WRF 4D-Var

The Weather Research and Forecasting model based 4-Dimensional Variational data assimilation system

Xiang-Yu Huang
National Center for Atmospheric Research, Boulder, Colorado

On leave from Danish Meteorological Institute, Copenhagen, Denmark.
The WRF 4D-Var Team

Xiang-Yu Huang, Qingnong Xiao, Wei Huang, Dale Barker, John Michalakes, John Bray, Xin Zhang, Zaizhong Ma, Yongrun Guo, Hui-Chuan Lin, Ying-Hwa Kuo

Acknowledgments. The WRF 4D-Var development has been primarily supported by the Air Force Weather Agency.
Outline

1. WRF
2. 4D-Var
3. Current status of WRF 4D-Var
4. Single ob experiments
5. Noise control
6. Typhoon (Haitang) forecasts
7. Work plan
8. Summary
WRF overview

• Eight-year, multi-agency collaboration to develop advanced community mesoscale model and data assimilation system with direct path to operations

• Current release WRFV2.1 (Next release 2.2 November 2006)
  – Two dynamical cores, numerous physics, chemistry
  – Variational Data Assimilation (3D-Var released) and Ensemble Kalman Filter (in development)

• Rapid community growth
  – More than 3,000 registered users
  – June 2005 Users Workshop: 219 participants, 117 inst., 65 countries
  – Scientific papers: real-time NWP, atmos. chemistry, data assimilation, climate, wildfires, mesoscale processes

• Operational capabilities implemented or planned
  – Air Force Weather Agency
  – National Centers for Environmental Prediction
  – BMB (Beijing), KMA (Korea), IMD (India), CWB (Taiwan), IAF (Israel), WSI (U.S.)
Observations are not enough for initializing NWP models:

- Observations have errors.
- Observations are not evenly distributed in time and/or in space.
- Many observations are indirect, e.g. radiance. (not “model variables”, e.g. p, T, u, v, q).
- …
Variational methods: 3D-Var and 4D-Var

Model state

Observations y

Forecast

Analysis

Background

(old forecast)

(initial condition for NWP)

the assimilation window

kth observation window

$t_0$ $t_1$ $t_2$ $\cdots$ $t_k$ $\cdots$ $t_K$ Time
4D-Var

\[ J = J_b + J_o \]

\[ J_b(x_0) = \frac{1}{2} \left[ (x_0 - x_b)^T B^{-1} (x_0 - x_b) \right] \]

\[ J_o(x_0) = \frac{1}{2} \sum_{k=1}^{K} \left[ (H_k x_k - y_k)^T R_k^{-1} (H_k x_k - y_k) \right] \]

\[ x_k = M \left( x_0 \right) \]
WRF 4D-Var

Black  – WRF-3DVar \([B, R, U=B^{1/2}, v^n=U^{-1}(x^n-x^{n-1})]\)

Green  – modification required

Blue    – existing (for 4DVar)

Red     – new development

\[ J'_{vn} = v^n + \sum_{i=1}^{n-1} v^i + U^T S_{W,V}^T \sum_{k=1}^{K} M_k^T S_{W,V}^T H_k^T R^{-1} [H_k S_{W,V} M_k S_{W,V}] U^{-1} v^n + H_k(M_k(x^{n-1})) - y_k \]

(Huang, et.al. 2006: Preliminary results of WRF 4D-Var. WRF users’ workshop, Boulder, Colorado.)
Necessary components of 4D-Var

• $H$ observation operator, including the tangent linear operator $H$ and the adjoint operator $H^T$.

• $M$ forecast model, including the tangent linear model $M$ and adjoint model $M^T$.

• $B$ background error covariance (N*N matrix).

• $R$ observation error covariance which includes the representative error (K*K matrix).
Why 4D-Var?

• Use observations over a time interval, which suits most asynoptic data.
• Use a forecast model as a constraint, which ensures the dynamic balance of the analysis.
• Implicitly use flow-dependent background errors, which ensures the analysis quality for fast developing weather systems.
A short 4D-Var review

- The idea: Le Dimet and Talagrand (1986); Lewis and Derber (1985)
- Implementation examples:
  - Courtier and Talagrand (1990); a shallow water model
  - Thepaut and Courtier (1991); a multi-level primitive equation model
  - Navon, et al. (1992); the NMC global model
  - Zupanski M (1993); the Eta model
  - Zou, et al. (1995); the MM5 model
  - Sun and Crook (1998); a cloud model
  - Rabier, et al. (2000); the ECMWF model
  - Huang, et al. (2002); the HIRLAM model
  - Zupanski M, et al. (2005); the RAMS model
  - Ishikawa, et al. (2005); the JMA mesoscale model
  - Huang, et al. (2005); the WRF model
- Operation: ECMWF, Meteo France, JMA, UKMO, MSC.
- Pre-operation: HIRLAM
Current status of WRF 4D-Var

- Necessary modifications to WRF 3D-Var have been completed.
- WRF tangent-linear and adjoint models have been developed.
- WRF 4D-Var framework has been developed.
- The prototype has been put together and can run. An implementation of it has been made at AFWA in Jan 2006.
The prototype: Use separate executables, communicate through I/O
Single observation experiment

The idea behind single ob tests:
The solution of 3D-Var should be
\[ x^a = x^b + BH^T[H^T+B^T]^{-1}[y - Hx^b] \]

Single observation
\[ x^a - x^b = B_i \left[ \sigma_b^2 + \sigma_o^2 \right]^{-1} [y_i - x_i] \]

3D-Var → 4D-Var: \( H \rightarrow HM; H^T \rightarrow M^TH^T \)
The solution of 4D-Var should be
\[ x^a = x^b + BM^TH^T[H(MBM^T)H^T + R]^{-1}[y - HMx^b] \]

Single observation, solution at observation time
\[ M(x^a - x^b) = (MBM^T)_i \left[ \sigma_b^2 + \sigma_o^2 \right]^{-1} [y_i - x_i] \]
500mb θ increments from 3D-Var at 00h and from 4D-Var at 06h due to a 500mb T observation at 06h.
500mb $\theta$ increments at 00, 01, 02, 03, 04, 05, 06h to a 500mb T ob at 06h
500mb θ difference at 00, 01, 02, 03, 04, 05, 06h from two nonlinear runs (one from background; one from 4D-Var)
500mb $\theta$ difference at 00, 01, 02, 03, 04, 05, 06h from two nonlinear runs (one from background; one from FGAT)
Noise

MSLP (hPa)  Surface pressure tendency (hPa/3h)

$\text{Noise}$

$\text{MSLP (hPa)}$  $\text{Surface pressure tendency (hPa/3h)}$

$t=0$

Hans Huang: WRF 4D-Var
Seminar at UCD 5th October 2006
Sea level pressure and surface pressure tendency at +6h
Evolution of the surface pressure tendency: DPSDT
Noise level

Grid-points: 74×61×28
Resolution: 30 km
Time step: 180 s
Initial state: 3DVAR analysis at 2000.01.25.00 (the second cycle)
DFI for WRF

X.-Y. Huang,
M. Chen, J.-W. Kim, W. Wang,
T. Henderson, W. Skamarock

NCAR, BMB, KMA

Project funded by KMA and BMB
Implemented options of DFI

**DFL:**
- Filtering
- Forecast

**DDFI:**
- Backward integration
- Filtering
- Forecast

**TDFI:**
- Backward integration
- Filtering
- Forecast
DFL test
The KMA domain 10 km : 12UTC 04 May ~ 12UTC 11 May 2006

The mean absolute Psfc tendency (KMA 10 km Domain)
JcDF in WRF 4D-Var

Xin Zhang, University of Hawaii
Hans Huang, NCAR

\[
J = J_b + J_o + J_c
\]

\[
J_b(x_0) = \frac{1}{2} \left[ (x_0 - x_b)^T B^{-1} (x_0 - x_b) \right]
\]

\[
J_o(x_0) = \frac{1}{2} \sum_{k=1}^{K} \left[ (H_k x_k - y_k)^T R_k^{-1} (H_k x_k - y_k) \right]
\]

\[
J_c(x_0) = \frac{\gamma_{df}}{2} \left[ (x_{N/2} - x_{DF}^{N/2})^T C^{-1} (x_{N/2} - x_{DF}^{N/2}) \right]
\]

\[
x_{DF}^{N/2} = \sum_{n=0}^{N} h_n x_n
\]
WRF 4D-Var

Black  – WRF-3DVar \( [\mathbf{B}, \mathbf{R}, \mathbf{U}=\mathbf{B}^{1/2}, \mathbf{v}^n=\mathbf{U}^{-1}(\mathbf{x}^n-\mathbf{x}^{n-1})] \)

Green  – modification required

Blue    – existing (for 4DVar)

Red     – new development

\[
J'_{\mathbf{v}_n} = \mathbf{v}^n + \sum_{i=1}^{n-1} \mathbf{v}^i + \mathbf{U}^T \mathbf{S}_{\mathbf{v}-\mathbf{w}}^T \sum_{k=1}^{K} \mathbf{M}_k \mathbf{S}_{\mathbf{w}-\mathbf{v}}^T \mathbf{H}_k^T \mathbf{R}^{-1} [\mathbf{H}_k \mathbf{S}_{\mathbf{w}-\mathbf{v}} \mathbf{M}_k \mathbf{S}_{\mathbf{v}-\mathbf{w}} \mathbf{U}^{-1} \mathbf{v}^n + H_k(M_k(\mathbf{x}^{n-1})) - \mathbf{y}_k]
\]

\[
+ \mathbf{U}^T \mathbf{S}_{\mathbf{v}-\mathbf{w}}^T \sum_{i=0}^{N} \mathbf{M}_i^T h_i \gamma_{df} \mathbf{C}^{-1} \left( \sum_{i=0}^{N} (h_i \mathbf{M}_i \mathbf{S}_{\mathbf{v}-\mathbf{w}} \mathbf{U} \mathbf{v}) \right)
\]
Jb, Jo and Jc in WRF
\[ \gamma = 10.0 \]
Typhoon Haitang experiments:

4 experiments, every 6 h, 00Z 16 July - 00 Z 18 July, 2005
Typhoon Haitang hit Taiwan 00Z 18 July 2005

1. FGS – forecast from the background [The background fields are 6-h WRF forecasts from National Center for Environment Prediction (NCEP) GFS analysis.]

2. AVN - forecast from the NCEP GPS analysis

3. 3DVAR – forecast from WRF 3D-Var

4. 4DVAR – forecast from WRF 4D-Var
## Observations used in 4DVAR and FGAT at 0000UTC 16 July 2005

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(At 0600UTC 16 July: GPS refractivity 2594, QuikScat u 2594, v 2605)
Typhoon (Haitang) forecasts
Typhoon (Haitang) forecasts

TRACK ERROR (KM)

FORECAST TIME

138.30 = POS
83.22 = AVN
92.22 = 3DREF
68.72 = 4DREF
The track error in km averaged over 48 h

48 hours forecasted typhoon track verification

- FGS
- AVN
- 3DREF
- 4DREF

KM

0 20 40 60 80 100 120 140 160 180 200

1512 1518 1600 1606 1612 1618 1700 1706 1712 1718 1800
The track error in km averaged over 48 h

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Typhoon (Haitang) forecasts
The central pressure error in hpa averaged over 48 h

48 hours forecasted typhoon MSLP verification

- FGS
- AVN
- 3DREF
- 4DREF
The central pressure error in hpa averaged over 48 h

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Cost issue (current status)

• Single processor - limited grid points.

The largest domain ever tested is: 91x73x17 and 45km
(This domain is large enough for a model on 271x220x17 and 15km
- realistic tests are possible.)

• Single processor + Disk I/O = slow.

With the largest domain and an operational data set over 6h,
40 iteration take: 20 h on a Mac G5
Work plan

1. On going work:
   - Case studies.
   - Code merging.
   - Parallelization.
   - JcDF
2. Near future plan: Multi-incremental; Simple physics;
3. Long term plan: lateral boundary control (J_bdy); more physics, extensive parallel runs.
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