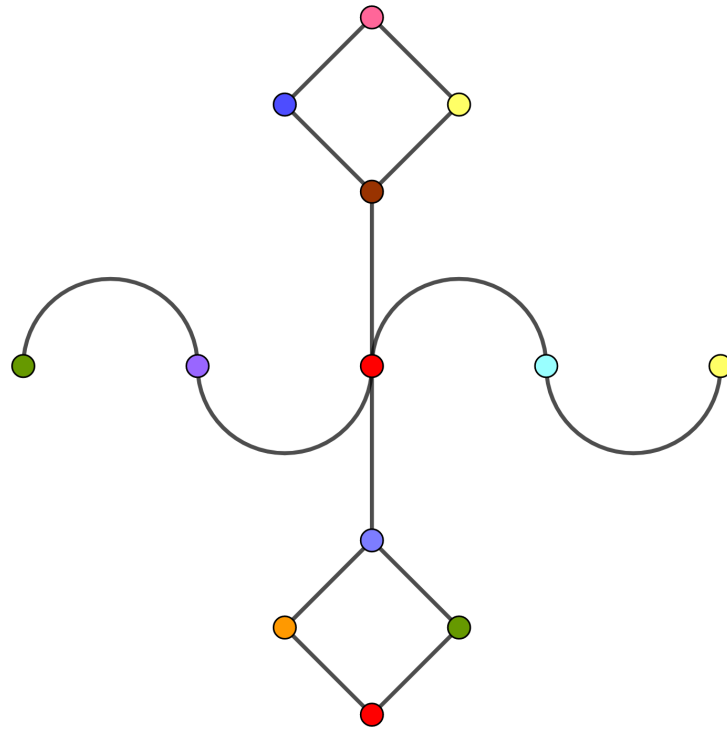
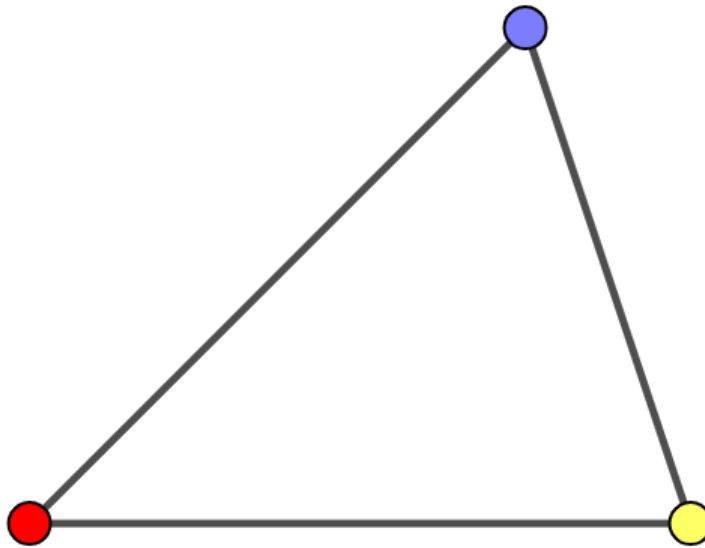


Graph Theory



A **graph** is a collection of **vertices**
connected by **edges**.

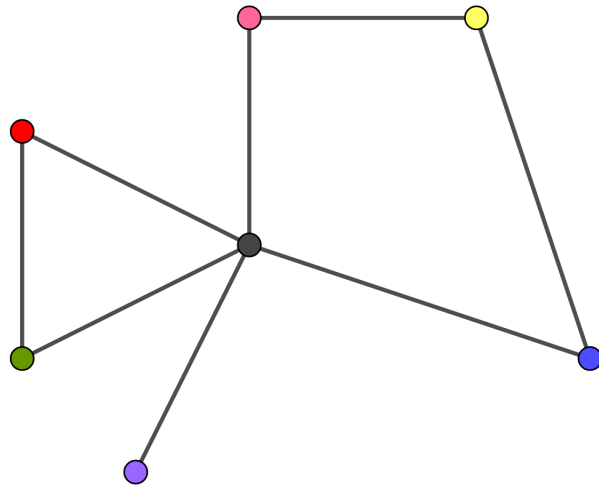


This graph has
3 Vertices
3 Edges
and it divides the page into
2 Regions

Leonhard Euler is a famous mathematician, who made an important discovery in graph theory. He discovered something interesting when he calculated:

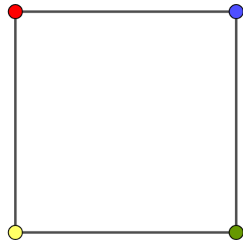
$$\begin{array}{ccccccc}
 \mathbf{V} & & + & & \mathbf{R} & & - & & \mathbf{E} \\
 \text{number of } \text{vertices} & & & & \text{number of } \text{regions} & & & & \text{number of } \text{edges}
 \end{array}$$

Let's calculate it for this graph:

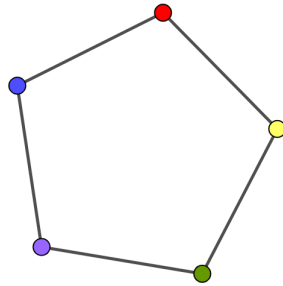


$$\begin{array}{ccccccc}
 \mathbf{V} & + & \mathbf{R} & - & \mathbf{E} & & \\
 \square & + & \square & - & \square & = & \square
 \end{array}$$

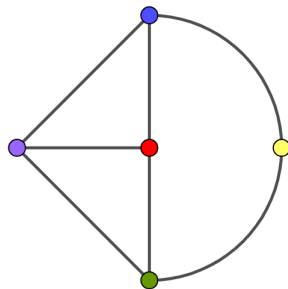
Now you try some!



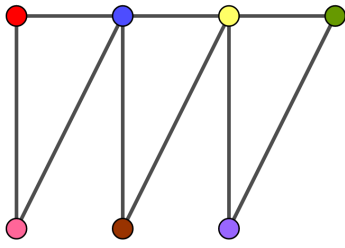
$$\begin{array}{ccccccc} \mathbf{V} & + & \mathbf{R} & - & \mathbf{E} & & \\ \square & + & \square & - & \square & = & \square \end{array}$$



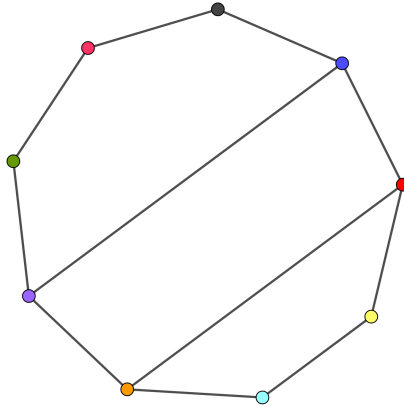
$$\begin{array}{ccccccc} \mathbf{V} & + & \mathbf{R} & - & \mathbf{E} & & \\ \square & + & \square & - & \square & = & \square \end{array}$$



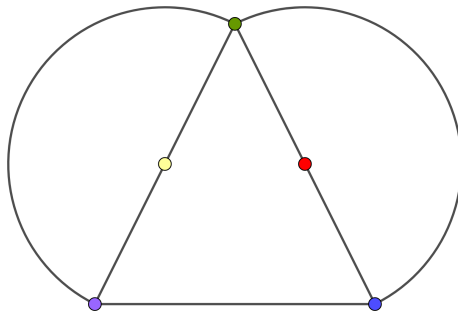
$$\begin{array}{ccccccc} \mathbf{V} & + & \mathbf{R} & - & \mathbf{E} & & \\ \square & + & \square & - & \square & = & \square \end{array}$$



$$\begin{array}{ccccccc}
 \mathbf{V} & + & \mathbf{R} & - & \mathbf{E} & & \\
 \square & + & \square & - & \square & = & \square
 \end{array}$$



$$\begin{array}{ccccccc}
 \mathbf{V} & + & \mathbf{R} & - & \mathbf{E} & & \\
 \square & + & \square & - & \square & = & \square
 \end{array}$$



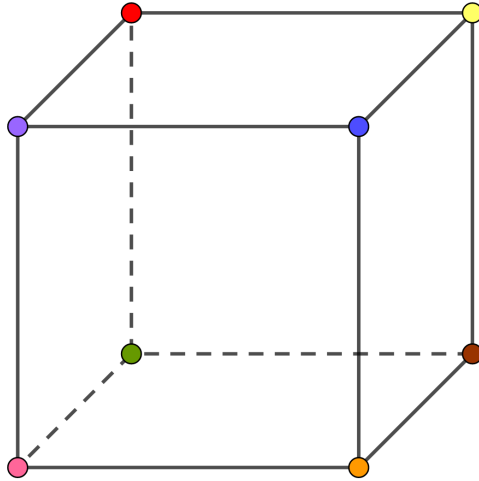
$$\begin{array}{ccccccc}
 \mathbf{V} & + & \mathbf{R} & - & \mathbf{E} & & \\
 \square & + & \square & - & \square & = & \square
 \end{array}$$

Design your own graph, and see if the pattern continues!

My graph:

$$\begin{array}{ccccccc} \mathbf{V} & + & \mathbf{R} & - & \mathbf{E} & = & \\ \square & + & \square & - & \square & = & \square \end{array}$$

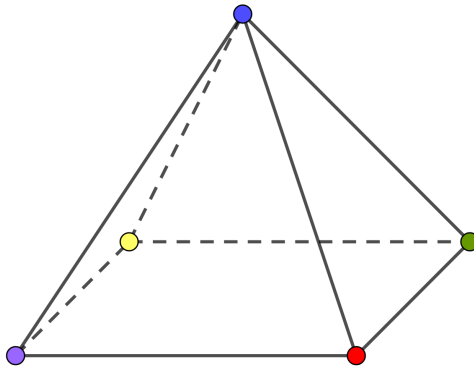
Let's try a 3D graph.
 For 3D graphs, instead of regions we use **faces**.



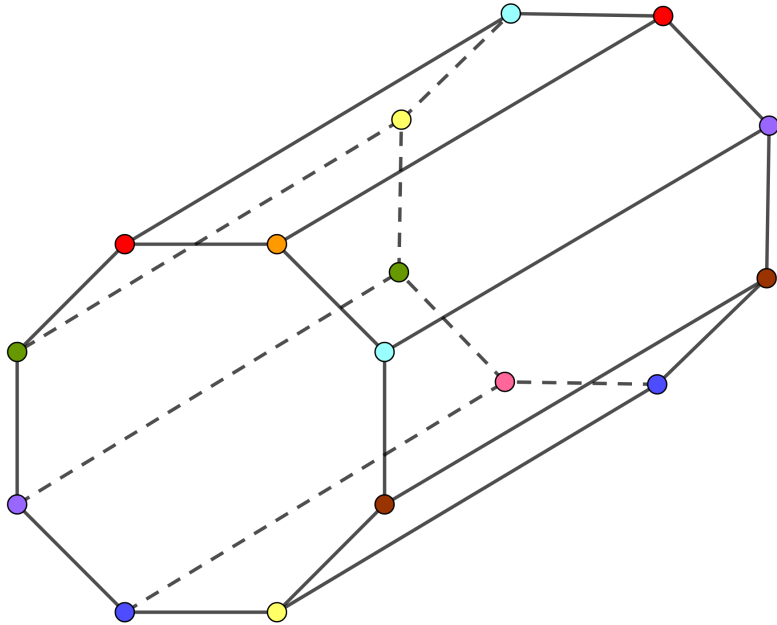
$$\begin{array}{ccccccc}
 \mathbf{V} & + & \mathbf{F} & - & \mathbf{E} & & \\
 \square & + & \square & - & \square & = & \square
 \end{array}$$

Does the pattern continue?

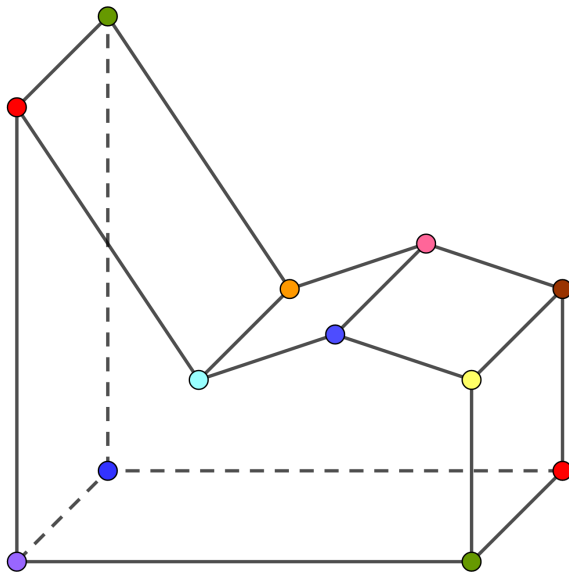
Try some more 3D graphs:



$$\begin{array}{ccccccc}
 \mathbf{V} & + & \mathbf{F} & - & \mathbf{E} & & \\
 \square & + & \square & - & \square & = & \square
 \end{array}$$



$$\begin{array}{ccccccc}
 \mathbf{V} & + & \mathbf{F} & - & \mathbf{E} & = & \\
 \square & + & \square & - & \square & = & \square
 \end{array}$$



$$\begin{array}{ccccccc}
 \mathbf{V} & + & \mathbf{F} & - & \mathbf{E} & = & \\
 \square & + & \square & - & \square & = & \square
 \end{array}$$