Geoboards & Pick's Theorem Lesson Plan

4th Class Maths

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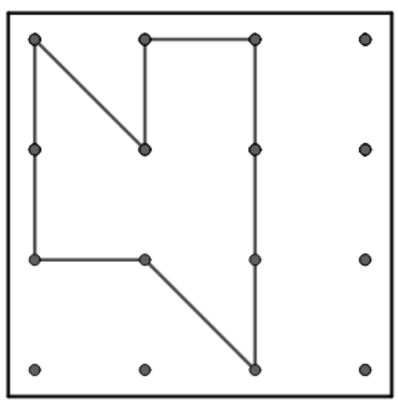


Abstract

We examine the structure and properties of 2D polygons using geoboards. Students can create and design different 2D shapes using these geoboards. Pick's theorem will be introduced. This theorem will enable students to calculate the area of different shapes on the geobards by using a formula that requires the application of division and other arithmetic techniques. Geogebra will be incorporated into the lesson where students can create their own shapes.

1 Learning Outcomes

According to the Curriculum documents (NCCA, 1999), 4th class students should be able to;

- Identify, describe and classify 2-D shapes: equilateral, isosceles and scalene triangle, parallelogram, rhombus, pentagon, octagon
- Explore, describe and compare the properties (sides, angles, parallel and non-parallel lines) of 2-D shapes
- Divide a one-digit or two-digit number by a one-digit number with and without remainders
- Solve and complete practical tasks and problems involving division of whole numbers
- Solve and complete practical tasks and problems involving 2-D shapes.

2 Learning Intentions

Upon successful completion of this lesson, students will be able to;

- Design shapes with different characteristics on a geoboard
- Incorporate knowledge of 2-D shape into their constructions (angles, edges, boundary point, interior point)
- Understand Pick's Theorem
- Apply this theorem to determine the area of different shapes on the geoboard
- Practice arithmetic techniques by subbing values into Pick's Theorem
- Create shapes that satisfy the criteria of Pick's theorem
- Design different 2-D shapes on Geogebra.

3 Lesson Rationale

3.1 Prior Knowledge

In previous years students have encountered the shape and space strand. According to the curriculum documents (NCCA, 1999) students should already be able to;

- Identify, describe and classify 2-D shapes: square, rectangle, triangle, hexagon, circle, semicircle, oval and irregular shapes
- Explore, describe and compare the properties (sides, angles, parallel and non-parallel lines) of 2-D shapes
- Construct and draw 2-D shapes use templates such stencils, geostrips, and geoboards
- Identify the use of 2-D shapes in the environment buildings, road signs, printing, household objects

3.2 Resources Required

- Geoboards
- Elastic Bands
- Geogebra / access to webpage
- Workbook

3.3 Common Student Misconceptions

- Counting boundary and interior points.
- Applying Pick's theorem to shapes that share a common vertex
- Splitting up shapes if we can't immediately apply Plick's Theorem
- Working with remainders and decimals in Pick's Theorem (The area will not always be a whole number).

3.4 How will learning be assessed?

- Geoboard Questions on PowerPoint
- Workbook exercises
- Listening to student approaches and conversations

4 Geoboards & Pick's Theorem

4.1 Terminology

Before commencing the lesson it is important to understand some key terms. Student friendly definitions that are complimented by visuals can be found in the lesson Powerpoint.

- A **geoboard** is a physical board with a number of nails or pins drilled into it that are normally a fixed distance apart. Elastic bands can be used to create shapes on a geoboard.
- **Lattices** are an arrangement of isolated points in space.
- Polygons whose vetrices lie on lattice points are known as **Lattice polygons**
- A **boundary point** is a lattice point that lies on an edge of a lattice polygon.
- An **interior point** is a point inside the shape that is surrounded by boundary points.

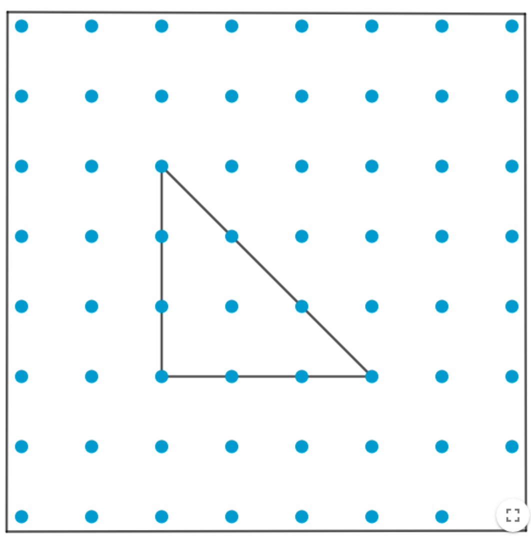


Figure 1: A Geoboard. The blue dots are known as lattice points and in this case the triangle formed is an example of a lattice polygon with 9 boundary points and 1 interior point

4.2 Pick's Theorem

Georg Alexander Pick was an Austrian born mathematician. His main field of study was complex analysis where Pick contributed to several results such as the Pick Matrix and Nevanlinna-Pick interpolation. Pick was a close aquaintance of Albert Einstein and introduced him to advanced differential calculus through the Italian mathematicians Gregorio Ricci-Curbastro and Tullio Levi-Civita. Pick was Jewish and as a result was persecuted by the Nazi's in WW2. He died in Theresienstadt concentration camp in 1942.

Pick's Theorem describes the area of a polygon whose vertices lie on lattice points. The theorem states that;

$$A = \frac{B}{2} + I - 1 \tag{1}$$

- A = Area of lattice polygon
- B = Number of Boundary Points
- I = Number of Interior Points

In order to understand Pick's theorem we will apply it to an example.

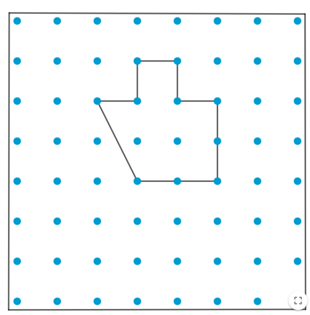


Figure 2: Example 1

The lattice polygon depicted in fig. 2, has 10 boundary points and two interior points.

- B = 10
- I = 2

Subbing these values into Pick's Theorem;

A = $\frac{10}{2}$ + 2 - 1

A = 6 square units

4.3 Comparing Pick's Theorem with known area formulae

In order to test the robustness of Pick's theorem we should test that the results agree with other theorems that we have to calculate the area of 2-D polygons. The real power of Pick's theorem lies in the ability to determine the area of irregular polygons.

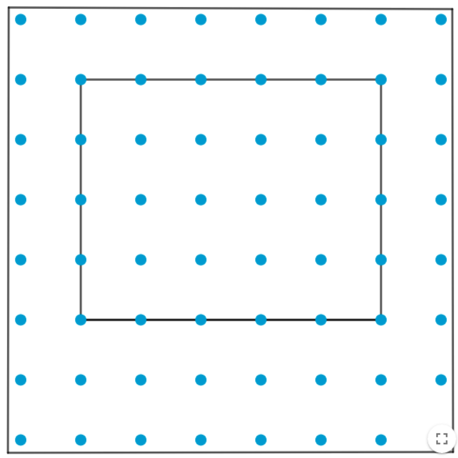


Figure 3: Rectangle

To determine the area of this rectangle we can use the trivial equation Area = Length x Width. This reveals that the area of the rectangle = 4 x 5 = **20 square units**.

Now let's apply Pick's Theorem.

- B = 18
- I = 12

By Pick's theorem, A = $\frac{18}{2} + 12 - 1 =$ **20 square units.**

Let's see if this also works for the area of a triangle. The area of a right angled triangle is often expressed as half the base length multiplied by the perpendicular height. Or $A = \frac{1}{2} \times b \times h$.

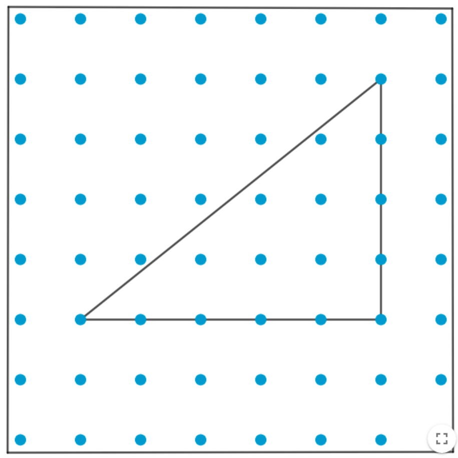


Figure 4: Triangle

This means that the area of the triangle shown above $= \frac{1}{2} \times 5 \times 4 =$ **10 square units**

Now, by Pick's Theorem;

- B = 10
- I = 6

Subbing these values into equation 1, $A = \frac{10}{2} + 6 - 1 =$ **10 square units**

4.4 Extended Examples and Rules

Pick's Theorem does not always work and it is important to note some rules. In order to apply Pick's theorem we must have a shape that shares no vertices with other polygons. As you can see in the figure below, two triangles share the same vertex.

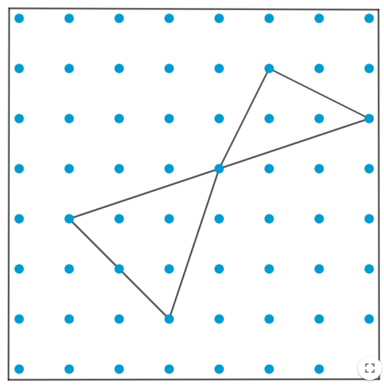
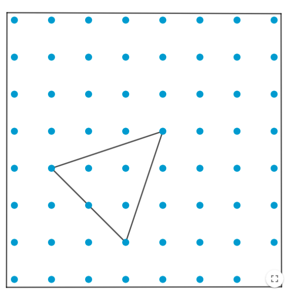
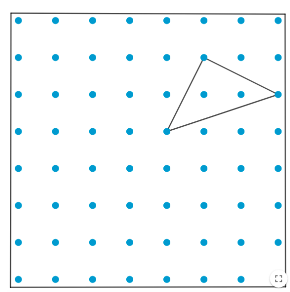


Figure 5: A shape that violates Pick's Theorem

In order to solve this problem we can separate the triangles and consider them independently using Pick's Theorem.



(a) Triangle 1



(b) Triangle 2

Figure 6: Splitting up the triangles

Applying Pick's Theorem to Triangle 1 we find that $A = \frac{4}{2} + 3 - 1 = 4$ square units.

Similarly, applying Pick's theorem to Triangle 2 we find $A = \frac{3}{2} + 2 - 1 = 2.5$ square units.

So the total area of the polygon in fig. 3 is **6.5 square units**.

5 Lesson Flow

Timing	Learning Activity	Notes for Teacher
10 - 15 mins	Introduce Geoboards Geoboard terms Introduce boundary points and interior points through examples	Show students different shapes that we can make on geoboards Link to prior knowledge of shapes... "Is this a hexagon or octagon?" Different shapes for students to count boundary and interior points
10 - 15 mins	Introduce Georg Alexander Plick Plick's Theorem The different terms in Plick's Theorem and what they mean simple Examples	Calculating areas of different shapes using Plick's theorem Comparing these results with counting boxes Areas of irregular Polygons Ensure students are not struggling with applying division
10 -15 mins	More complex examples Examples where we need intermediate steps before applying the Theorem Designing different shapes on a geoboard and calculating the area Reflect on learning	"Can we split this shape up into two simpler shapes?" Design a shape with 8 boundary points and two interior points What did we learn today