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The theory of the energy distribution of blackbody radiation was developed by Planck and first appeared in 1901.

Planck postulated that energy can be absorbed or emitted only in discrete units or *photons* with energy

$$E = h\nu = \hbar\omega$$

The constant of proportionality is $h = 6.626 \times 10^{-34} \text{J s}$.

Planck showed that the intensity of radiation emitted by a black body is given by

$$B_{\lambda} = \frac{c_1 \lambda^{-5}}{\exp(c_2/\lambda T) - 1}$$

where c_1 and c_2 are constants

$$c_1 = 2\pi h c^2 = 3.74 \times 10^{-16} \text{ W m}^{-2} \quad \text{and} \quad c_2 = \frac{hc}{k} = 1.44 \times 10^{-2} \text{ m K}.$$

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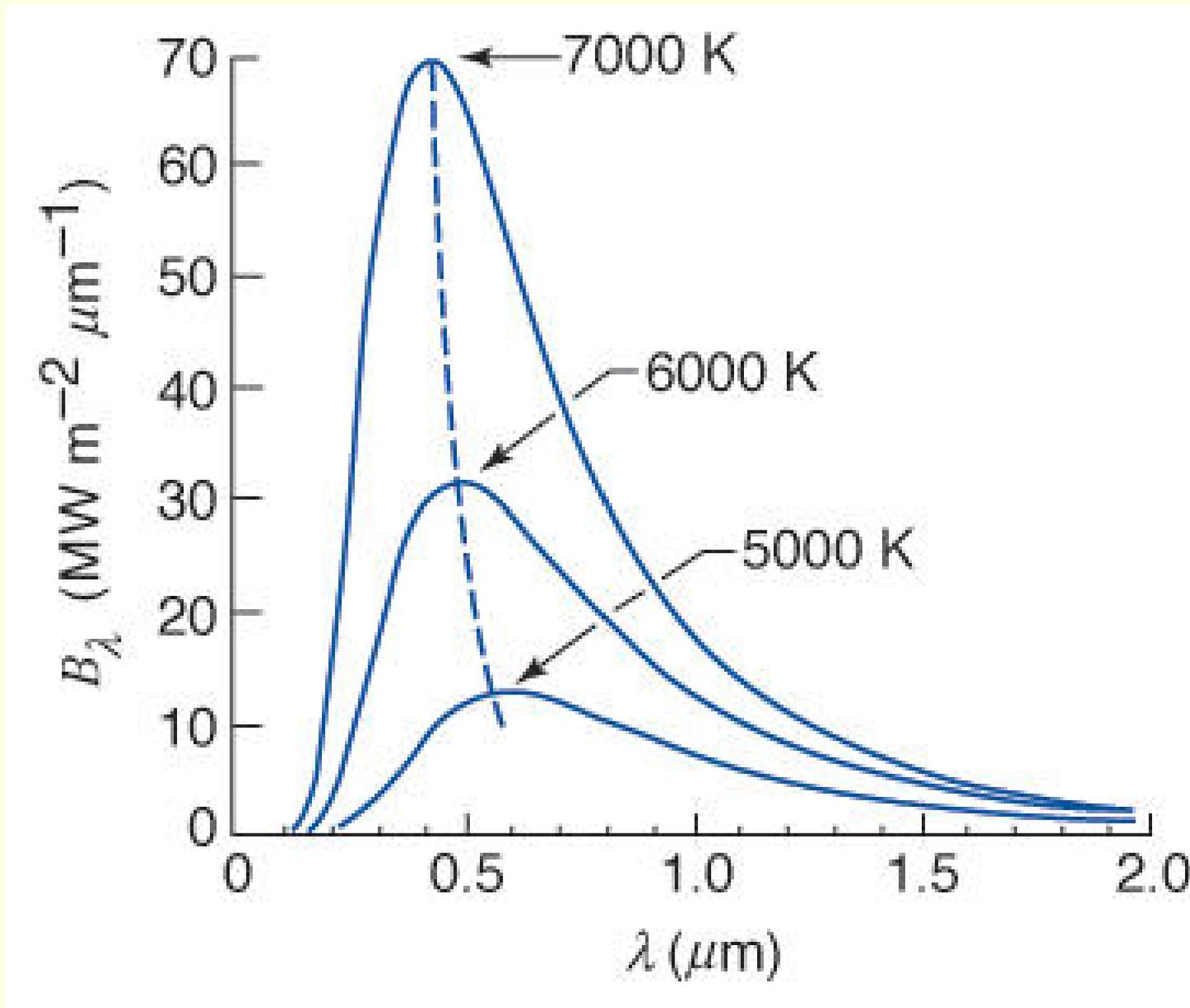
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Blackbody radiation is isotropic.

When $B_{\lambda}(T)$ is plotted as a function of wavelength on a linear scale the resulting spectrum of monochromatic intensity exhibits the shape illustrated as shown next.



Blackbody emission (the Planck function) for absolute temperatures as indicated, plotted as a function of wavelength on a linear scale.

Wien's Displacement Law

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Differentiating Planck's function and setting the derivative equal to zero yields the wavelength of peak emission for a blackbody at temperature T

$$\lambda_m \approx \frac{2900}{T}$$

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On the basis of this equation, it is possible to estimate the temperature of a radiation source from a knowledge of its emission spectrum, as illustrated in an example below.

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For values of interest in atmospheric and solar science, the exponential term is much larger than unity. Assuming this, we may write

$$B_{\lambda} = \frac{c_1 \lambda^{-5}}{\exp(c_2/\lambda T)} = c_1 \times \lambda^{-5} \times \exp(-c_2/\lambda T)$$

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$$\log B_\lambda = \log c_1 - 5 \log \lambda - \frac{c_2}{\lambda T}$$

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At the maximum, we have

$$\frac{dB_{\lambda}}{d\lambda} = 0 \quad \text{or} \quad \frac{d \log B_{\lambda}}{d\lambda} = 0$$

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Since $c_2 = 1.44 \times 10^{-2} \text{ m K}$, we have $c_2/5 \approx 0.0029$, so

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MATLAB *Exercise:*

- Plot B_λ as a function of λ for $T = 300$ and $T = 6000$.
Use the range $\lambda \in (0.1 \mu\text{m}, 100 \mu\text{m})$.

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Use the range $\lambda \in (0.1 \mu\text{m}, 100 \mu\text{m})$.
- Plot B_λ for $T = 300$ and also the *approximation* obtained by assuming $\exp(c_2/\lambda T) \gg 1$ (as used above).

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Hence

$$T = \frac{2900}{\lambda_m} = \frac{2900}{0.475} = 6100 \text{ K}$$

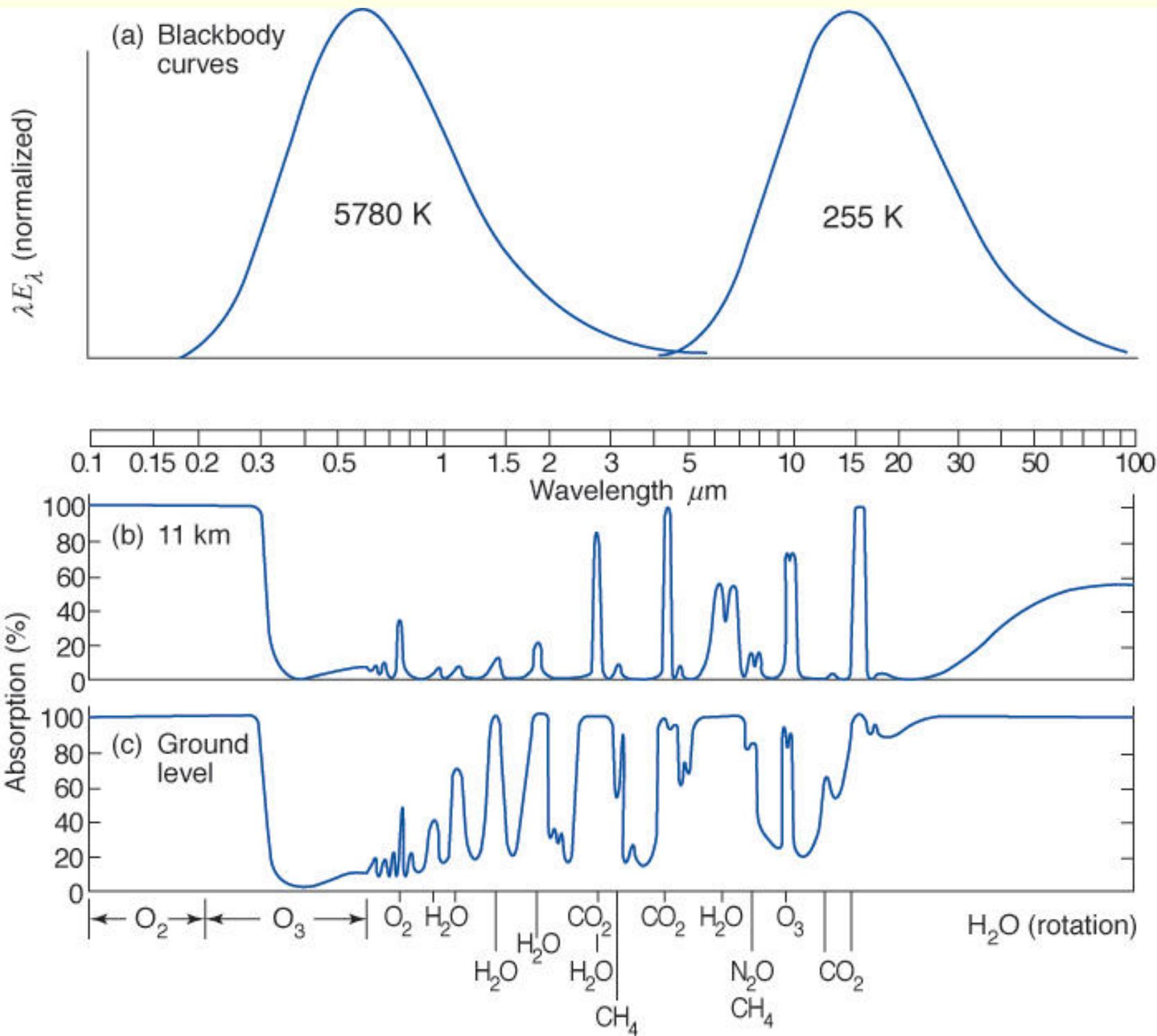
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Wien's displacement law explains why solar radiation is concentrated in the UV, visible and near infrared regions of the spectrum, while radiation emitted by planets and their atmospheres is largely confined to the infrared, as shown in the following figure.



Key to above figure

- (a) Blackbody spectra representative of the sun (left) and the earth (right). The wavelength scale is logarithmic rather than linear, and the ordinate has been multiplied by wavelength in order to make area under the curve proportional to intensity. The intensity scale for the right hand curve has been stretched to make the areas under the two curves the same.

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- (c) the atmospheric absorptivity (for flux density) for parallel mean solar ($\lambda < 4 \mu\text{m}$) radiation for a solar zenith angle of 50° and isotropic terrestrial ($(\lambda > 4 \mu\text{m})$) radiation.

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- (b) as in (c) but for the upper atmosphere defined as levels above 10 km.

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$$F = \sigma T^4$$

where σ is a constant equal to $5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$.

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Applications of the Stefan Boltzmann Law and the concept of equivalent blackbody temperature are illustrated in the following problems.

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Solution: We first calculate the flux density at the top of the layer, making use of the inverse square law:

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Therefore

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So, the equivalent temperature is

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That this value is slightly lower than the sun's colour temperature estimated in the previous exercise is evidence that the spectrum of the sun's emission differs slightly from the blackbody spectrum prescribed by Planck's law.

Exercise: Calculate the equivalent blackbody temperature of the earth assuming a planetary albedo of 0.30.

Assume that the earth is in radiative equilibrium with the sun: i.e., that there is no net energy gain or loss due to radiative transfer.

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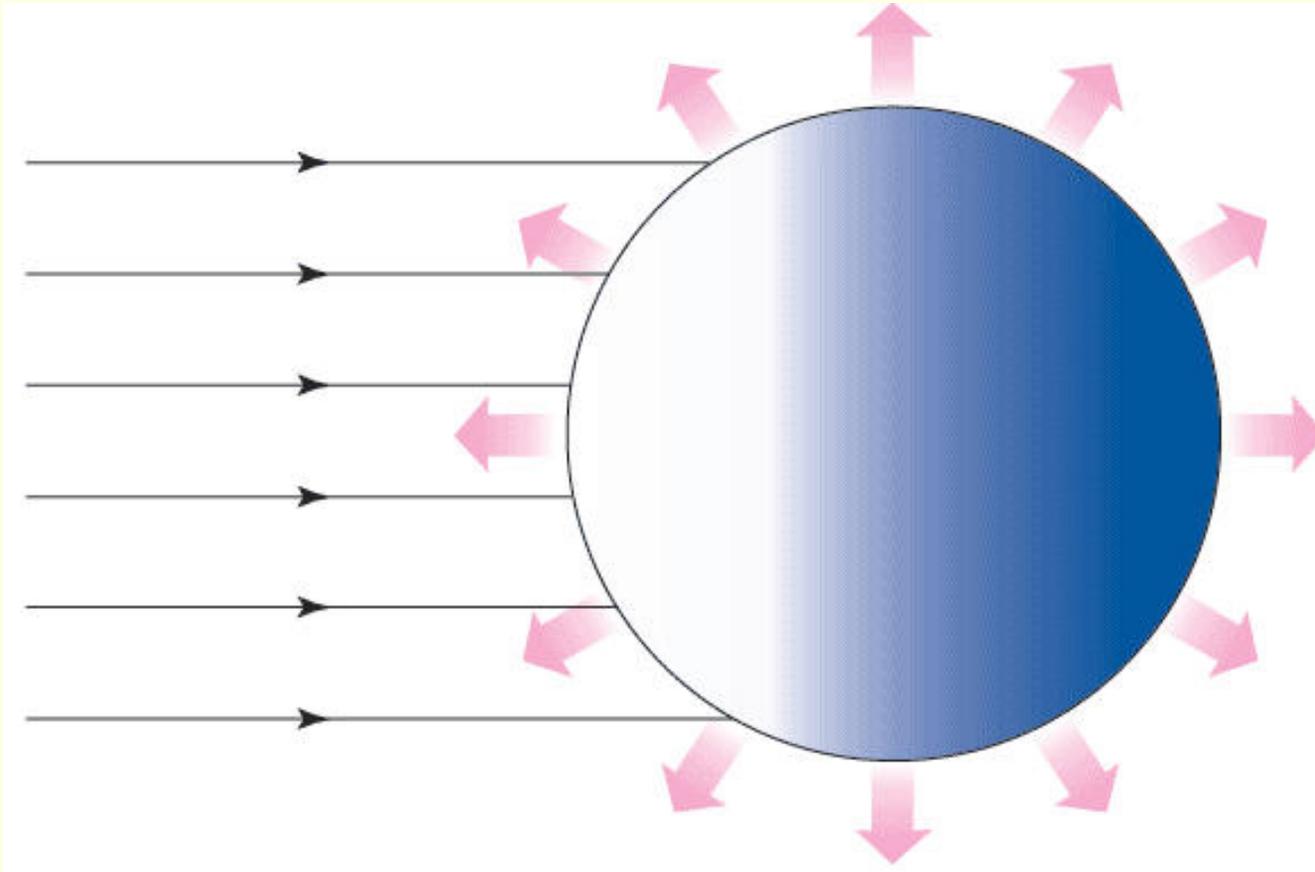
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and
- A the planetary albedo of the earth.

Calculate the earth's equivalent blackbody temperature T_E :

$$F_E = \sigma T_E^4 = \frac{(1 - A)F_S}{4} = \frac{0.7 \times 1370}{4} = 240 \text{ W m}^{-2}$$

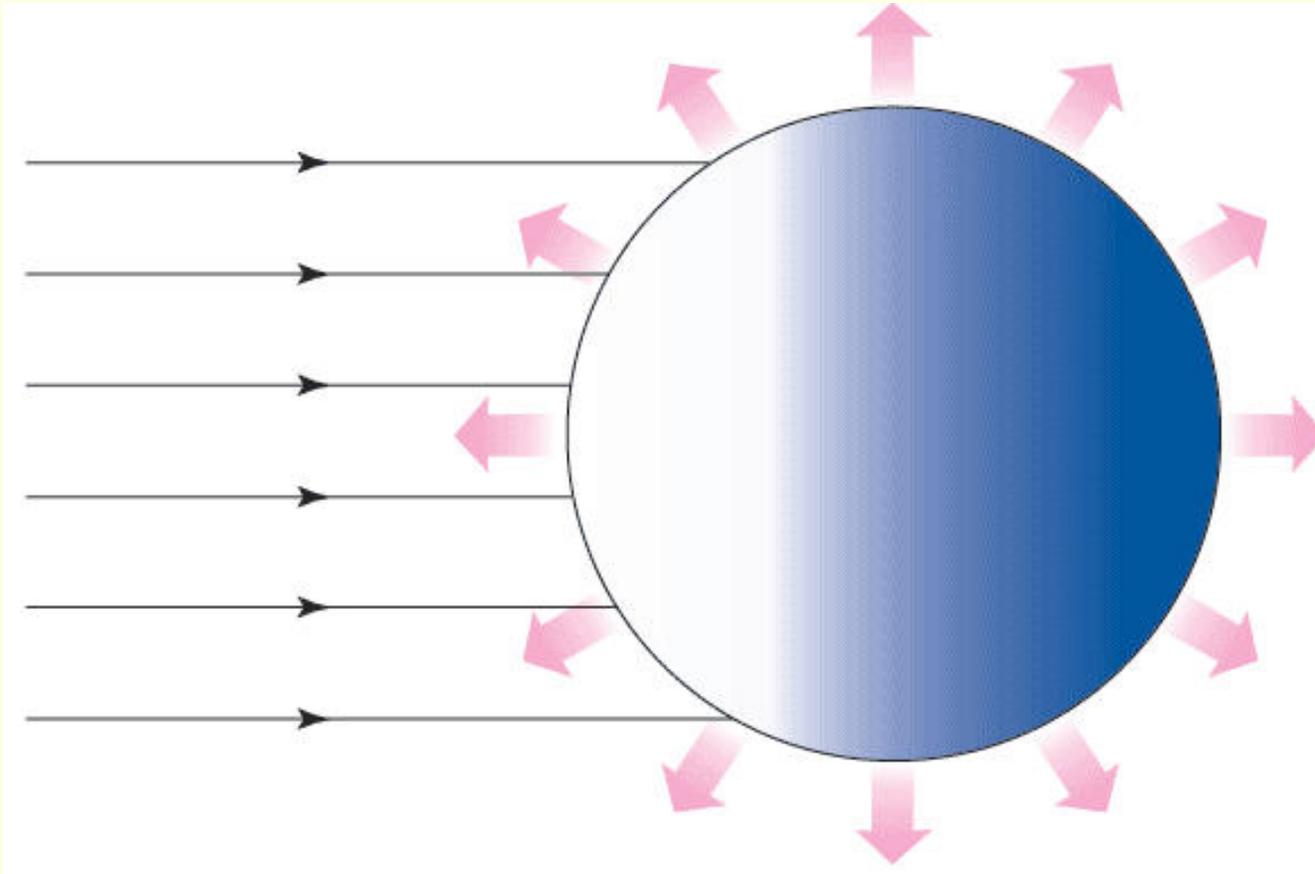
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Solving for T_E , we obtain

$$T_E = \sqrt[4]{F_E/\sigma} = 255 \text{ K} = -18^\circ\text{C}$$

Equivalent blackbody temperature of some of the planets, based on the assumption that they are in radiative equilibrium with the sun.

Planet	Dist. from sun	Albedo	TE (K)
Mercury	0.39 AU	0.06	442
Venus	0.72 AU	0.78	227
Earth	1.00 AU	0.30	255
Mars	1.52 AU	0.17	216
Jupiter	5.18 AU	0.45	105

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$$\varepsilon_{\lambda} = \frac{I_{\lambda}(\text{emitted})}{B_{\lambda}(T)}$$

The *absorptivity* is the fraction of the incident monochromatic intensity that is absorbed

$$A_{\lambda} = \frac{I_{\lambda}(\text{absorbed})}{I_{\lambda}(\text{incident})}$$

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Likewise, a body which is a poor absorber at a given wavelength is also a poor emitter at that wavelength.

End of §4.2