Reynolds Averaging
Reynolds Averaging

We separate the dynamical fields into slowly varying mean fields and rapidly varying turbulent components.
Reynolds Averaging

We separate the dynamical fields into slowly varying mean fields and rapidly varying turbulent components.

For example

$$\theta = \bar{\theta} + \theta'$$
We separate the dynamical fields into slowly varying mean fields and rapidly varying turbulent components.

For example

\[ \theta = \bar{\theta} + \theta' \]

We assume the mean variable is constant over the period of averaging. By definition, the mean of the perturbation vanishes:

\[ \bar{\theta}' = 0 \quad \bar{\theta} = \bar{\theta} \quad \bar{\theta} \theta' = \bar{\theta} \]
Reynolds Averaging

We separate the dynamical fields into slowly varying mean fields and rapidly varying turbulent components.

For example

$$\theta = \bar{\theta} + \theta'$$

We assume the mean variable is constant over the period of averaging. By definition, the mean of the perturbation vanishes:

$$\bar{\theta}' = 0 \quad \bar{\theta} = \bar{\theta} \quad \bar{\theta}\theta' = \bar{\theta}$$

Thus, the mean of a product has two components:

$$\bar{w\theta} = \frac{(\bar{w} + w')(\bar{\theta} + \theta'\bar{\theta})}{\bar{\theta}'}$$

$$= \frac{(\bar{w}\theta + \bar{w}\theta' + w'\bar{\theta} + w'\theta')}{\bar{\theta}'}$$

$$= \bar{w}\theta + \bar{w}\theta' + w'\bar{\theta} + w'\theta'$$

$$= \bar{w}\theta + w'\theta'$$
Thus, we have:

\[ w\theta = \overline{w\theta} + w'\theta' \]
Thus, we have:

\[ w\theta = \overline{w\theta} + w'\theta' \]

We assume that the flow is nondivergent (see Holton §5.1.1):

\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \]
Thus, we have:

\[ w\theta = \overline{w}\theta + w'\theta' \]

We assume that the flow is nondivergent (see Holton §5.1.1):

\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \]

The total time derivative of \( u \) is

\[
\frac{du}{dt} = \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} + u \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right) \\
= \frac{\partial u}{\partial t} + \frac{\partial uu}{\partial x} + \frac{\partial uv}{\partial y} + \frac{\partial uw}{\partial z}
\]
Thus, we have:

$$\overline{w\theta} = \overline{w\theta} + \overline{w'\theta'}$$

We assume that the flow is nondivergent (see Holton §5.1.1):

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

The total time derivative of $u$ is

$$\frac{du}{dt} = \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} + u \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right)$$

$$= \frac{\partial u}{\partial t} + \frac{\partial uu}{\partial x} + \frac{\partial uv}{\partial y} + \frac{\partial uw}{\partial z}$$

Writing sums of mean and eddy parts and averaging:

$$\overline{\frac{du}{dt}} = \frac{\partial \overline{u}}{\partial t} + \frac{\partial}{\partial x} \left( \overline{uu} + \overline{u'u'} \right) + \frac{\partial}{\partial y} \left( \overline{uv} + \overline{u'v'} \right) + \frac{\partial}{\partial z} \left( \overline{uw} + \overline{u'w'} \right)$$
Repeat:

\[
\frac{d\bar{u}}{dt} = \frac{\partial \bar{u}}{\partial t} + \frac{\partial}{\partial x} \left( \bar{u} \bar{u} + u'u' \right) + \frac{\partial}{\partial y} \left( \bar{u} \bar{v} + u'v' \right) + \frac{\partial}{\partial z} \left( \bar{u} \bar{w} + u'w' \right)
\]
\[
\frac{\overline{du}}{dt} = \frac{\partial \overline{u}}{\partial t} + \frac{\partial}{\partial x} \left( \overline{u} \overline{u} + \overline{u'u'} \right) + \frac{\partial}{\partial y} \left( \overline{u} \overline{v} + \overline{u'v'} \right) + \frac{\partial}{\partial z} \left( \overline{u} \overline{w} + \overline{u'w'} \right)
\]

But the mean flow is nondivergent, so
\[
\frac{\partial}{\partial x} (\overline{u} \overline{u}) + \frac{\partial}{\partial y} (\overline{u} \overline{v}) + \frac{\partial}{\partial z} (\overline{u} \overline{w}) = \overline{u} \frac{\partial \overline{u}}{\partial x} + \overline{v} \frac{\partial \overline{u}}{\partial y} + \overline{w} \frac{\partial \overline{u}}{\partial z}
\]
Repeat:
\[
\frac{\overline{du}}{dt} = \frac{\partial \overline{u}}{\partial t} + \frac{\partial}{\partial x} (\overline{u} \overline{u} + \overline{u'}u') + \frac{\partial}{\partial y} (\overline{u} \overline{v} + \overline{u'}v') + \frac{\partial}{\partial z} (\overline{u} \overline{w} + \overline{u'}w')
\]

But the mean flow is nondivergent, so
\[
\frac{\partial}{\partial x} (\overline{u} \overline{u}) + \frac{\partial}{\partial y} (\overline{u} \overline{v}) + \frac{\partial}{\partial z} (\overline{u} \overline{w}) = \overline{u} \frac{\partial \overline{u}}{\partial x} + \overline{v} \frac{\partial \overline{u}}{\partial y} + \overline{w} \frac{\partial \overline{u}}{\partial z}
\]

We define the mean operator
\[
\frac{\overline{d}}{dt} = \frac{\partial}{\partial t} + \overline{u} \frac{\partial}{\partial x} + \overline{v} \frac{\partial}{\partial y} + \overline{w} \frac{\partial}{\partial z}
\]
Repeat:

\[
\frac{\overline{du}}{dt} = \frac{\partial \overline{u}}{\partial t} + \frac{\partial}{\partial x} \left( \overline{u} \overline{u} + \overline{u}' \overline{u}' \right) + \frac{\partial}{\partial y} \left( \overline{u} \overline{v} + \overline{u}' \overline{v}' \right) + \frac{\partial}{\partial z} \left( \overline{u} \overline{w} + \overline{u}' \overline{w}' \right)
\]

But the mean flow is nondivergent, so

\[
\frac{\partial}{\partial x} \left( \overline{u} \overline{u} \right) + \frac{\partial}{\partial y} \left( \overline{u} \overline{v} \right) + \frac{\partial}{\partial z} \left( \overline{u} \overline{w} \right) = \overline{u} \frac{\partial \overline{u}}{\partial x} + \overline{v} \frac{\partial \overline{u}}{\partial y} + \overline{w} \frac{\partial \overline{u}}{\partial z}
\]

We define the mean operator

\[
\frac{\overline{d}}{dt} = \frac{\partial}{\partial t} + \overline{u} \frac{\partial}{\partial x} + \overline{v} \frac{\partial}{\partial y} + \overline{w} \frac{\partial}{\partial z}
\]

Then we have

\[
\frac{\overline{du}}{dt} = \frac{\partial \overline{u}}{\partial t} + \frac{\partial}{\partial x} \left( \overline{u}' \overline{u}' \right) + \frac{\partial}{\partial y} \left( \overline{u}' \overline{v}' \right) + \frac{\partial}{\partial z} \left( \overline{u}' \overline{w}' \right)
\]
Now we can write the momentum equations as

\[
\begin{align*}
\frac{\overline{du}}{dt} - f\overline{v} + \frac{1}{\rho_0} \frac{\partial \overline{p}}{\partial x} + \left[ \frac{\partial}{\partial x} \left( \overline{u'u'} \right) + \frac{\partial}{\partial y} \left( \overline{v'u'} \right) + \frac{\partial}{\partial z} \left( \overline{w'u'} \right) \right] & = 0 \\
\frac{\overline{dv}}{dt} + f\overline{u} + \frac{1}{\rho_0} \frac{\partial \overline{p}}{\partial y} + \left[ \frac{\partial}{\partial x} \left( \overline{v'u'} \right) + \frac{\partial}{\partial y} \left( \overline{v'v'} \right) + \frac{\partial}{\partial z} \left( \overline{v'w'} \right) \right] & = 0
\end{align*}
\]
Now we can write the momentum equations as

\[
\frac{\overline{du}}{dt} - f\overline{v} + \frac{1}{\rho_0} \frac{\partial \overline{p}}{\partial x} + \left[ \frac{\partial}{\partial x} \left( \overline{u'u'} \right) + \frac{\partial}{\partial y} \left( \overline{v'u'} \right) + \frac{\partial}{\partial z} \left( \overline{w'u'} \right) \right] = 0
\]

\[
\frac{\overline{dv}}{dt} + f\overline{u} + \frac{1}{\rho_0} \frac{\partial \overline{p}}{\partial y} + \left[ \frac{\partial}{\partial x} \left( \overline{v'u'} \right) + \frac{\partial}{\partial y} \left( \overline{v'v'} \right) + \frac{\partial}{\partial z} \left( \overline{v'w'} \right) \right] = 0
\]

The thermodynamic equation may be expressed in a similar way:

\[
\frac{\overline{d\theta}}{dt} + \overline{w} \frac{d\theta_0}{dz} + \left[ \frac{\partial}{\partial x} \left( \overline{u'\theta'} \right) + \frac{\partial}{\partial y} \left( \overline{v'\theta'} \right) + \frac{\partial}{\partial z} \left( \overline{w'\theta'} \right) \right] = 0
\]
Now we can write the momentum equations as

\[
\frac{d\bar{u}}{dt} - f\bar{v} + \frac{1}{\rho_0} \frac{\partial \bar{p}}{\partial x} + \left[ \frac{\partial}{\partial x} (u' u') + \frac{\partial}{\partial y} (v' u') + \frac{\partial}{\partial z} (w' u') \right] = 0
\]

\[
\frac{d\bar{v}}{dt} + f\bar{u} + \frac{1}{\rho_0} \frac{\partial \bar{p}}{\partial y} + \left[ \frac{\partial}{\partial x} (v' u') + \frac{\partial}{\partial y} (v' v') + \frac{\partial}{\partial z} (v' w') \right] = 0
\]

The thermodynamic equation may be expressed in a similar way:

\[
\frac{d\bar{\theta}}{dt} + \bar{w} \frac{d\theta_0}{dz} + \left[ \frac{\partial}{\partial x} (u' \theta') + \frac{\partial}{\partial y} (v' \theta') + \frac{\partial}{\partial z} (w' \theta') \right] = 0
\]

The terms in square brackets are the \textit{turbulent fluxes}.
Now we can write the momentum equations as

\[
\frac{d\bar{u}}{dt} - f\bar{v} + \frac{1}{\rho_0} \frac{\partial \bar{p}}{\partial x} + \left[ \frac{\partial}{\partial x} (u'u') + \frac{\partial}{\partial y} (v'u') + \frac{\partial}{\partial z} (w'u') \right] = 0
\]

\[
\frac{d\bar{v}}{dt} + f\bar{u} + \frac{1}{\rho_0} \frac{\partial \bar{p}}{\partial y} + \left[ \frac{\partial}{\partial x} (v'u') + \frac{\partial}{\partial y} (v'v') + \frac{\partial}{\partial z} (v'w') \right] = 0
\]

The thermodynamic equation may be expressed in a similar way:

\[
\frac{d\bar{\theta}}{dt} + \bar{w} \frac{d\theta_0}{dz} + \left[ \frac{\partial}{\partial x} (u'\theta') + \frac{\partial}{\partial y} (v'\theta') + \frac{\partial}{\partial z} (w'\theta') \right] = 0
\]

The terms in square brackets are the turbulent fluxes.

For example, \(u'w'\) is the vertical turbulent flux of zonal momentum, and \(w'\theta'\) is the vertical turbulent heat flux.
Now we can write the momentum equations as

\[
\frac{\overline{du}}{dt} - f\overline{v} + \frac{1}{\rho_0} \frac{\partial p}{\partial x} + \left[ \frac{\partial}{\partial x} \left( \overline{u'u'} \right) + \frac{\partial}{\partial y} \left( \overline{v'u'} \right) + \frac{\partial}{\partial z} \left( \overline{w'u'} \right) \right] = 0
\]

\[
\frac{\overline{dv}}{dt} + f\overline{u} + \frac{1}{\rho_0} \frac{\partial p}{\partial y} + \left[ \frac{\partial}{\partial x} \left( \overline{v'u'} \right) + \frac{\partial}{\partial y} \left( \overline{v'v'} \right) + \frac{\partial}{\partial z} \left( \overline{v'w'} \right) \right] = 0
\]

The thermodynamic equation may be expressed in a similar way:

\[
\frac{\overline{d\theta}}{dt} + \overline{w} \frac{d\theta_0}{dz} + \left[ \frac{\partial}{\partial x} \left( \overline{u'\theta'} \right) + \frac{\partial}{\partial y} \left( \overline{v'\theta'} \right) + \frac{\partial}{\partial z} \left( \overline{w'\theta'} \right) \right] = 0
\]

The terms in square brackets are the **turbulent fluxes**.

For example, \( \overline{u'w'} \) is the vertical turbulent flux of zonal momentum, and \( \overline{w'\theta'} \) is the vertical turbulent heat flux.

In the free atmosphere, the terms in square brackets are sufficiently small that they can be neglected.
Now we can write the momentum equations as

\[
\frac{d\bar{u}}{dt} - f\bar{v} + \frac{1}{\rho_0} \frac{\partial \bar{p}}{\partial x} + \left[ \frac{\partial}{\partial x} \left( \bar{u}'u' \right) + \frac{\partial}{\partial y} \left( \bar{v}'u' \right) + \frac{\partial}{\partial z} \left( \bar{w}'u' \right) \right] = 0
\]

\[
\frac{d\bar{v}}{dt} + f\bar{u} + \frac{1}{\rho_0} \frac{\partial \bar{p}}{\partial y} + \left[ \frac{\partial}{\partial x} \left( \bar{v}'u' \right) + \frac{\partial}{\partial y} \left( \bar{v}'v' \right) + \frac{\partial}{\partial z} \left( \bar{v}'w' \right) \right] = 0
\]

The thermodynamic equation may be expressed in a similar way:

\[
\frac{d\bar{\theta}}{dt} + \bar{w} \frac{d\theta_0}{dz} + \left[ \frac{\partial}{\partial x} \left( \bar{u}'\theta' \right) + \frac{\partial}{\partial y} \left( \bar{v}'\theta' \right) + \frac{\partial}{\partial z} \left( \bar{w}'\theta' \right) \right] = 0
\]

The terms in square brackets are the turbulent fluxes. For example, \( \bar{u}'w' \) is the vertical turbulent flux of zonal momentum, and \( \bar{w}'\theta' \) is the vertical turbulent heat flux.

In the free atmosphere, the terms in square brackets are sufficiently small that they can be neglected.

Within the boundary layer, the turbulent flux terms are comparable in magnitude to the remaining terms, and must be included in the analysis.
In the boundary layer, vertical gradients are generally orders of magnitude larger than variations in the horizontal. Thus, it is possible to omit the $x$ and $y$ derivative terms in the square brackets.
In the boundary layer, vertical gradients are generally orders of magnitude larger than variations in the horizontal. Thus, it is possible to omit the $x$ and $y$ derivative terms in the square brackets.

Then the complete system of equations becomes

\[
\begin{align*}
\frac{\overline{du}}{dt} - f\overline{v} + \frac{1}{\rho_0} \frac{\partial \overline{p}}{\partial x} + \frac{\partial}{\partial z}(w'u') &= 0 \\
\frac{\overline{dv}}{dt} + f\overline{u} + \frac{1}{\rho_0} \frac{\partial \overline{p}}{\partial y} + \frac{\partial}{\partial z}(w'v') &= 0 \\
\frac{\overline{d\theta}}{dt} + w\overline{d\theta_0} + \frac{\partial}{\partial z}(w'\theta') &= 0 \\
\frac{\partial \overline{p}}{\partial z} + g\rho_0 &= 0 \\
\frac{\partial \overline{u}}{\partial x} + \frac{\partial \overline{v}}{\partial y} + \frac{\partial \overline{w}}{\partial z} &= 0
\end{align*}
\]
We note that this system of five equations, for the variables $(\bar{u}, \bar{v}, \bar{w}, \bar{p}, \bar{\theta})$, also contains the variables $(u', v', w', \theta')$ in the turbulent fluxes.
We note that this system of five equations, for the variables $(\bar{u}, \bar{v}, \bar{w}, \bar{p}, \bar{\theta})$, also contains the variables $(u', v', w', \theta')$ in the turbulent fluxes. Thus, the system is not self contained. To make it soluble, we must make some *closure assumption*, in which the fluxes are parameterized in terms of the mean fields.
We note that this system of five equations, for the variables \((\bar{u}, \bar{v}, \bar{w}, \bar{p}, \bar{\theta})\), also contains the variables \((u', v', w', \theta')\) in the turbulent fluxes.

Thus, the system is not self contained. To make it soluble, we must make some closure assumption, in which the fluxes are parameterized in terms of the mean fields.

Atmospheric modellers make various closure assumptions in designing parameterization schemes for numerical models.
We note that this system of five equations, for the variables \((\bar{u}, \bar{v}, \bar{w}, \bar{p}, \bar{\theta})\), also contains the variables \((u', v', w', \theta')\) in the turbulent fluxes.

Thus, the system is not self contained. To make it soluble, we must make some \textit{closure assumption}, in which the fluxes are \textit{parameterized} in terms of the mean fields.

Atmospheric modellers make various closure assumptions in designing parameterization schemes for numerical models.

Next, we will consider a simple parameterization of the vertical momentum flux, \(\partial (w'u')/\partial z\).
Parameterization of Eddy Fluxes
Parameterization of Eddy Fluxes

The momentum equations will be taken in the form

\[
\frac{du}{dt} - f v + \frac{1}{\rho_0} \frac{\partial p}{\partial x} + \frac{\partial}{\partial z} \left( w'u' \right) = 0
\]

\[
\frac{dv}{dt} + f u + \frac{1}{\rho_0} \frac{\partial p}{\partial y} + \frac{\partial}{\partial z} \left( w'v' \right) = 0
\]
The momentum equations will be taken in the form

\[
\frac{du}{dt} - fv + \frac{1}{\rho_0} \frac{\partial p}{\partial x} + \frac{\partial}{\partial z} \left( \overline{w'u'} \right) = 0
\]

\[
\frac{dv}{dt} + fu + \frac{1}{\rho_0} \frac{\partial p}{\partial y} + \frac{\partial}{\partial z} \left( \overline{w'v'} \right) = 0
\]

We have dropped the overbars on the mean quantities.
Parameterization of Eddy Fluxes

The momentum equations will be taken in the form

\[
\frac{du}{dt} - fv + \frac{1}{\rho_0} \frac{\partial p}{\partial x} + \frac{\partial}{\partial z} \left( w'u' \right) = 0
\]

\[
\frac{dv}{dt} + fu + \frac{1}{\rho_0} \frac{\partial p}{\partial y} + \frac{\partial}{\partial z} \left( w'v' \right) = 0
\]

We have dropped the overbars on the mean quantities.

In the free atmosphere, scale analysis shows that the inertial terms and turbulent flux terms are small compared to the Coriolis term and pressure gradient terms.
Parameterization of Eddy Fluxes

The momentum equations will be taken in the form

$$\frac{du}{dt} - fv + \frac{1}{\rho_0} \frac{\partial p}{\partial x} + \frac{\partial}{\partial z} \left( w' u' \right) = 0$$

$$\frac{dv}{dt} + fu + \frac{1}{\rho_0} \frac{\partial p}{\partial y} + \frac{\partial}{\partial z} \left( w' v' \right) = 0$$

We have dropped the overbars on the mean quantities.

In the free atmosphere, scale analysis shows that the inertial terms and turbulent flux terms are small compared to the Coriolis term and pressure gradient terms.

This leads us to the relation for geostrophic balance:

$$u \approx u_G \equiv -\frac{1}{f \rho_0} \frac{\partial p}{\partial y}, \quad v \approx v_G \equiv \frac{1}{f \rho_0} \frac{\partial p}{\partial x}$$
In the boundary layer, it is no longer appropriate to neglect the turbulent flux terms. However, the inertial terms may still be assumed to be relatively small.
In the boundary layer, it is no longer appropriate to neglect the turbulent flux terms. However, the inertial terms may still be assumed to be relatively small.

Thus we get a three-way balance between the forces:

\[-fv + \frac{1}{\rho_0} \frac{\partial p}{\partial x} + \frac{\partial}{\partial z} \left( w'u' \right) = 0\]

\[+fu + \frac{1}{\rho_0} \frac{\partial p}{\partial y} + \frac{\partial}{\partial z} \left( w'v' \right) = 0\]
In the boundary layer, it is no longer appropriate to neglect the turbulent flux terms. However, the inertial terms may still be assumed to be relatively small.

Thus we get a three-way balance between the forces:

\[-f v + \frac{1}{\rho_0} \frac{\partial p}{\partial x} + \frac{\partial}{\partial z} \left( w' u' \right) = 0\]

\[+ f u + \frac{1}{\rho_0} \frac{\partial p}{\partial y} + \frac{\partial}{\partial z} \left( w' v' \right) = 0\]
Using the definition of the geostrophic winds, we write the momentum equations as

\[-f(v - v_G) + \frac{\partial}{\partial z} \left( w'u' \right) = 0\]

\[+f(u - u_G) + \frac{\partial}{\partial z} \left( w'v' \right) = 0\]
Using the definition of the geostrophic winds, we write the momentum equations as

\[-f(v - v_G) + \frac{\partial}{\partial z} \left( w'u' \right) = 0\]

\[+f(u - u_G) + \frac{\partial}{\partial z} \left( w'v' \right) = 0\]

To progress, we need some means of representing the turbulent fluxes in terms of the mean variables.
Using the definition of the geostrophic winds, we write the momentum equations as

\[-f(v - v_G) + \frac{\partial}{\partial z} \left( \overline{w'u'} \right) = 0\]

\[+f(u - u_G) + \frac{\partial}{\partial z} \left( \overline{w'v'} \right) = 0\]

To progress, we need some means of representing the turbulent fluxes in terms of the mean variables.

The traditional approach to this closure problem is to assume that the turbulent eddies act in a manner analogous to molecular diffusion.
Using the definition of the geostrophic winds, we write the momentum equations as

\[ -f(v - v_G) + \frac{\partial}{\partial z} \left( \overline{w'u'} \right) = 0 \]
\[ +f(u - u_G) + \frac{\partial}{\partial z} \left( \overline{w'v'} \right) = 0 \]

To progress, we need some means of representing the turbulent fluxes in terms of the mean variables.

The traditional approach to this closure problem is to assume that the turbulent eddies act in a manner analogous to molecular diffusion.

Thus, the flux of momentum is assumed to be proportional to the vertical gradient of the mean momentum. Then we can write

\[ \overline{u'w'} = -K \left( \frac{\partial \overline{u}}{\partial z} \right) \]

where \( K \) is called the eddy viscosity coefficient.
The negative sign ensures that a positive vertical shear yields downward momentum flux.
The negative sign ensures that a positive vertical shear yields downward momentum flux.

This closure scheme is often referred to as K-theory. Alternatively, we may call it the Flux-gradient theory.
The negative sign ensures that a positive vertical shear yields downward momentum flux.

This closure scheme is often referred to as K-theory. Alternatively, we may call it the Flux-gradient theory.

We will assume that $K$ is constant. Thus, for example,

$$\frac{\partial}{\partial z} \left( w'u' \right) = - \frac{\partial}{\partial z} \left( K \frac{\partial u}{\partial z} \right) = -K \frac{\partial^2 u}{\partial z^2}$$
The negative sign ensures that a positive vertical shear yields downward momentum flux.

This closure scheme is often referred to as K-theory. Alternatively, we may call it the Flux-gradient theory.

We will assume that $K$ is constant. Thus, for example,

$$\frac{\partial}{\partial z} \left( w'u' \right) = -\frac{\partial}{\partial z} \left( K \frac{\partial u}{\partial z} \right) = -K \frac{\partial^2 u}{\partial z^2}$$

The momentum equations then become

$$-f(u - u_G) - K \frac{\partial^2 u}{\partial z^2} = 0$$

$$+f(u - u_G) - K \frac{\partial^2 v}{\partial z^2} = 0$$

In the following lecture, we will use these equations to model the **Ekman Layer**.
End of §5.3