Graph Theory Lesson Plan

1st Class Maths

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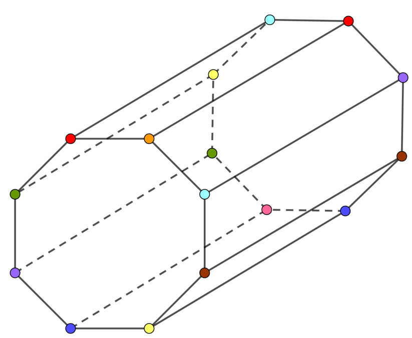


Abstract

We hope to use graph theory to build on students understanding of geometry through the lens of a computational framework. This lesson is an opportunity to expose children to the practicality of using computers to solve and display problems. By incorporating Scratch into the lesson we demonstrate how such software can be used to display geometrical objects and their properties (vertices, edges and regions). We also hope to build on students basic arithmetic (addition and subtraction) through the introduction of Euler's characteristic formula. We envision students coding on scratch and creating simple shapes by 4th class. Hence, it is very important that we introduce the software to students through practical examples in the preceding years.

1 Learning Outcomes

Building on the curriculum documents for primary school mathematics (NCCA, 1999), the *Shape and Space* strand states that students should be able to;

- Sort, describe, compare and name 2-D shapes including the square, rectangle, triangle, circle, semicircle
- Construct and draw 2-D shapes
- Recognise halves of 2-D shapes and different shapes in the environment
- Describe, compare and name 3-D shapes, including the cube, cuboid, cylinder and sphere
- Solve and complete practical tasks and problems involving 2-D and 3-D shapes
- Explore the relationship between 2-D and 3-D shapes
- identify line symmetry in shapes and in the environment
- Explore and recognise angles in the environment

2 Learning Intentions

Upon successful completion of this lesson, students will be able to...

- Identify and name different 2-D and 3-D shapes.
- Explain what is meant by a graph, side, region and vertex.
- Count the number of sides vertices and regions for different 2-D shapes.
- Design 2-D and 3-D shapes with different features.
- Analyze the properties of three dimensional shapes.
- Recall some facts about Leonard Euler, his work and the formula required to calculate the Euler characteristic.
- Calculate the Euler characteristic for different shapes.
- Recognise a planar graph and a graph that is not connected. Comment on the implications of this on the Euler Characteristic.

3 Lesson Rationale

3.1 Prior Knowledge

In junior and senior infants, students have engaged with the strand; Shape and Space. According to the curriculum documents (NCCA, 1999); students should be able to identify and sort different 2-D shapes. Students should also be familiar with determining if a shape is round and identifying how many corners different shapes have.

3.2 Resources Required

- Colouring Pencils, Pencils, Rulers
- Workbooks (See our provided workbook)
- Scratch and Geogebra

3.3 Common Student Misconceptions

- Forgetting to count the outside of a shape as a region.
- Counting vertices, edges and faces of 3-D shapes.
- Recognizing if a graph is planar.
- Identifying if a graph is connected or not.

3.4 How will learning be assessed?

- Teacher Questioning
- Listening to students conversations as they approach problems
- Workbook Exercises
- Scratch Exercises

Suggested Timing	**Learning Context (Task or activity as related to learning intentions)**	**Student activity (Anticipated responses or thinking**	**Teacher activity (prompts, questions, additional examples if necessary)**
5 - 10 mins	Revise some of the more basic 2-D shapes covered in Junior and Senior infants (rectangle, square, circle) and ask students to identify how many corners and edges each shape has. Introduce the vertex terminology.	Students should identify that a rectangle has four corners.	Share learning objectives with the students. Give out relevant materials (paper, colouring pencils, workbooks). Link the terminology of corners and vertices. How many vertices does a circle have?
10 mins	Introduction to Euler and his work. Definition of a graph and a region. Discuss the characteristic equation through some examples. Designing shapes and more difficult examples.	Students should enjoy the historic background and the facts about Euler. Students should solve problems with the formula. Some students may forget to consider the exterior as a region!	Go through some examples with the students. Give out the workbook. How many vertices does this shape have? Is this shape a 2-Dimensional shape or a 3-Dimensional shape? Why? What's the difference between and edge and a vertex?
10 mins	Introduce the concept of region and connected graphs. Define planar graph. Ask students to follow the rules to design a shape of their own and calculate the Euler characteristic. Applying the same knowledge to 3-D shapes. 3-D Exercises	Students should enjoy the scratch visuals. The software will hopefully clear up any confusion between the terms in Euler's characteristic formula. Students may find counting edges, faces and vertices more difficult for 3-D shapes.	What's the formula for the Euler Characteristic? What does each term in this formula mean? Cube Example How many faces does a cube have?
10 mins	Quiz on Scratch. Reflection on Learning	By recapping the main points of the lesson students should evaluate if they have met the learning objectives. Students can design and colour their workbook.	In three sentences summarize what we learned today.

4 The Life and Work of Leonhard Euler

Figure 1: Leanhard Euler 1707-1783

4.1 Some facts about Euler

Leonhard Euler was a Swiss mathematician. He was born in 1707 and died in 1783. He went to university at the early age of thirteen where Johann Bernouli quickly discovered his amazing talent for mathematics. Euler is widely renowned as one of the most influential mathematicians of all time. He worked in almost all areas of mathematics and contributed to geomerty, number theory calculus and lunar theory. He suffered many problem with his eyesight and at one stage went almost totally blind! However, this never stopped him from working on mathematics as he had an exceptional memory and could do almost all of the calculations in his head!

4.2 The Euler Characteristic in 2-D

A **Graph** is a collection of vertices joined by edges. This graph has 5 **V**ertices, and 8 **E**dges (sides). The graph splits the plane (page) into 5 **R**egions.

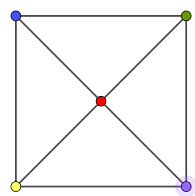


Figure 2: An Example of a Graph

In 2-Dimensions the Euler characteristic is defined as;

$$\chi = \mathbf{V} + \mathbf{R} - \mathbf{E} \tag{1}$$

Amazingly, Euler discovered that χ is always $= 2$ for planar connected graphs. Let us confirm that this works in the example provided.

$$\boxed{\text{V}} + \boxed{\text{R}} - \boxed{\text{E}} = \boxed{\chi}$$

$$\boxed{5} + \boxed{5} - \boxed{8} = \boxed{2}$$

4.3 The Euler Characteristic in Higher Dimensions

This theory can also be extended to higher dimensions. Let us now look at the example of a 3-Dimensional cube. We replace the number of regions in the 2-D formula with number of faces and define a new formula for the 3-D case.

$$\chi = \mathbf{V} + \mathbf{F} - \mathbf{E} \tag{2}$$

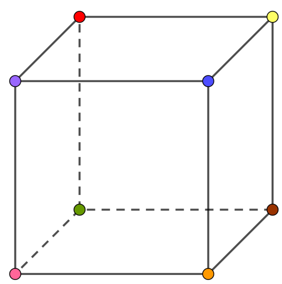


Figure 3: The Cube

This Cube has 8 **V**ertices, 12 **E**dges (sides) and 6 **F**aces. Subbing these values into Eulers formula;

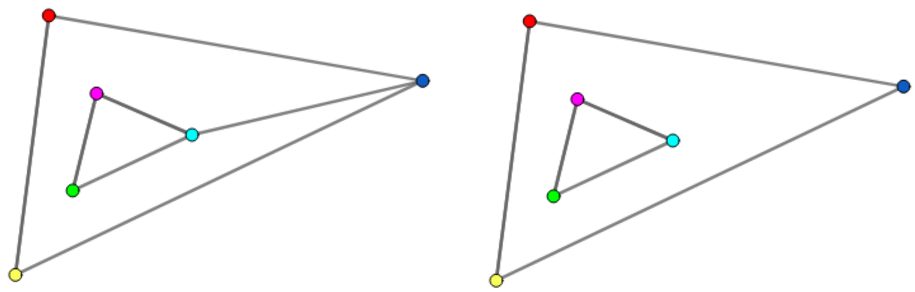
$$\boxed{\text{V}} + \boxed{\text{F}} - \boxed{\text{E}} = \boxed{\chi}$$

$$\boxed{8} + \boxed{6} - \boxed{12} = \boxed{2}$$

Again we see that the answer is 2. This begs the question, does the Euler characteristic formula always give us 2?

4.4 Does the Euler Characteristic always equal two?

In order for the Euler characteristic formula to return 2 we must have a planar connected graph. By considering the following two figures we can appreciate what it means for a graph to be connected.



(a) Connected Graph

(b) Unconnected Graph

Figure 4: A comparison of Connected and Unconnected graphs

Similarly, the graph must also be planar. This means that we must be able to draw the graph in a plane without any of the edges crossing (except at vertices).

(a) Planar Graph

(b) Non Planar Graph

Figure 5: A comparison of planar and non-planar graphs

An Example where $\chi \neq 2$

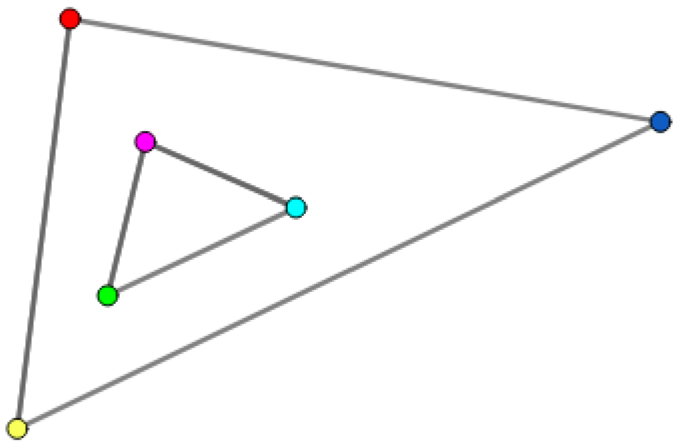


Figure 6: Unconnected Graph

$$\boxed{V} + \boxed{R} - \boxed{E} = \boxed{\chi}$$

$$\boxed{6} + \boxed{3} - \boxed{6} = \boxed{3}$$