Seminar at the UCD Meteorology and Climate Centre
8 March 2007

MANUALLY ADJUSTING A NUMERICAL WEATHER ANALYSIS: APPLICATION TO A CASE OF RAPID CYCLOGENESIS

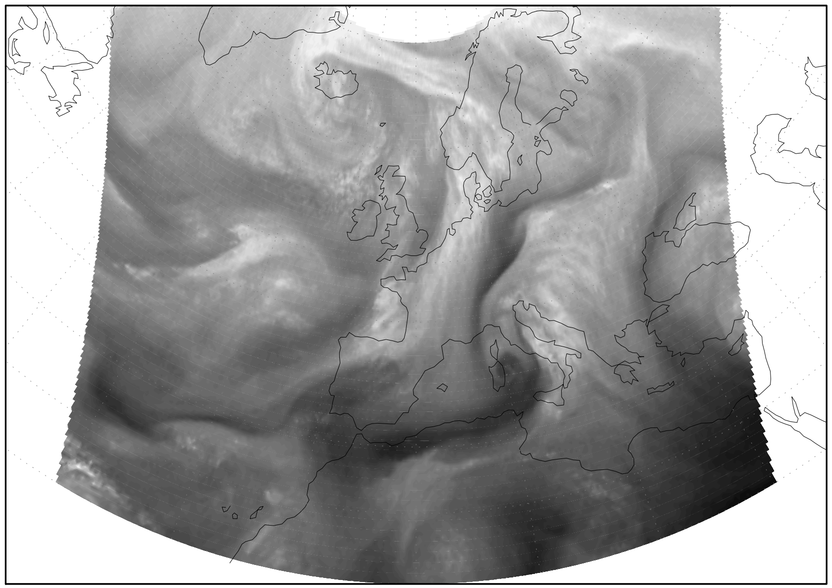
Astrid Manders, Wim Verkley, Toon Moene and Jos Diepeveen

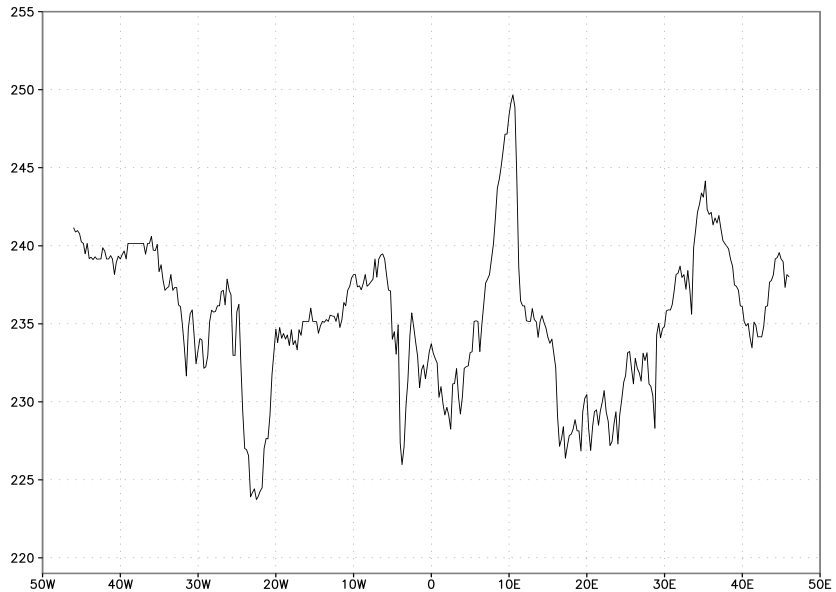
OVERVIEW

1. Radiative transfer
2. Potential vorticity
3. Manual intervention
4. Rapid cyclogenesis

1. RADIATIVE TRANSFER

Example of a satellite image in the water vapour channel. Left: brightness temperature (K) in terms of gray shades. Right: brightness temperature (K) at 48 N plotted as a function of longitude.





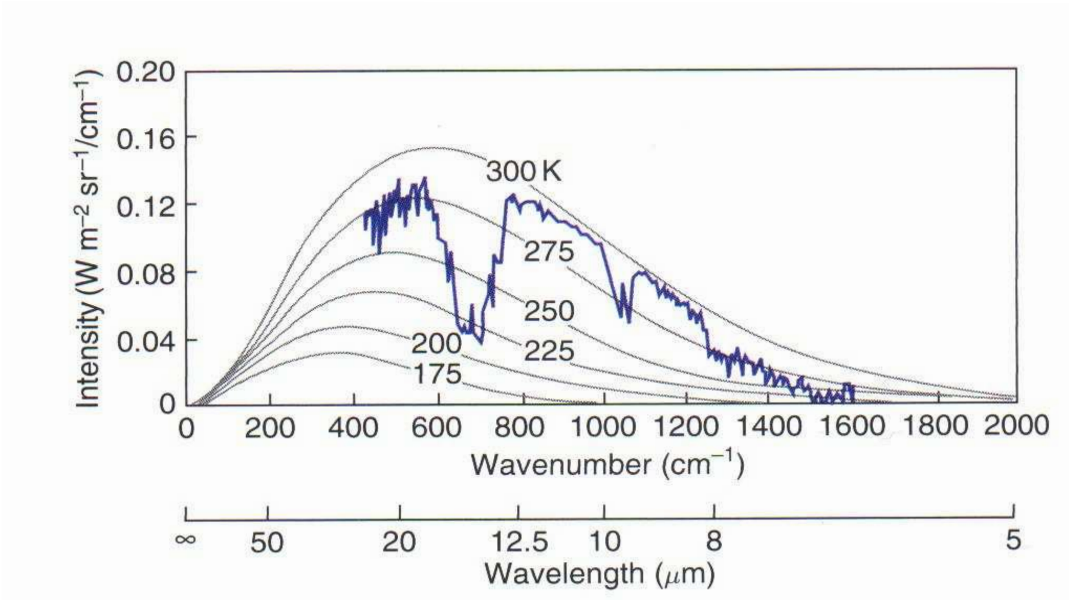
In the simplified context of radiation that travels vertically upwards Schwarzschild's equation for radiative transfer reads:

$$\frac{dI_\nu(z)}{dz} = -k_\nu \rho_v [I_\nu(z) - B_\nu(z)].$$

Here I_ν is the upward radiance ($\mathrm{Wm^{-2}sr^{-1}cm}$). The function B_ν is the Planck function: the radiation emitted by a so-called black body. The Planck function depends on the wavenumber ν and on the temperature T and is given by:

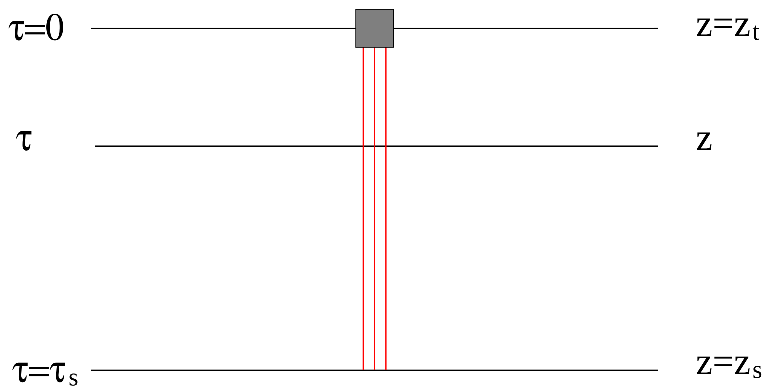
$$B_\nu(T) = \frac{2h\nu^3c^2}{\exp(hc\nu/kT) - 1}.$$

The SEVIRI radiometer on the Meteosat-8 satellite has two water vapour channels: one centered at 6.2μm (1612.9 cm^{-1}) and another centered at 7.3μm (1369.9 cm^{-1}).



Using the Planck function, each radiance I_ν can be identified with a temperature T_ν: the brightness temperature.

The radiative transfer equation assumes a very simple form by using the optical depth as a vertical coordinate.



The optical depth τ, with respect to the height z_t of the radiometer, is defined as:

$$\tau(z) = \int_{z}^{z_t} k_\nu(z')\rho_v(z')dz'.$$

Because we have that $d\tau = -k_\nu(z)\rho_v(z)dz$, the radiative transfer equation becomes:

$$\frac{dI_\nu(\tau)}{d\tau} = I_\nu(\tau) - B_\nu(\tau).$$

It can be verified straightforwardly that the solution is:

$$I_\nu(\tau) = B_\nu(\tau_s)\,\mathrm{exp}(\tau - \tau_s) + \int_\tau^{\tau_s} B_\nu(\tau')\,\mathrm{exp}(\tau - \tau')d\tau',$$

where τ_s is the optical depth corresponding to the surface. Identifying z_t with the height of the satellite, so that $\tau = 0$ at the satellite, we find for the radiance received by the satellite:

$$I_\nu(0) = B_\nu(\tau_s)\,\mathrm{exp}(-\tau_s) + \int_0^{\tau_s} B_\nu(\tau')\,\mathrm{exp}(-\tau')d\tau'.$$

From the general expression just derived,

$$I_\nu(0) = B_\nu(\tau_s)\,\mathrm{exp}(-\tau_s) + \int_0^{\tau_s} B_\nu(\tau')\,\mathrm{exp}(-\tau')d\tau',$$

we can derive the ‘topographic interpretation’, given by:

$$I_\nu(0) \approx B_\nu(\tau_m),$$

where τ_m is the value of the optical depth at the upper boundary of the layer that contains water vapour.

If we assume that the stratosphere is completely dry, the radiance gives us information on the temperature of the tropopause.

2. POTENTIAL VORTICITY

For the hydrostatic primitive equations we have for the continuity equation and the equation for absolute vorticity:

$$\frac{1}{\rho}\frac{D\rho}{Dt} + \mathcal{D} + \frac{1}{\rho}\frac{\partial[\rho w]}{\partial z} = 0,$$

$$\frac{1}{\omega}\frac{D\omega}{Dt} + \mathcal{D} + \frac{1}{\omega}\mathbf{k}\cdot\nabla\times[\frac{\partial \mathbf{v}}{\partial z}w - \mathbf{F}] = -\frac{\nabla p \times \nabla \rho}{\rho^3},$$

where $\rho = -1/g\partial p/\partial z$ is density and $\omega = f + \zeta$ absolute vorticity. To derive the equation for potential vorticity it is convenient to use the potential temperature $\theta = T(p_r/p)^\kappa$ as a vertical coordinate. Because $\rho = p_r/(R\theta)(p/p_r)^{1-\kappa}$, we have that

$$\frac{\nabla p \times \nabla \rho}{\rho^3} = 0,$$

simplifying the equation for absolute vorticity.

With potential temperature as a vertical coordinate (isentropic coordinates) we thus have for the continuity equation and the equation for absolute vorticity:

$$\frac{1}{\sigma}\frac{D\sigma}{Dt} + \mathcal{D} + \frac{1}{\sigma}\frac{\partial[\sigma\dot{\theta}]}{\partial\theta} = 0,$$

$$\frac{1}{\omega}\frac{D\omega}{Dt} + \mathcal{D} + \frac{1}{\omega}\mathbf{k}\cdot\nabla\times[\frac{\partial\mathbf{v}}{\partial\theta}\dot{\theta} - \mathbf{F}] = 0,$$

where $\sigma = -1/g\partial p/\partial\theta$ and $\omega = f + \zeta$. Note that all horizontal and time derivates are now taken with θ fixed instead of z fixed. The vertical velocity in terms of θ is directly given in terms of the heating rate Q:

$$\dot{\theta} = \frac{\theta}{c_p T}Q.$$

Diabatic processes therefore come in very directly if isentropic coordinates are used.

From the continuity equation and the equation for absolute vorticity,

$$\frac{1}{\sigma}\frac{D\sigma}{Dt} + \mathcal{D} + \frac{1}{\sigma}\frac{\partial[\sigma\dot{\theta}]}{\partial\theta} = 0,$$

$$\frac{1}{\omega}\frac{D\omega}{Dt} + \mathcal{D} + \frac{1}{\omega}\mathbf{k}\cdot\nabla\times[\frac{\partial\mathbf{v}}{\partial\theta}\dot{\theta} - \mathbf{F}] = 0,$$

we can straightforwardly eliminate the horizontal divergence $\mathcal{D}$ to obtain:

$$\frac{DP}{Dt} + \frac{1}{\sigma}[\mathbf{k}\cdot\nabla\times(\frac{\partial\mathbf{v}}{\partial\theta}\dot{\theta} - \mathbf{F})] - P\frac{\partial}{\partial\theta}(\sigma\dot{\theta})] = 0.$$

Here

$$P = \frac{\omega}{\sigma}$$

is the potential vorticity.

We used isentropic coordinates to simplify the derivation, but the resulting expression

$$P = \frac{\omega}{\sigma}$$

can always be transformed to a system with an other vertical coordinate, such as the η-coordinate of HIRLAM. In that case we have:

$$P = -g \left(\omega \frac{\partial \theta}{\partial p} - \frac{1}{a \cos \phi} \frac{\partial v}{\partial p} \left[\frac{\partial \theta}{\partial \lambda} \right] + \frac{1}{a} \frac{\partial u}{\partial p} \left[\frac{\partial \theta}{\partial \phi} \right] \right),$$

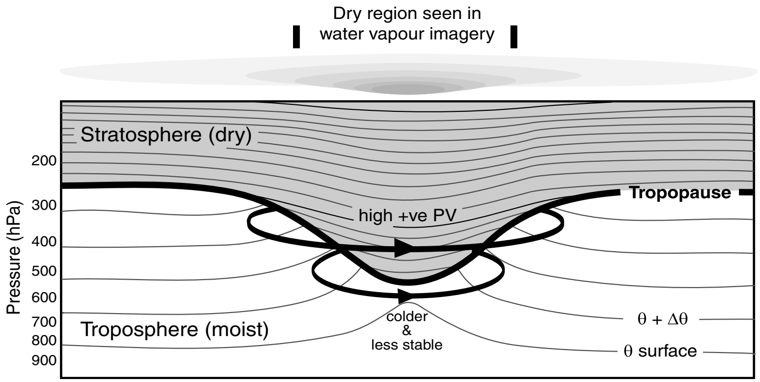
where now all horizontal derivatives are to be evaluated with η kept constant. However, in displaying the potential vorticity we often interpolate back to an isentropic surface, that is, to a surface on which θ is constant. The expression of potential vorticity is evaluated numerically by using simple centered finite differences for the horizontal and vertical derivatives.

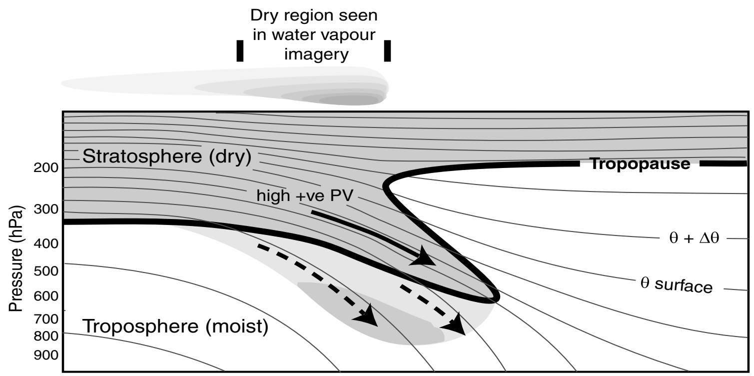
Potential vorticity is materially conserved if $\dot{\theta}$ and $\mathbf{F}$ are zero.

It is furthermore invertible if the atmosphere is hydrostatically and geostrophically balanced.

It increases with height with a sharp jump at the dynamical tropopause around $P = 2$ PVU.

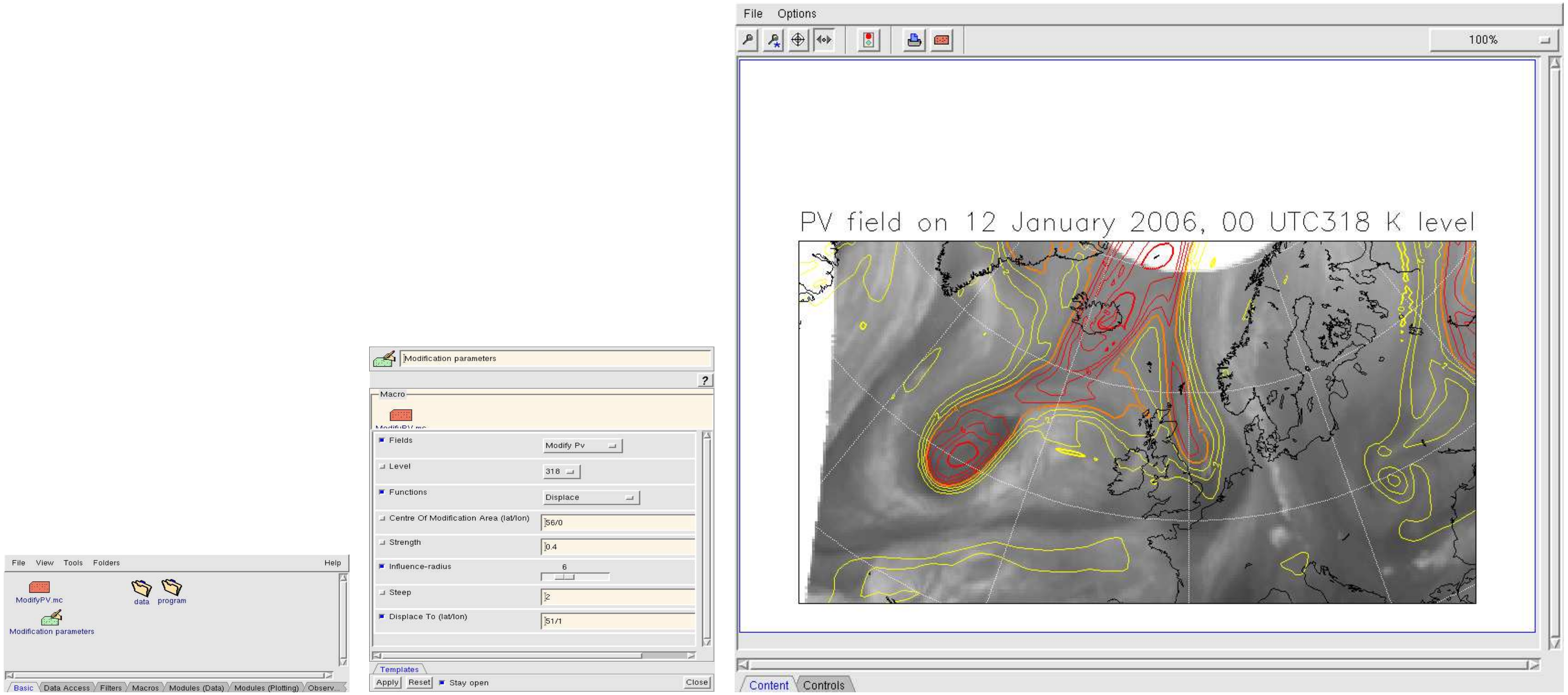
Water vapour imagery can thus be related to the potential vorticity field and therewith entails dynamical information:





3. MANUAL INTERVENTION

We have developed a graphical interface with which the potential vorticity field of a numerical weather analysis can be modified:



To incorporate a modified potential vorticity field into the analysis, we use three-dimensional variational data-assimilation and treat the modified potential vorticity field as a field of observations with the current analysis as the background.

Using standard terminology, this means that we minimize the following cost function:

$$J = J_b + J_p,$$

where the terms J_b (background) and J_p (potential vorticity) are:

$$J_b = \frac{1}{2}[\mathbf{x} - \mathbf{x}_b]^\top \mathbf{B}^{-1}[\mathbf{x} - \mathbf{x}_b], \quad J_p = \frac{1}{2}[\mathcal{F}(\mathbf{x}) - \mathbf{z}]^\top \mathbf{W}^{-1}[\mathcal{F}(\mathbf{x}) - \mathbf{z}].$$

Here $\mathbf{x}$ is the model state, $\mathbf{x}_b$ the background state, $\mathcal{F}(\mathbf{x})$ the operator that calculates the potential vorticity field from the model state $\mathbf{x}$ and $\mathbf{z}$ is the modified potential vorticity field.

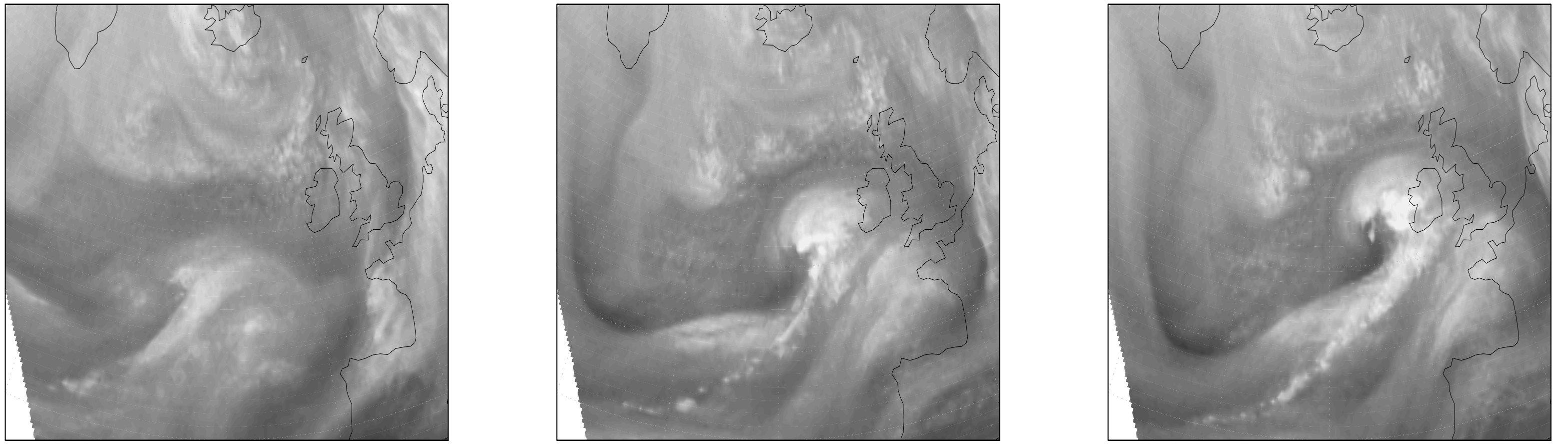
For the background covariance matrix $\mathbf{B}$ we used the same matrix as the one used in the assimilation of conventional observations. For the background covariance matrix $\mathbf{W}$ we used a simple diagonal matrix with variances proportional to the squared potential vorticity field of the background.

By giving the term J_p a much larger weight than the term J_b in the total cost function J the modified analysis can be made to have a potential vorticity field that is very close to the modified potential vorticity field.

The system was implemented in version 6.3.5 of HIRLAM, operational until autumn of last year.

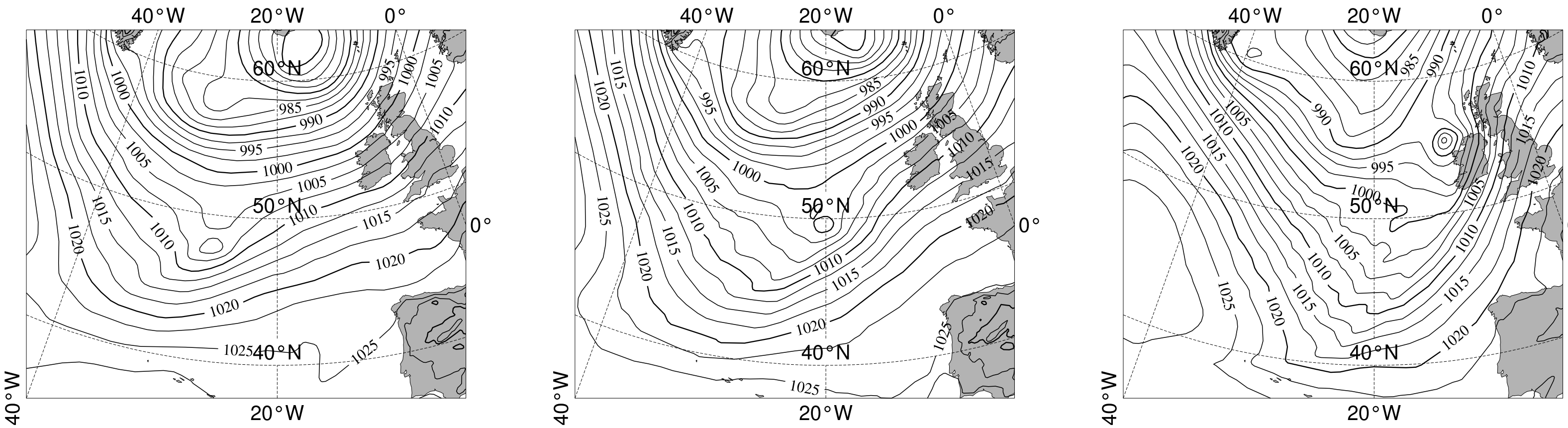
4. RAPID CYCLOGENESIS

The following pictures show the water vapour satellite images of 7 november 2005, 01 UTC, 09 UTC and 12 UTC:



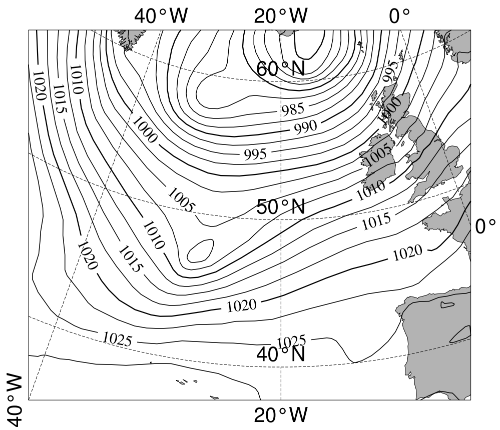
This is a rather typical case of rapid cyclogenesis, leading to a deep depression to the south-west of Ireland of 974 hPa at 18 UTC.

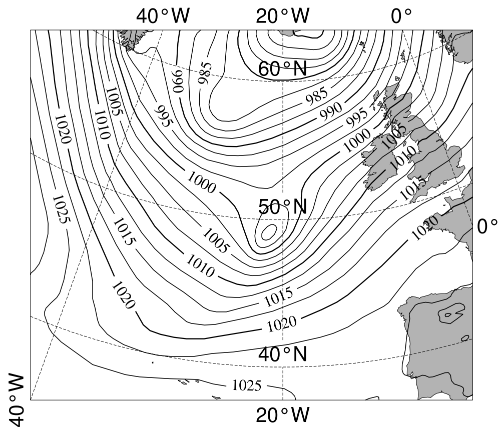
HIRLAM did not predict the evolution of the surface pressure very well. The following pictures show the HIRLAM analysis at 00 UTC and the forecast at 06 and 18 UTC:

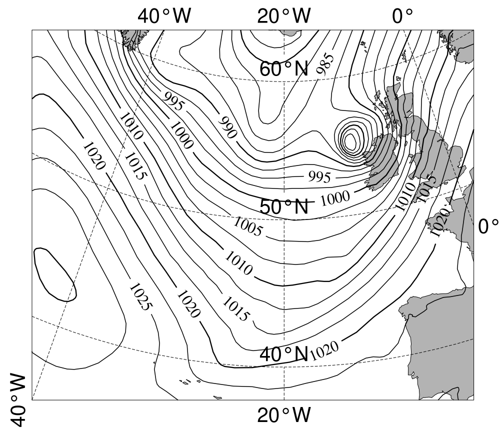


The surface pressure at 18 UTC in the HIRLAM forecast is 985 hPa instead of 974 hPa.

The ECMWF model predicted the evolution better. The following pictures show the ECMWF analysis at 00 UTC and the forecast at 06 and 18 UTC:

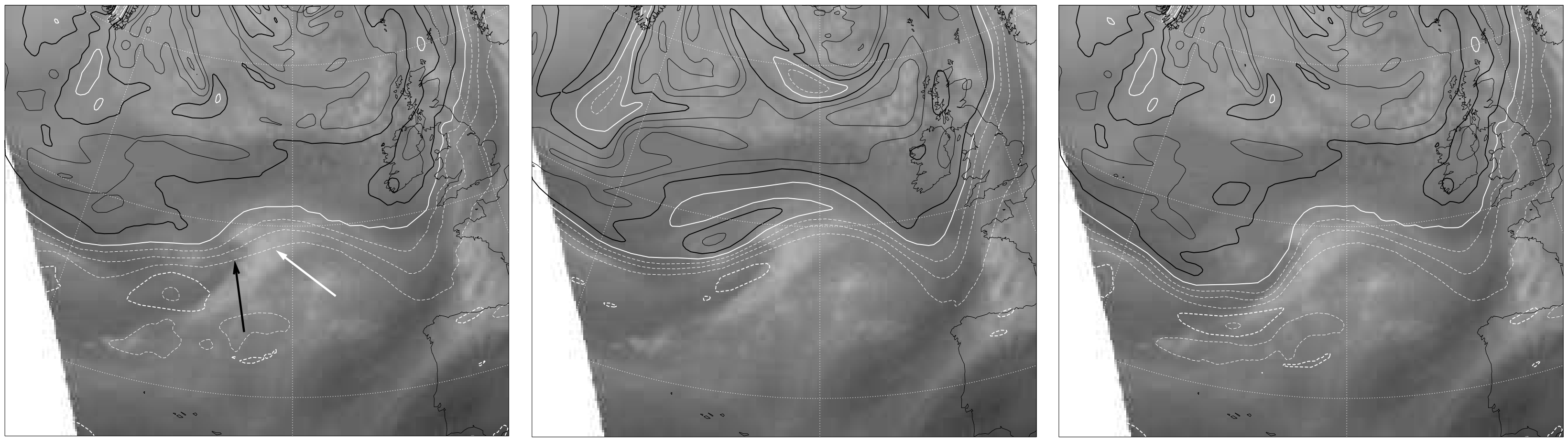




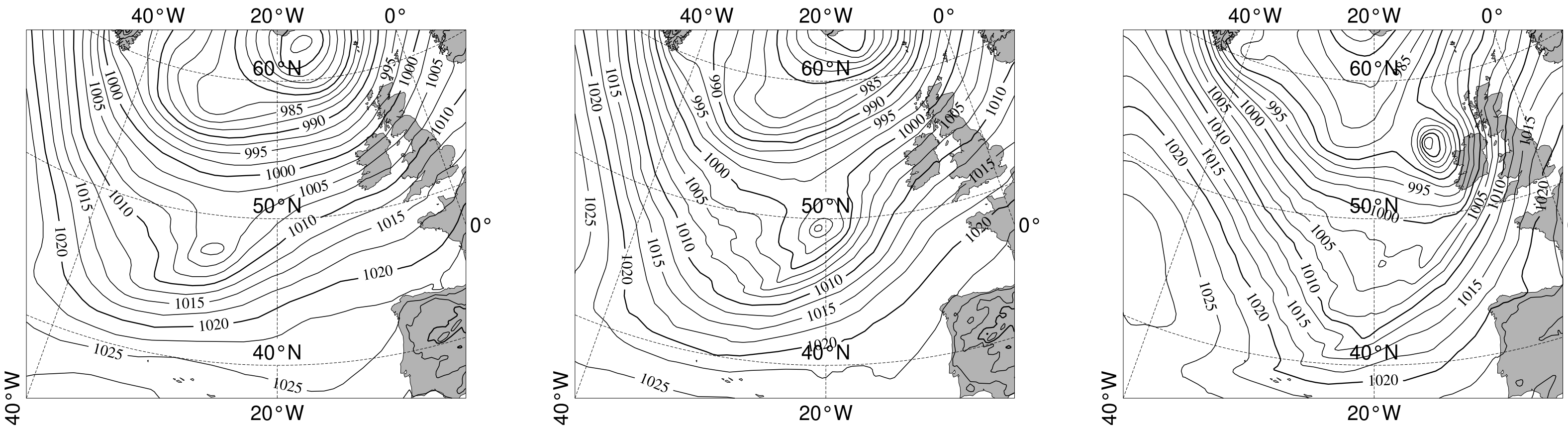


The surface pressure at 18 UTC in the ECMWF forecast is 973 hPa instead of 974 hPa.

The left figure shows the water vapour satellite image at 01 UTC, in combination with the potential vorticity field at 315 K. The arrows denote possible mismatches. The middle figure shows the ECMWF analysis. The right figure shows the HIRLAM analysis after we made a modification that should bring the potential vorticity field closer to the water vapour image.

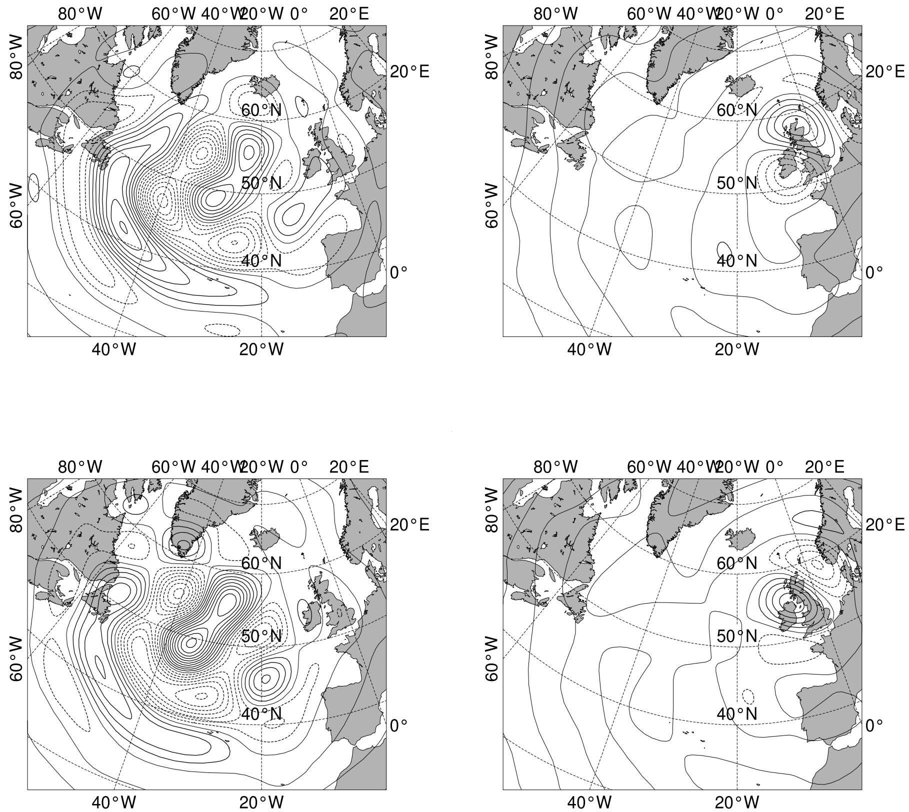


The modified HIRLAM analysis predicted the evolution better. The following pictures show the modified HIRLAM analysis at 00 UTC and the resulting modified forecast at 06 and 18 UTC:

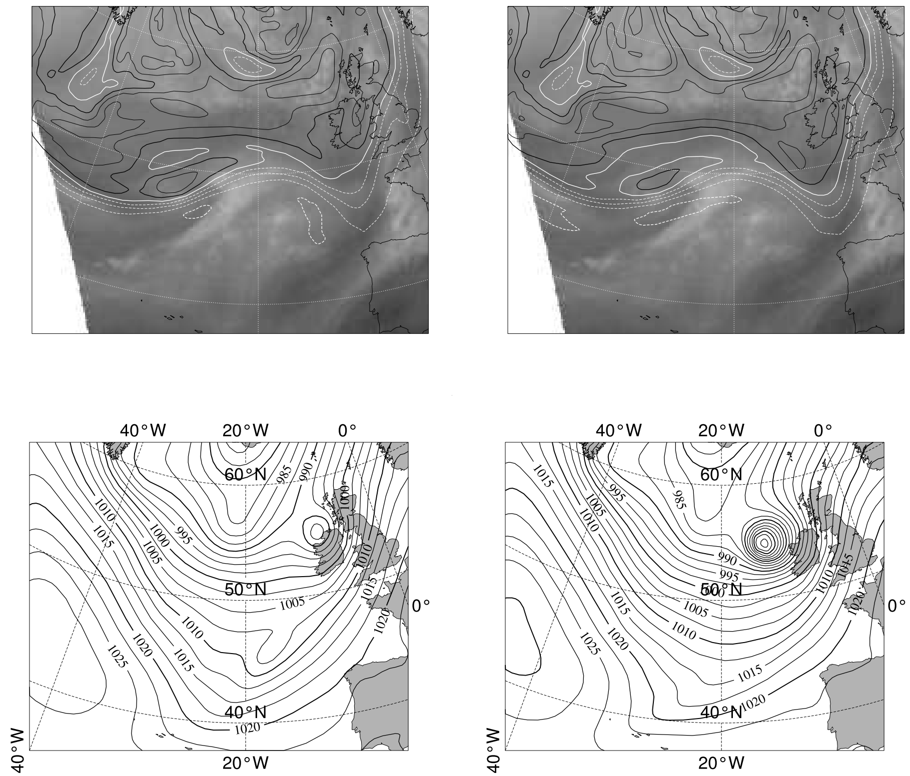


The surface pressure at 18 UTC in the modified HIRLAM forecast is now 976 hPa.

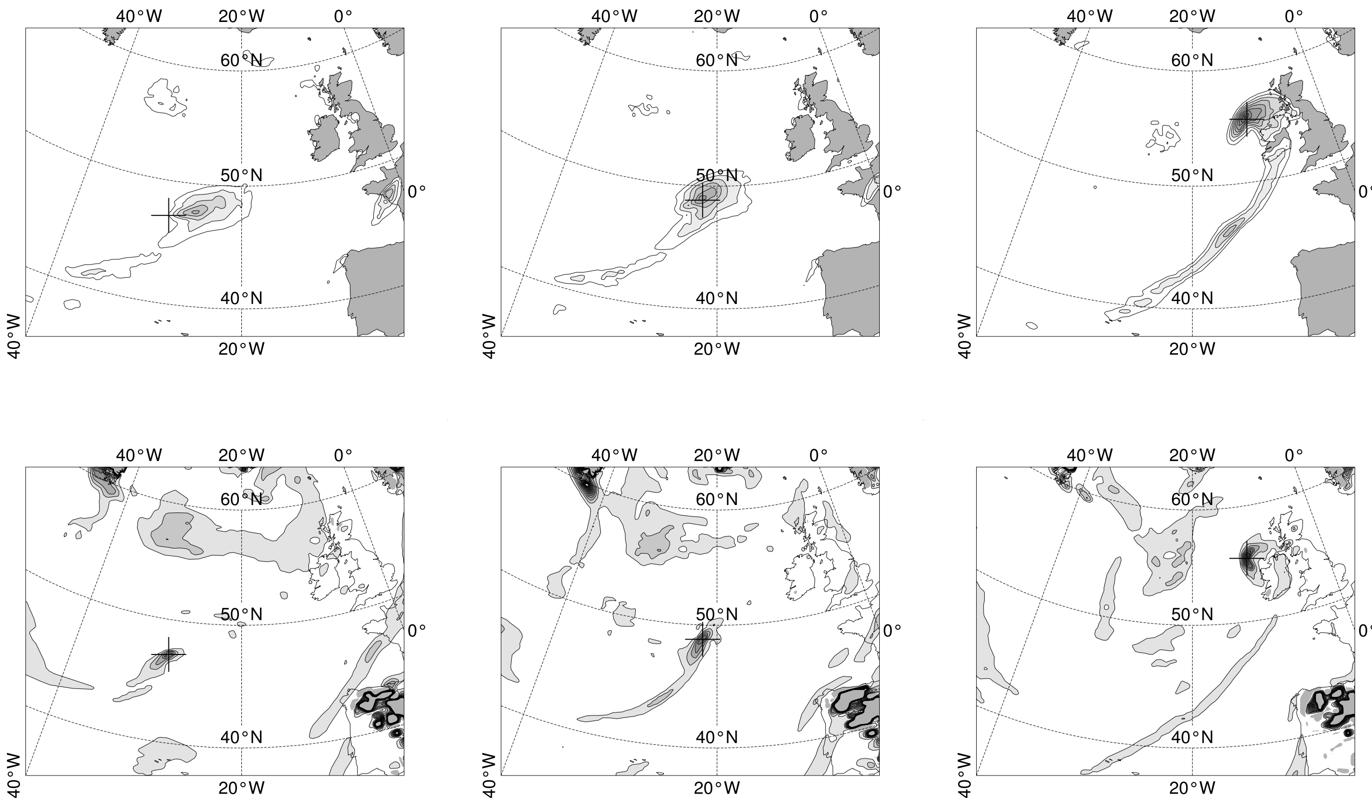
Finding the right modification was not an easy matter. The ECMWF singular vectors helped us very much:



We also noted that two members of the ECMWF ensemble (with opposite signs of the initial perturbations) gave the different extremes of the evolution:



Latent heat release seems to have played a major role. The figures below show accumulated precipitation over three hours and low level (925 hPa) potential vorticity for the ECMWF run:



It seems that our modification of the potential vorticity has placed the jet stream and the initial low level perturbation in a better relative position to produce rapid growth. Both from the perspective of operational practice and theoretical insight, it is useful to obtain a better understanding of this case as it is probably representative of many other cases of rapid cyclogenesis.

At the moment a very similar method of manual intervention in the numerical weather prediction system is being tested operationally by the Norwegian Meteorological Service. It is part of a project aimed at exploring the future role of the human forecaster in the weather prediction process.