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BOOK OF ABSTRACTS / TALKS

INVITED TALKS

Global integrability of supertemperatures

Hiroaki Aikawa

Hokkaido University, Japan

Ever since Armitage showed that every nonnegative superharmonic function on a bounded domain of bounded curvature ($= C^{1,1}$ domain) in $\mathbb{R}^n$ is L^p-integrable up to the boundary for $0 < p < n/(n-1)$, the global integrability of nonnegative supersolutions has attracted many mathematicians.

In this talk we consider a parabolic counterpart. We study the global integrability of nonnegative supertemperatures on the cylinder $D \times (0, T)$, where D is a Lipschitz domain or a John domain. We show that the integrability depends on the lower estimate of the Green function for the Dirichlet Laplacian on D.

In particular, if D is a bounded C^1-domain, then every nonnegative supertemperature on $D \times (0, T)$ is L^p-integrable over $D \times (0, T')$ for any $0 < T' < T$, provided $0 < p < (n+2)/(n+1)$. The bound $(n+2)/(n+1)$ is sharp. This talk is based on a joint work with Hara and Hirata.

Optimal polynomial approximants: Zeros and Limit Points

Catherine Bénéteau

University of South Florida, USA

In this talk, I will be interested in polynomials that approximate, in some optimal sense, inverses of functions in certain Hilbert spaces of analytic functions of the disk. These polynomials are closely related to classical objects of function theory such as orthogonal polynomials and weighted reproducing kernels. I will be interested in the following questions:

(1) Does there exist a Jentzsch-type theorem for these polynomials, namely, can the zeros of the optimal approximants of a given function converge to any point of the unit circle?

(2) Given a point on the unit circle, and given a function f, what is the set of limit points of the optimal approximants to $1/f$ at that point?

Fine potential theory via analysis on metric spaces

Jana Björn

Lindköping University, Sweden

Fine potential theory and the fact that superharmonic functions are finely continuous have during the last half century lead to studying p-harmonic functions on finely open sets in Euclidean spaces. This requires Sobolev spaces and a notion of gradient on such non-open sets. Defining those notions is not quite trivial and has been solved in different ways by various authors.

In this talk we shall see how analysis based on upper gradients on metric spaces naturally leads to the fine topology, thus bringing new light on the classical theory, even in Euclidean spaces. In particular, we discuss p-harmonic functions on bad sets, and compare upper gradients with respect to different underlying spaces. We characterize sets, for which these notions are nontrivial. Fine potential theory and the above results are discussed in the setting of metric spaces and illustrated by examples.

The sharp constant in the Sobolev-Poincaré inequality for a region

Tom Carroll

University College Cork, Ireland

Let Ω be a bounded region in $\mathbb{R}^n$, let $r \in [1, n)$ and $p \in [1, r^*]$, where $r^* = \frac{nr}{n-r}$. There is a finite constant $S_{p,r}(\Omega)$ such that

$$\|u\|_{L^p(\Omega)} \leqslant S_{p,r}(\Omega)\|\nabla u\|_{L^r(\Omega)}$$

for any function u in the Sobolev space $W_0^{1,r}(\Omega)$, so that $S_{p,r}(\Omega)$ is the sharp constant in the Sobolev-Poincaré inequality for the region Ω.

Set

$$\mathcal{C}_{p,r}(\Omega) = \inf\left\{ \frac{\int_\Omega |\nabla u|^r \, d\mu}{\left(\int_\Omega |u|^p \, d\mu\right)^{r/p}} : u \in \mathcal{C}_0^\infty(\Omega), u \not\equiv 0 \right\}.$$

Then

$$S_{p,r}(\Omega) = \mathcal{C}_{p,r}(\Omega)^{-1/r}$$

and, from a classical point of view, $\mathcal{C}_{2,2}(\Omega)$ is the classical Rayleigh quotient for the principal frequency or bass note of the region Ω in two dimensions while $4/\mathcal{C}_{1,2}$ corresponds to its torsional rigidity, both important physical concepts in the context of solid mechanics. Moreover, $\mathcal{C}_{p,p}(\Omega)$ is the eigenvalue of the Dirichlet p-Laplacian corresponding to a positive eigenfunction.

In this talk, we will describe some of what is known about the eigenvalues $\mathcal{C}_{p,r}(\Omega)$ and their corresponding eigenfunctions, focusing in particular on isoperimetric inequalities, reverse-Hölder inequalities for the eigenfunctions, and estimates for the rate of change of the eigenvalue for a domain that moves outward. This is joint work with Moustapha Fall and Jesse Ratzkin.

Nearly hyperharmonic functions and Jensen measures

Wolfhard Hansen

University of Bielefeld, Germany

Let $(X, \mathcal{H})$ be a $\mathcal{P}$-harmonic space and assume for simplicity that constants are harmonic (as in the special case of classical potential theory). Given a numerical function φ on X which is locally lower bounded, let

$$J_\varphi(x) := \sup\{\int^* \varphi \, d\mu(x) \colon \mu \in \mathcal{J}_x(X)\}, \qquad x \in X,$$

where $\mathcal{J}_x(X)$ denotes the set of all Jensen measures μ for x, that is, μ is a compactly supported measure on X satisfying $\int u \, d\mu \leq u(x)$ for every hyperharmonic function u on X.

The main goal is to show that, assuming quasi-universal measurability of φ, the function J_φ is the smallest nearly hyperharmonic function majorizing φ and that $J_\varphi = \varphi \vee \hat{J}_\varphi$, where $\hat{J}_\varphi$ is the lower semicontinuous regularization of J_φ. So, in particular, J_φ turns out to be at least "as measurable as" φ.

This is based on joint work with Ivan Netuka and improves our recent results, where the axiom of polarity was assumed. Preliminaries on nearly hyperharmonic functions in the framework of balayage spaces are closely related to the study of strongly supermedian functions triggered by J.-F. Mertens more than forty years ago.

What is an "inner function"?

Dmitry Khavinson

University of South Florida, USA

The concept of an inner function has been a focal point of function theoretic operator theory since the celebrated Beurling theorem characterizing invariant (with respect to the unilateral shift) subspaces in Hardy spaces in the unit disk.

In the 1990s, it was extended to Bergman spaces where however Beurling's theorem fails. Since the 1960s, many plausible ways of extending the notion of an inner function were pursued in the context of Hardy, Bergman and other spaces of analytic functions in a more general setting than the unit disk: multiply connected domains, Riemann surfaces and several variables. Yet, the "big picture" is far from clear. This talk is based on a joint research project with C. Bénéteau, M. Fleeman, C. Liaw and A. Sola.

Polynomial and entire solutions to the Dirichlet problem

Erik Lundberg

Florida Atlantic University, USA

It is well-known that the classical Dirichlet problem for Laplace's equation in the ball with polynomial boundary data always has a polynomial solution. It is less well-known (but still a classical fact) that the same holds true for ellipsoids. According to the Khavinson-Shapiro conjecture (1992) this property characterizes ellipsoids. I will discuss this problem along with the broader theme of analytic continuation of solutions to the Dirichlet problem.

Behaviour of optimal polynomial approximants on the unit circle

Myrto Manolaki

University of South Florida, USA

The notion of optimal polynomial approximants to reciprocals of functions in Dirichlet-type spaces $\mathcal{D}_\alpha$ was introduced to examine the phenomenon of cyclicity. In particular, a function f in $\mathcal{D}_\alpha$ is cyclic, if and only if the optimal polynomial approximants p_n to $1/f$ satisfy $\|p_n f - 1\|_\alpha \to 0$ as $n \to \infty$. In this talk, we will discuss the limiting behaviour of the sequence (p_n) on the unit circle, focusing on the special cases of Hardy and Bergman spaces. (Joint work with Catherine Bénéteau and Daniel Seco.)

Generic boundary behaviour of Taylor series

Jürgen Müller

University of Trier, Germany

Let $H(\mathbb{D})$ denote the space of functions holomorphic in the unit disc $\mathbb{D}$ endowed with the topology of locally uniform convergence. For $f \in H(\mathbb{D})$ we write $S_n f$ for n-th partial sum of the Taylor series $f(z) = \sum_{\nu=0}^{\infty} a_\nu z^\nu$ about 0. It is known that, generically in $H(\mathbb{D})$, the behaviour of the sequence $(S_n f)$ on the unit circle $\mathbb{T}$ is extremely erratic e.g. in the sense that all continuous functions on $\mathbb{T}$ are realised as pointwise limit functions. According to a result of Gardiner and

Manolaki, the situation changes in a significant way if $f \in H(\mathbb{D})$ has nontangential limits $f^*(\zeta)$ for ζ in subsets of $\mathbb{T}$ of positive arc length measure: In this case there is a clear preference for the limit function f^*. We consider the question to which extent spurious limit functions on small subsets of $\mathbb{T}$ still appear generically in spaces of holomorphic functions where nontangential limits are guaranteed.

From universality to generic non-extendability and totally unbounded functions in new localized function spaces

Vassili Nestoridis

University of Athens, Greece

J.-P. Kahane conjectured that any universal Taylor series on the disc defines a nowhere extendable holomorphic function. The positive answer to this conjecture led me to use the techniques of Universality and prove that in various spaces of holomorphic functions on rather general domains G the set of non-extendable functions is a dense G_δ set under the natural topology of the function space. Later Montel's theorem was combined with Baire's theorem and gave a very simple proof of the above result in a very general setting in one, finitely many or infinitely many complex variables. The existence, for every w in the boundary $Fr(G)$ of G, of a function f_w in the function space $X = X(G)$ non-extendable near w is sufficient for the above fact to hold. A stronger notion is that of totally unbounded function, where similar facts have been proven. If X contains only bounded functions, then often some derivative is generically totally unbounded, hence non-extendable.

Most of the classical function spaces X satisfy the above mentioned facts. Often X is defined as the set of holomorphic functions f on the domain G satisfying some property P when we approach from G the whole boundary $Fr(G)$ of G. One can define localized versions of the above spaces, by requiring that the holomorphic on G function f satisfies a denumerable set of properties P_n, $n = 1, 2, ...$ when we approach (relatively open) subsets J_n of $Fr(G)$, respectively. Then it is possible to define the topology on these new spaces X in such a way that all the above facts remain valid.

What are the properties of the elements of these new spaces? Are there non-tangential limits? What about the zeros of their elements? Does rectifiability of an arc of $Fr(G)$ imply that the derivative of the Riemann map onto G belongs to the localized Hardy space H^1 on the disc? For function spaces X containing only functions extending continuously on J_n, does the the generic function f is nowhere differentiable on J_n? What about the growth of such functions, their derivatives or their Taylor coefficients? Are polynomials or rational functions dense in these new spaces X?

Stephen J. Gardiner's contribution to potential theory

Ivan Netuka

Charles University Prague, Czech Republic

This talk expands on the contribution of Stephen J. Gardiner to the area of Potential Theory over the last decades.

Chebychev quadrature formulas in algebraic manifolds

JOAQUIM ORTEGA-CERDA

University of Barcelona, Spain

We present a generalization of the classical Bernstein inequality for polynomials on algebraic manifolds and see some of its applications, namely the construction of algebraic designs, building upon a similar result of Bondarenko, Radchenko and Viazovska on spherical designs.

Weak factorization of Bergman and Hardy spaces

JORDI PAU

University of Barcelona, Spain

We obtain a weak factorization for Bergman spaces A^p on the unit ball for $p > 1$. Moreover, we extend this result to vector-valued Bergman spaces. For the obtention of the results, a characterization of the boundedness of small Hankel operators with vector-valued holomorphic symbols is obtained. Finally, we discuss the open problem of the weak factorization for Hardy spaces H^p with $p > 1$.

A uniform boundedness principle in pluripotential theory

THOMAS RANSFORD

Université Laval, Canada

Whilst attempting to construct a counterexample to an old conjecture concerning the definition of plurisubharmonic function, we have run up against an obstruction which may be of interest in its own right. This obstruction can be formulated as a kind of uniform boundedness principle. Loosely stated, it says that a family of continuous plurisubharmonic functions is locally uniformly bounded above provided that it locally uniformly bounded above in each variable separately. In this talk, I shall discuss this result and some of its ramifications.

This is joint work with Łukasz Kosiński (Krakow) and Étienne Martel (Laval).

Energy Bounds for Minimizing Riesz and Gauss Configurations

EDWARD SAFF

Vanderbilt University, USA

Utilizing frameworks developed by Delsarte, Yudin and Levenshtein, we deduce linear programming lower bounds (as $N \to \infty$) for the Riesz energy of N-point configurations on the d-dimensional unit sphere in the so-called hypersingular case; i.e, for non-integrable Riesz kernels of the form $|x - y|^{-s}$ with $s > d$. As a consequence, we immediately get (thanks to the Poppy-seed bagel theorem) lower estimates for the large N limits of minimal hypersingular Riesz energy on compact d-rectifiable sets. Furthermore, for the Gaussian potential $\exp(-\alpha|x-y|^2)$ on $\mathbb{R}^p$, we obtain lower bounds for the energy of infinite configurations having a prescribed density.

Hardy and BMO spaces of Dirichlet series

Kristian Seip

Norwegian University of Science and Technology, Trondheim, Norway

The theory of Hardy spaces of Dirichlet series that has emerged in recent years, differs in many aspects from its classical counterpart on the unit disc. Unforeseen phenomena appear, some related to the complicated structure of the dual of such Hardy spaces and others arising from interaction with number theory. In this talk, I will describe this situation, with emphasis on recent progress and open problems.

Free boundaries on Lattice, and their scaling limits

Henrik Shahgholian

Royal Institute of Technology Stockholm, Sweden

Probably the most well-known fact in classical potential theory is the mean value property for harmonic functions over spherical shells or balls. We shall discuss similar properties for harmonic functions on the lattice $s\mathbb{Z}^2$, and show (through numerics) that interesting new objects may appear when the size of lattice s tends to zero.

These objects have been studied in two recent works in collaboration with Hayk Aleksanyan.

Some applications of Partial Balayage

Tomas Sjödin

Lindköping University, Sweden

Partial balayage is an operation with the aim of constructing a so called quadrature domain for subharmonic functions with respect to a given measure. It can be defined in several ways, where perhaps the most natural one from a potential theoretic view is by solving a type of obstacle problem. One can also say that the construction leads in a natural way to a Laplacian growth problem connected to e.g. Hele-Shaw flow. Although the original motivation for its study was to prove existence of these quadrature domains, during the last decade the construction has turned out to be useful to answer other questions in potential theory.

In the talk we will briefly introduce the partial balayage contruction and then discuss how it can be useful. In particular we will look at an application to the exterior inverse problem where we, together with Stephen Gardiner, made progress by proving uniqueness in case one of the domains in question is convex, and the solution of a conjecture regarding so called analytic content which was resolved together with Stephen Gardiner and Marius Ghergu.

PARALLEL TALKS

Connections between the Dirichlet and the Neumann problem

LUCIAN BEZNEA

Simion Stoilow Institute of Mathematics of the Romanian Academy and University of Bucharest, Romania

We give a representation of the solution of the Neumann problem for the Laplace operator on the unit ball in $\mathbb{R}^n$, $n \geq 1$, in terms of the solution of an associated Dirichlet problem. The representation is suitable for extensions to other operators besides the Laplacian, smooth planar domains, and the infinite dimensional case. It also holds in the case of integrable boundary data. We derive an explicit formula for the Dirichlet-to-Neumann operator, and provide an explicit solution of the generalized solution of the Neumann problem. The talk is based on joint works with Mihai N. Pascu and Nicoale R. Pascu.

Selected approximation results on L-regular sets and on algebraic varieties

LEOKADIA BIALAS-CIEZ

Jagiellonian University in Krakow, Poland

and

AGNIESZKA KOWALSKA

Pedagogical University of Krakow, Poland

For large enough compact subsets of $\mathbb{C}^N$ we have significant approximation results, e.g. Bernstein-Walsh-Siciak theorem, Markov inequality, division estimate etc. The situation is quite different if we consider compact subsets of an algebraic variety. This case requires specific definitions and particular methods. We present selected approximation results obtained for this kind of sets during the last years.

The talk is partially based on a joint work with Jean-Paul Calvi, Toulouse, France.

Convergences of row sequences of simultaneous Padé-Faber approximants

NATTAPONG BOSUWAN

Mahidol University, Thailand

We investigate convergences of vector valued Padé-Faber approximations on their row sequences. Two types of these approximations are considered. We prove two Montessus de Ballore type theorems for the first type of simultaneous Padé-Faber approximation. For the second type, we introduce a new concept of vector valued Padé-Faber approximation. In this talk, we prove not only a Montessus de Ballore type theorem for this new approximation but also its inverse result on row sequences. In particular, we give necessary and sufficient conditions for the convergence with geometric rate of the common denominators of the sequence of these new simultaneous Padé-Faber approximants. Moreover, the exact rate of convergence of these denominators is provided.

On a Problem of Beurling

James Brennan

University of Kentucky, USA

Let Ω be a bounded simply connected domain in the complex plane $\mathbb{C}$, and denote by Ω_∞ the unbounded complementary component of its closure. We shall assume that $\partial\Omega \setminus \partial\Omega_\infty \neq \emptyset$, as would be the case if, for example, Ω is obtained from the open unit disk D by introducing a cut or slit in the form of a spiral (not necessarily smooth) extending outward and accumulating along the entire length of ∂D. In his Collected Works (Birkhäuser 1989) Beurling has considered the following generalization of the classic *Bernstein problem* for weighted polynomial approximation on the real line: Let $w(z) > 0$ be a bounded continuous function on Ω with $w(z) \to 0$ at each point of $\partial\Omega$, and let $C_w(\Omega)$ be the Banach space consisting of all complex-valued functions f such that the product $f(z)w(z)$ is continuous on $\bar{\Omega}$ and vanishes on $\partial\Omega$, with the norm being defined by $\|f\|_w = \sup_{\Omega} |f|w$. Evidently,

$$A_w(\Omega) = \{f \in C_w(\Omega) : f \text{ is analytic in } \Omega\}$$

is a closed linear subspace of $C_w(\Omega)$. Beurling's problem is to give a complete description of those weights w for which the polynomials are dense in $A_w(\Omega)$. Under the assumption that $w = w(g)$ depends only on Green's function g, thereby ensuring that the problem is conformally invariant, Beurling obtained a solution for a restricted class of domains. Retaining the assumption that w depends only on Green's function, the solution for an arbitrary simply connected domain Ω makes extensive use of deep *potential theoretic* ideas, and in particular, the theory of *asymptotically holomorphic* functions in a form that grew out of the work of E. M. Dyn'kin and A. L. Vol'berg, and was not available to Beurling.

Weighted generalized potential and Hardy-type operators in Morrey-type spaces

Evgeniya Burtseva

Luleå University, Sweden

In this talk we present our recent results on the boundedness of the weighted generalized Hardy and potential operators from generalized Morrey spaces $L^{p,\varphi}(\mathbb{R}^n)$ to Orlicz-Morrey spaces $L^{\Phi,\varphi}(\mathbb{R}^n)$. Due to the classes of the weights we use in our study, the weighted potential operator can be estimated by non-weighted one and weighted Hardy-type operators. We prove the boundedness of the weighted potential operator.

We also consider generalized potential operators with radial quasi-monotone weights and prove the weighted and non-weighted boundedness of such operators from the generalized Morrey spaces to Orlicz-Morrey spaces. For the study of non-weighted boundedness of potential operators we use "Hedberg's trick". Then we consider the weighted boundedness of generalized potential operator. We prove some pointwise estimates for weighted generalized potential operators via generalized Hardy operators. Then we obtain the weighted boundedness for the generalized potential operators in Morrey-type spaces.

The talk is based on the joint papers with Professor N. Samko.

Bounds on the norms of Chebyshev polynomials

Jacob Christiansen

Lund University, Sweden

In the talk, I will discuss upper and lower bounds on the norms of Chebyshev Polynomials. In particular, I'll explain which sets (in $\mathbb{R}$ and $\mathbb{C}$) saturate these bounds.

The talk is based on joint work with B. Simon (Caltech) and M. Zinchenko (UNM).

Asymptotic behaviour of pth means of subharmonic and analytic functions in the unit disc

Igor Chyzhykov

Ivan Franko National University of Lviv, Ukraine
and University of Warmia and Mazury in Olsztyn, Poland

Let SH^∞ be the class of subharmonic functions bounded from above in the unit disc $\mathbb{D}$. If $u \in SH^\infty$ such that $u(z) \le 0$ and $u(z)$ is harmonic in a neighborhood of the origin, then

$$u(z) = \int_{\mathbb{D}} \ln \frac{|z-\zeta|}{|1-z\bar{\zeta}|} d\mu_u(\zeta) - \frac{1}{2\pi} \int_{\partial\mathbb{D}} \frac{1-|z|^2}{|\zeta - z|^2} d\psi(\zeta),$$

where μ_u is the Riesz measure of u, ψ is a Borel measure on $\partial\mathbb{D}$.

For a Borel subset $M \subset \overline{\mathbb{D}}$ such that $M \cap \partial\mathbb{D}$ is Borel measurable on $\partial\mathbb{D}$ the *complete measure* λ_u *of* u *in the sense of Grishin* λ_u is defined [1, 2] (cf. [3]) by

$$\lambda_u(M) = \int_{\mathbb{D}\cap M} (1-|\zeta|)\, d\mu_u(\zeta) + \psi(M \cap \partial\mathbb{D}).$$

For $p > 0$ we define

$$m_p(r,u) = \left(\frac{1}{2\pi} \int_0^{2\pi} |u(re^{i\theta})|^p \, d\theta \right)^{\frac{1}{p}}, \quad 0 < r < 1.$$

Criteria for boundedness of pth integral means, $1 \le p < \infty$, of $\log|B|$ and $\log B$ are established by Ya.V. Mykytyuk and Ya.V. Vasyl'kiv in 2000 ([4]). Let $C(\varphi,\delta) = \{\zeta \in \overline{\mathbb{D}} : |\zeta| \ge 1-\delta, |\arg\zeta - \varphi| \le \pi\delta\}$.

Theorem. *Let $u \in SH^\infty$, $\gamma \in (0,2)$, $p \in [1,\infty)$. Let λ be the complete measure of u. In order that*

$$m_p(r,u) = O((1-r)^{\gamma-1}), \quad r \uparrow 1,$$

hold it is necessary and sufficient that

$$\left(\int_0^{2\pi} \lambda^p(C(\varphi,\delta))\, d\varphi \right)^{\frac{1}{p}} = O(\delta^\gamma), \quad 0 < \delta < 1.$$

We also prove sharp upper estimates of pth means of analytic and subharmonic functions of finite order in the unit disc. A multidimensional counterpart for M-subharmonic functions in the unit ball in $\mathbb{C}^n$ is proved as well [5].

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Approximation properties of polynomials with unimodular coefficients

Giorgios Costakis

University of Crete, Greece

Let $\sum_{n=0}^{\infty} a_n z^n$ be a power series in the complex plane with unimodular coefficients, i.e. $|a_n| = 1$ for every $n = 0, 1, 2, \ldots$. What can be said about the limiting behavior of the sequence of partial sums for such power series on given subsets K lying outside the open unit disk $\{z : |z| < 1\}$? We will see that interesting phenomena occur when: a) the set K is contained in the annulus $\{z : 1 \leqslant |z| < 3\}$, b) the size of K is small in a certain sense, c) uniform convergence of partial sums with respect to K is considered. We also touch on some real valued analogues of these problems where interestingly enough some number theoretic issues arise. This is work in progress.

A new boundary property of bounded analytic functions

Arthur A. Danielyan

University of South Florida, USA

By Fatou's fundamental theorem of 1906, a bounded analytic function f in the open unit disc D has radial (even non tangential) limits at the points of the unit circle T except a subset E of measure zero. It is a well-known elementary (topological) fact that the exceptional set E is a $G_{\delta\sigma}$. But if f has unrestricted limits at each point of $T \setminus E$, then obviously E becomes just an F_σ set. We show that the converse statement is true as well. Namely, we prove the following:

Theorem 1. Let E be a subset on T. There exists a bounded analytic function in D which has no radial limits on E but has unrestricted limits at each point of $T \setminus E$ if and only if E is an F_σ set of measure zero.

An obvious corollary of Theorem 1 is the Lohwater-Piranian theorem of 1957: If E is an F_σ set of measure zero on T then there exists a bounded analytic function in D which has no radial limits exactly on E. In 1994 Kolesnikov has proved the following remarkable converse of Fatou's theorem: There exists a bounded analytic function in D which has no radial limits exactly on the given set E (on T) if and only if E is a $G_{\delta\sigma}$ set of measure zero. Both Theorem 1 and Kolesnikov's theorem extend the Lohwater-Piranian theorem up to necessary and sufficient results, *but in different directions*. The method of the proof of Theorem 1 is *completely elementary* and it offers some simplification even for the proof of Kolesnikov's theorem.

On the complex Łojasiewicz inequality with parameter

Maciej P. Denkowski

Jagiellonian University in Krakow, Poland

We prove a continuity property in the sense of currents of a continuous family of holomorphic functions which allows us to obtain a Łojasiewicz inequality with an effective exponent independent of the parameter. Time allowing, we will further discuss a parameter version of the Łojasiewicz gradient inequality.

Integral representations of holomorphic functions

Seán Dineen

University College Dublin, Ireland

We obtain, using the Grothendieck-Pietsch criterion for nuclearity and Minlos' Theorem on the Fourier Transform of Gaussian measures, an integral representation for holomorphic functions satisfying a growth condition on certain infinite dimensional locally convex spaces.

Analogues of Whittaker's theorem for Laplace-Stieltjes integrals

Markian S. Dobushovskyy

Lviv Franko National University of Lviv, Ukraine

For an entire function $g(z)=\sum\limits_{n=0}^{\infty} a_n z^{\lambda_n}$ let ϱ and λ be the order and the lower order of g correspondingly. J.M. Whittaker (1933) has proved that $\lambda \le \varrho\beta$, where $\beta = \varliminf\limits_{n\to+\infty} (\ln \lambda_n)/\ln \lambda_{n+1}$. Here we investigate similar problems for the integrals of Laplace-Stieltjes.

Let V be a class of nonnegative nondecreasing unbounded continuous on the right functions F on $[0,+\infty)$. For a nonnegative function f on $[0,+\infty)$ the integral $I(\sigma)=\int\limits_0^{\infty} f(x)e^{x\sigma}dF(x)$ is called of Laplace-Stieltjes. Let $\mu(\sigma,I)=\sup\{f(x)e^{x\sigma} : x\ge 0\}$ be the maximum of the integrand and σ_μ be its abscissa of existence.

The most used characteristics of growth for integrals $I(\sigma)$ with $\sigma_\mu=+\infty$ are R-order $\varrho_R[I]$ and lower R-order $\lambda_R[I]$ which are defined by formulas $\varrho_R[I]=\varlimsup\limits_{\sigma\to+\infty}\dfrac{\ln\ln I(\sigma)}{\sigma}$, $\lambda_R[I]=\varliminf\limits_{\sigma\to+\infty}\dfrac{\ln\ln I(\sigma)}{\sigma}$. For example, the following theorem is true.

Theorem. Let $F\in V$, $\sigma_\mu=+\infty$ and $X=(x_k)$ be a some sequence of positive numbers increasing to $+\infty$. Suppose that f is nonincreasing function and has regular variation in regard to F.

If $\ln F(x) = O(x)$ as $x \to +\infty$ and $\ln f(x_k) = (1 + o(1)) \ln f(x_{k+1})$ as $k \to \infty$ then

$$\lambda_R[I] \le \beta \varrho_R[I], \quad \beta = \varliminf_{k\to\infty} \frac{\ln x_k}{\ln x_{k+1}}.$$

Asymptotics of extremal polynomials on several arcs

Benjamin Eichinger

Lund University, Sweden

The nth Chebyshev polynomial associated to an infinite compact subset, $K \subset \mathbb{C}$, is the unique monic polynomial of degree n which deviates the least from zero on K. Due to results of Szegö, it is clear that potential theory plays a crucial role in the study of Chebyshev polynomials. In 1969, Widom gave a complete description of the asymptotics of the extremal polynomials when K consists of finitely many Jordan regions. But, if K also includes Jordan arcs, the asymptotics are still unknown. Exceptional are $E \subset \mathbb{R}$. Recently, Christiansen, Simon, Yuditskii and Zinchenko described the asymptotics of Chebyshev polynomials associated to real sets, $E \subset \mathbb{R}$, which are homogeneous in the sense of Carleson.

In order to develop the theory further for complex sets, we propose to study a more general problem. Namely, to investigate asymptotics of the upper envelope of polynomials uniformly bounded on a given set. We have proven that in the case of one arc of the unit circle, $E \subset \mathbb{T}$, the asymptotics of the extremal value can be given as the diagonal of a certain reproducing kernel of analytic functions in $\overline{\mathbb{C}} \setminus E$. A similar result holds for the so-called Ahlfors problem, i.e., maximizing the first derivative at a given point instead of the point evaluation. The main goal of this talk is to discuss extensions to sets, $E \subset \mathbb{T}$, which are a union of several arcs.

The talk is based on joint work with P. Yuditskii.

Stable geometric properties of logharmonic mappings

Layan El Hajj

American University in Dubai, UAE

We consider logharmonic mappings which are stable univalent, stable starlike, stable with positive real part, stable close to starlike and stable typically real. We prove that the mappings $f_\lambda = zh(z)\overline{g(z)}^\lambda$ are starlike logharmonic (resp. logharmonic univalent, close to starlike logharmonic, typically real logharmonic) for all $|\lambda| = 1$ if and only if the mappings $\varphi_\lambda = \frac{zh}{g^\lambda}$ are starlike analytic (resp. analytic univalent, close to starlike analytic, typically real analytic) for all $|\lambda| = 1$. We also prove growth distortion theorems for these family of functions.

Hyponormal Toeplitz Operators with non-harmonic symbol acting on the Bergman space

MATTHEW FLEEMAN

Baylor University, USA

The Toeplitz operator acting on the Bergman space $A^2(\mathbb{D})$, with symbol φ is given by $T_\varphi f = P(\varphi f)$, where P is the projection from $L^2(\mathbb{D})$ onto the Bergman space. We present some history on the study of hyponormal Toeplitz operators acting on $A^2(\mathbb{D})$, as well as give results for when φ is a non-harmonic polynomial. Particular attention is given to unusual hyponormality behavior that arises due to the extension of the class of allowed symbols. This is joint work with Constanze Liaw.

Nevanlinna domains and density of polyanalytic polynomial moduli

RICHARD FOURNIER

University of Montreal, Canada

We give a new proof and minor refinements, in the polynomial case, of Jack's lemma. This proof is essentially based on the Bernstein's inequality for polynomials on the unit circle. We also discuss the cases of equality of

$$\frac{\partial^2}{\partial\theta^2} \ln |f(e^{i\theta})| \leqslant 0,$$

where f is analytic in the unit disc and in a neighbourhood of $e^{i\theta}$ where $|f(e^{i\theta})| = \sup_{|z|=1} |f(z)|$.

Weighted Beurling density and sampling in certain spaces of band-limited limited functions

JEAN-PIERRE GABARDO

McMaster University, Canada

The concepts of upper and lower Beurling density play an important role in the sampling theory for square-integrable, band-limited functions on $\mathbb{R}^d$ with spectrum contained in a bounded set. In this talk, we will consider the problem of defining appropriate notions of density for Hilbert spaces of band-limited functions whose norm are defined by the integral of the square of the Fourier transform multiplied by a certain weight which we assume to be moderate and tempered. We will use the theory of frames to extend the classical density results of H. Landau valid for the usual L^2-norm to this more general setting.

Sharp growth of frequently hypercyclic harmonic functions

Clifford Gilmore

University of Manchester, UK

The notion of frequent hypercyclicity, which has fundamental connections to ergodic theory, was introduced by Bayart and Grivaux in 2004. Many natural continuous linear operators are frequently hypercyclic, for instance the differentiation operator on the space of entire functions and the partial differentiation operator acting on the space of harmonic functions on $\mathbb{R}^N$, where $N \geqslant 2$.

Aldred and Armitage identified growth rates, in terms of the L^2-norm on spheres, of harmonic functions that are hypercyclic for the partial differentiation operator. In this talk we consider the frequently hypercyclic case and we identify sharp growth rates of frequently hypercyclic harmonic functions. This answers a question posed by Blasco, Bonilla and Grosse-Erdmann.

This is joint work with Eero Saksman and Hans-Olav Tylli.

Overconvergence properties of harmonic homogeneous polynomial expansions

Mayya Golitsyna

University College Dublin, Ireland

A harmonic function on a domain Ω in $\mathbb{R}^N$ may be expanded locally as a series of harmonic homogeneous polynomials. Such an expansion is said to be universal if subsequences of its partial sums approximate arbitrary harmonic functions on compact sets K in the complement of Ω, where K has connected complement. In this talk I will explore the relationship between overconvergence properties of series of harmonic homogeneous polynomials and their gap structure. This leads to new insights concerning the existence of universal polynomial expansions of harmonic functions. This work was inspired by known results for universal Taylor series. However, new phenomena arise for harmonic functions in higher dimensions.

Disjoint hypercyclicity for the Taylor shift Operator

Deguang Han

University of Central Florida, USA

Given a finite group G and a projective unitary representation π, we consider the problem of characterizing all the frame generators x such that $\{\pi(g)x : g \in G\}$ does phase retrieval for the representation space. A complete characterization will be discussed for irreducible projective representations for finite abelian groups. A special case covers some recent results for Gabor measurements.

The $\overline{\partial}$-Neumann problem and Schrödinger operators

Friedrich Haslinger

Universität Wien, Austria

We apply methods from complex analysis, in particular the $\overline{\partial}$-Neumann operator, to investigate spectral properties of Schrödinger operators with magnetic field (Pauli operators). For this purpose we consider the weighted $\overline{\partial}$-complex on $\mathbb{C}^n$ with a plurisubharmonic weight function φ, let $1 \leq q \leq n-1$:

$$L^2_{(0,q-1)}(\mathbb{C}^n, e^{-\varphi}) \underset{\overline{\partial}^*_\varphi}{\overset{\overline{\partial}}{\rightleftarrows}} L^2_{(0,q)}(\mathbb{C}^n, e^{-\varphi}) \underset{\overline{\partial}^*_\varphi}{\overset{\overline{\partial}}{\rightleftarrows}} L^2_{(0,q+1)}(\mathbb{C}^n, e^{-\varphi})$$

and $\Box_{\varphi,q} = \overline{\partial}\,\overline{\partial}^*_\varphi + \overline{\partial}^*_\varphi \overline{\partial}$.

We derive a necessary condition for compactness of the corresponding $\overline{\partial}$-Neumann operator (the inverse of $\Box_{\varphi,q}$) and a sufficient condition, both are not sharp. So far, a characterization can only be given in the complex 1-dimensional case.

The Pauli operators appear at the beginning and at the end of the weighted $\overline{\partial}$-complex. It is also of importance to know whether a related Bergman space of entire functions $A^2(\mathbb{C}^n, e^{-\varphi}) = \{f : \mathbb{C}^n \longrightarrow \mathbb{C}\,\text{entire} : \int_{\mathbb{C}^n} |f|^2 e^{-\varphi} d\lambda < \infty\}$ is of infinite dimension.

In addition we consider the ∂-complex, let $1 \leq p \leq n-1$:

$$A^2_{(p-1,0)}(\mathbb{C}^n, e^{-\varphi}) \underset{\partial^*_\varphi}{\overset{\partial}{\rightleftarrows}} A^2_{(p,0)}(\mathbb{C}^n, e^{-\varphi}) \underset{\partial^*_\varphi}{\overset{\partial}{\rightleftarrows}} A^2_{(p+1,0)}(\mathbb{C}^n, e^{-\varphi}).$$

On the space of pluriregular compact subsets of $\mathbb{C}^N$

Marta Kosek

Jagiellonian University in Krakow, Poland

Consider the space of pluriregular polynomially convex compact subsets of $\mathbb{C}^N$ with the metric defined by the means of the pluricomplex Green function. We list some properties of the space. We consider a family of contractive similarities defined by regular polynomial mappings $\mathbb{C}^N \longrightarrow \mathbb{C}^N$. We consider approximation of some Julia type sets and, in the case of $N = 1$, approximation by Julia sets.

Reflection principles in several complex variables

Mika Koskenoja

University of Helsinki, Finland

We study reflection principle for some central objects in several complex variables and pluripotential theory. First we show that the odd reflected function gives an extension for pluriharmonic functions over a flat boundary. Then we show that the even reflected function gives an extension for positive plurisubharmonic functions. In particular cases odd and/or even reflected functions give extensions for classical solutions of the homogeneous complex Monge–Ampère equation. Finally, we state reflection principle for maximal plurisubharmonic functions.

Quadrature Domains in Several Complex Variables

ALAN LEGG

Indiana University-Purdue University Ft. Wayne, USA

A quadrature domain for square-integrable holomorphic functions is a domain on which integration of a function in the Bergman space coincides with a finite linear combination of point evaluations of the function and some of its derivatives. Planar quadrature domains have an elegant theory, but quadrature domains in several dimensions are not as well understood. I'll use the Bergman kernel to gain some beginning insights into how multi-dimensional quadrature domains might look.

The Krzyż Conjecture and an Entropy Conjecture

JOHN E. MCCARTHY

Washington University in St. Louis, USA

In 1969, J. Krzyż conjectured that, over all holomorphic functions

$$f(z) = \sum_{n=0}^{\infty} \hat{f}(n) z^n$$

that map the unit disk $\mathbb{D}$ to $\mathbb{D} \setminus \{0\}$, we have

$$\sup |\hat{f}(n)| \ = \ \frac{2}{e}$$

for every positive integer n. The conjecture has been proved up to $n = 5$, but is open for larger n.

The entropy conjecture is that for all non-constant polynomials p that have all their roots on the unit circle $\mathbb{T}$ and with L^2-norm 1 there, we have

$$\int_{\mathbb{T}} |p|^2 \log |p|^2 \ \geqslant \ 1 - \log(2).$$

We will discuss how these two conjectures are related, show how the entropy conjecture implies the Krzyż conjecture provided a non-degeneracy condition is satisfied, and prove a special case of the entropy conjecture.

The talk is based on joint work with J. Agler.

Complex symmetric operators

PAWEL MLECZKO

Adam Mickiewicz University in Poznań, Poland

In the course of a talk I will discuss interpolation properties of complex symmetric operators on Hilbert spaces and show applications to the study of Toeplitz operators on weighted Hardy–Hilbert spaces of analytic functions on the unit disc. The talk is based on a joint work with Radoslaw Szwedek from Adam Mickiewicz University in Poznań.

The convex combination problem for Herglotz-Nevanlinna functions

Mitja Nedic

Stockholm University, Sweden

Herglotz-Nevanlinna functions are holomorphic functions defined in the poly-upper half-plane having non-negative imaginary part. Any such function admits an integral representation formula involving a real number, a vector of non-negative numbers and a positive Borel measure on $\mathbb{R}^n$ satisfying certain properties. These parameters are of great interest as they completely characterize this class of functions.

The convex combination problem for Herglotz-Nevanlinna functions asks whether we can relate the parameters of a Herglotz-Nevanlinna function in one variable to the parameters of a Herglotz-Nevanlinna function in several variables, if the several-variable function was built out of the one-variable function by replacing the independent variable with a convex combination of independent variables. In this talk, we present a completely explicit solution to this problem.

Boundary values of bounded holomorphic functions on domains in $\mathbb{C}^n$

Sofía Ortega Castillo

Centro de Investigación en Matemáticas, Mexic

I will talk about the boundary behavior of bounded holomorphic functions on pseudoconvex domains in $\mathbb{C}^n$. Through the construction of smooth solutions to the Cauchy-Riemann equations with continuous extension to a boundary point of strong pseudoconvexity, I will show you that the limit behaviour at such point is the expected in terms of a cluster value theorem, which is a weak form of a Corona theorem. I will also discuss an example of a domain in $\mathbb{C}^2$ without empty Corona that satisfies a cluster value theorem at every point other than an element in the boundary.

Differences of Composition operators on Large weighted Bergman spaces

Inyoung Park

Pusan National University, Republic of Korea

There are many results on the theory of composition operators in the standard Bergman spaces due to useful tools, but it has been relatively not so much known in the large Bergman spaces. In this paper, we characterize when the differences of composition operators acting on the Bergman spaces with exponential type weights is compact by using the Riemannian distance defined newly.

Julia Sets of Orthogonal Polynomials

Henrik Laurberg Pedersen

University of Copenhagen, Denmark

For a probability measure with compact and non-polar support in the complex plane we relate dynamical properties of the associated sequence of orthogonal polynomials $\{P_n\}$ to properties of the support. More precisely we relate the Julia set of P_n to the outer boundary of the support, the filled Julia set to the polynomial convex hull K of the support, and the Green's function associated with P_n to the Green's function for the complement of K.

The talk is based on joint work of Jacob Stordal Christiansen (Lund University, Sweden), Christian Henriksen (Technical University of Denmark), Henrik Laurberg Pedersen (University of Copenhagen, Denmark) and Carsten Lunde Petersen (Roskilde University, Denmark).

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Markov's inequality and polynomial mappings

Rafal Pierzchala

Jagiellonian University in Krakow, Poland

One of the most important polynomial inequalities is the following Markov's inequality (1889). If P is a polynomial of one variable, then

$$\|P'\|_{[-1,\,1]} \leqslant (\deg P)^2 \|P\|_{[-1,\,1]} \, .$$

Moreover, this inequality is optimal, because for the Chebyshev polynomials T_n, we have $T_n'(1) = n^2$ and $\|T_n\|_{[-1,\,1]} = 1$.

It is natural to ask about similar inequalities if we replace the interval $[-1,\,1]$ by another compact set in $\mathbb{R}^N$ or $\mathbb{C}^N$. In the talk, we will address this issue. In particular, we will give a solution to an old problem, studied among others by Baran and Pleniak, and concerning the invariance of Markov's inequality under polynomial mappings.

ŁS condition for filled Julia sets in $\mathbb{C}$

Frédéric Protin

INSA de Toulouse, France

We propose to present a recent work in which we derive an inequality of Łojasiewicz-Siciak type for certain sets arising in the context of the complex dynamics in dimension 1. More precisely, if we denote by *dist* the euclidian distance in $\mathbb{C}$, we show that the Green function G_K of the filled Julia set K of a polynomial such that $\mathring{K} \neq \emptyset$, satisfies the so-called *LS condition* $G_A \geqslant c{\cdot}dist(\cdot, K)^{c'}$ in a neighborhood of K, for some constants $c, c' > 0$. Relatively few examples of compact sets satisfying the ŁS condition are known. Our result highlights an interesting class of compact sets fulfilling this condition. The fact that filled Julia sets satisfy the ŁS condition may seem surprising, since they are in general very irregular. However, we provide an explicit example of a curve which has a cusp and satisfies the ŁS condition.

In order to prove our main result, we define and study the set of obstruction points to the ŁS condition. We also prove, in dimension $n \geqslant 1$, that for a polynomially convex and L-regular compact set of non empty interior, these obstruction points are rare, in a sense which will be specified.

Nevanlinna counting function and Carleson function of analytic self-maps on finitely connected domains

Michal Rzeczkowski

Jagiellonian University in Krakow, Poland

In the complex analysis there are many important concepts. In the talk we consider two of them, Carleson function and Nevanlinna counting function. Carleson measures (which we use to define Carleson function) have appeared in the famous characterization of Borel measures μ on the unit disc $\mathbb{D}$ for which the canonical inclusion map $j_\mu \colon H^p(\mathbb{D}) \to L^p(\mathbb{D}, \mu)$ is bounded. This result was obtained by Carleson and applied to solve the famous *corona problem.* As it turned out Carleson measures were a useful tool in the theory of spaces of analytic functions. In the eighties MacCluer obtained a characterization of compact composition operators on Hardy spaces using Carleson measures.

The Nevanlinna counting function occurred in the paper of Littlewood when he shows that each composition operator on H^2 is bounded. Over sixty years later Shapiro used it to compute the essential norm of composition operators on H^2.

Recently Lefèvre, Li, Queffélec and Rodríguez-Piazza characterized compact composition operators on Hardy–Orlicz spaces on the unit disc and obtained that Carleson function is equivalent to Nevanlinna counting function, that is, the following inequalities

$$1/K \ \rho_\varphi(h/K) \leqslant \sup_{|w| \geqslant 1-h} N_\varphi(w) \leqslant K \rho_\varphi(Kh)$$

are satisfied for some positive constant K and every analytic self-map φ of the unit disc and sufficiently small h.

Our aim is to present the analogous equivalence for Carleson and Nevanlinna functions generated by analytic self-maps $\varphi \colon \Omega \to \Omega$ where Ω is finitely connected domain and some applications of this result.

In order to prove our main result, we define and study the set of obstruction points to the ŁS condition. We also prove, in dimension $n \geqslant 1$, that for a polynomially convex and L-regular compact set of non empty interior, these obstruction points are rare, in a sense which will be specified.

On a two-phase flow driven by a change in the gap of the Hele-Shaw cell in the presence of sinks and sources

Tatiana Savina

Ohio University, USA

A Hele-Shaw cell is a pair of parallel plates separated by a small gap. The motion of the fluids sandwiched between the plates could be driven by a pressure gradient, gravity, fluid injection, and an external potential fields. This has been the subject of numerous investigations. The reason for the intense interest is the mathematical relation to modeling of several applied problems in material science and fluid dynamics, as well as to modeling of biological processes involving moving fronts of populations or tumors. These latter processes include cancer, biofilms, wound healing, granulomas, and atherosclerosis.

In this talk we consider two fluids with different viscosities in a Hele-Shaw cell. The evolution of the interface, separating the fluids, is driven by a uniform change in the gap width of the cell as well as by the presence of some special distributions of sinks and sources located in both the interior and exterior domains The effect of surface tension is neglected.

Using the Schwarz function approach, we give examples of exact solutions when the interface belongs to a certain family of algebraic curves.

The talk is based on joint work with Lanre Akinyemi and Avital Savin.

Sets of Minimal Logarithmic Capacity

Klaus Schiefermayr

University of Applied Sciences Upper Austria

Let f be analytic in a neighborhood of ∞, and consider the following problem: Find an extremal domain $D_0(f) \subseteq \overline{\mathbb{C}} := \mathbb{C} \cup \{\infty\}$ to which the function f can be extended in an analytic and single-valued manner. The domain $D_0(f)$ is extremal in that way that the complement $K_0(f) := \overline{\mathbb{C}} \setminus D_0(f)$ has minimal logarithmic capacity. Existence and uniqueness were proved by Stahl in the first and second part of [2], respectively, see also [3]. In this talk, we give a complete description of the set $K_0(f)$ in case when f is of the form

$$f(z) := \frac{1}{\sqrt{(z-a_1)(z-a_2)(z-a_3)(z-a_4)}}$$

with given points $a_1, a_2, a_3, a_4 \in \mathbb{C}$. It turns out that $K_0(f)$ consists either of two analytic Jordan arcs (possibly intersecting) or of three analytic Jordan arcs with one common endpoint. The Jordan arcs, the intersecting points, and the corresponding Green function for $D_0(f)$ are all given in terms of Jacobi's elliptic and theta functions. The results are strongly based on [1].

[1] F. Peherstorfer and K. Schiefermayr, *Description of inverse polynomial images which consist of two Jordan arcs with the help of Jacobi's elliptic functions*, Comput. Methods Funct. Theory **4** (2004), 355–390.

[2] H. Stahl, *Extremal domains associated with an analytic function. I, II*, Complex Variables Theory Appl. **4** (1985), 311–324, 325–338.

[3] H. Stahl, *Sets of extremal capacity and extremal domains*, http://arxiv.org/abs/1205.3811, 2012.

Level curve portraits of rational inner functions

Alan Sola

Stockholm University, Sweden

We analyze rational inner functions on the unit bidisk near singularities on the distinguished boundary via unimodular level sets. We show that these level sets admit smooth parametrizations, and deduce a number of analytic and algebraic results concerning boundary behavior of rational inner functions.

This reports on joint work with Kelly Bickel (Bucknell University, Lewisburg, PA) and James Pascoe (Washington University in St Louis, MO).

Some quantitative aspects of approximation of planar sets by polynomial Julia sets

MARGARET STAWISKA-FRIEDLAND

Mathematical Reviews, Ann Arbor, USA

We revisit approximation of nonempty compact planar sets by filled-in Julia sets of polynomials proposed recently by K. Lindsey and M. Younsi and show that using slightly modified fundamental Lagrange interpolation polynomials with certain nodes having subexponential growth of Lebesgue constants improves the approximation rate. To this end we investigate interpolation properties of some arrays of points in $\mathbb{C}$ and prove subexponential growth of Lebesgue constants for pseudo Leja sequences with bounded Edrei growth on finite unions of quasiconformal arcs. We also discuss relations with recent results in interpolation theory by V. Andrievskii, M. Ounaies, A. Irigoyen and others. Finally, for some classes of sets we estimate more precisely the rate of approximation by filled-in Julia sets in Hausdorff and Klimek metrics.

This talk is based on joint work with Leokadia Bialas-Ciez and Marta Kosek from Jagiellonian University, Krakow, Poland.

Universal Taylor series for non-simply connected domains

NIKOS TSIRIVAS

University of Crete, Greece

It is known that, for any simply connected proper subdomain Ω of the complex plane and any point j in Ω, there are holomorphic functions on Ω that have universal Taylor series expansions about j; that is, partial sums of the Taylor series approximate arbitrary polynomials on arbitrary compacta in $\mathbb{C} \setminus \Omega$ that have connected complement. This talk shows that this phenomenon can break down for non-simply connected domains Ω, even when $\mathbb{C} \setminus \Omega$ is compact. This answers a question of Melas and disproves a conjecture of Muller, Vlachou and Yaviran.

We note that this result is a very specific case of a result of Stephen Gardiner. Work joint with Stephen Gardiner.

Disjoint hypercyclicity for the Taylor shift Operator

VAGIA VLACHOU

University of Patras, Greece

The last decade the notion of disjoint hypercyclicity has been introduced and studied by several authors. Very recently H. Klaja and A. Mouze proved that the Taylor shift in $C^{\infty}(\mathbb{R})$ is doubly universal. We will present that the notion can be generalized for more than two sequences of operators. Finally we will talk about an interpolation problem. This work is part of the PhD thesis of N. Chatzigiannakidou.

Domains of existence for finely holomorphic functions

Jan Wiegerinck

University of Amsterdam, The Netherlands

Finely holomorphic functions are the natural generalisation of holomorphic functions in the setting of the fine topology. We recall definitions and properties of the fine topology and finely holomorphic functions and will study what remains of the well-known theorem of Weierstrass that every domain U in $\mathbf{C}$ is a domain of existence. Roughly speaking, this says that every domain admits a holomorphic function that can nowhere be extended beyond U.

We will discuss joint work with Alan Groot and Bent Fuglede, showing that fine domains in $\mathbb{C}$ with the property that they are Euclidean F_σ and G_δ, are in fact fine domains of existence for finely holomorphic functions. Moreover *regular* fine domains are also fine domains of existence. However, fine domains such as $\mathbb{C} \setminus \mathbb{Q}$ or $\mathbb{C} \setminus (\mathbb{Q} \times i\mathbb{Q})$, more specifically fine domains V with the property that their complement contains a non-empty polar set E that is of the first Baire category in its Euclidean closure K and that $(K \setminus E) \subset V$, are *not* fine domains of existence.
