Climate Sensitivity

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- Compute the equilibrium temperature of the surface and of the atmosphere.
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**Problems:**

- Compute the equilibrium temperature of the surface and of the atmosphere.
- Investigate the effects of *changing parameters* on the temperature.
Exercise:
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Assume that the atmosphere can be regarded as a thin layer with an absorbtivity of $a_S = 0.1$ for shortwave (solar) radiation and $a_L = 0.8$ for longwave (terrestrial) radiation.

Assume the Earth’s *albedo* is $A = 0.3$ and the *solar constant* is $F_{\text{solar}} = 1370 \text{ W m}^{-2}$.

Assume that the earth’s surface radiates as a blackbody at all wavelengths.

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Assume the Earth’s albedo is $A = 0.3$ and the solar constant is $F_{\text{solar}} = 1370 \text{ W m}^{-2}$.

Assume that the earth’s surface radiates as a blackbody at all wavelengths.

Calculate the radiative equilibrium temperature $T_E$ of the surface and the sensitivity of $T_E$ to changes in the following parameters:

- Absorbtivity of the atmosphere to shortwave radiation
- Absorbtivity of the atmosphere to longwave radiation
- Planetary albedo
- Solar constant
Solution:
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The net solar irradiance $F_S$ *absorbed* by the earth-atmosphere system is equal to the solar constant *reduced by* the albedo and by the areal factor of four:

$$F_S = \left( \frac{1 - A}{4} \right) F_{\text{solar}} = \frac{0.7}{4} \times 1370 = 240 \text{ W m}^{-2}$$
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The absorbtivity for solar radiation is $a_S = 0.1$.

We define the transmissivity as $\tau_S = 1 - a_S$. 
Solution:
The net solar irradiance $F_S$ absorbed by the earth-atmosphere system is equal to the solar constant reduced by the albedo and by the areal factor of four:

$$F_S = \left(1 - \frac{A}{4}\right) F_{\text{solar}} = \frac{0.7}{4} \times 1370 = 240 \text{ W m}^{-2}$$

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The downward flux of short wave radiation at the surface is the incoming flux multiplied by the transmissivity, $\tau_S F_S$. 

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The net solar irradiance $F_S$ absorbed by the earth-atmosphere system is equal to the solar constant reduced by the albedo and by the areal factor of four:

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Let $F_E$ be the longwave flux emitted upwards by the surface.
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The downward flux of short wave radiation at the surface is the incoming flux multiplied by the transmissivity, $\tau_S F_S$.

Let $F_E$ be the longwave flux emitted upwards by the surface.

Since the absorbtivity for terrestrial radiation is $a_L = 0.8$, the longwave transmissivity is $\tau_L = 1 - a_L = 0.2$. 
Thus, there results an upward flux at the top of the atmosphere of $\tau_L F_E$. 
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Let $F_L$ be the long wave flux emitted upwards by the atmosphere; this is also the long wave flux emitted \textit{downwards}.
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Let $F_L$ be the long wave flux emitted upwards by the atmosphere; this is also the long wave flux emitted downward. Thus, the total downward flux at the surface is $\tau_S F_S + F_L$.
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Thus, the \textbf{total downward flux at the surface is $\tau_S F_S + F_L$}

Radiative balance at the surface (upward flux equal to downward flux) gives:

$$F_E = \tau_S F_S + F_L$$
Thus, there results an upward flux at the top of the atmosphere of $\tau_L F_E$.

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Radiative balance at the surface (upward flux equal to downward flux) gives:

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The upward and downward fluxes at the top of the atmosphere must also be in balance, which gives us the relation

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The upward and downward fluxes at the top of the atmosphere must also be in balance, which gives us the relation

$$F_S = \tau_L F_E + F_L$$

To find $F_E$ and $F_L$, we solve the simultaneous equations

$$F_E - F_L = \tau_S F_S$$
$$\tau_L F_E + F_L = F_S$$
Repeat: to find $F_E$ and $F_L$, we must solve

$$F_E - F_L = \tau_S F_S$$

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F_E - F_L = \tau_S F_S
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This gives the values

\[
F_E = \left(\frac{1 + \tau_S}{1 + \tau_L}\right) F_S \quad F_L = \left(\frac{1 - \tau_S \tau_L}{1 + \tau_L}\right) F_S
\]

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Assuming that the Earth radiates like a blackbody, the Stefan-Boltzman Law gives

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\sigma T_{\text{surface}}^4 = F_E
\]
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Assuming that the Earth radiates like a blackbody, the Stefan-Boltzmann Law gives

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\sigma T_{\text{surface}}^4 = F_E
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using the expressions derived for $F_S$ and $F_E$, this is

\[
\sigma T_{\text{surface}}^4 = \frac{1 + \tau_S}{1 + \tau_L} \left( \frac{1 - A}{4} \right) F_{\text{solar}}
\]
Again,

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Taking logarithms, we have

\[ \log \sigma + 4 \log T_{\text{surface}} = \log(1 + \tau_S) - \log(1 + \tau_L) + \log(1 - A) - \log 4 + \log F_{\text{solar}} \]
Again,
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\sigma T_{\text{surface}}^4 = \frac{1 + \tau_S}{1 + \tau_L} \left( \frac{1 - A}{4} \right) F_{\text{solar}}
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Now differentiate:
\[
4 \frac{dT_{\text{surface}}}{T_{\text{surface}}} = \frac{d\tau_S}{1 + \tau_S} - \frac{d\tau_L}{1 + \tau_L} - \frac{dA}{1 - A} + \frac{dF_{\text{solar}}}{F_{\text{solar}}}
\]
Again,
\[ \sigma T_{\text{surface}}^4 = \frac{1 + \tau_S}{1 + \tau_L} \left( \frac{1 - A}{4} \right) F_{\text{solar}} \]

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This equation enables us to investigate the sensitivity of the surface temperature to changes in various parameters.
Again,

$$\sigma T_{\text{surface}}^4 = \frac{1 + \tau_S}{1 + \tau_L} \left( \frac{1 - A}{4} \right) F_{\text{solar}}$$

Taking logarithms, we have

$$\log \sigma + 4 \log T_{\text{surface}} = \log(1 + \tau_S) - \log(1 + \tau_L) + \log(1 - A) - \log 4 + \log F_{\text{solar}}$$

Now differentiate:

$$\frac{4}{T_{\text{surface}}} \frac{dT_{\text{surface}}}{dT_{\text{surface}}} = \frac{d\tau_S}{1 + \tau_S} - \frac{d\tau_L}{1 + \tau_L} - \frac{dA}{1 - A} + \frac{dF_{\text{solar}}}{F_{\text{solar}}}$$

This equation enables us to investigate the sensitivity of the surface temperature to changes in various parameters.

For reference, let’s call this the Blue Equation.

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Sensitivity to Shortwave Absorption
Suppose that some change causes an *increase in the absorption* of *solar radiation* by the atmosphere.
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For example, an *increase in ozone concentration in the stratosphere* would result in greater *absorption* of incoming solar radiation.
Sensitivity to Shortwave Absorbtion

Suppose that some change causes an *increase in the absorption* of *solar radiation* by the atmosphere.

For example, an *increase in ozone concentration in the stratosphere* would result in greater absorption of incoming solar radiation.

So,

\[ a_S \Rightarrow a_S + da_S \]
\[ \tau_S \Rightarrow \tau_S + d\tau_S \]

Clearly, if \( da_S > 0 \) then \( d\tau_S = -da_S < 0 \).
Suppose that some change causes an *increase in the absorption* of *solar radiation* by the atmosphere. For example, an *increase in ozone concentration in the stratosphere* would result in greater absorption of incoming solar radiation.

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Suppose the transmissivity decreases from 0.9 to 0.8. Then \( \tau_S = 0.9 \) and \( d\tau_S = -0.1 \).
Again, 

\[ 4 \frac{dT_{\text{surface}}}{T_{\text{surface}}} = \frac{d\tau_S}{1 + \tau_S} \]

Suppose the transmissivity decreases from 0.9 to 0.8. Then \( \tau_S = 0.9 \) and \( d\tau_S = -0.1 \).

Suppose also that the equilibrium temperature of the Earth with \( \tau_S = 0.9 \) is 288 K (as we found above).
Again,

\[ 4 \frac{dT_{\text{surface}}}{T_{\text{surface}}} = \frac{d\tau_S}{1 + \tau_S} \]

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Suppose also that the equilibrium temperature of the Earth with \( \tau_S = 0.9 \) is 288 K (as we found above).

Then

\[ 4 \frac{dT_{\text{surface}}}{288} = \left( \frac{-0.1}{1.9} \right) \quad \text{or} \quad dT_{\text{surface}} = \left( \frac{-0.1}{1.9} \right) \frac{288}{4} = -3.8 \, K \]
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Thus, the assumed increase in shortwave absorbtivity has resulted in a decrease in surface temperature of about 4°C.
Sensitivity to Longwave Absorption
Suppose next that some change causes an increase in the absorption of terrestrial radiation by the atmosphere.
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The Blue Equation reduces to

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\]

Suppose the longwave transmissivity decreases from 0.2 to 0.1. Then \( \tau_L = 0.2 \) and \( d\tau_S = -0.1 \).
Again,

\[ 4 \frac{dT_{\text{surface}}}{T_{\text{surface}}} = - \frac{d\tau_L}{1 + \tau_L} \]

Suppose the longwave transmissivity decreases from 0.2 to 0.1. Then \( \tau_L = 0.2 \) and \( d\tau_S = -0.1 \).

Suppose once more that the equilibrium temperature of the Earth with \( \tau_L = 0.2 \) is 288 K.
Again,

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Suppose the longwave transmissivity decreases from 0.2 to 0.1. Then \( \tau_L = 0.2 \) and \( d\tau_S = -0.1 \).

Suppose once more that the equilibrium temperature of the Earth with \( \tau_L = 0.2 \) is 288 K.

Then

\[ 4 \frac{dT_{\text{surface}}}{288} = - \left( \frac{-0.1}{1.2} \right) \quad \text{or} \quad dT_{\text{surface}} = \left( \frac{0.1}{1.2} \right) \frac{288}{4} = 6.0 \, K \]
Again,

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Suppose the longwave transmissivity decreases from 0.2 to 0.1. Then \( \tau_L = 0.2 \) and \( d\tau_S = -0.1 \).

Suppose once more that the equilibrium temperature of the Earth with \( \tau_L = 0.2 \) is 288 K.

Then

\[ 4 \frac{dT_{\text{surface}}}{288} = - \left( \frac{-0.1}{1.2} \right) \quad \text{or} \quad dT_{\text{surface}} = \left( \frac{0.1}{1.2} \right) \frac{288}{4} = 6.0 \, K \]

Thus, the assumed increase in longwave absorptivity has resulted in an increase in surface temperature of about 6°C.
Sensitivity to Planetary Albedo
Suppose next that some change causes an increase in the albedo or reflectivity of the atmosphere.
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Suppose next that some change causes an increase in the albedo or reflectivity of the atmosphere.

For example, an increase in condensation nuclei might result in a greater coverage of high-level cirrus cloud, which could reflect a higher proportion of incoming solar radiation.
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So,

\[ A \Rightarrow A + dA \]
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For example, an increase in condensation nuclei might result in a greater coverage of high-level cirrus cloud, which could reflect a higher proportion of incoming solar radiation.

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The Blue Equation reduces to

\[ 4 \frac{dT_{\text{surface}}}{T_{\text{surface}}} = -\frac{dA}{1 - A} \]
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$$4 \frac{dT_{\text{surface}}}{T_{\text{surface}}} = -\frac{dA}{1 - A}$$

Suppose the albedo increases from 0.3 to 0.4. Then $A = 0.3$ and $dA = 0.1$. 
Again,

\[ 4 \frac{dT_{\text{surface}}}{T_{\text{surface}}} = -\frac{dA}{1 - A} \]

Suppose the albedo increases from 0.3 to 0.4. Then \( A = 0.3 \) and \( dA = 0.1 \).

Suppose once more that the equilibrium temperature of the Earth with \( A = 0.3 \) is 288 K.
Again,

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Suppose the albedo increases from 0.3 to 0.4. Then \( A = 0.3 \) and \( dA = 0.1 \).

Suppose once more that the equilibrium temperature of the Earth with \( A = 0.3 \) is 288 K.

Then

\[ 4 \frac{dT_{\text{surface}}}{288} = - \left( \frac{0.1}{0.7} \right) \quad \text{or} \quad dT_{\text{surface}} = - \left( \frac{0.1}{0.7} \right) \frac{288}{4} = -10.3 \, K \]
Again,
\[ 4 \frac{dT_{\text{surface}}}{T_{\text{surface}}} = - \frac{dA}{1 - A} \]

Suppose the albedo increases from 0.3 to 0.4. Then \( A = 0.3 \) and \( dA = 0.1 \).

Suppose once more that the equilibrium temperature of the Earth with \( A = 0.3 \) is 288 K.

Then
\[ 4 \frac{dT_{\text{surface}}}{288} = - \left( \frac{0.1}{0.7} \right) \quad \text{or} \quad dT_{\text{surface}} = - \left( \frac{0.1}{0.7} \right) \frac{288}{4} = -10.3 K \]

Thus, the assumed increase in albedo has resulted in an decrease in surface temperature of about 10°C.
Sensitivity to Solar Constant
Suppose next that the solar energy flux varies.
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\[ F_{\text{solar}} \Rightarrow F_{\text{solar}} + dF_{\text{solar}} \]
Suppose next that the \textit{solar energy flux varies}.

This could be the result of a major solar anomaly, or due to a secular or cyclic variation associated, for example, with the sun-spot cycle.

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The Blue Equation reduces to

\[ 4 \frac{dT_{\text{surface}}}{T_{\text{surface}}} = \frac{dF_{\text{solar}}}{F_{\text{solar}}} \]
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Suppose the solar output increases by 1%. Then \( dF_{\text{solar}}/F_{\text{solar}} = 0.01 \).
Again,

\[ 4 \frac{dT_{\text{surface}}}{T_{\text{surface}}} = \frac{dF_{\text{solar}}}{F_{\text{solar}}} \]

Suppose the solar output increases by 1%. Then \( dF_{\text{solar}}/F_{\text{solar}} = 0.01 \).

Suppose once more that the equilibrium temperature of the Earth is 288 K.
Again,

\[ 4 \frac{dT_{\text{surface}}}{T_{\text{surface}}} = \frac{dF_{\text{solar}}}{F_{\text{solar}}} \]

Suppose the solar output increases by 1%. Then \( dF_{\text{solar}}/F_{\text{solar}} = 0.01 \).

Suppose once more that the equilibrium temperature of the Earth is 288 K.

Then

\[ 4 \frac{dT_{\text{surface}}}{288} = \left( \frac{0.01 F_{\text{solar}}}{F_{\text{solar}}} \right) \quad \text{or} \quad dT_{\text{surface}} = 0.01 \times \frac{288}{4} = 0.7 K \]
Again,

\[ 4 \frac{dT_{\text{surface}}}{T_{\text{surface}}} = \frac{dF_{\text{solar}}}{F_{\text{solar}}} \]

Suppose the solar output increases by 1%. Then \( dF_{\text{solar}}/F_{\text{solar}} = 0.01 \).

Suppose once more that the equilibrium temperature of the Earth is 288 K.

Then

\[ 4 \frac{dT_{\text{surface}}}{288} = \left( \frac{0.01 F_{\text{solar}}}{F_{\text{solar}}} \right) \quad \text{or} \quad dT_{\text{surface}} = 0.01 \times \frac{288}{4} = 0.7 \text{ K} \]

Thus, the assumed 1% increase in solar flux has resulted in an increase in surface temperature of less than 1°C.
Review of Sensitivities
An increase in shortwave absorbtivity of 0.1 resulted in a decrease in surface temperature of about 4°C.
• An increase in shortwave absorbtivity of 0.1 resulted in a decrease in surface temperature of about $4^\circ$C.

• An increase in longwave absorbtivity of 0.1 resulted in an increase in surface temperature of about $6^\circ$C.
• An increase in shortwave absorbtivity of 0.1 resulted in a decrease in surface temperature of about 4°C.
• An increase in longwave absorbtivity of 0.1 resulted in an increase in surface temperature of about 6°C.
• An increase in albedo of 0.1 resulted in a decrease in surface temperature of about 10°C.
Review of Sensitivities

- An increase in shortwave absorptivity of 0.1 resulted in a decrease in surface temperature of about 4°C.
- An increase in longwave absorptivity of 0.1 resulted in an increase in surface temperature of about 6°C.
- An increase in albedo of 0.1 resulted in a decrease in surface temperature of about 10°C.
- A 1% increase in solar flux has resulted in an increase in surface temperature of less than 1°C.
• An increase in shortwave absorptivity of 0.1 resulted in a decrease in surface temperature of about 4°C.

• An increase in longwave absorptivity of 0.1 resulted in an increase in surface temperature of about 6°C.

• An increase in albedo of 0.1 resulted in a decrease in surface temperature of about 10°C.

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In general, all parameters undergo small changes. Moreover, we have completely neglected the effects of water vapour and liquid water.
Review of Sensitivities

- An increase in shortwave absorbtivity of 0.1 resulted in a decrease in surface temperature of about $4^\circ$C.
- An increase in longwave absorbtivity of 0.1 resulted in an increase in surface temperature of about $6^\circ$C.
- An increase in albedo of 0.1 resulted in a decrease in surface temperature of about $10^\circ$C.
- A 1% increase in solar flux has resulted in an increase in surface temperature of less than $1^\circ$C.

In general, all parameters undergo small changes. Moreover, we have completely neglected the effects of water vapour and liquid water.

The results give an indication of how difficult it is to gauge the consequences for climate of any changes which may occur.
End of §4.5