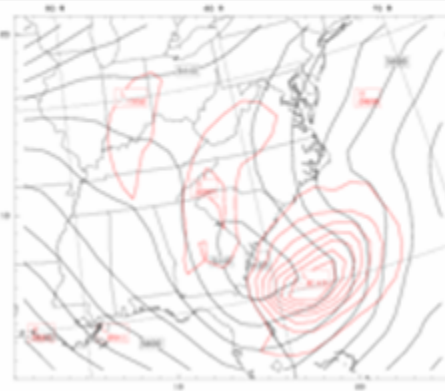
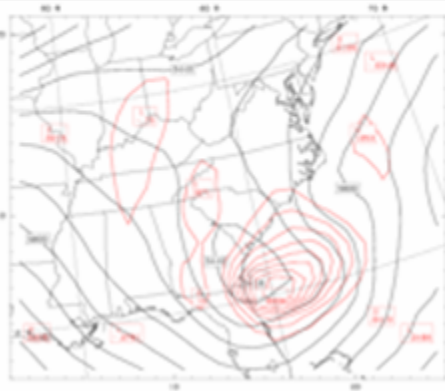
03h



04h

05h

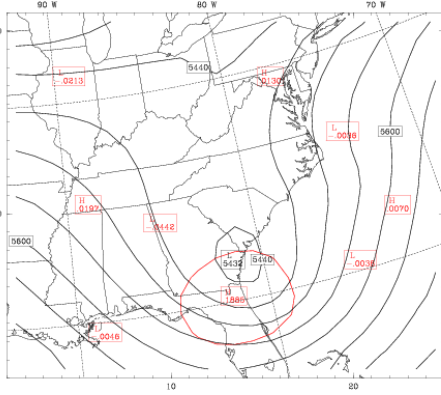
06h (4D-Var structure function)



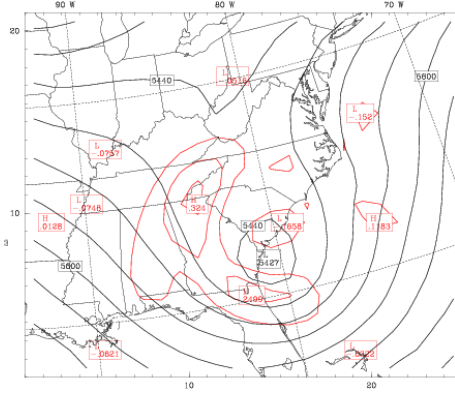
Obs

500mb θ difference at 00,01,02,03,04,05,06h from two nonlinear runs (one from background; one from 4D-Var)

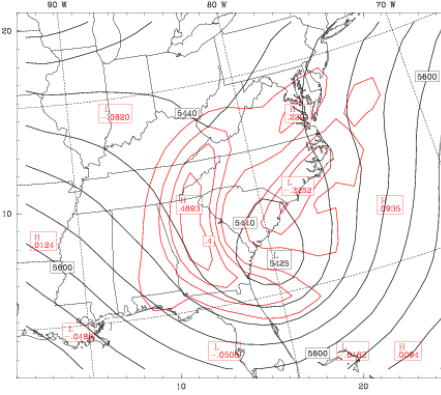
00h



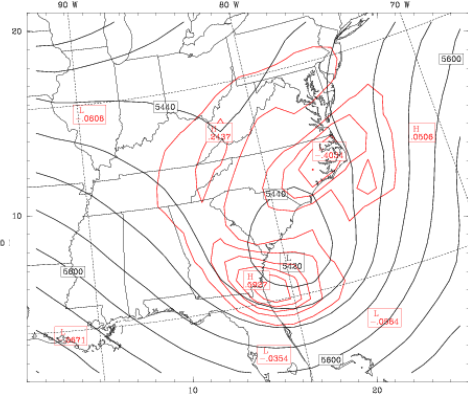
01h



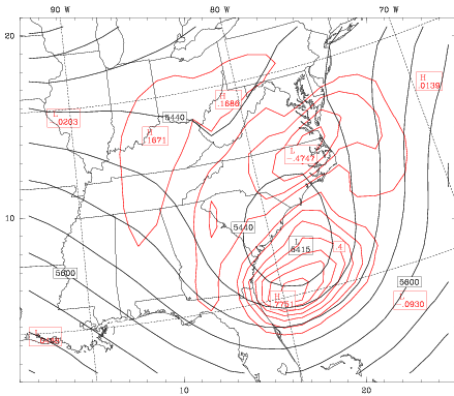
02h



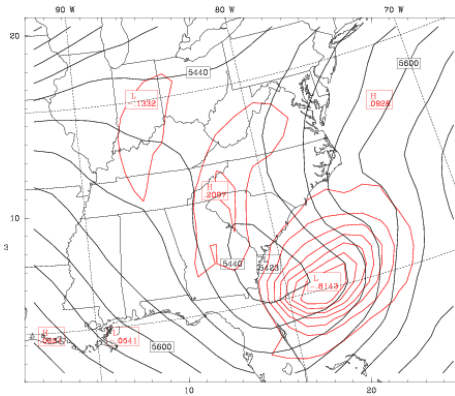
03h



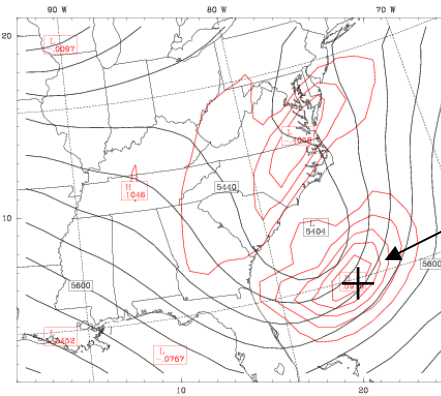
04h



05h



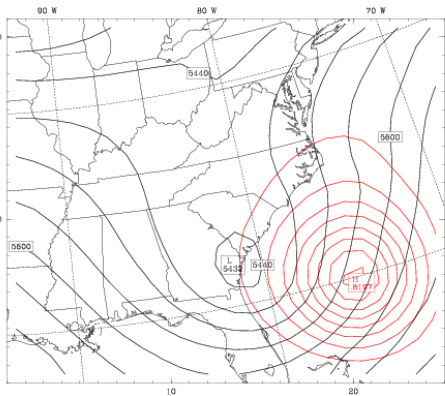
06h



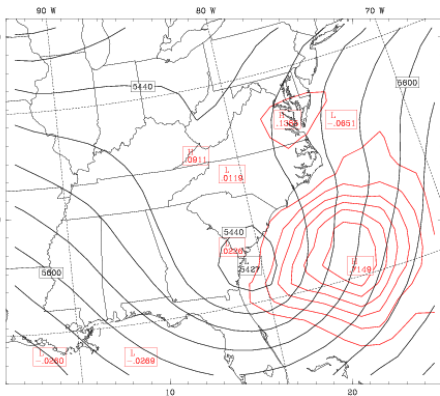
Obs

500mb θ difference at 00,01,02,03,04,05,06h from two nonlinear runs (one from background; one from FGAT)

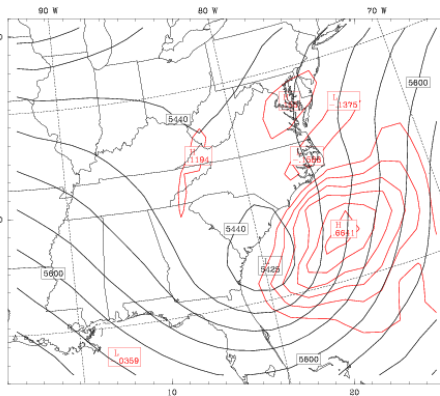
00h



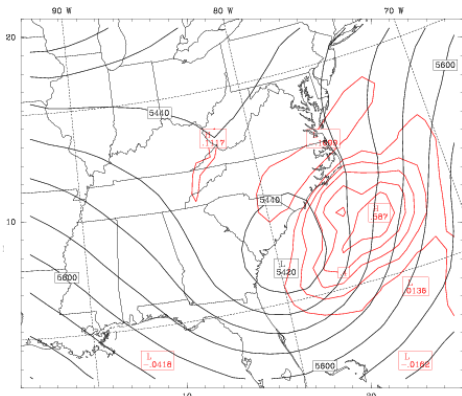
01h



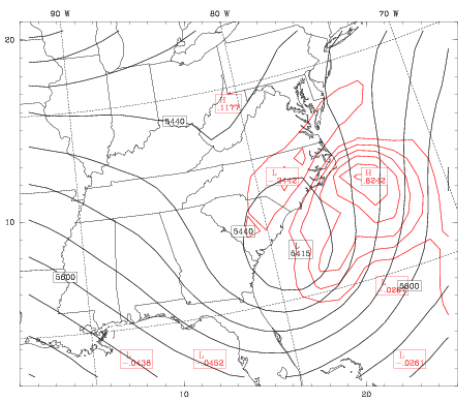
02h



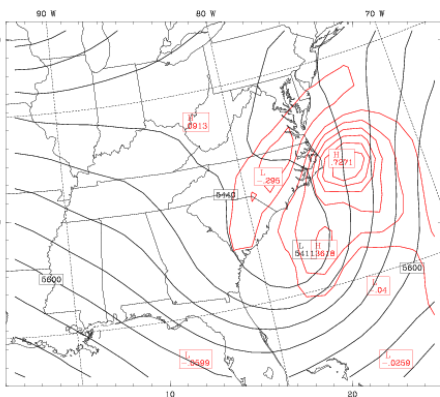
03h



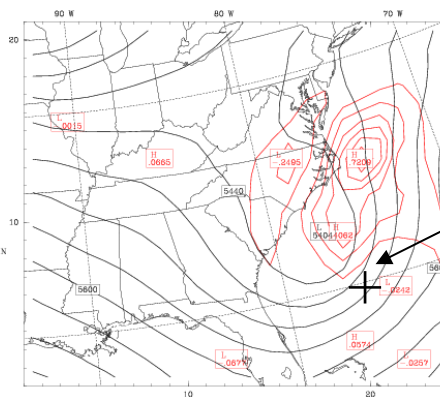
04h



05h



06h



Noise

Dataset: mytest RIP: rip
Fest: 0.10
Sea-level pressure

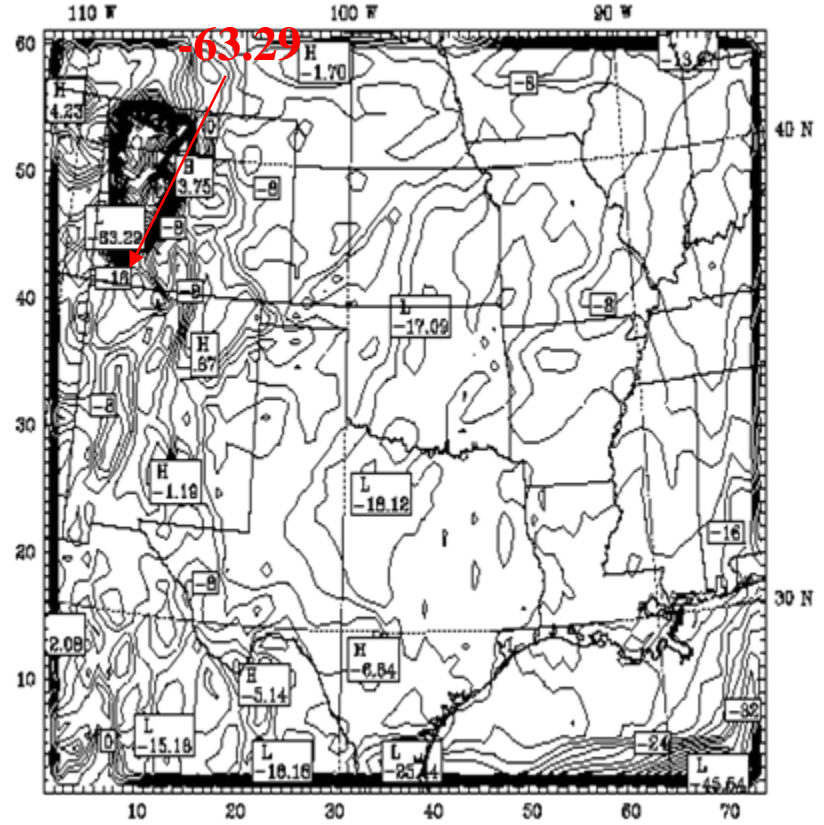
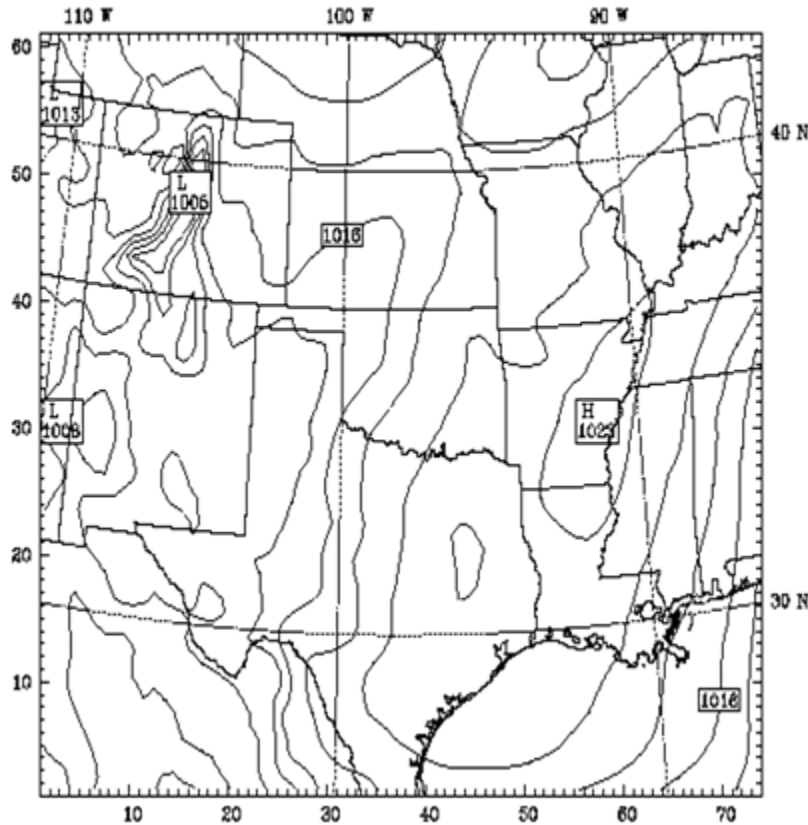
Init: 0000 UTC Tue 25 Jan 00
Valid: 0006 UTC Tue 25 Jan 00 (1706 MST Mon 24 Jan 00)

Dataset: mytest RIP: rip
Fest: 0.10
surface pressure tendency

Init: 0000 UTC Tue 25 Jan 00
Valid: 0006 UTC Tue 25 Jan 00 (1706 MST Mon 24 Jan 00)

MSLP (hPa)

Surface pressure tendency (hPa/3h)



t=0

CONTOUR: UNITS-hPa LOW= 1006.0 HIGH= 1022.0 INTERVAL= 2.000
Model info: V2.0.3 Eals-F-Stra YSU PBL WSM Sclase 30 km, 27 levels, 180 sec

CONTOUR: UNITS-hPa/3h LOW= -62.000 HIGH= 18.000 INTERVAL= 2.000
Model info: V2.0.3 Eals-F-Stra YSU PBL WSM Sclase 30 km, 27 levels, 180 sec

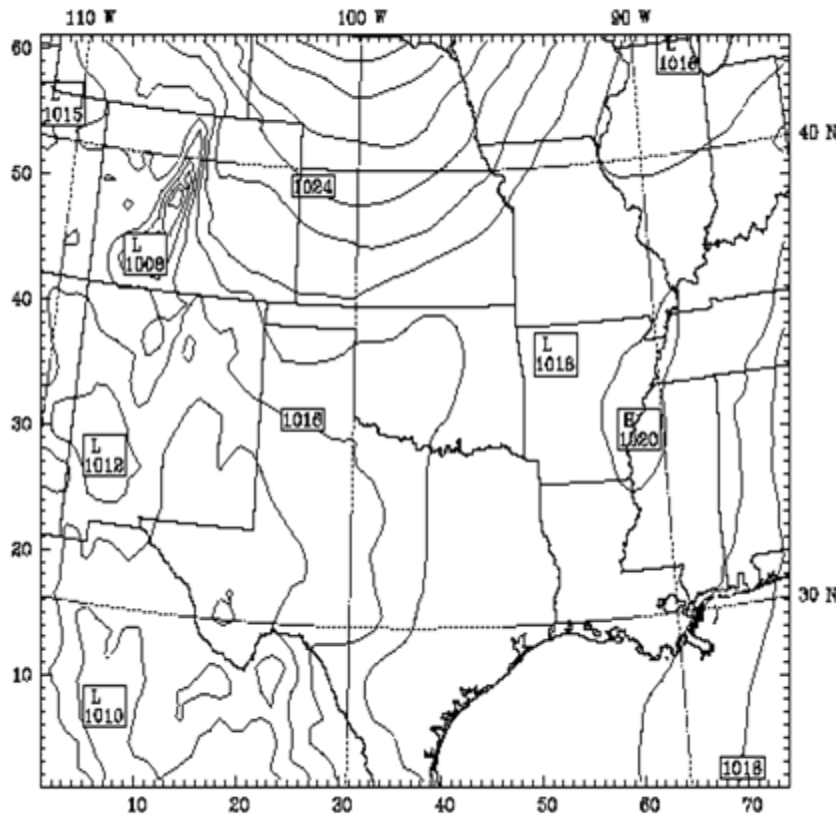
Sea level pressure and surface pressure tendency at +6h

Dataset: mytest RIP: rip
 Fcst: 6.00
 Sea-level pressure

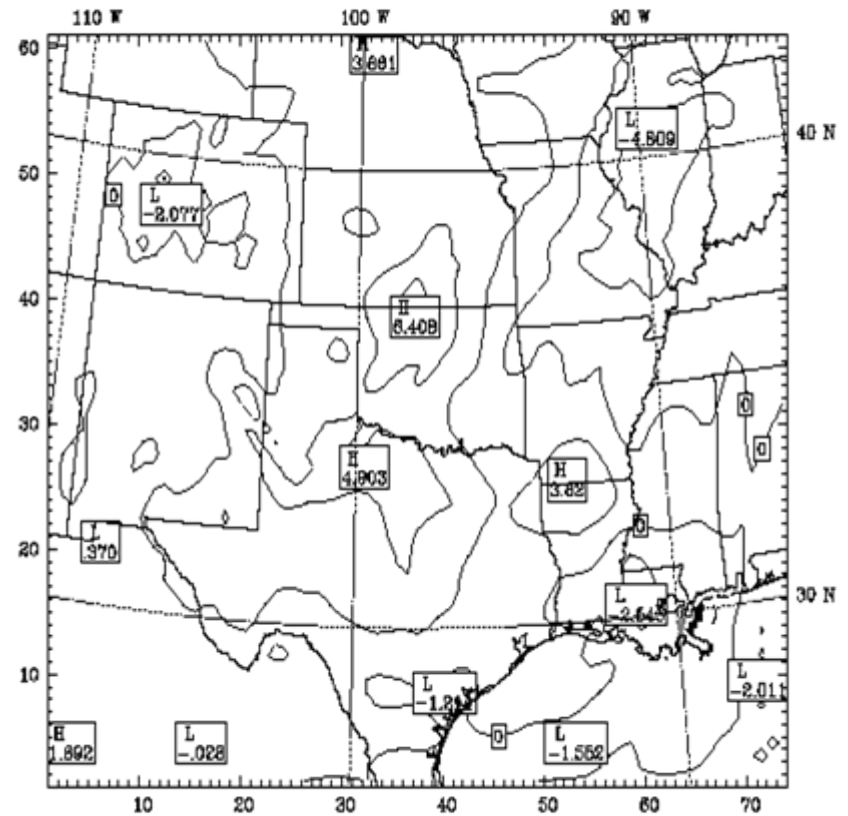
Init: 0000 UTC Tue 25 Jan 00
 Valid: 0600 UTC Tue 25 Jan 00 (2300 MST Mon 24 Jan 00)

Dataset: mytest RIP: rip
 Fcst: 6.00
 surface pressure tendency

Init: 0000 UTC Tue 25 Jan 00
 Valid: 0600 UTC Tue 25 Jan 00 (2300 MST Mon 24 Jan 00)



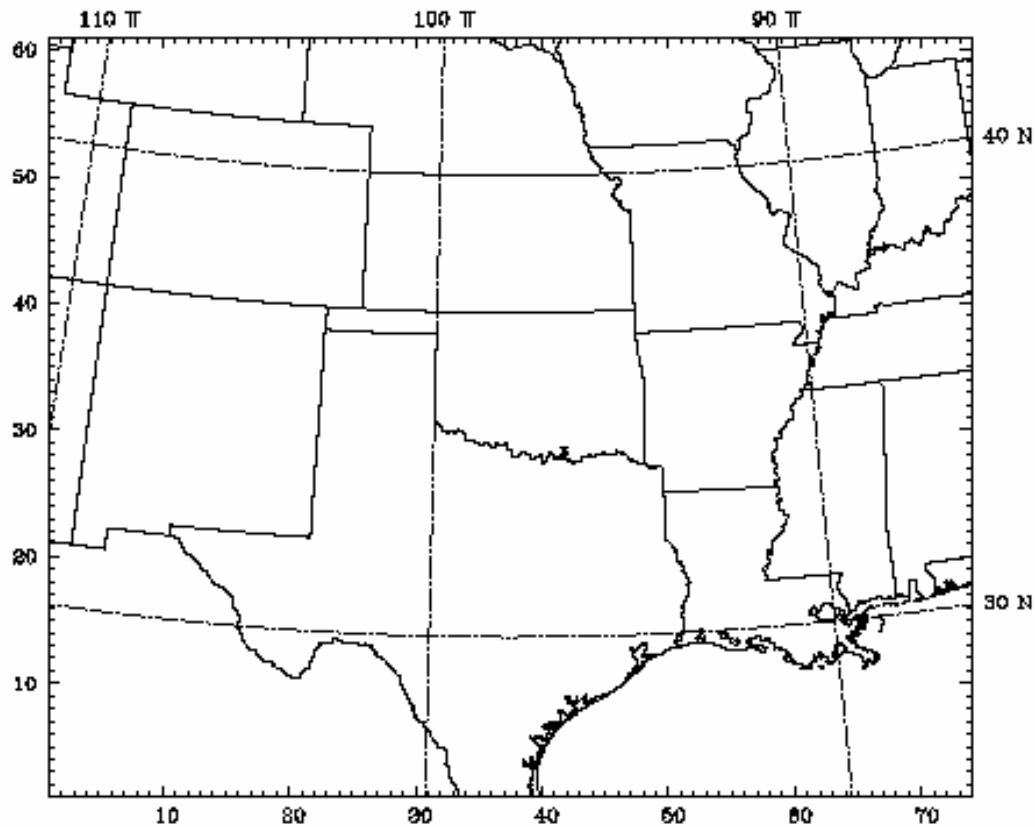
CONTOURS: UNITS-hPa LOW- 1006.0 HIGH- 1036.0 INTERVAL- 2.0000
 Model info: V3.0.3 Kaln-F-Eta YSU PBL WSM Sclass 30 km, 27 levels, 180 sec



CONTOURS: UNITS-hPa/3h LOW- -4.0000 HIGH- 6.0000 INTERVAL- 2.0000
 Model info: V3.0.3 Kaln-F-Eta YSU PBL WSM Sclass 30 km, 27 levels, 180 sec

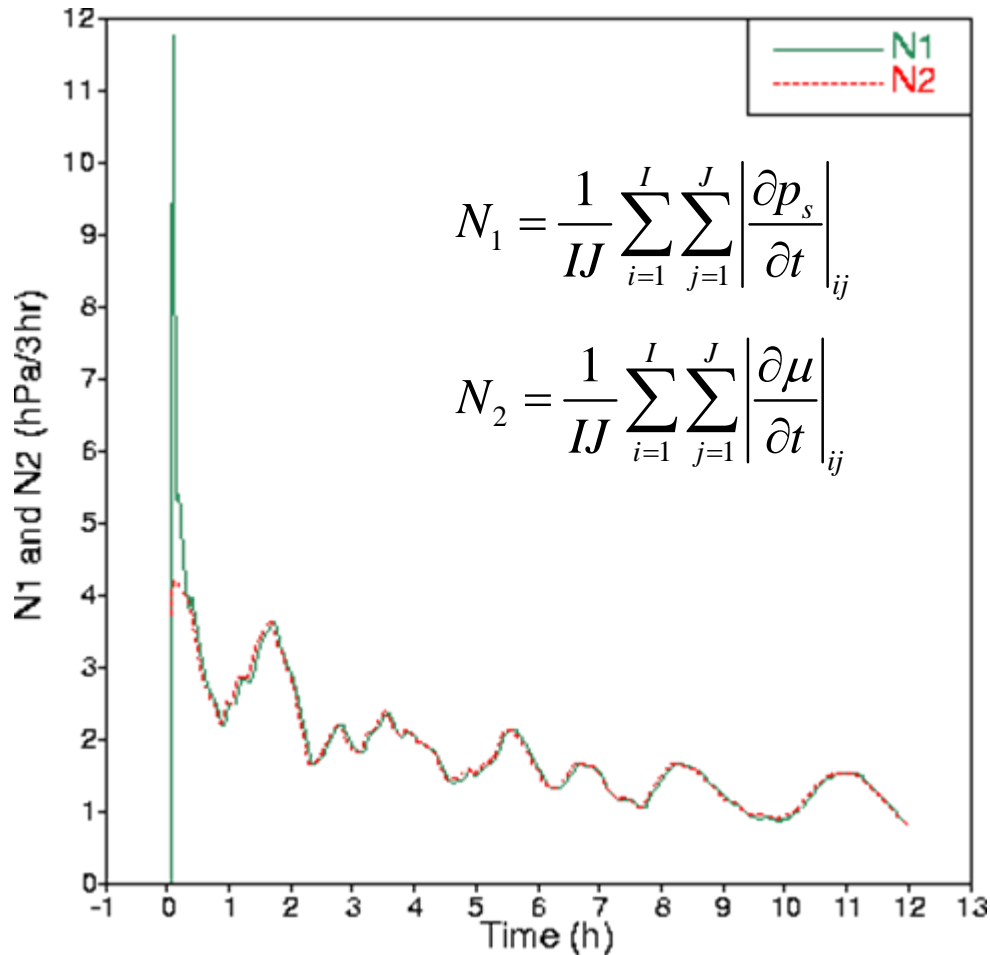
Evolution of the surface pressure tendency: DPSDT

Dataset: mytest RIP: rip Init: 0000 UTC Tue 25 Jan 00
Fcast: 0.00 Valid: 0000 UTC Tue 25 Jan 00 (1700 MST Mon 24 Jan 00)
surface pressure tendency



Model info: V2.0.3 Kain-F-Eta YSU PBL TSM Scheme 30 km, 27 levels, 180 sec

Noise level



Grid-points: 74×61×28

Resolution: 30 km

Time step: 180 s

**Initial state: 3DVAR
analysis at 2000.01.25.00
(the second cycle)**

DFI for WRF

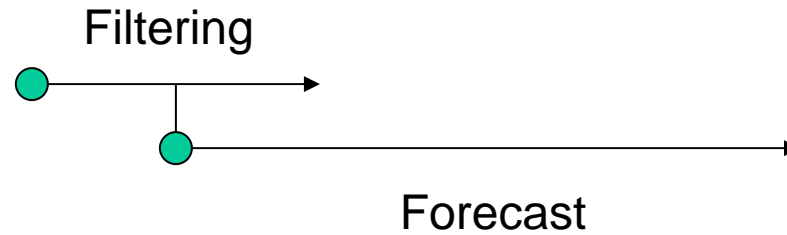
X.-Y. Huang,
M. Chen, J.-W. Kim, W. Wang,
T. Henderson, W. Skamarock

NCAR, BMB, KMA

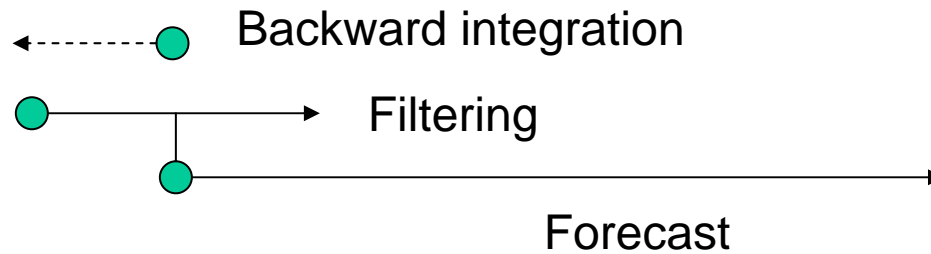
Project funded by KMA and BMB

Implemented options of DFI

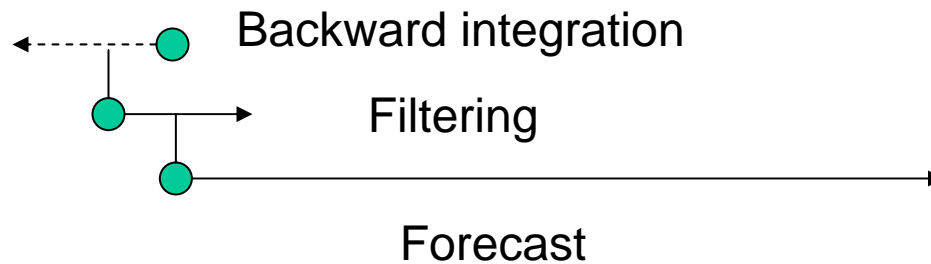
DFL:



DDFI:



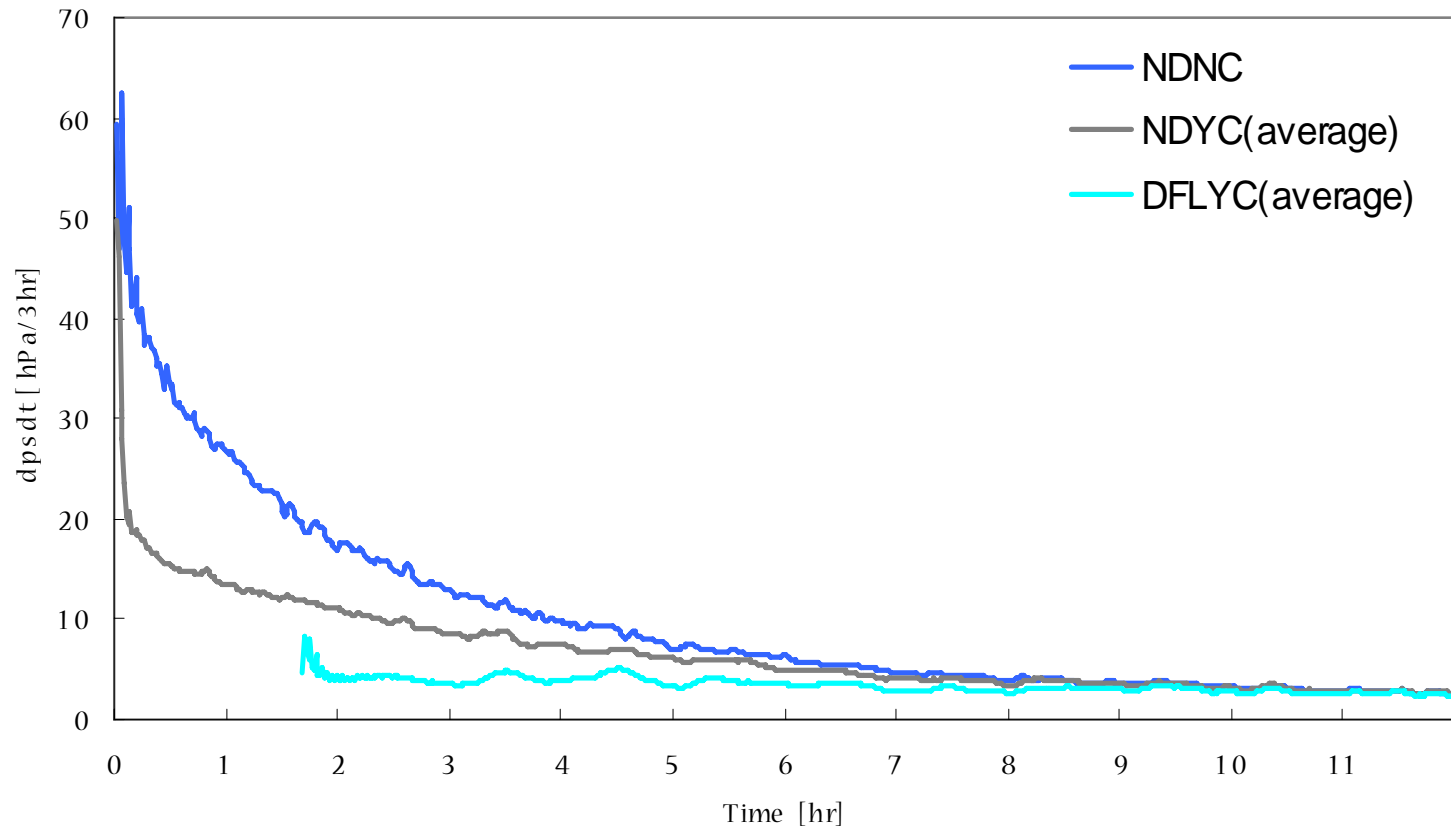
TDFI:



DFL test

The KMA domain 10 km : 12UTC 04 May ~ 12UTC 11 May 2006

The mean absolute Psfc tendency(KMA 10km Domain)



JcDF in WRF 4D-Var

Xin Zhang, University of Hawaii

Hans Huang, NCAR

$$J = J_b + J_o + J_c$$

$$J_b(\mathbf{x}_0) = \frac{1}{2} \left[(\mathbf{x}_0 - \mathbf{x}_b)^T \mathbf{B}^{-1} (\mathbf{x}_0 - \mathbf{x}_b) \right]$$

$$J_o(\mathbf{x}_0) = \frac{1}{2} \sum_{k=1}^K \left[(H_k \mathbf{x}_k - \mathbf{y}_k)^T \mathbf{R}_k^{-1} (H_k \mathbf{x}_k - \mathbf{y}_k) \right]$$

$$J_c(\mathbf{x}_0) = \frac{\gamma_{df}}{2} \left[(\mathbf{x}_{N/2} - \mathbf{x}_{N/2}^{DF})^T \mathbf{C}^{-1} (\mathbf{x}_{N/2} - \mathbf{x}_{N/2}^{DF}) \right]$$

$$\mathbf{x}_{N/2}^{DF} = \sum_{n=0}^N h_n \mathbf{x}_n$$

WRF 4D-Var

Black – WRF-3DVar [\mathbf{B} , \mathbf{R} , $\mathbf{U}=\mathbf{B}^{1/2}$, $\mathbf{v}^n=\mathbf{U}^{-1}(\mathbf{x}^n-\mathbf{x}^{n-1})$]

Green – modification required

Blue – existing (for 4DVar)

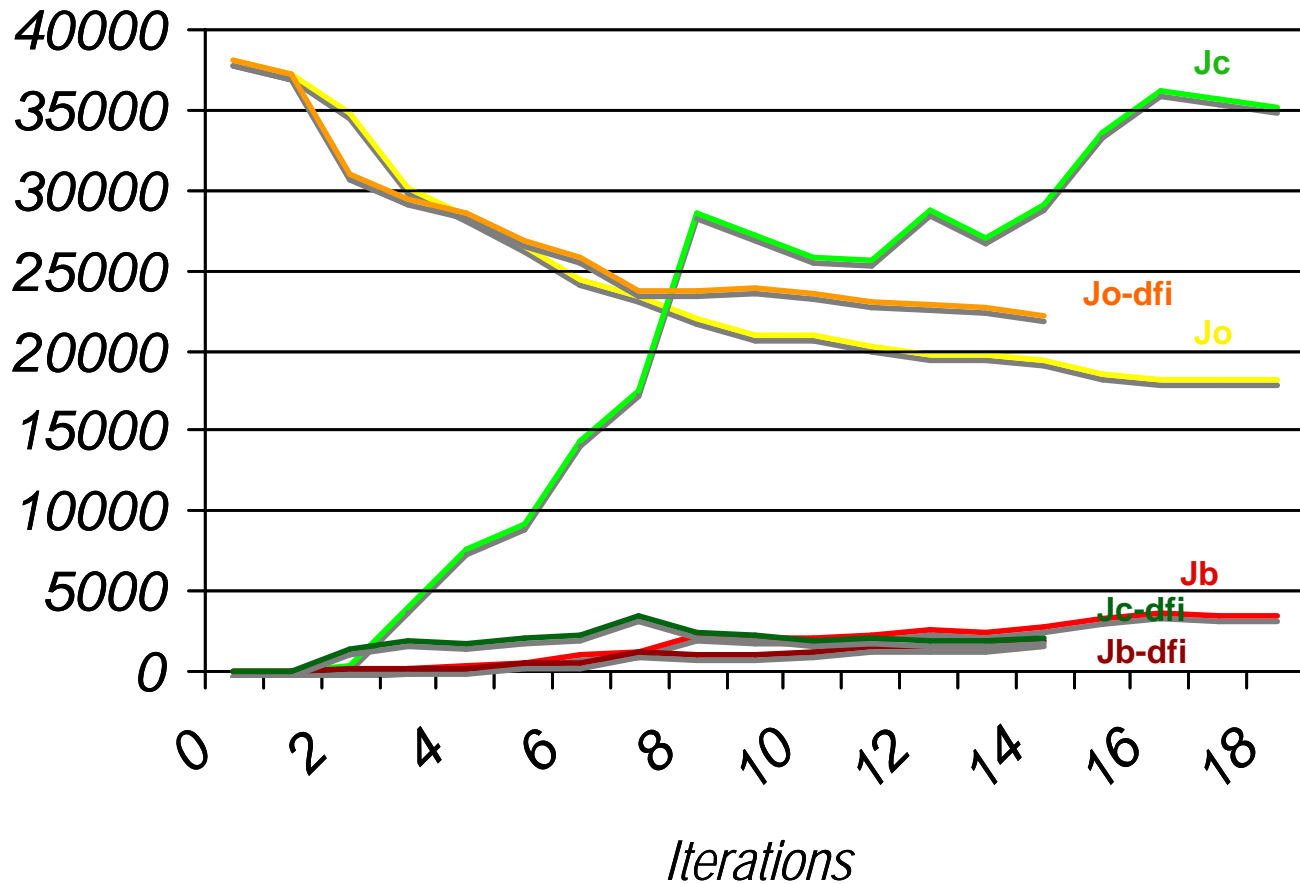
Red – new development

$$J'_{\mathbf{v}^n} = \mathbf{v}^n + \sum_{i=1}^{n-1} \mathbf{v}^i + \mathbf{U}^T \mathbf{S}_{v-w}^T \sum_{k=1}^K \mathbf{M}_k^T \mathbf{S}_{w-v}^T \mathbf{H}_k^T \mathbf{R}^{-1} [\mathbf{H}_k \mathbf{S}_{w-v} \mathbf{M}_k \mathbf{S}_{v-w} \mathbf{U}^{-1} \mathbf{v}^n + H_k(\mathbf{M}_k(\mathbf{x}^{n-1})) - \mathbf{y}_k]$$

$$+ \mathbf{U}^T \mathbf{S}_{v-w}^T \sum_{i=N}^0 \mathbf{M}_i^T h_i \gamma_{df} \mathbf{C}^{-1} \left(\sum_{i=0}^N (h_i \mathbf{M}_i \mathbf{S}_{v-w} \mathbf{U} \mathbf{v}) \right)$$

Jb, Jo and Jc in WRF

$\gamma=10.0$



Typhoon Haitang experiments:

4 experiments, every 6 h, 00Z 16 July - 00 Z 18 July, 2005

Typhoon Haitang hit Taiwan 00Z 18 July 2005

1. **FGS** – forecast from the background [The background fields are 6-h WRF forecasts from National Center for Environment Prediction (NCEP) GFS analysis.]
2. **AVN**- forecast from the NCEP GPS analysis
3. **3DVAR** – forecast from WRF 3D-Var
4. **4DVAR** – forecast from WRF 4D-Var

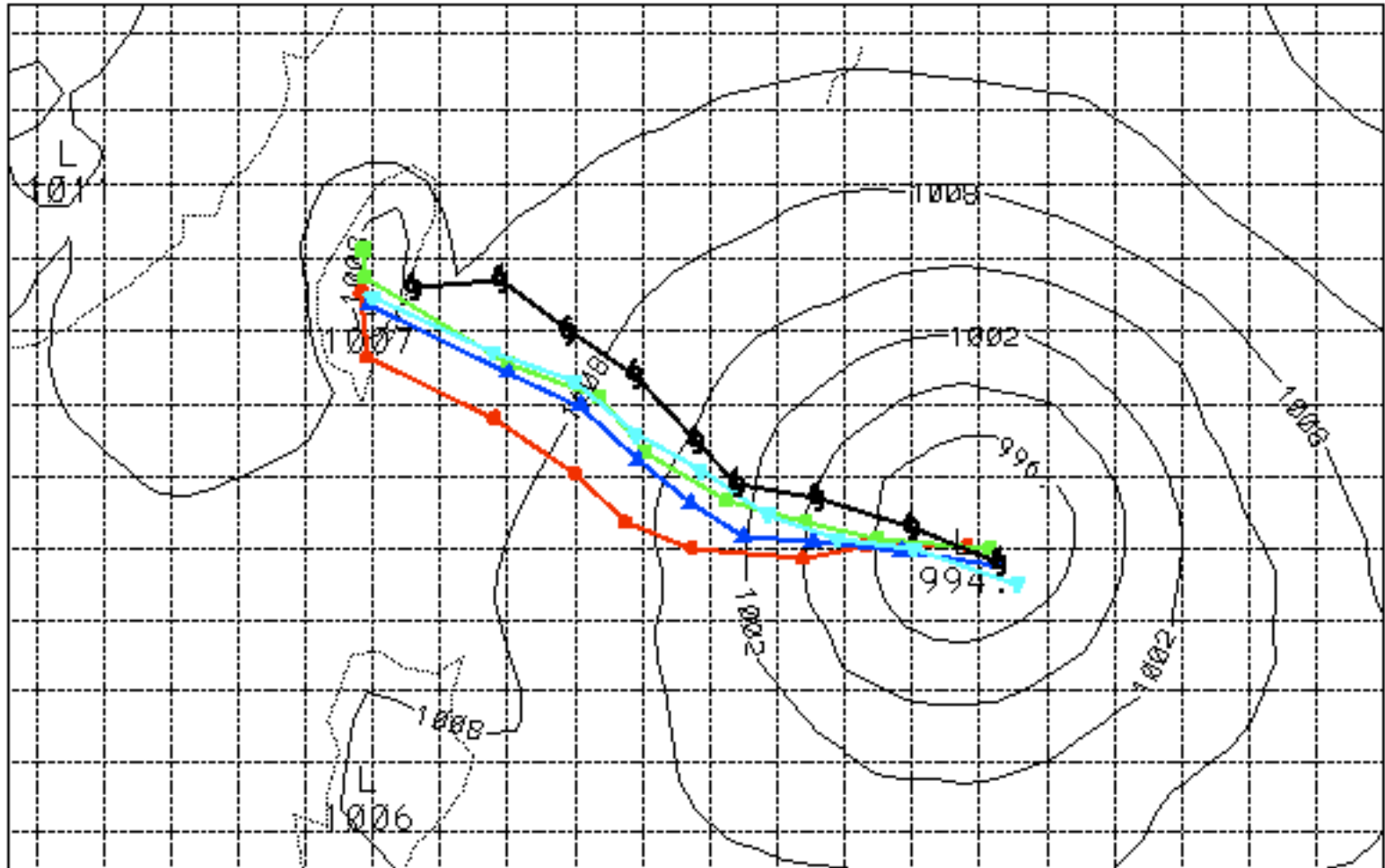
Observations used in 4DVAR and FGAT at 0000UTC 16 July 2005

	u	v	T	p	q	dZ
TEMP	727	724	869		697	
TEMPsurf	6	8	8	8	8	
SYNOP	199	218	237	226	236	
SATOB	3187	3182				
AIREP	923	930	939			
PILOT	159	160				
METAR	167	191	216	0	200	
SHIP	69	70	77	79	73	
SATEM						511
BUOY	67	67	0	64	0	
BOGUS	1200	1200	788	788	80	

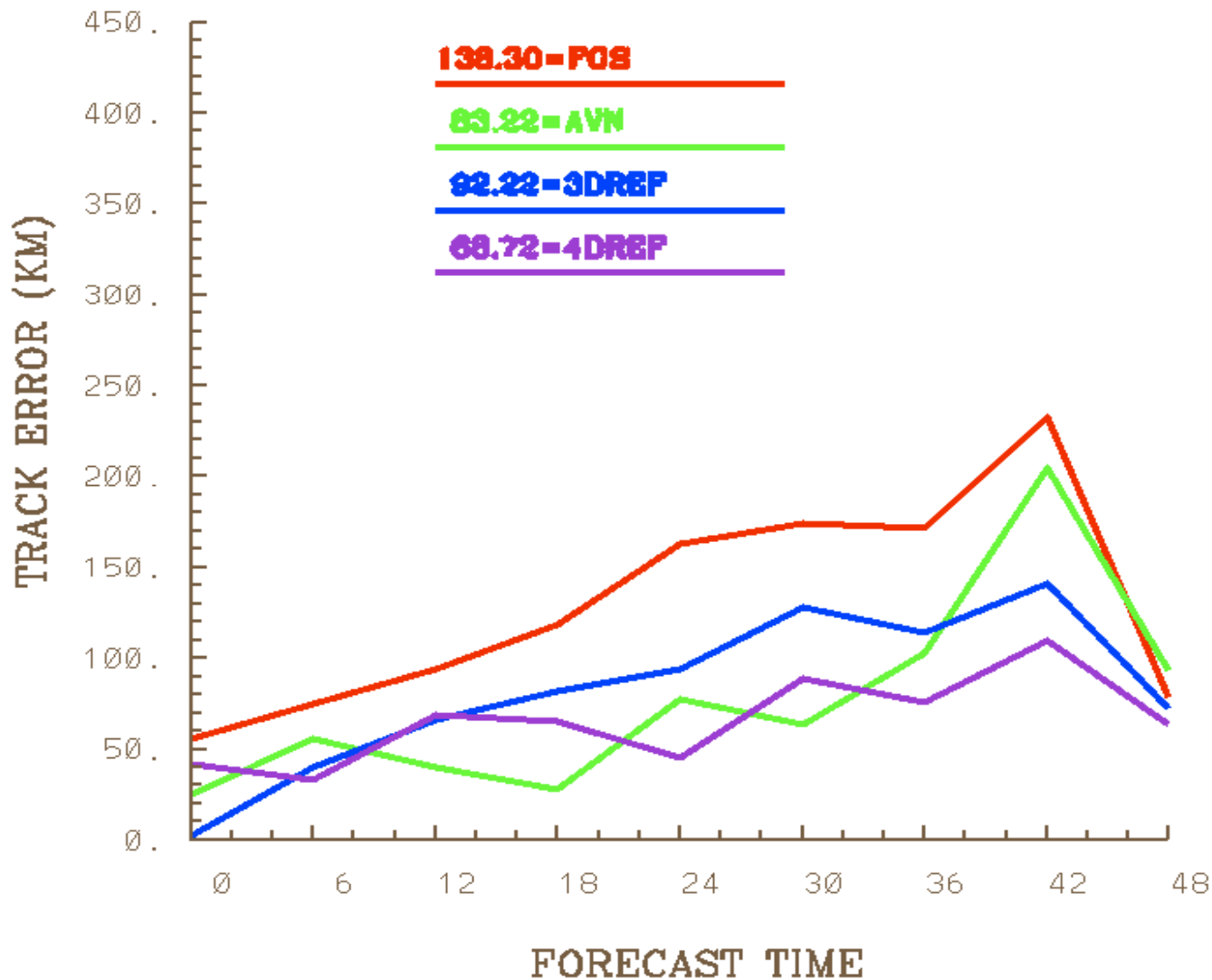
(At 0600UTC 16 July: GPS refractivity 2594, QuikScat u 2594, v 2605)

Typhoon (Haitang) forecasts

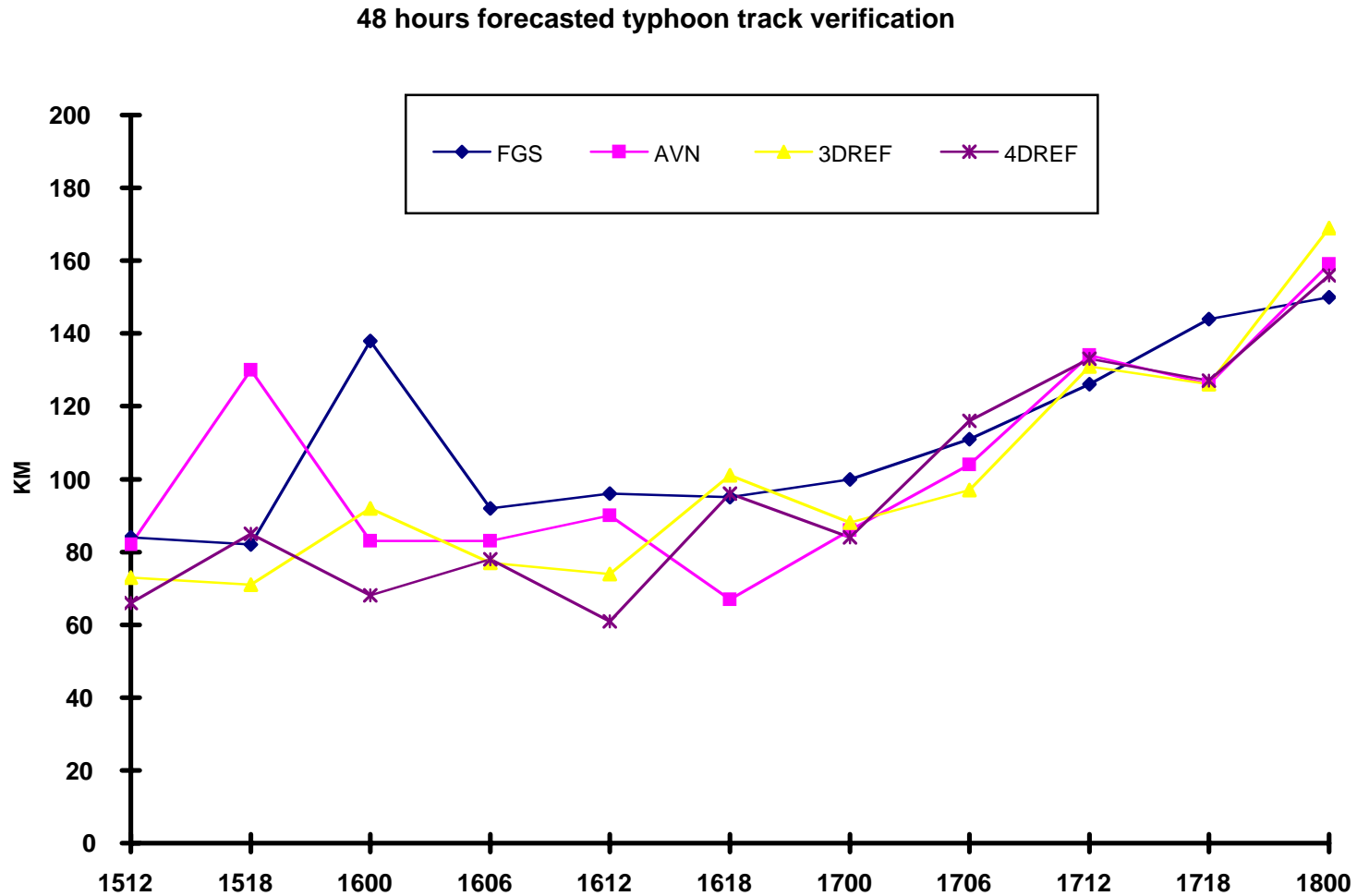
OBS TRACK FGS AVN 3DREF 4DREF



Typhoon (Haitang) forecasts



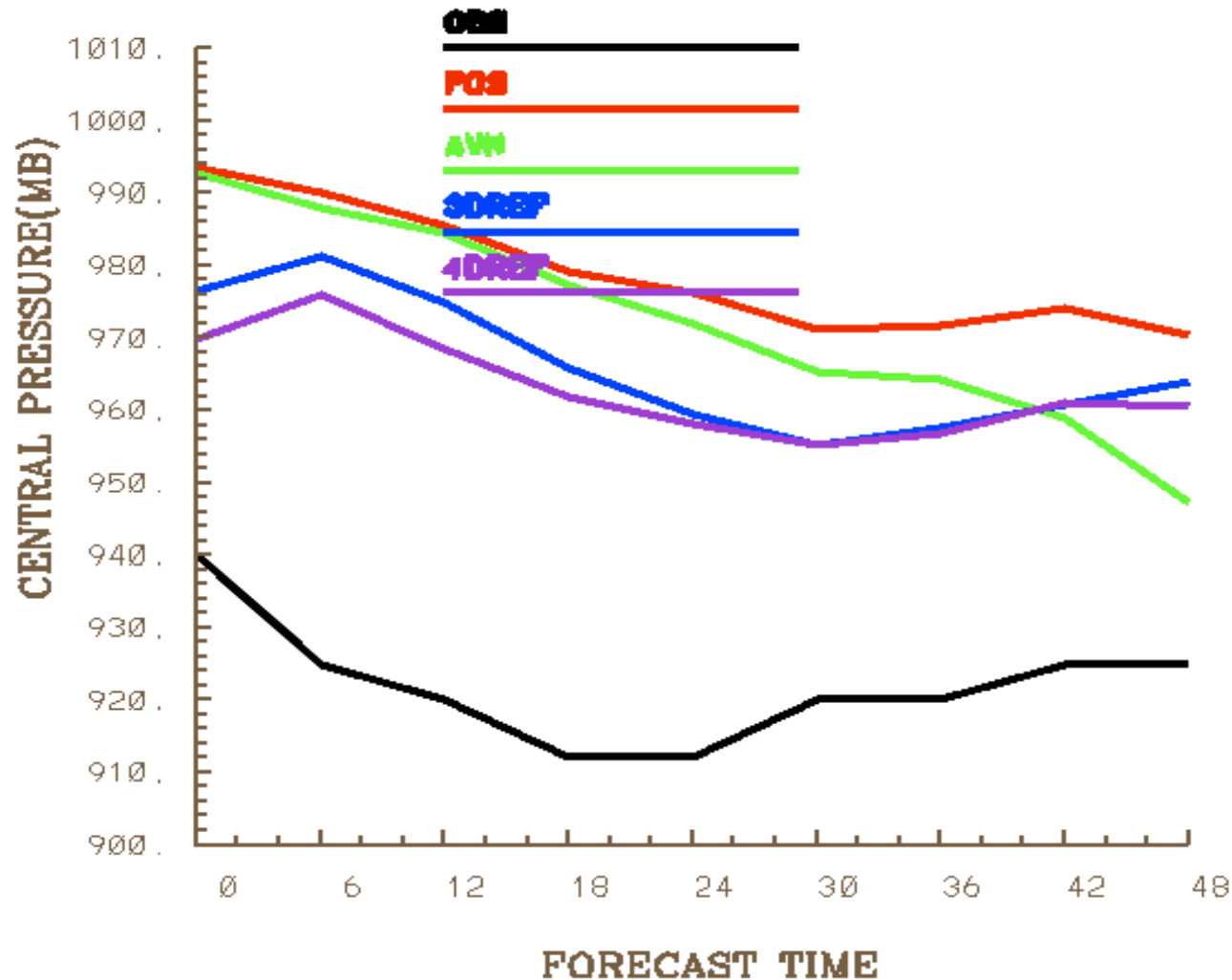
The track error in km averaged over 48 h



The track error in km averaged over 48 h

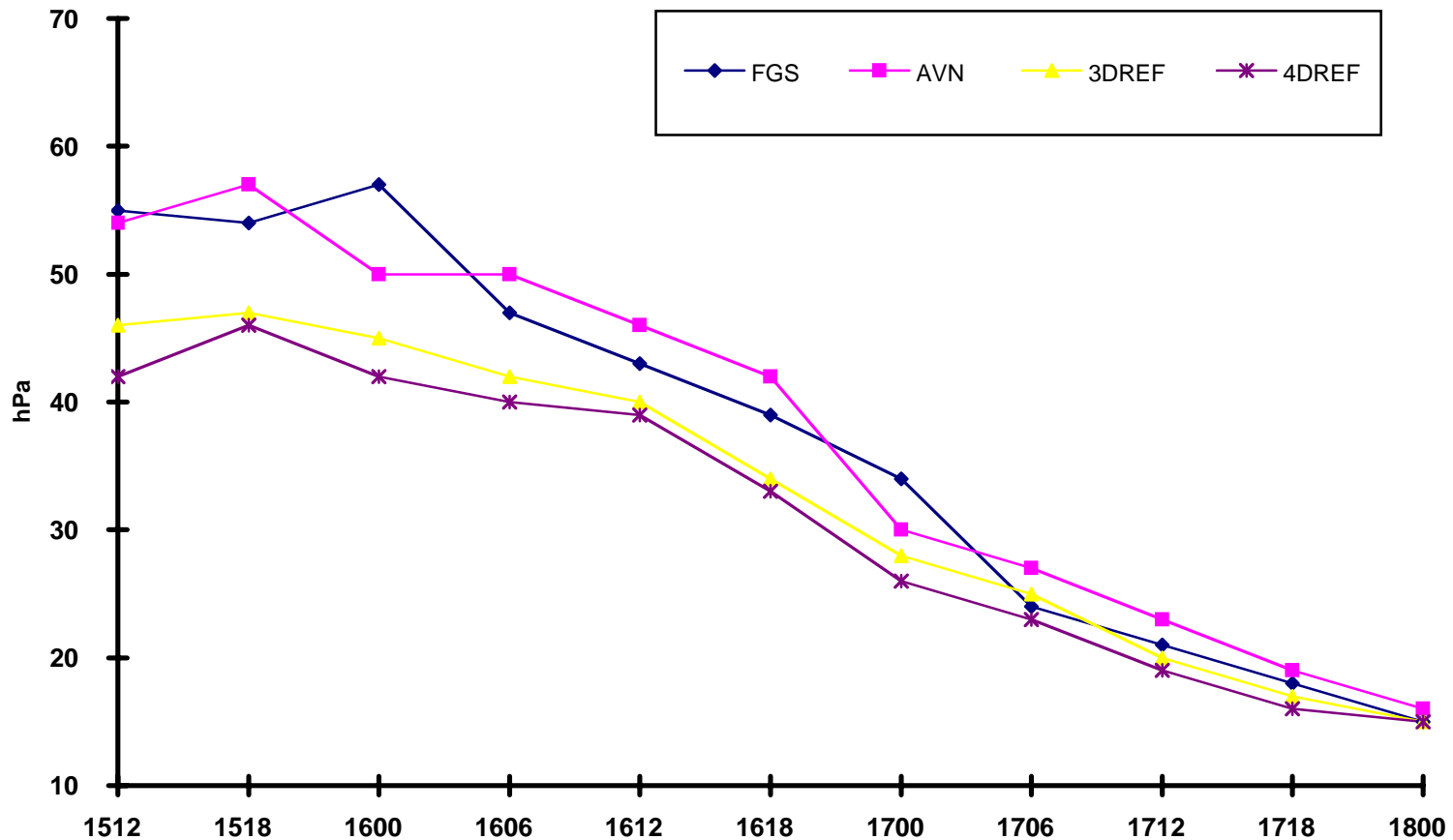
Time	FGS	AVN	3DREF	4DREF
1512	84	82	73	66
1518	82	130	71	85
1600	138	83	92	68
1606	92	83	77	78
1612	96	90	74	61
1618	95	67	101	96
1700	100	86	88	84
1706	111	104	97	116
1712	126	134	131	133
1718	144	126	126	127
1800	150	159	169	156
Average	110.7	104.0	99.9	97.3

Typhoon (Haitang) forecasts



The central pressure error in hpa averaged over 48 h

48 hours forecasted typhoon MSLP verification



The central pressure error in hpa averaged over 48 h

Time	FGS	AVN	3DREF	4DREF
1512	55	54	46	42
1518	54	57	47	46
1600	57	50	45	42
1606	47	50	42	40
1612	43	46	40	39
1618	39	42	34	33
1700	34	30	28	26
1706	24	27	25	23
1712	21	23	20	19
1718	18	19	17	16
1800	15	16	15	15
Average	37.0	37.6	32.6	31.0

Cost issue (current status)

- Single processor - limited grid points.

The largest domain ever tested is: **91x73x17 and 45km**

(This domain is large enough for a model on **271x220x17 and 15km**

- realistic tests are possible.)

- Single processor + Disk I/O = slow.

With the largest domain and an operational data set over 6h,
40 iteration take: **20 h** on a Mac G5

Work plan

1. On going work:
 - Case studies.
 - Code merging.
 - Parallelization.
 - JcDF
2. Near future plan: Multi-incremental; Simple physics;
3. Long term plan: lateral boundary control (J_bdy); more physics, extensive parallel runs.

Summary

1. WRF
2. 4D-Var
3. Current status of WRF 4D-Var
4. Single ob experiments
5. Noise control
6. Typhoon (Haitang) forecasts
7. Work plan
8. Summary