International Linear Algebra & Matrix Theory Workshop

at UCD

on the occasion of Thomas J. Laffey's 75th birthday

Dublin, 23-24 May 2019

All talks will take place in Room 128 Science North (Physics)

Sponsored by

- *Science Foundation Ireland*
- *Irish Mathematical Society*
- *School of Mathematics and Statistics, University College Dublin*
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- *Optum Technology*

BOOK OF ABSTRACTS

Linear Algebra and Matrix Theory Workshop May 23 & 24 2019

Speakers' Timetable – All talks in Room 128 Physics (Science North)

Keynote speakers (40 minutes + 5 minutes for questions)
Short Invited Talks (20 minutes + 5 minutes for questions)

Time	Thursday May 23rd	Friday May 24th
9.00-9.45	Alexander Guterman[1]	Volker Mehrmann[5]
9.45-10.10	John Sheekey	Rachel Quinlan
10.10-10.55	Damjana Kokol Bukovšek	Joao Queiro
10.55-11.25	Coffee Break	Coffee Break
11.25-11.50	Robert Shorten[2]	Richard Ellard[6]
11.50-12.15	Helena Smigoc	Tom Laffey
12.15-12.40	Conor Finnegan	Cian O'Brien
12.40-1.45	Lunch (Common Room)	Lunch (Common Room)
1.45-2.30	Olga Markova[3]	Sepideh Stewart[7]
2.30-2.55	Paul Barry	Raja Mukherji
2.55-3.20	Coffee Break	Coffee Break
3.20-3.45	Oliver Mason[4]	Charlie Johnson I[8]
3.45-4.30	Sergey Sergeev	Charlie Johnson II
4.30-4.55	Rod Gow	Dmitry Kudryavtsev
5.00	Testimonials (Rod, G Lessells, P Guiry)	Conference Close
7.00	Dinner at Beaufield Mews	

Session chairs

1,5 - Anthony Cronin
2,8 - Kevin Jennings
3 - John Sheekey
4 - Rachel Quinlan
6 - Helena Smigoc
7 - Oliver Mason

INVITED TALKS

Centralizing centralizers and beyond

Alexander E. Guterman

Lomonosov Moscow State University, Russia

For a matrix $A \in M_n(\mathbb{F})$ its centralizer $\mathcal{C}(A) = \{X \in M_n(\mathbb{F}) | \ AX = XA\}$ is the set of all matrices commuting with A. For a set $S \subseteq M_n(\mathbb{F})$ its centralizer $\mathcal{C}(S) = \{X \in M_n(\mathbb{F}) | \ AX = XA$ for every $A \in S\} = \cap_{A \in S}\mathcal{C}(A)$ is the intersection of centralizers of all its elements. Centralizers are important and useful both in fundamental and applied sciences.

A non-scalar matrix $A \in M_n(\mathbb{F})$ is *minimal* if for every $X \in M_n(\mathbb{F})$ with $\mathcal{C}(A) \supseteq \mathcal{C}(X)$ it follows that $\mathcal{C}(A) = \mathcal{C}(X)$. A non-scalar matrix $A \in M_n(\mathbb{F})$ is *maximal* if for every non-scalar $X \in M_n(\mathbb{F})$ with $\mathcal{C}(A) \subseteq \mathcal{C}(X)$ it follows that $\mathcal{C}(A) = \mathcal{C}(X)$. We investigate and characterize minimal and maximal matrices over arbitrary fields. Our results are illustrated by applications to the theory of commuting graphs of matrix rings, to the preserver problems, namely to characterize of commutativity preserving maps on matrices, and to the centralizers of high orders.

Similar problems for q-centralizers will be discussed. In particular, we examine the existence of an analogue of the classical Double Centralizing Theorem and describe matrices with extremal centralizers over algebraically closed fields, once commutativity is substituted by commutativity up to a factor. This relation is important in quantum physics.

The talk is based on our recent joint works with G. Dolinar, B. Kuzma, O. Markova, and P. Oblak.

Spectral Theory of Tridiagonal Matrices

Charles Johnson

College of William and Mary, Virginia, USA

We discuss the possible spectra of tridiagonal matrices over a field. Time permitting, we will contrast this with the spectral structure of “arrow matrices”, both in the irreducible case.

Recent Results on the NIEP

Charles Johnson

College of William and Mary, Virginia, USA

a) A spectrum is said to be universally “realizable” if, for every Jordan canonical form (JCF) that it permits, there is a nonnegative realization. We describe some recent results on the coarsening of JCF in realizations, as well as their implications for universal realizability.
b) The doubly stochastic single eigenvalue problem remains unresolved (in contrast to the row stochastic one). We discuss recent results and their implication for a likely conjecture.

Commuting graph of the algebra of matrices

Damjana Kokol Bukovšek

University of Ljubljana, Slovenia

Let $\mathcal{A}$ be a groupoid, i.e. a nonempty set equipped with an inner operation, written as product ab. The essence of commutativity relation on $\mathcal{A}$ is captured in its commuting graph $\Gamma(\mathcal{A})$. By definition, this is a simple graph whose vertices are all noncentral elements of $\mathcal{A}$ and where two distinct vertices a, b are connected if they commute in $\mathcal{A}$, i.e., if $ab = ba$. One of the basic properties of a graph is its diameter and connectedness. This question turned out to be surprisingly hard for the commuting graphs.

We will consider the commuting graph of $M_n(\mathbb{F})$, the algebra of n-by-n matrices over a field $\mathbb{F}$. If $\mathbb{F}$ is algebraically closed and $n \geq 3$ then the diameter of the commuting graph $\Gamma(M_n(\mathbb{F}))$ is 4. The diameter of a connected commuting graph of $M_n(\mathbb{F})$ is bounded above by 6. The commuting graph of $M_n(\mathbb{Q})$ is disconnected for any $n \geq 2$ (here, $\mathbb{Q}$ denotes the field of rational numbers), but for each prime $p \geq 7$ there exist two similar matrices $A, B \in M_{2p}(\mathbb{Q})$ at maximal possible distance 6. If $\mathbb{F}$ is a finite field then $\Gamma(M_n(\mathbb{F}))$ is a finite graph and it has the following properties: if $n \geq 4$ is even, then its diameter is 4, if n is a prime, then it is disconnected, and if odd n is neither a prime nor a square of a prime (the smallest such n is 15), then its diameter is at most 5.

Length of non-associative algebras

Dmitry Kudryavtsev

Lomonosov Moscow State University, Russia

The problem of computing the length of an algebra was firstly explored in the works of Spencer and Rivlin for the algebra of 3×3 matrices in the context of applied mechanics. Length as a function or combinatorial invariant was investigated later for associative algebras, and, in particular, for the matrix algebra, by Paz, Pappacena, Laffey, et al. In the present talk the case of non-associative algebras will be discussed. Firstly we show that the algebra of real quaternions has length 2 and the octonion algebra has length 3. Among our other results we have the following theorems.

Theorem 1. *Let A be an arbitrary unital non-associative algebra of the dimension n. Then the length $l(A) \leq 2^{n-2}$, and there are algebras of the length 2^{n-2} for all n.*

A is a *locally complex algebra*, if it is finitely generated non-associative algebra over the real field $\mathbb{R}$, such that any 1-generated subalgebra of A, which is generated by an element of $A \backslash \mathbb{R}$, is isomorphic to the field of complex numbers.

Theorem 2. *Let A be a locally complex algebra of the dimension n. Then the length $l(A) \leq F_n$ (n-th Fibonacci number), and there are algebras of the length F_n for all n.*

To prove these statements we introduced integer sequences associated with the generating sets of algebras, so-called characteristic sequences, and provided their complete characterization. It turns out that these sequences belong to the class of so-called additive chains known in combinatorics. The talk is based on the joint work with A.E. Guterman.

On the length of the full matrix algebra and its generating sets

Olga Markova

Lomonosov Moscow State University, Russia

By the length of a finite system of generators for a finite-dimensional algebra over an arbitrary field we mean the least positive integer k such that the products of length not exceeding k span this algebra (as a vector space). The maximum length for the systems of generators of an algebra is referred to as the length of the algebra.

The length evaluation can be a difficult problem, since, for example, the length of the full matrix algebra is still unknown (Paz's Problem, 1984). Paz conjectured that the length of any generating set for the algebra of n by n matrices is at most $2n-2$. In this talk we will show that this conjecture holds under the assumption that the generating set contains a nonderogatory matrix or a matrix with minimal polynomial of degree $n-1$. We will also present linear bounds for the length of generating sets that include a matrix with some restrictions on its Jordan normal form.

This talk is based on a joint work with Alexander Guterman (Moscow State University), Thomas Laffey and Helena Smigoc (University College Dublin).

Port-Hamiltonian Systems and Linear Algebra: A great relationship

Volker Mehrmann

Technische Universität Berlin, Germany

Port-Hamiltonian systems are an important class of control systems that arise in all areas of science and engineering. When the system is linearized arround a stationary solution one gets a linear port-Hamiltonian system. Despite the fact that the system looks very unstructured at first sight, it has remarkable properties. Stability and passivity are automatic, Jordan structures for purely imaginary eigenvalues, eigenvalues at infinity, and even singular blocks in the Kronecker canonical form are very restricted and furthermore the structure leads to fast and efficient iterative solution methods for associated linear systems.

This is joint work with Christian Mehl and Michal Wojtylak.

Some invariant factor theorems following R. Thompson and E. M. Sá

João Queiró

University of Coimbra, Portugal

From the 1970s to the 1990s, Robert C. Thompson and Eduardo Marques de Sá were two of the main protagonists in the study of fundamental questions concerning invariant factors of matrices over special types of rings.

The main problems addressed were the behaviour of invariant factors under sums of matrices, products of matrices and passage to submatrices.

In this talk we will briefly survey these problems, the advances made and relations to other questions in mathematics, as well as some recent results.

Tropical matrix powers and their periodicity transients

Sergey Sergeev

University of Birmingham, UK

We study sequences of matrix powers in tropical linear algebra. These matrix powers, while resembling the nonnegative matrix powers, also represent the weights of optimal walks on weighted directed graphs. The sequences of such matrices have been long known to be ultimately periodic in important special cases, and the transients of such periodicity have been of interest. Our contribution is a new general approach to these transients based on what we call CSR-expansions. These expansions allow us to write a tropical matrix power as a finite tropical sum of special terms that are tropical matrix decompositions of the form "CSR", and the ultimate periodicity starts as soon as the first of these terms starts to dominate the rest of them.

We make the most use of the weak CSR expansion, which represents a matrix power as a tropical sum of the first CSR term and a matrix power of a "subordinate" matrix. The periodicity transients are then naturally written as maximum of 1) a transient after which the weak CSR expansion starts to hold, 2) a transient after which the CSR term starts to dominate. The bounds on the first kind of transients are quadratic in the matrix dimension and closely related to the known bounds on the indices of periodicity of unweighted digraphs due to Wielandt, Dulmage-Mendelsohn, Schwarz, Kim and Gregory-Kirkland-Pullman. Transients of the second kind have to depend on matrix entries and can be arbitrarily big if the matrix dimension is fixed.

We will also briefly discuss an extension of the notion of periodicity transient to inhomogeneous products of tropical matrices.

The talk is based on several articles in collaboration with Hans Schneider, Glenn Merlet, Thomas Nowak, Arthur Kennedy-Cochran-Patrick, and Stefan Berezny.

Moving between the worlds of mathematical thinking in Linear Algebra: Mathematicians deliberations and students' perspectives

Sepideh Stewart

University of Oklahoma, USA

Linear algebra is one of the first theoretical mathematics topics that students encounter at the university level. The concepts are often introduced through definitions, theorems and proofs. A mixture of matrices and their symbolic manipulations as well as some geometry are often present.

In this talk, employing Tall's (2013) model of three worlds (embodied, symbolic, formal) of mathematical thinking, we present some studies that capture mathematics instructors' deliberations in moving between the worlds. In addition, we show that sometimes students resist the move or may not have the desire or need to move.

Our working hypothesis is that mathematicians and university teachers who have gone through their own mathematical journey know the path well. Hence, understanding when and why the teachers move between the worlds is a key issue. Furthermore, we believe creating opportunities for students to move will encourage them to consider multiple modes of thinking which result in richer conceptual understanding.

Matrix discovery through algebraic geometry

Paul Barry

Waterford Institute of Technology, Ireland

Algebraic geometry can be a rich source of interesting matrices. In this talk, we use Hilbert's Syzygy Theorem to construct a sequence of infinite lower-triangular matrices whose properties we explore. The initial triangle of this sequence is Pascal's triangle. The investigation shows links to a special Catalan matrix, and to translates of this and of Pascal's triangle. We find interesting decompositions and a further family of square matrices with interesting properties. Links to sequences of combinatorial significance are uncovered.

Newton-like inequalities and their relation to the work of Tom Laffey

Richard Ellard

School of Mathematical Sciences, Technological University Dublin, Ireland

The problem of characterising the spectra of nonnegative matrices—the co-called *Nonnegative Inverse Eigenvalue Problem*—was first posed seventy years ago and it remains unsolved. In order to decide if a given list $\sigma := (\rho, -\lambda_1, -\lambda_2, \ldots, -\lambda_n)$ of complex numbers is *realisable* as the spectrum of some nonnegative matrix with Perron eigenvalue ρ (and if so, to determine the possible properties of the realising matrix), it is useful to establish some inequalities for the coefficients of the polynomial

$$\prod_{i=1}^{n}(x+\lambda_i) = x^n + b_1x^{n-1} + b_2x^{n-2} + \cdots + b_n.$$

In particular, if the λ_i are real and nonnegative, then so are the b_i, and Newton's famous inequalities state that $b_k^2 \geqslant b_{k-1}b_{k+1}$ for each k. We have given families of "Newton-like" inequalities for the case when $\lambda_1, \lambda_2, \ldots, \lambda_n$ are self-conjugate complex numbers with nonnegative real parts. These are the first known inequalities of this type which are independent of the proximity of the λ_i to the real axis.

In this talk, I will discuss these inequalities and outline some combinatorial and matrix-theoretic proofs. I will also discuss how we have applied these inequalities to the Nonnegative Inverse Eigenvalue Problem, and, in particular, how this relates to an important result by Laffey and Šmigoc.

The projective characters of metacyclic p-groups

Conor Finnegan

University College Dublin, Ireland

The projective characters of a group can provide us with important information regarding the structure and properties of the group. In this talk, I will give a brief overview of some of the fundamental methods and results in projective representation theory, assuming some prior knowledge of ordinary representation theory. I will then discuss how these methods can be applied in order to find the projective character tables of metacyclic p-groups of positive type.

Mutually unbiased bases, Hadamard matrices and group actions on spreads

Rod Gow

University College Dublin, Ireland

A real $n \times n$ matrix H, each of whose entries is ± 1, is said to be a Hadamard matrix if $HH^T = nI$, where H^T is the transpose of H and I is the $n \times n$ identity matrix. It is clear that if H is an $n \times n$ Hadamard matrix, $H/\sqrt{n}$ is orthogonal and each of its entries is $\pm 1/\sqrt{n}$.

If H and H_1 are $n \times n$ Hadamard matrices, it can happen that HH_1/n is also an Hadamard matrix, although this closure property with respect to multiplication does not occur in general.

We are interested in an extreme form of this closure property, related to representations of finite groups, which we will describe in our talk. The main result is as follows. Let G be any finite group of odd order. Then there is an even power of 2, 2^{2m} say, where m is a positive integer dependent on $|G|$, and a (faithful) representation R, say, of G by $2^{2m} \times 2^{2m}$ real orthogonal matrices such that for each non-identity element g of G, $2^m R(g)$ is an Hadamard matrix. Thus G is represented by suitably scaled real Hadamard matrices.

The proof uses techniques from the theory of orthogonal geometry over the field $\mathbb{F}_2$ of order 2, especially the idea of a partial spread. We also use an important and unusual representation over $\mathbb{F}_2$ of the symmetric group, called the spin representation, whose degree is a power of 2.

Our findings can also be expressed in terms of the concept of real mutually unbiased bases.

Thomas J. Laffey's contribution to Linear Algebra & Matrix Theory

Rod Gow

University College Dublin, Ireland

This talk expands on the contribution of Thomas J. Laffey to the area of Linear Algebra & Matrix Theory, among other achievements, over the last decades.

A question of Monov related to the nonnegative inverse eigenvalue problem

Thomas J. Laffey

University College Dublin, Ireland

This is joint work with Anthony Cronin (UCD) and Helena Ŝmigoc (UCD).

The algebraic Riccati & Sylvester equations and the positive observer problem

Oliver Mason

Maynooth University, Ireland

In this talk, I will first give a general overview of properties of the algebraic Riccati and Sylvester equations before discussing results obtained recently concerning positive systems and diagonal solutions. I will also describe a number of applications to stability theory and to the design of positive observers subject to privacy constraints.

Centralizers and Idealizers of Integer Matrices

Raja Mukherji

Distinguished Engineer in Artificial Intelligence at UnitedHealth Group, Ireland

In this talk, I show that the centralizer of an integer matrix is isomorphic to the idealizer of its corresponding ideal class under the Latimer-MacDuffee thereom. I also provide several additional results relating to idealizers and centralizers of integer matrices.

Alternating Signed Bipartite Graph Colourings

Cian O'Brien

National University of Ireland, Galway

This talk concerns a class of bipartite graphs that arise from alternating sign matrices. To an alternating sign matrix, we may associate an alternating signed bipartite graph, which has a vertex for each row and column of the matrix. Vertex r_i is connected to vertex c_j by a blue edge if there is a 1 in the (i, j) position of the matrix, and by a red edge if there is a -1. In this talk, we present results on when a given graph G admits an edge colouring c such that the coloured graph G^c is an alternating signed bipartite graph.

Rank distributions for entry pattern matrices

Rachel Quinlan

National University of Ireland, Galway

In an *entry pattern matrix*, every entry is an indeterminate. The same indeterminate may appear in multiple positions, but different indeterminates are independent. Examples of matrix classes that are defined by entry patterns include symmetric matrices, Toeplitz matrices and Hankel matrices.

For a field $\mathbb{F}$ and an entry pattern matrix A, an $\mathbb{F}$-completion of A is obtained by assigning a value from $\mathbb{F}$ to each indeterminate in A. The set of all $\mathbb{F}$-completions of A is a vector space whose dimension is the number of distinct indeterminates in A. We will discuss the distribution of ranks in the space of completions. In particular, we consider the following questions:

- when the maximum rank of an $\mathbb{F}$-completion of an entry pattern matrix A differs from its *generic rank*, which is its rank when considered as a matrix in $\mathbb{F}(x_1, \ldots, x_k)$, where $x_1, \ldots, x_k$ are the indeterminates appearing in A;
- for a field $\mathbb{F}$, what is the maximum possible number of indeterminates in an entry pattern matrix A whose every non-uniform $\mathbb{F}$-completion is nonsingular. A uniform completion is one in which all entries are equal.

This is joint work with Hieu Ha Van.

Subspaces of Matrices over Finite Fields with Restricted Rank

John Sheekey

University College Dublin, Ireland

In this talk we address the following questions: How large can a subspace of matrices be, if we require that all its nonzero elements have rank at least k? Or exactly k? Can we classify subspaces with these properties? Over a finite field, this question has been studied in part due to its connections with nonassociative algebra (semifields) and finite geometry. It has become a hot topic in recent years due to new applications which have arisen in coding theory; in particular in random network coding, the process of communicating information through large unpredictable networks of devices, which is crucial in our increasingly connected world. We will give an overview of these connections, the known constructions and classifications, and present some remaining open problems.

Distributed ledger technology, cyberphysics, and social compliance: Mathematics, Applications, and Open Questions

Robert Shorten

University College Dublin, Ireland

This talk describes how Distributed Ledger Technologies (DLT) can be used to design a class of cyber-physical systems, as well as to enforce social contracts and to orchestrate the behaviour of agents trying to access a shared resource. The first part of the paper analyses the advantages and disadvantages of using Distributed Ledger Technologies architectures to implement certain control systems in an Internet of Things (IoT) setting, and then focuses on a specific type of DLT based on a Directed Acyclic Graph. In this setting we propose a set of delay differential equations to describe the dynamical behaviour of the Tangle, an IoT-inspired Directed Acyclic Graph designed for the cryptocurrency IOTA. The second part proposes an application of Distributed Ledger Technologies as a mechanism for dynamic deposit pricing, wherein the deposit of digital currency is used to orchestrate access to a network of shared resources. The pricing signal is used as a mechanism to enforce the desired level of compliance according to a predetermined set of rules. After presenting an illustrative example, we analyze the control system and provide sufficient conditions for the stability of the network.

Diagonalizable realizability in the Nonnegative Inverse Eigenvalue Problem

Helena Ŝmigoc

University College Dublin, Ireland

We will show that if a list of nonzero complex numbers

$$\sigma = (\lambda_1, \lambda_2, \ldots, \lambda_k)$$

is the nonzero spectrum of a diagonalizable entrywise nonnegative matrix, then σ is the nonzero spectrum of a diagonalizable nonnegative matrix of order $k + k^2$. We will also present a solution to the Nonnegative Inverse Elementary Divisors Problem for the classical example $\sigma(t) = (3 + t, 3 - t, -2, -2, -2)$. Results presented are joint work with Thomas Laffey.