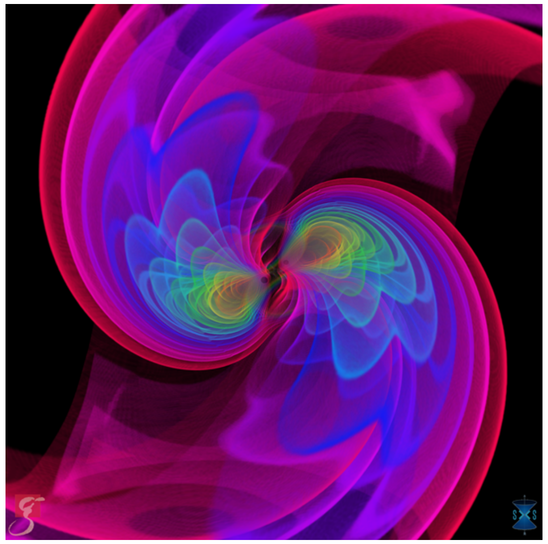  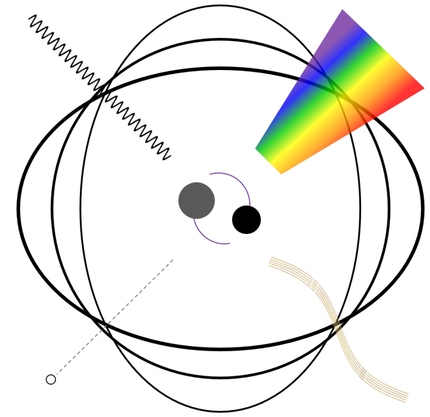 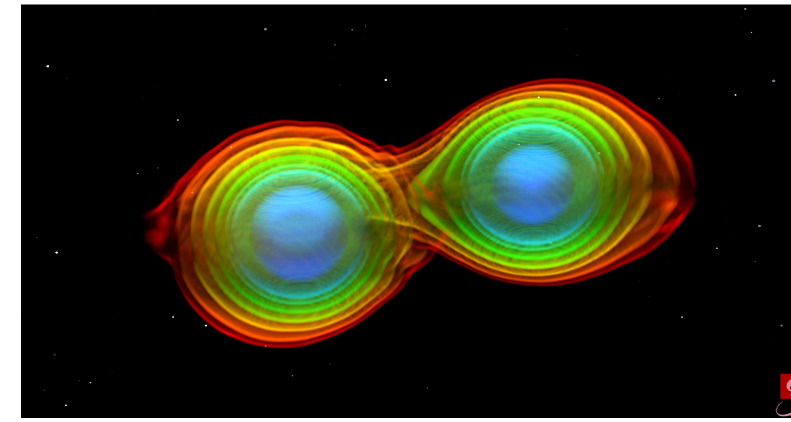

Effective-One-Body Theory & Applications to Gravitational-Wave Observations

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"Dublin School on Gravitational-Wave School Modeling", Dublin **June 11-12, 2018**

Outline

- **Lectures I-II:** Basics of gravitational waves

- **Lectures III-IV:** Motivations and development of effective-one-body (EOB) theory (two-body dynamics and waveforms)

- **Lecture V:** Using waveform models to infer astrophysical and cosmological information through gravitational-wave observations

- **Lecture VI:** Using waveform models to probe dynamical gravity and extreme matter with gravitational-wave observations

(NR simulation: Ossokine, AB & SXS @AEI)

(visualization credit: Benger @ Airborne Hydro Mapping Software & Haas @AEI)

References:

- **M. Maggiore's books**: "Gravitational Waves Volume I: Theory and Experiments" (2007) & "Gravitational Waves Volume II: Astrophysics and Cosmology" (2018).
- **E. Poisson & C. Will's book**: "Gravity" (2015).
- **E.E. Flanagan & S.A. Hughes' review**: arXiv:0501041.
- **AB's Les Houches School Proceedings**: arXiv:0709.4682.
- **AB & B. Sathyaprakash's review**: arXiv:1410.7832.
- **UMD/AEI graduate course on GW Physics & Astrophysics** taught in Winter-Spring 2017: http://www.aei.mpg.de/2000472.

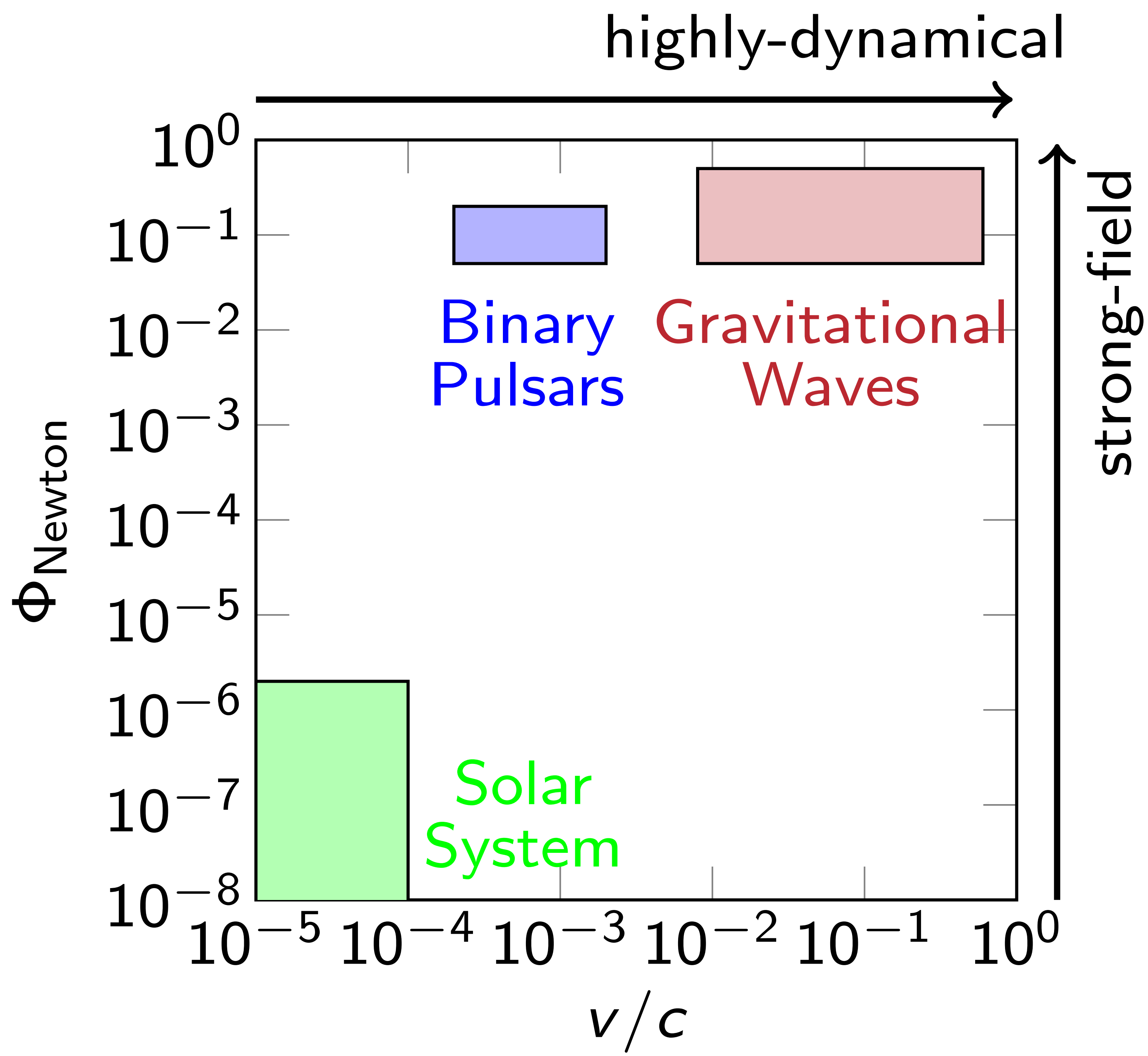
Extreme gravity, dynamical spacetime: tests of General Relativity

Solar system: $\frac{v}{c} \sim 10^{-5} - 10^{-4}$

Binary pulsars: $\frac{v}{c} \sim 10^{-3}$

LIGO/Virgo: $\frac{v}{c} \geq 0.1$

- Given **current tight constraints** on GR (e.g., Solar system, binary pulsars), can **any GR deviation be observed with GW detectors**?



(credit: Sennett)

PN templates in stationary phase approximation: TaylorF2

$$\tilde{h}(f) = \mathcal{A}_{\rm SPA}(f)\, e^{i\psi_{\rm SPA}(f)}$$

0PN

graviton with non zero mass

1PN

-1PN

dipole radiation

$$\begin{aligned}\psi_{\rm SPA}(f) \;=\;& 2\pi f t_c - \Phi_c - \pi/4 + \frac{3}{128}(\pi\mathcal{M}f)^{-5/3}\,\{1+ \\ & -\frac{5\hat{\alpha}^2}{336\omega_{\rm BD}}\nu^{2/5}(\pi\mathcal{M}f)^{-2/3} - \frac{128}{3}\frac{\pi^2 D\,\mathcal{M}}{\lambda_g^2(1+z)}(\pi\mathcal{M}f)^{2/3} \\ & + \left(\frac{3715}{756}+\frac{55}{9}\nu\right)\nu^{-2/5}(\pi\mathcal{M}f)^{2/3} - 16\pi\,\nu^{-3/5}(\pi\mathcal{M}f) + 4\beta\,\nu^{-3/5}(\pi\mathcal{M}f) \\ & + \left(\frac{15293365}{508032}+\frac{27145}{504}\nu+\frac{3085}{72}\nu^2\right)\nu^{-4/5}(\pi\mathcal{M}f)^{4/3} - 10\sigma\,\nu^{-4/5}(\pi\mathcal{M}f)^{4/3}\}\end{aligned}$$

spin-orbit

1PN 1.5PN 1.5PN

2PN 2PN

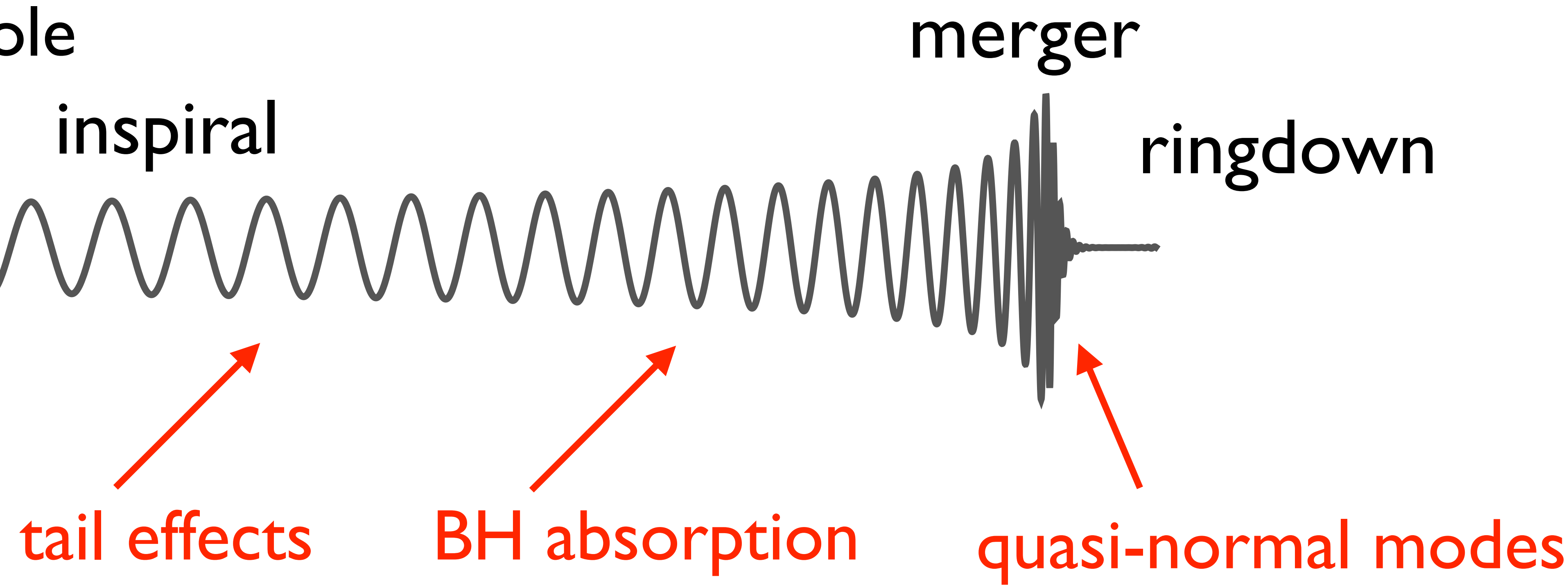
spin-spin

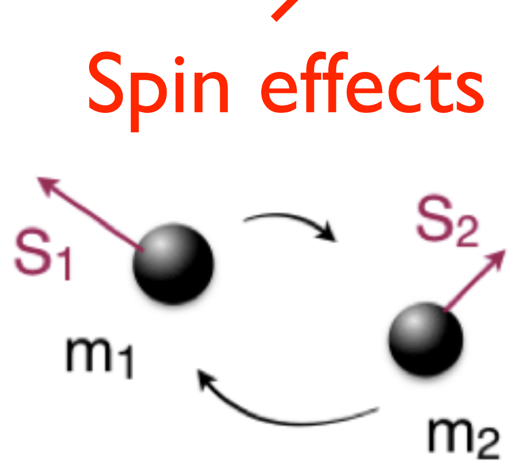
$$\beta = \frac{1}{12}\sum_{i=1}^{2}\chi_i\left[113\frac{m_i^2}{M^2}+75\nu\right]\hat{\boldsymbol{L}}\cdot\hat{\boldsymbol{S}}_i\,, \qquad \sigma = \frac{\nu}{48}\chi_1\chi_2\left(-27\hat{\boldsymbol{S}}_1\cdot\hat{\boldsymbol{S}}_2 + 721\hat{\boldsymbol{L}}\cdot\hat{\boldsymbol{S}}_1\hat{\boldsymbol{L}}\cdot\hat{\boldsymbol{S}}_2\right)$$

$$\chi_i = \frac{S_i}{m_i^2}$$

Waveforms encode plethora of physical effects

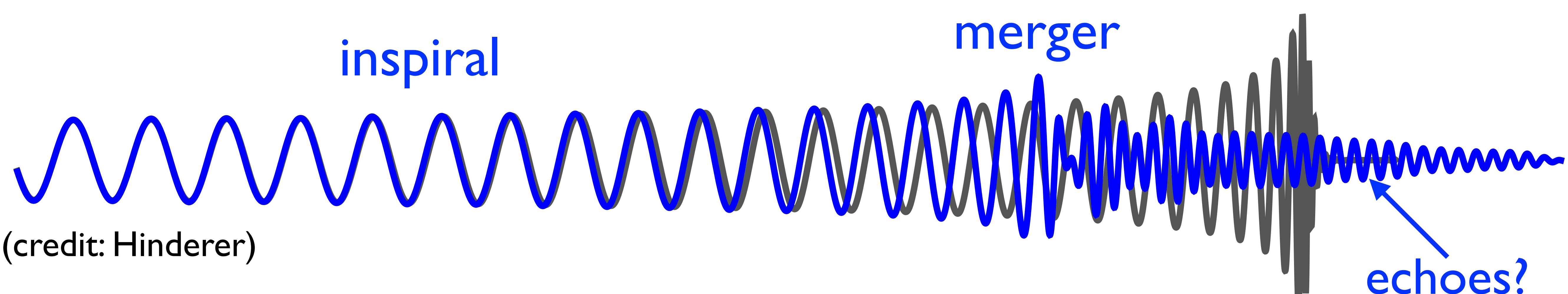
- Binary black hole





(credit: Hinderer)

- **Compact-object binary** with **matter** or in **modified theory** to GR?



(credit: Hinderer)

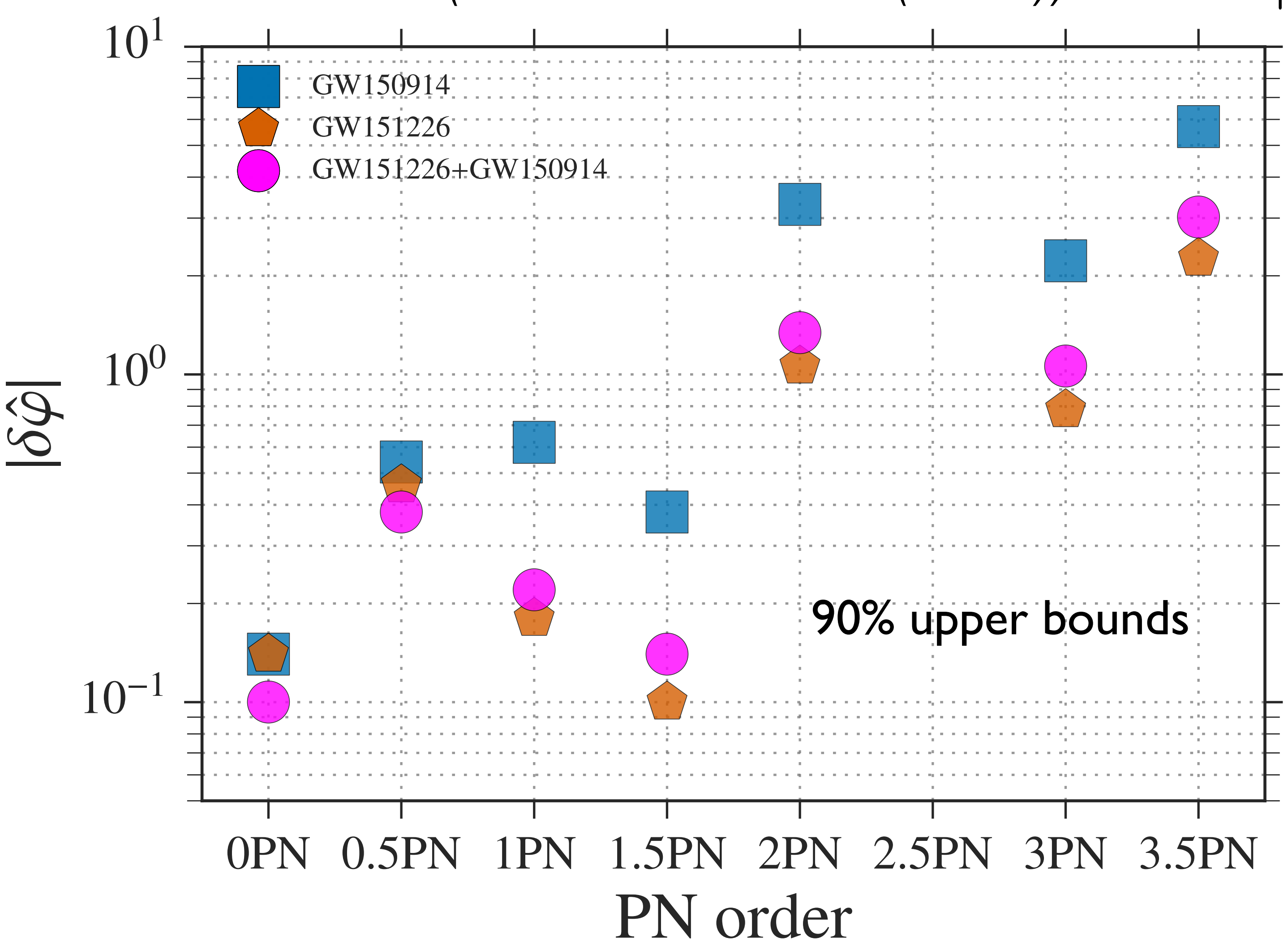
Bounding PN parameters: inspiral

- GW150914/GW122615's **rapidly varying orbital periods** allow us to **bound higher-order PN coefficients** in gravitational phase.

$$\tilde{h}(f) = \mathcal{A}(f) e^{i\varphi(f)}$$

$$\varphi(f) = \varphi_{\rm ref} + 2\pi f t_{\rm ref} + \varphi_{\rm Newt}(Mf)^{-5/3} + \varphi_{0.5\rm PN}(Mf)^{-4/3} + \varphi_{1\rm PN}(Mf)^{-3/3} + \varphi_{1.5\rm PN}(Mf)^{-2/3} + \cdots$$

(Abbott et al. PRX6 (2016))



(Arun et al. 06 , Mishra et al. 10, Yunes & Pretorius 09, Li et al. 12)

- **PN parameters** describe: **tails** of radiation due to backscattering, **spin-orbit** and **spin-spin** couplings.

- **PN parameters** take **different values** in **modified theories** to GR.

First tests of General Relativity in dynamical, strong field

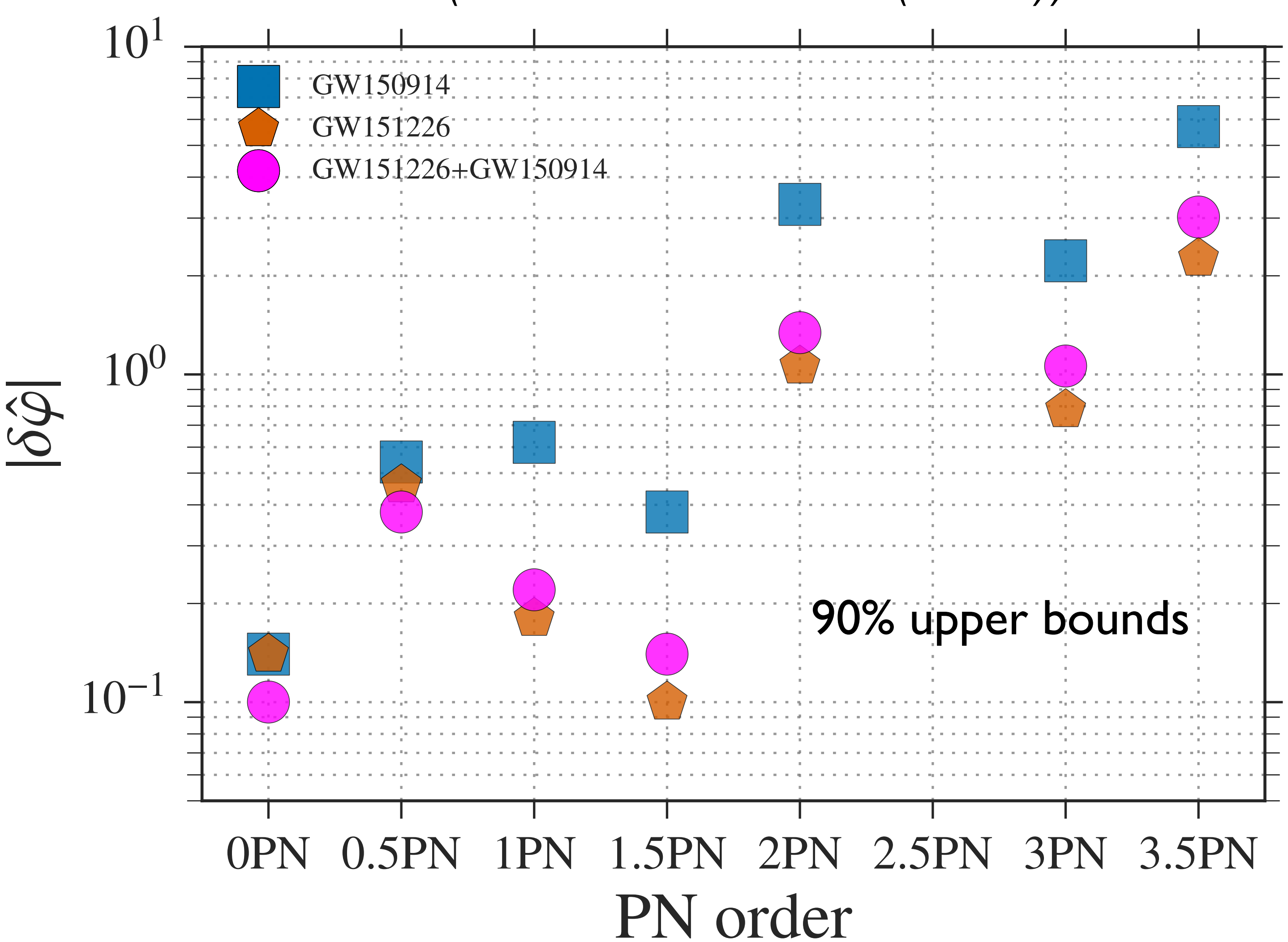
- GW150914/GW122615's **rapidly varying orbital periods** allow us to **bound higher-order PN coefficients** in gravitational phase.

$$\tilde{h}(f) = \mathcal{A}(f) e^{i\varphi(f)}$$

$$v = (2Mf)^{1/3}$$

$$\varphi(f) = \varphi_{\rm ref} + 2\pi f t_{\rm ref} + \tilde{\varphi}_{\rm Newt} v^{-5} \left[1 + \tilde{\varphi}_{0.5{\rm PN}}\, v + \tilde{\varphi}_{1{\rm PN}}\, v^2 + \tilde{\varphi}_{1.5{\rm PN}}\, v^3 + \cdots \right]$$

(Abbott et al. PRX6 (2016))



(Arun et al. 06 , Mishra et al. 10, Yunes & Pretorius 09, Li et al. 12)

- **PN parameters** describe: **tails** of radiation due to backscattering, **spin-orbit** and **spin-spin** couplings.

- **PN parameters** take **different values** in **modified theories** to GR.

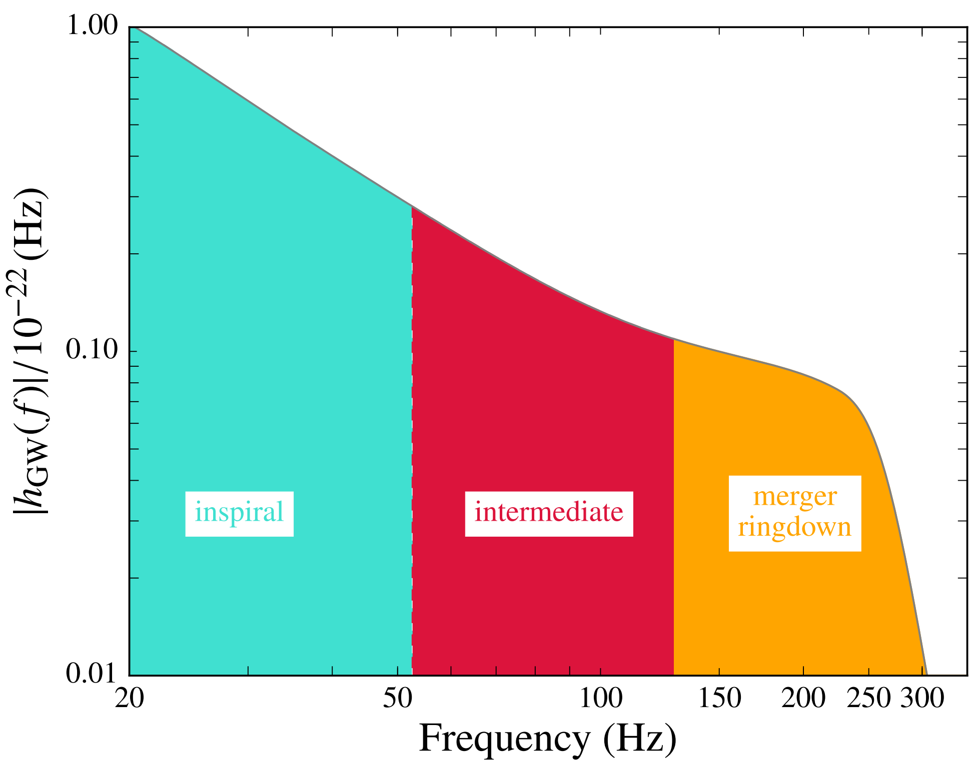
Some modified theories to General Relativity

Theory	$\alpha_{\rm ppE}$	$a_{\rm ppE}$	$\beta_{\rm ppE}$	$b_{\rm ppE}$
Jordan–Fierz–Brans–Dicke	$-\frac{5}{96}\frac{S^2}{\omega_{\rm BD}}\eta^{2/5}$	-2	$-\frac{5}{3584}\frac{S^2}{\omega_{\rm BD}}\eta^{2/5}$	-7
Dissipative Einstein-Dilaton-Gauss–Bonnet gravity	0	·	$-\frac{5}{7168}\zeta_3\eta^{-18/5}\delta_m^2$	-7
Massive Graviton	0	·	$-\frac{\pi^2 D\mathcal{M}_c}{\lambda_g^2(1+z)}$	-3
Lorentz Violation	0	·	$-\frac{\pi^{2-\gamma_{\rm LV}}}{(1-\gamma_{\rm LV})}\frac{D_{\gamma_{\rm LV}}}{\lambda_{\rm LV}^{2-\gamma_{\rm LV}}}\frac{\mathcal{M}_c^{1-\gamma_{\rm LV}}}{(1+z)^{1-\gamma_{\rm LV}}}$	$-3\gamma_{\rm LV}-3$
$G(t)$ Theory	$-\frac{5}{512}\dot{G}\mathcal{M}_c$	-8	$-\frac{25}{65536}\dot{G}_c\mathcal{M}_c$	-13
Extra Dimensions	·	·	$-\frac{75}{2554344}\frac{dM}{dt}\eta^{-4}(3-26\eta+24\eta^2)$	-13
Non-Dynamical Chern–Simons Gravity	$\alpha_{\rm PV}$	3	$\beta_{\rm PV}$	6
Dynamical Chern–Simons Gravity	0	·	$\beta_{\rm dCS}$	-1

(Yunes & Siemens 2013)

Bounding phenom parameters: intermediate/merger-RD

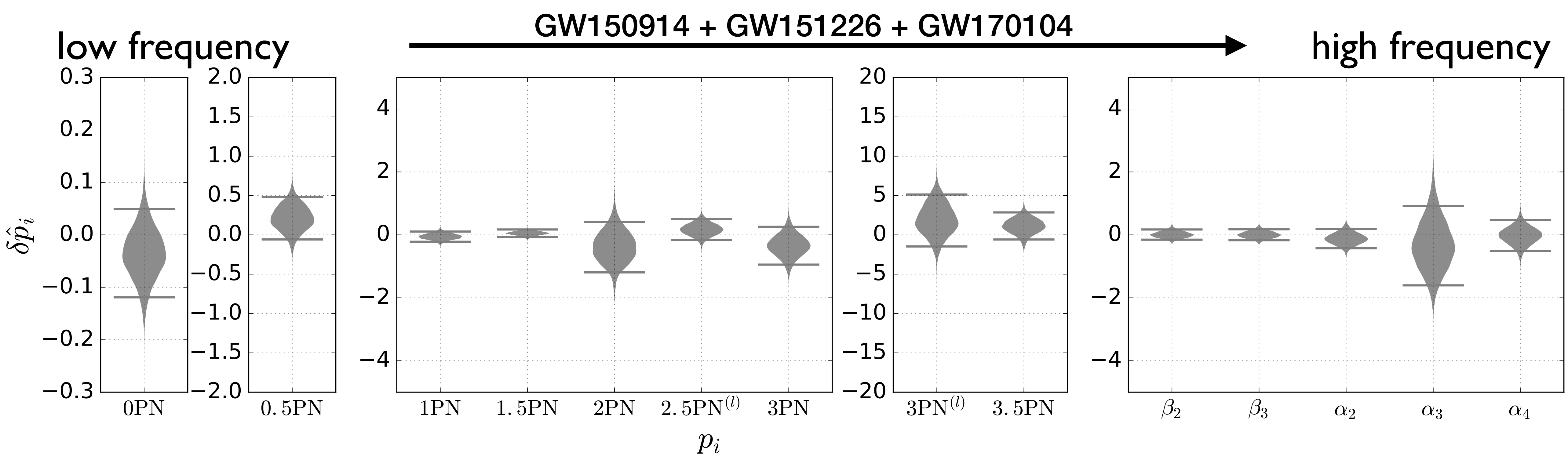
(Abbott et al. PRL 116 (2016) 221101)



$$
\begin{aligned}
\varphi(f) =& \varphi_{\rm ref} + 2\pi f t_{\rm ref} + \varphi_{\rm Newt}(Mf)^{-5/3} \\
& + \varphi_{0.5\rm PN}(Mf)^{-4/3} + \varphi_{1\rm PN}(Mf)^{-1} \\
& + \varphi_{1.5\rm PN}(Mf)^{-2/3} + \cdots + \beta_2 \log(Mf) \\
& + \cdots + \alpha_4 \tan^{-1}(aMf + b)
\end{aligned}
$$

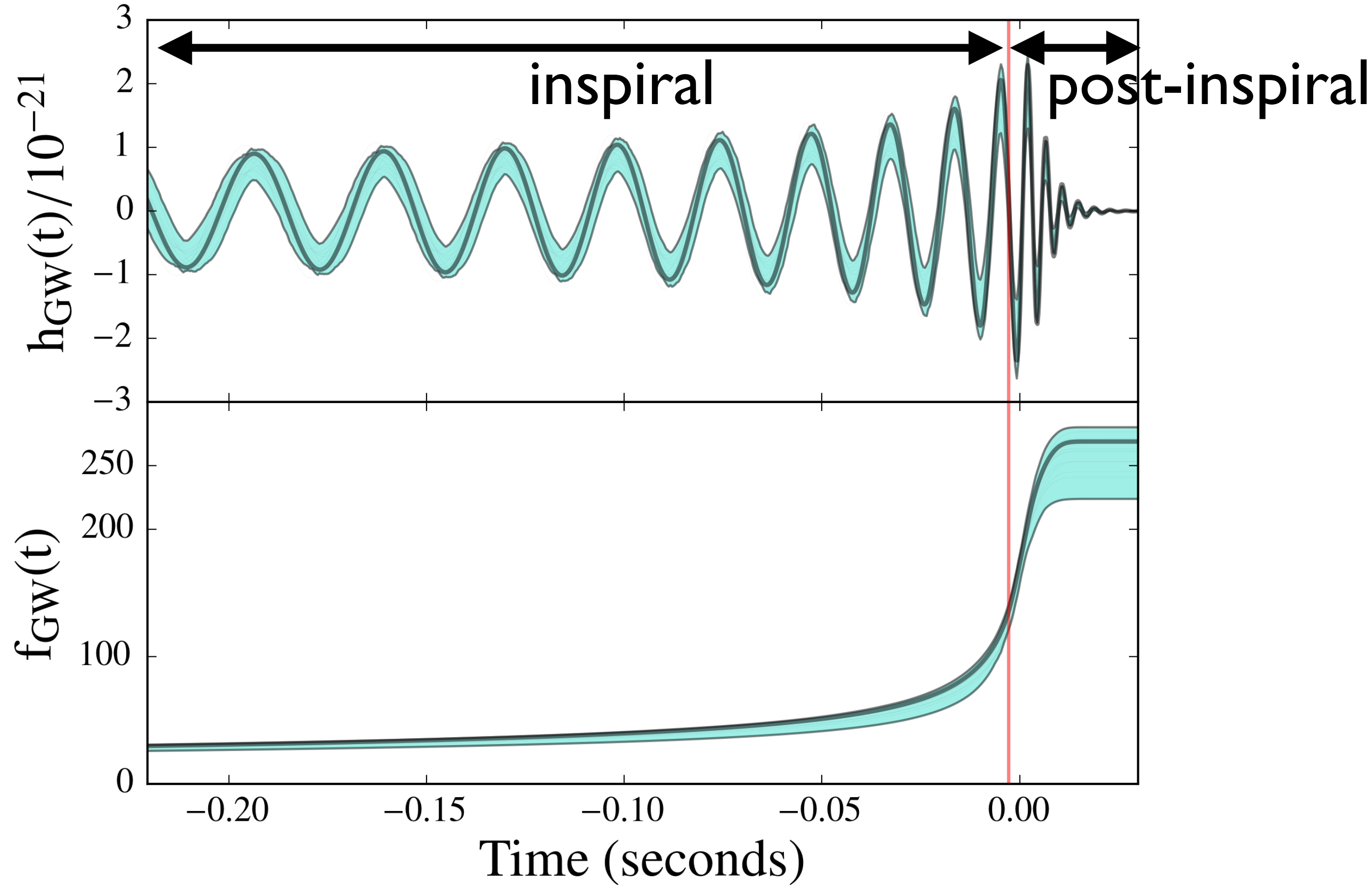
- Merger-ringdown **phenomenological parameters** (β_i and α_i) not yet **expressed in terms** of relevant **parameters in GR** and **modified theories of GR.**

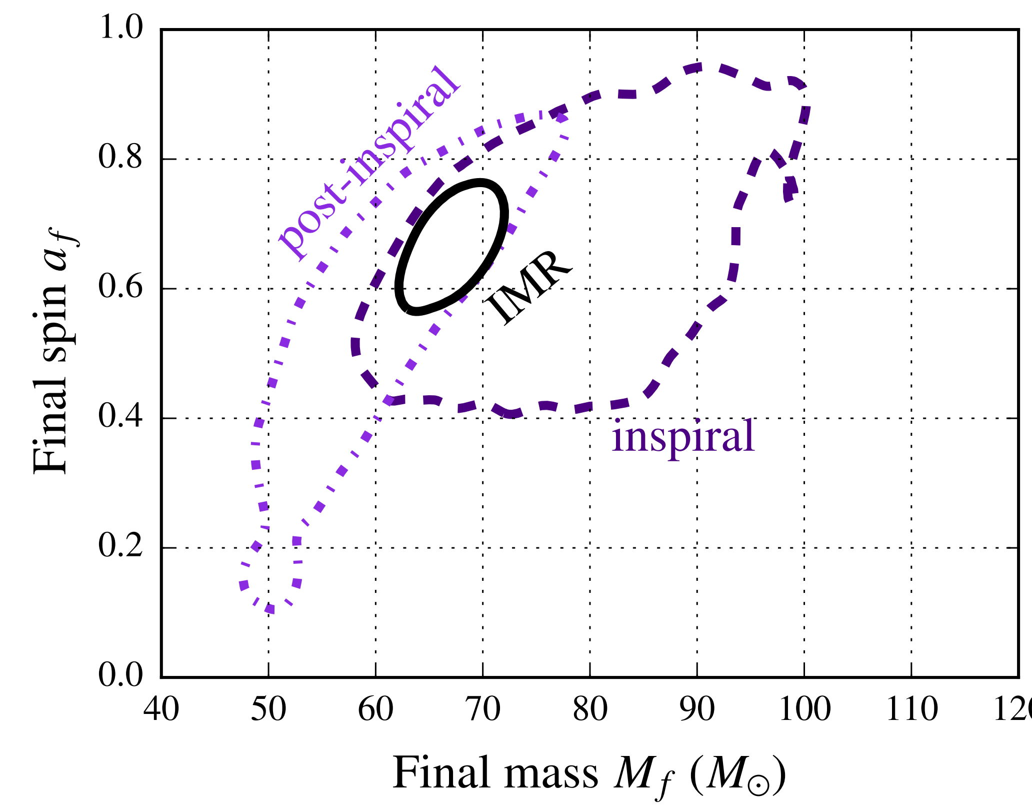
(Abbott et al. PRL 118 (2017) 221101)

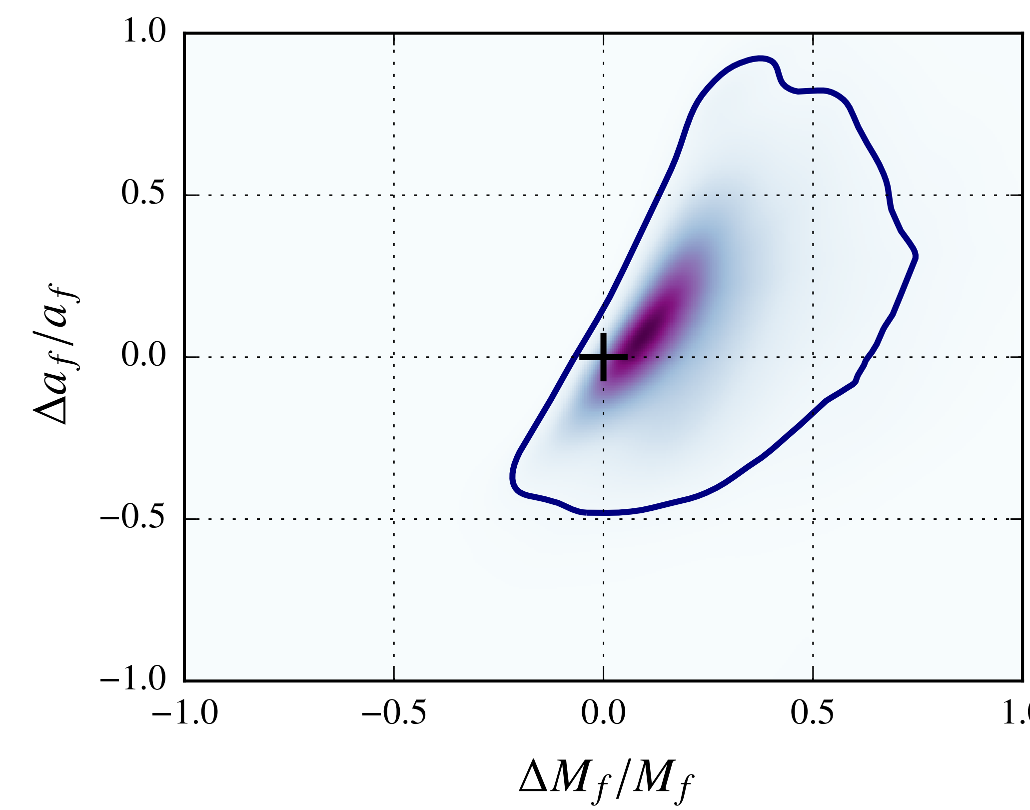


Inspiral-merger-ringdown consistency test: GW150914

(Abbott et al. PRL 116 (2016) 221101)





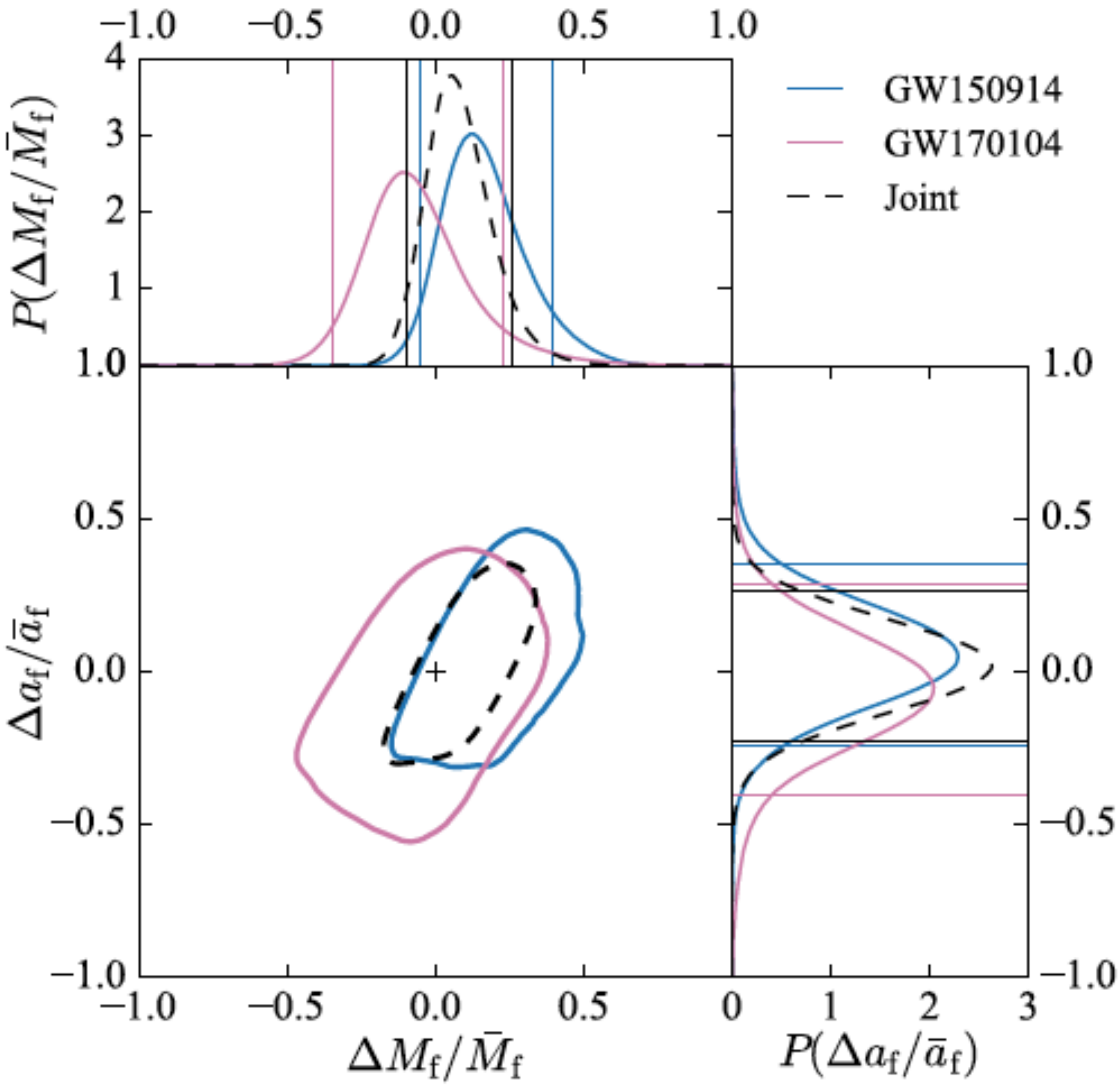


- **Using NR formulae** and **posterior distributions for full signal**, final black-hole's mass and spin are related to those of binary from which it formed.

- **Remnant's mass** and **spin** determined **from inspiral agree** with those from **post inspiral.**

Inspiral-merger-ringdown consistency test: GW150914 & GW170104

(Abbott et al. PRL118 (2017) 221101)



How to test GR and probe nature of compact objects: building deviations from GR & BHs/NSs

- Do **current** GR waveform models include **all physical effects**? Not yet.

- Will GR deviations be fully captured in **perturbative-like descriptions** during **merger-ringdown** stage? Likely not. *(e.g., Yunes & Pretorius 09, Li et al. 12, Endlich et al. 17)*

- Need **NRAR waveforms** in **modified theories of GR**: scalar-tensor theories, Einstein-Aether theory, dynamical Chern-Simons, Einstein-dilaton Gauss-Bonnet theory, massive gravity theories, etc. *(e.g., Stein et al. 17, Cayuso et al.17, Hirschmann et al. 17)*

- Need NRAR waveforms of binaries composed of **exotic objects** (BH & NS mimickers), such as boson stars, gravastar, etc. *(e.g., Palenzuela et al. 17)*

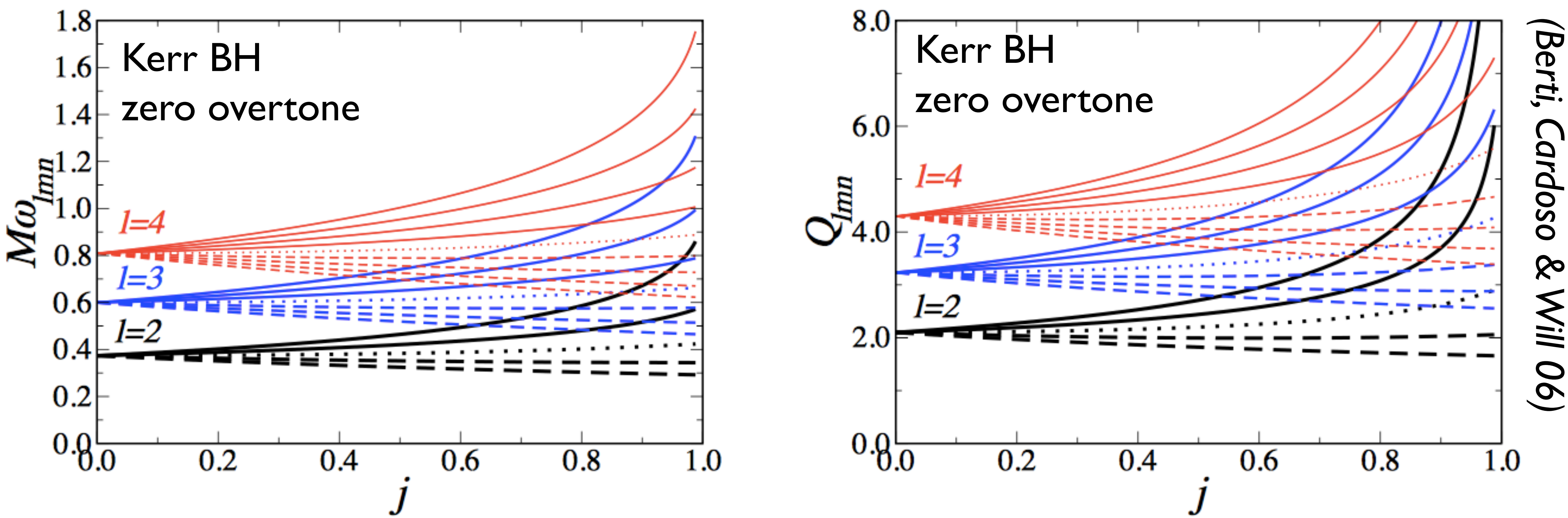
- Including **deviations from GR** in **EOB formalism.**

  (Julie & Deruelle 17, Julie 17, Khalil et al. in prep 18)

Probing nature of remnant through quasi-normal modes

- **One frequency** and **damping time** (or quality factor) of ringdown signal **cannot determine** values of **(l,m,n)** corresponding to **mode detected**, because there are **several values of** parameters **(M, j, l, m, n)** that **yield same frequencies** and **damping times**.

$$Q_{\ell mn} = \omega_{\ell mn}\, \tau_{\ell mn}/2$$



- **Multipole frequencies** and **decay times** will be **smoking gun** that **Nature's black holes** are black holes of **GR. We can** test **no-hair conjecture** and **second-law black-hole mechanics.** *(Israel 69, Carter 71; Hawking 71, Bardeen 73)*

- Note that those conjectures refer to **isolated, stationary black holes**, not to **dynamical back holes** (in a binary, merging).

Black-hole spectroscopy using damped sinusoids

- **BH spectroscopy:** measuring multipole QNMs.

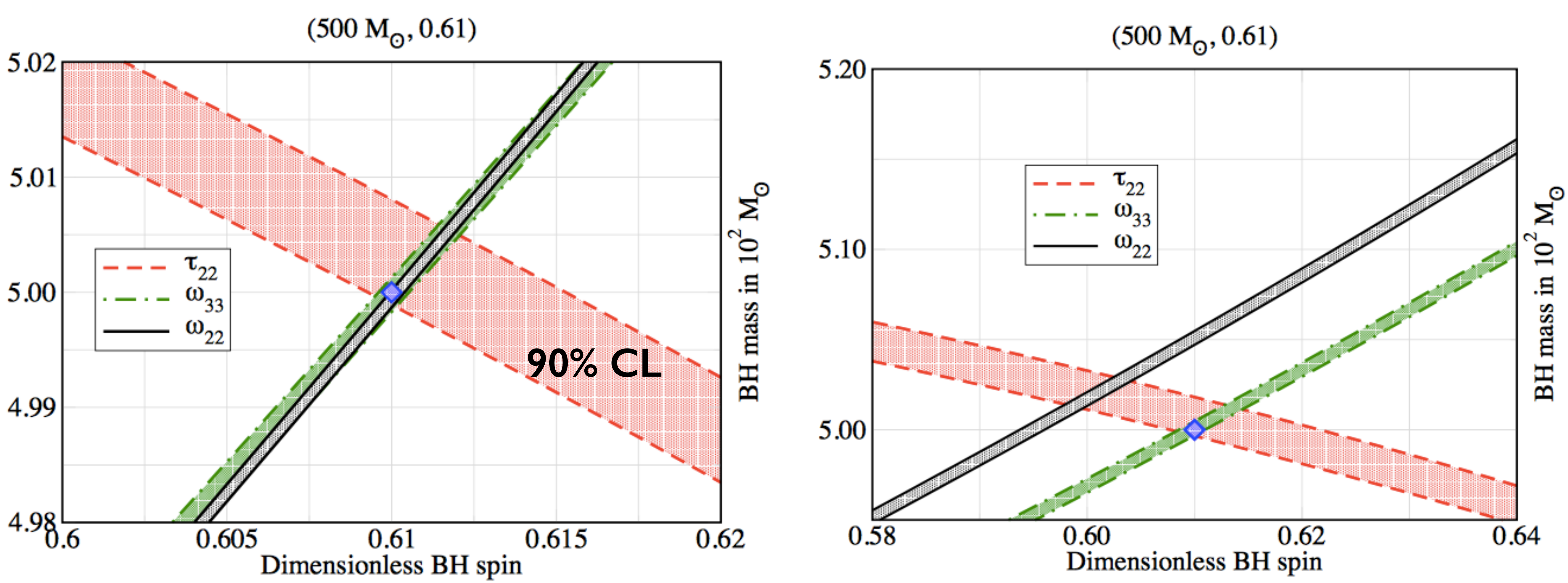
  (Dreyer et al. 2004, Berti et al. 2006, Gossan et al. 2012, Meidam et al. 2014, Bhagwat et al. 2017, Yang et al. 2017, Brito, AB, Raymond 18, Carullo et al. 18)

- **Plausible assumptions:** likely we detect zero overtone & $\ell = 2, 3.$ *(Gossan et al. 2012)*

$$\omega_{\ell mn} = \omega_{\ell mn}^{\rm GR}\,(1 + \delta\omega_{\ell mn})$$

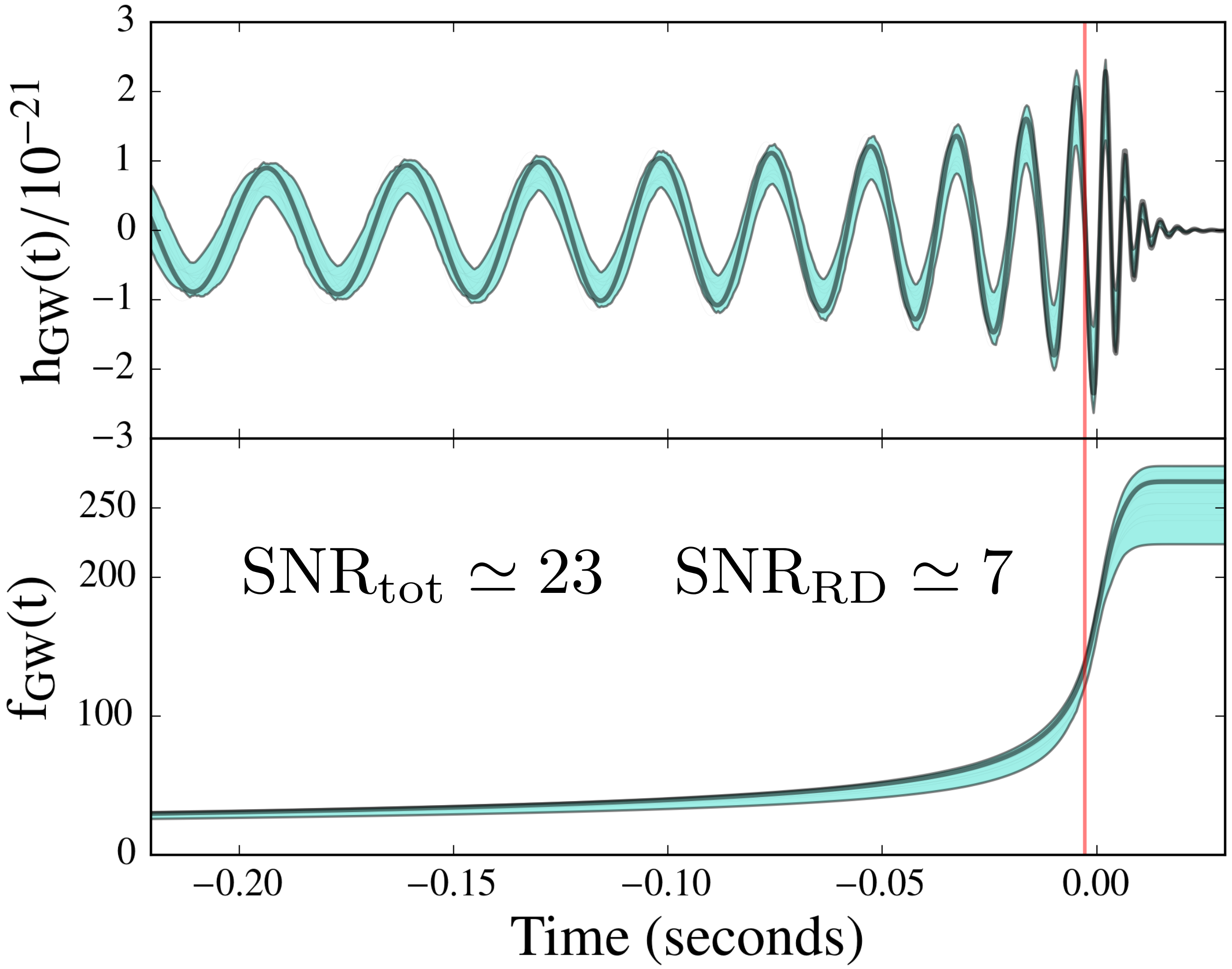
$$\tau_{\ell mn} = \tau_{\ell mn}^{\rm GR}\,(1 + \delta\tau_{\ell mn})$$

GR mock signal | **non-GR mock signal**



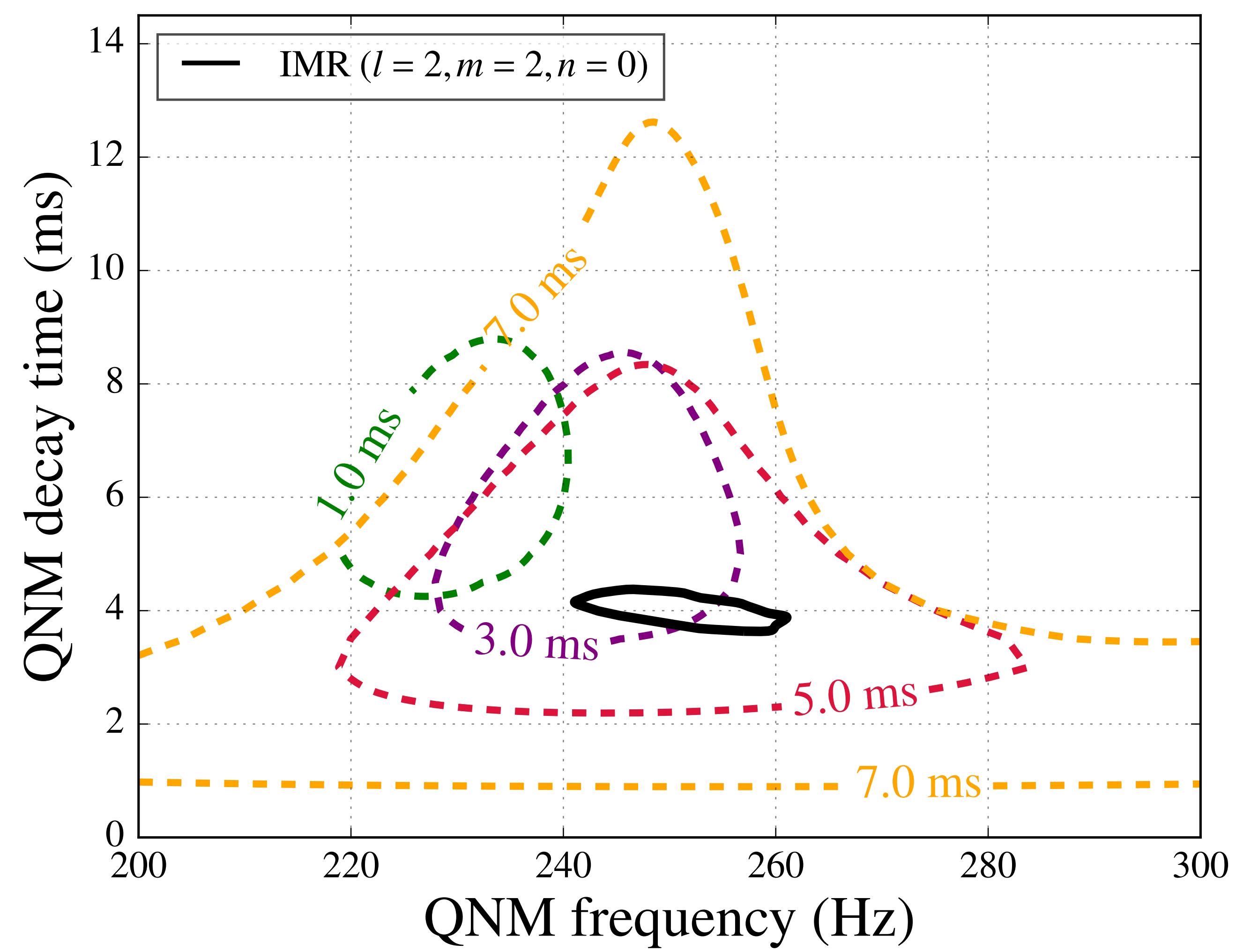
(using Einstein Telescope or third-generation ground-based detectors) *(Gossan et al. 2012)*

Probing remnant of GW150914 through quasi-normal modes



(Abbott et al. PRL 116 (2016) 221101)

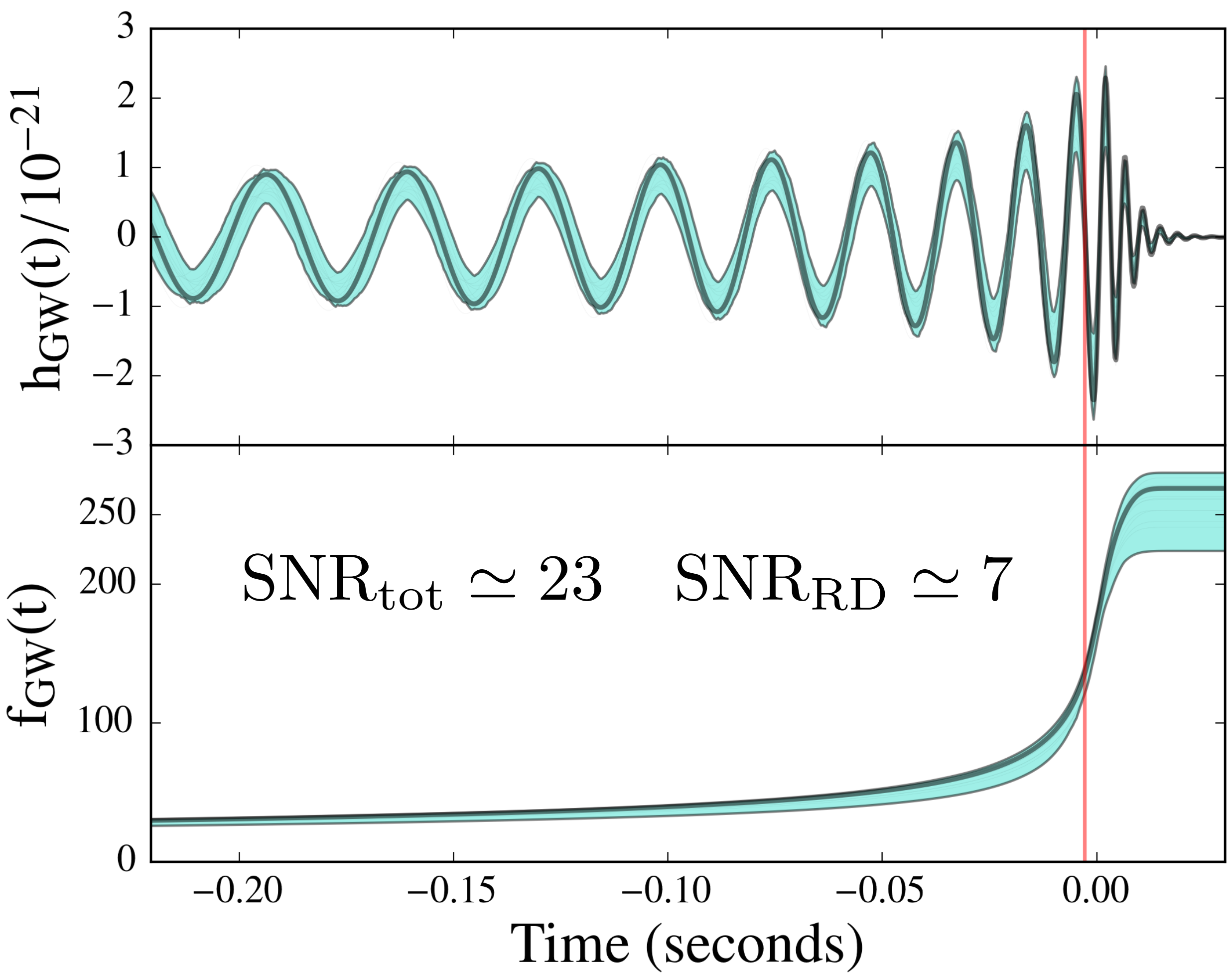
- Bayesian analysis with **damped-sinusoid template** to extract **frequency** and **decay time**, **starting at different times** after merger.



- Starting **from 5 msec** after merger, posterior distributions of **frequencies** and **decay times** from **damped sinusoid** and **IMR waveform** are **consistent**.

- First (low-accuracy) verification of **black hole uniqueness** properties (?)

Probing remnant of GW150914 through quasi-normal modes

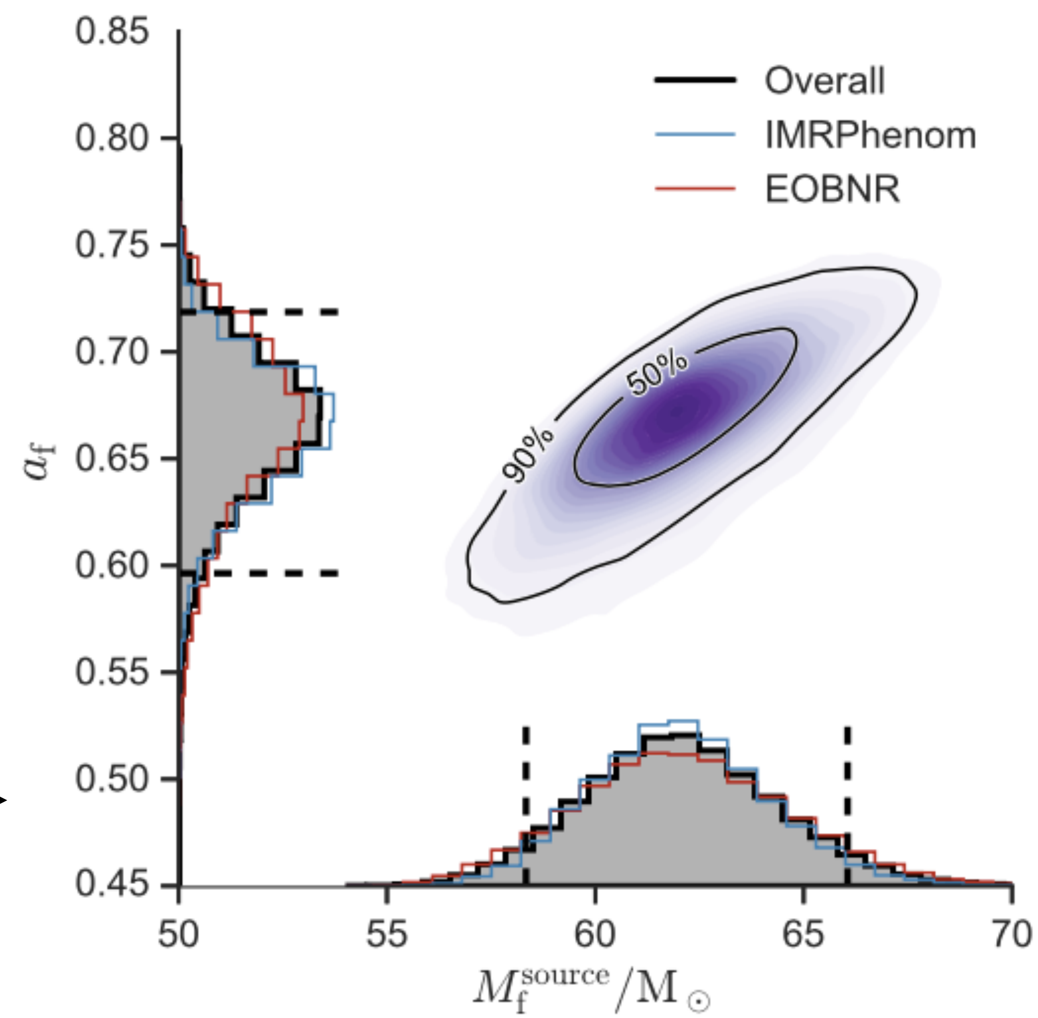


(*Abbott et al. PRL 116 (2016) 221101*)

- Bayesian analysis with **damped-sinusoid template** to extract **frequency** and **decay time**, **starting at different times** after merger.

(*Abbott et al. PRL 116 (2016) 241102*)

- IMR (l = 2, m=2) posterior obtained from **full Bayesian analysis of GW150914**, plus **information from NR** to obtain **final mass and spin** from component masses and spins.

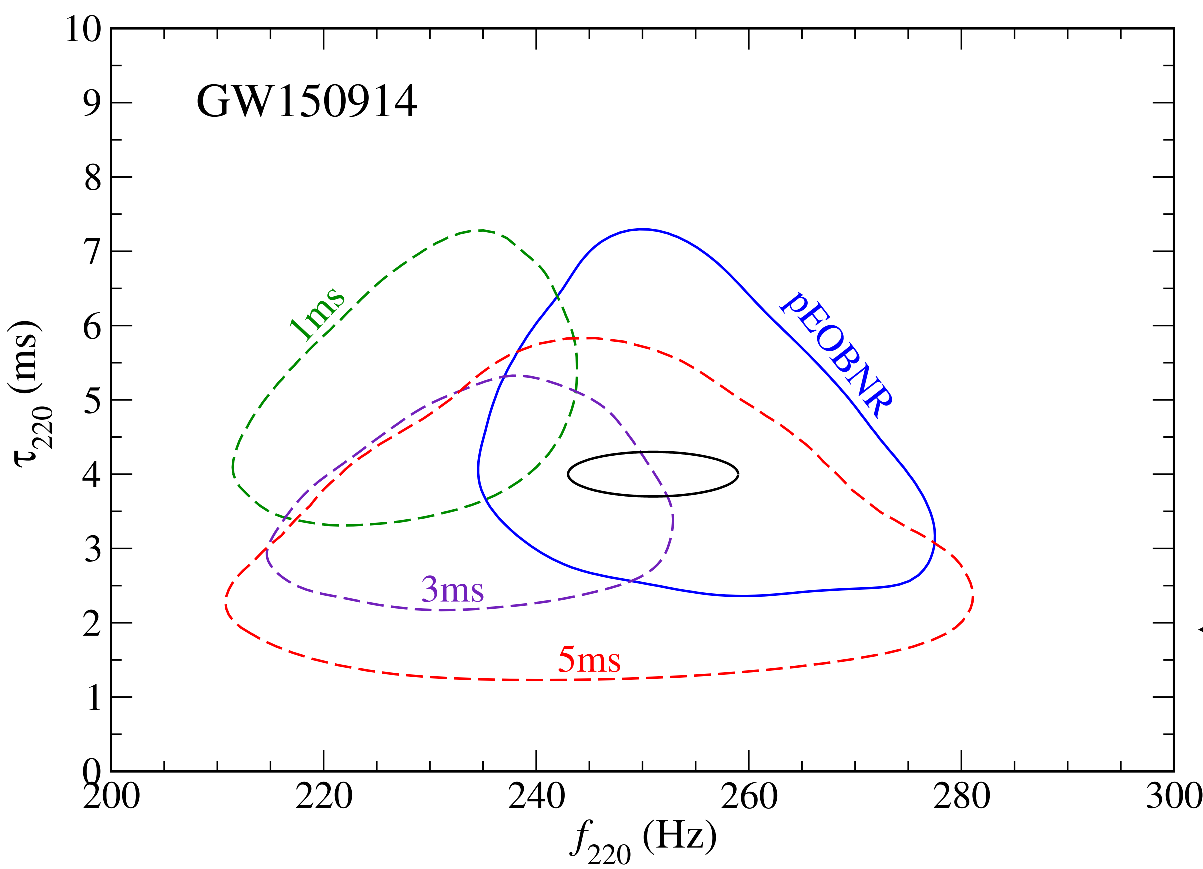


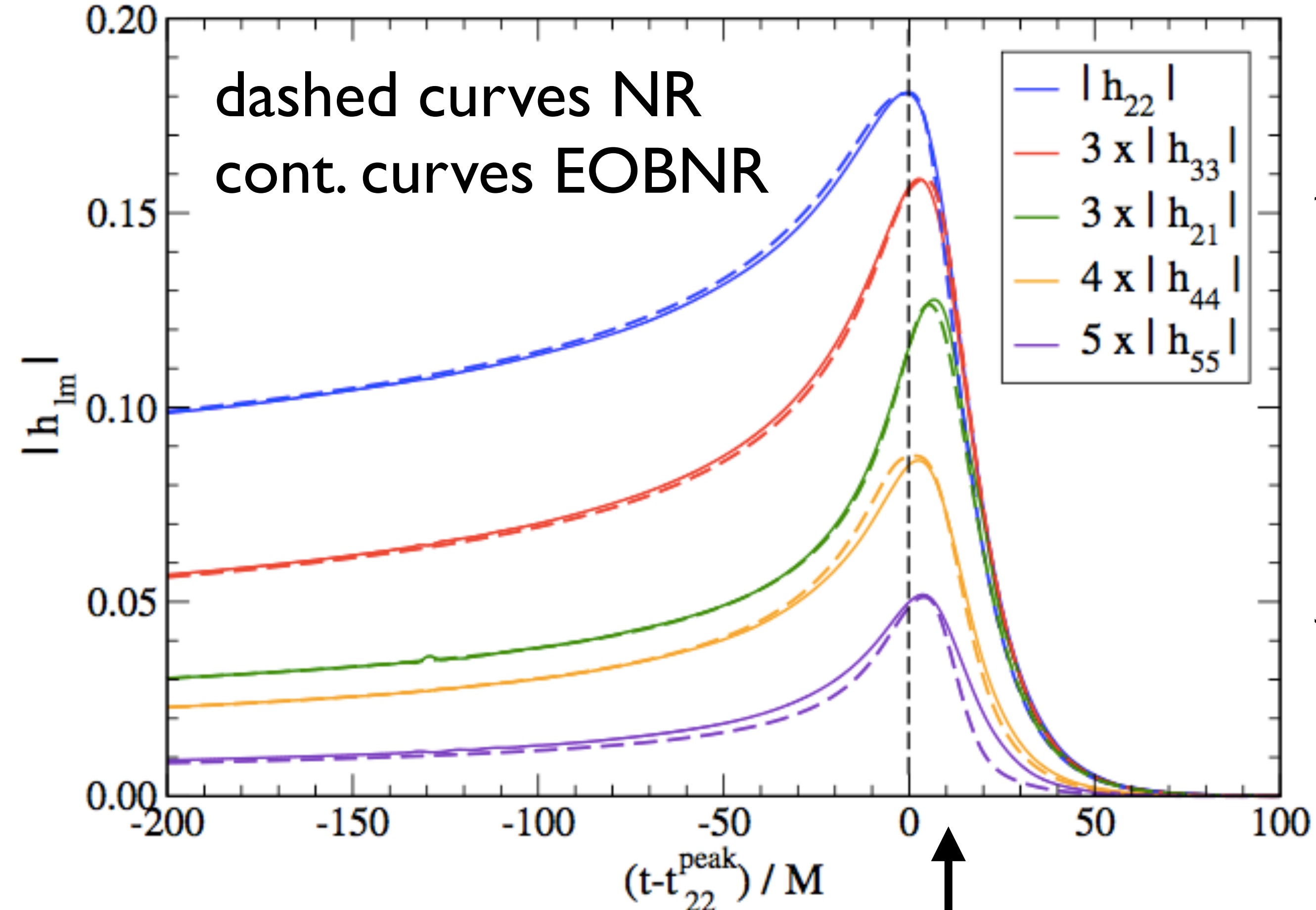
- First (low-accuracy) verification of **black hole uniqueness** properties (?)

Black-hole spectroscopy by making *full use* of GW modeling

(Brito, AB & Raymond 18)

- We build **parametrized inspiral-merger-ringdown** waveforms (pEOBNR):
  - QNM's **frequencies and decay times** are **free parameters**;
  - **(2,2), (2,1), (3,3), (4,4) & (5,5)** modes are **present**.





(Pan, AB et al. 12)

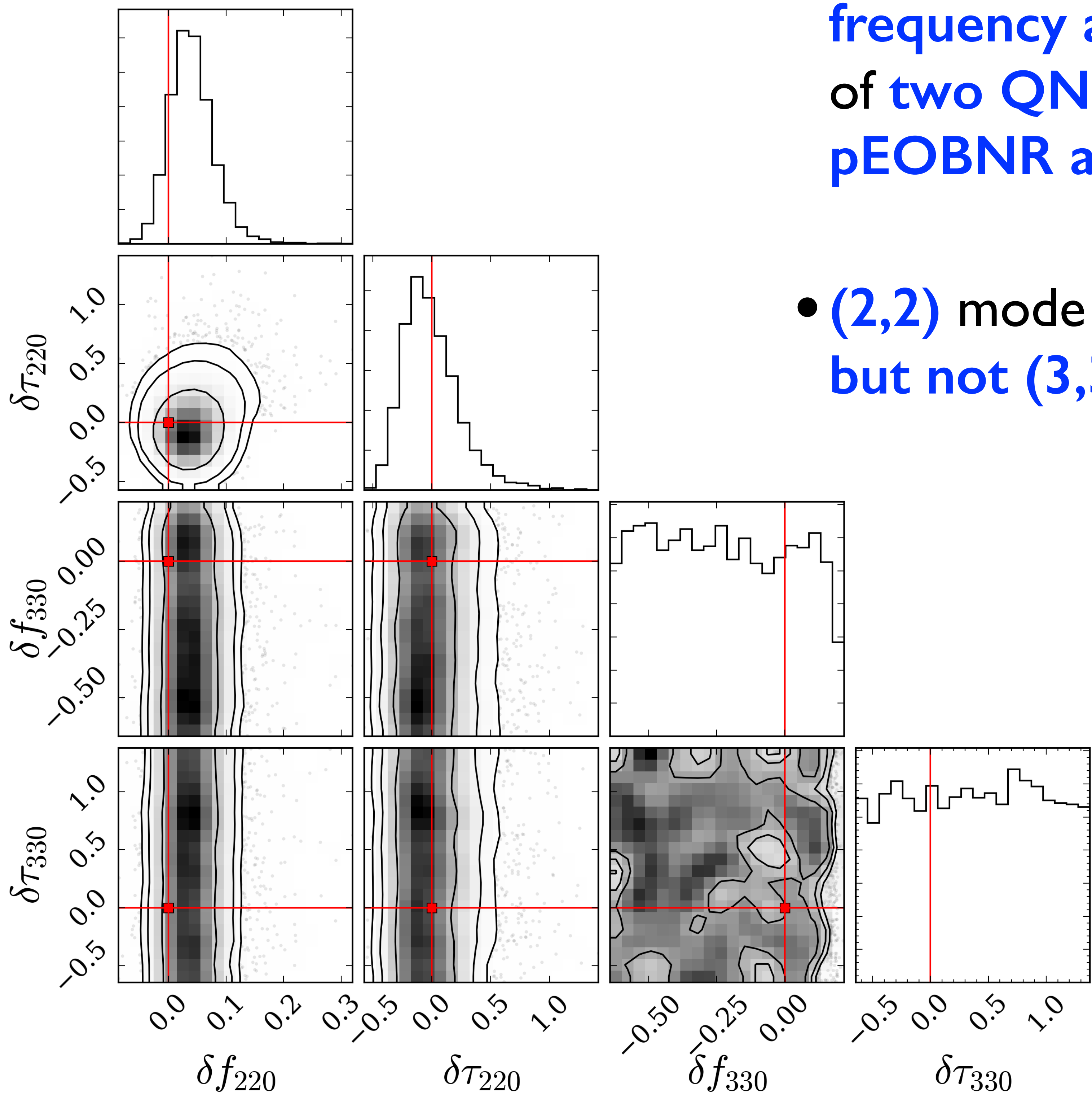
- Merger-ringdown EOBNR model **reproduces time & phase shifts** between **NR modes' at peak.**

- **GW150914**'s frequency and decay time **recovered "without ambiguity"** on *a priori* **unknown starting time** of QNM **ringing**.

Trying to extract 2 quasi-normal modes from GW150914 event

(Brito, AB & Raymond 18)

GW150914



- Posterior distributions of **frequency and decay times** of **two QNMs** employing **pEOBNR against data**.

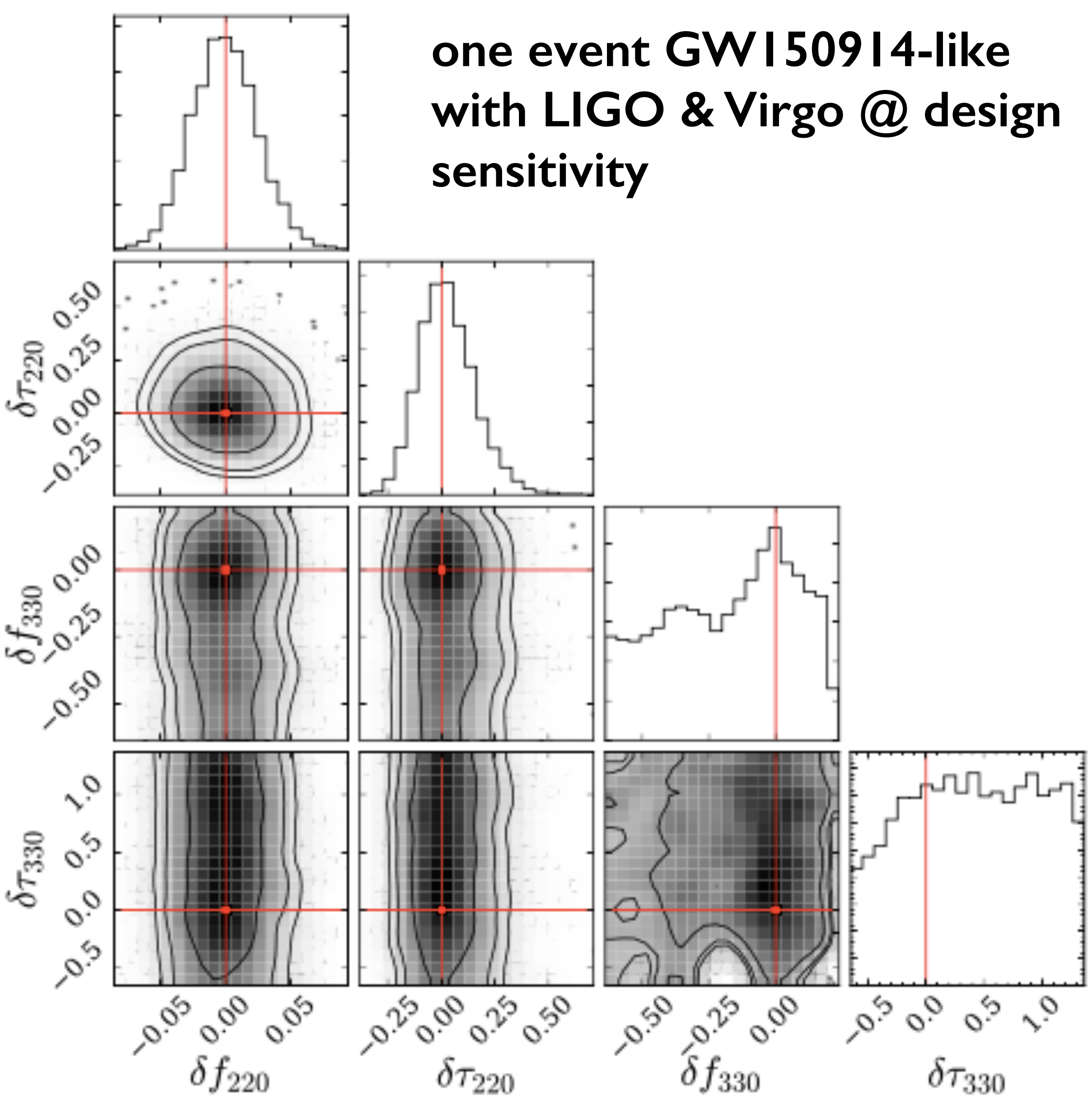
- **(2,2)** mode **resolved, but not (3,3)** mode.

Measuring at least 2 QNMs with LIGO & Virgo

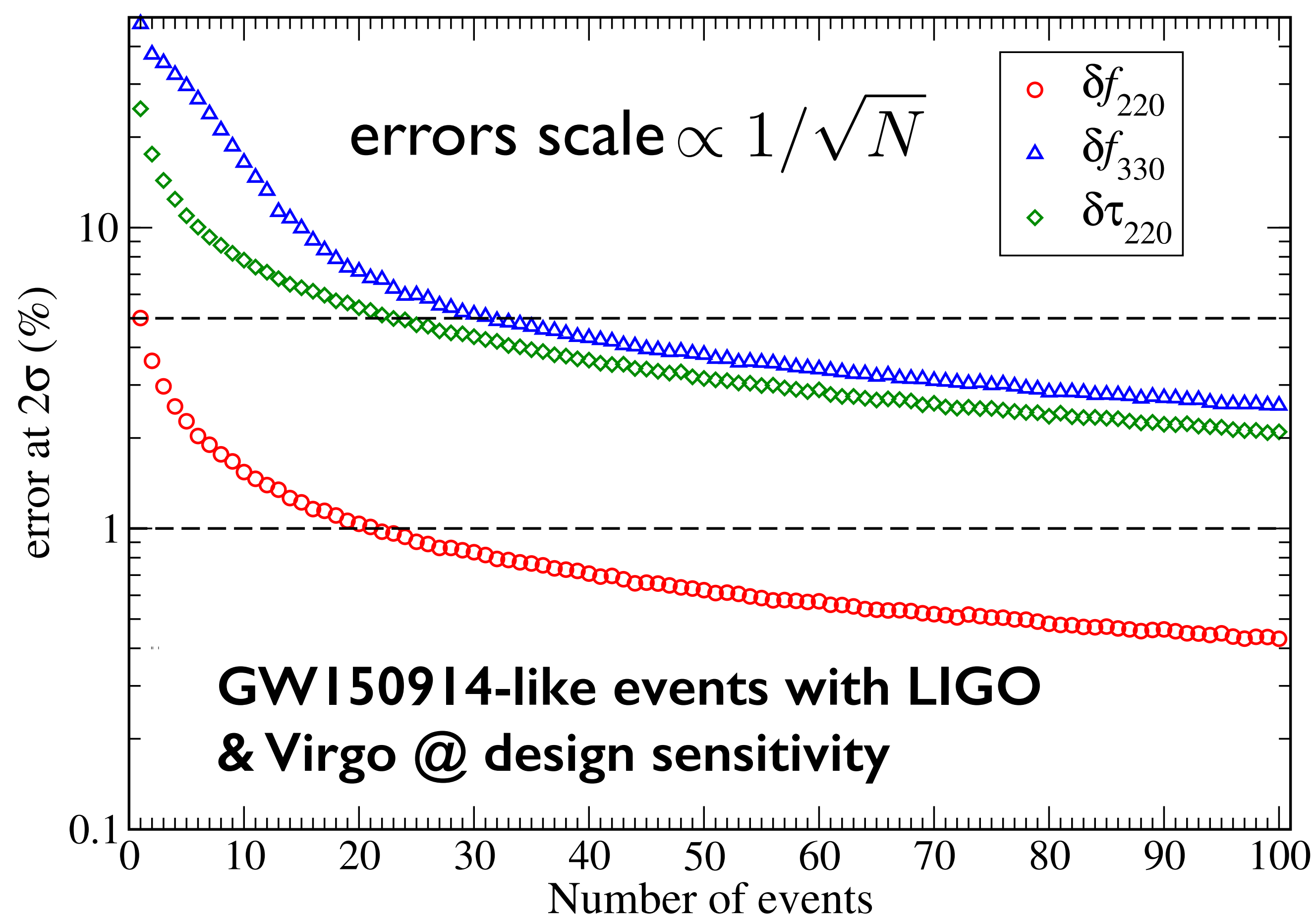
- Let us assume we had GW observations and **did not find deviations from GR**.
- We can bound **quasi-normal mode frequencies & decay times** by **combining several BH observations.**

$$\omega_{\ell mn} = \omega_{\ell mn}^{\rm GR}\,(1+\delta\omega_{\ell mn}) \quad \tau_{\ell mn} = \tau_{\ell mn}^{\rm GR}\,(1+\delta\tau_{\ell mn})$$

(Brito, AB & Raymond 18)



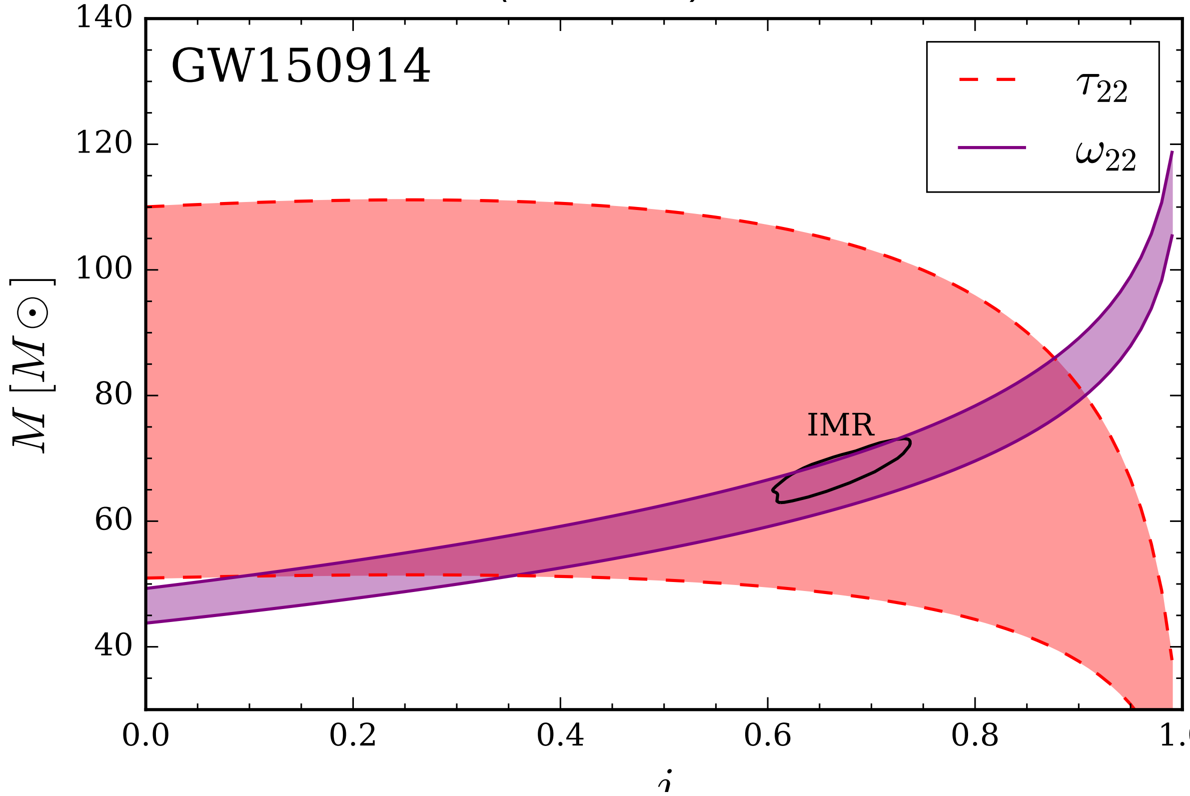
one event GW150914-like with LIGO & Virgo @ design sensitivity



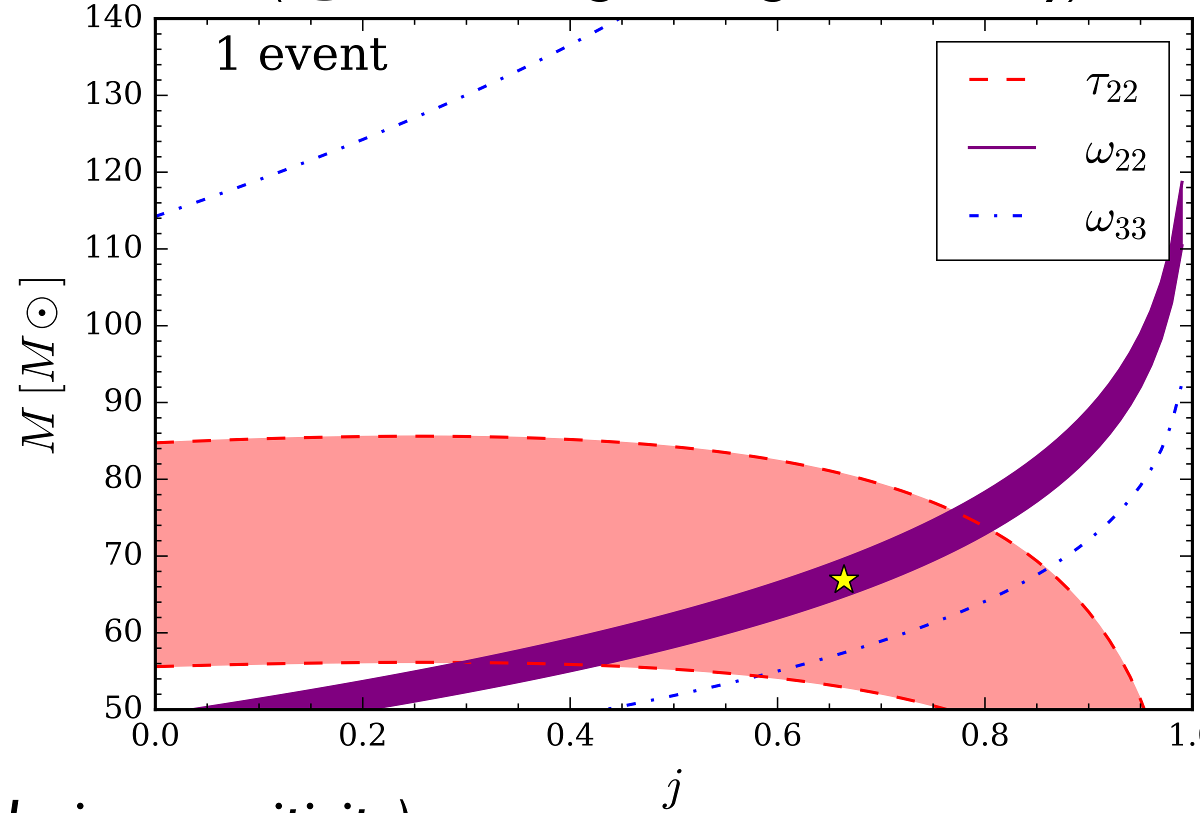
- About **30 GW150914-like events** are needed **to achieve errors of 5%.** We will **soon verify** more **accurately black hole uniqueness** properties.

Testing no-hair conjecture with several events @ design sensitivity

(O1 run)

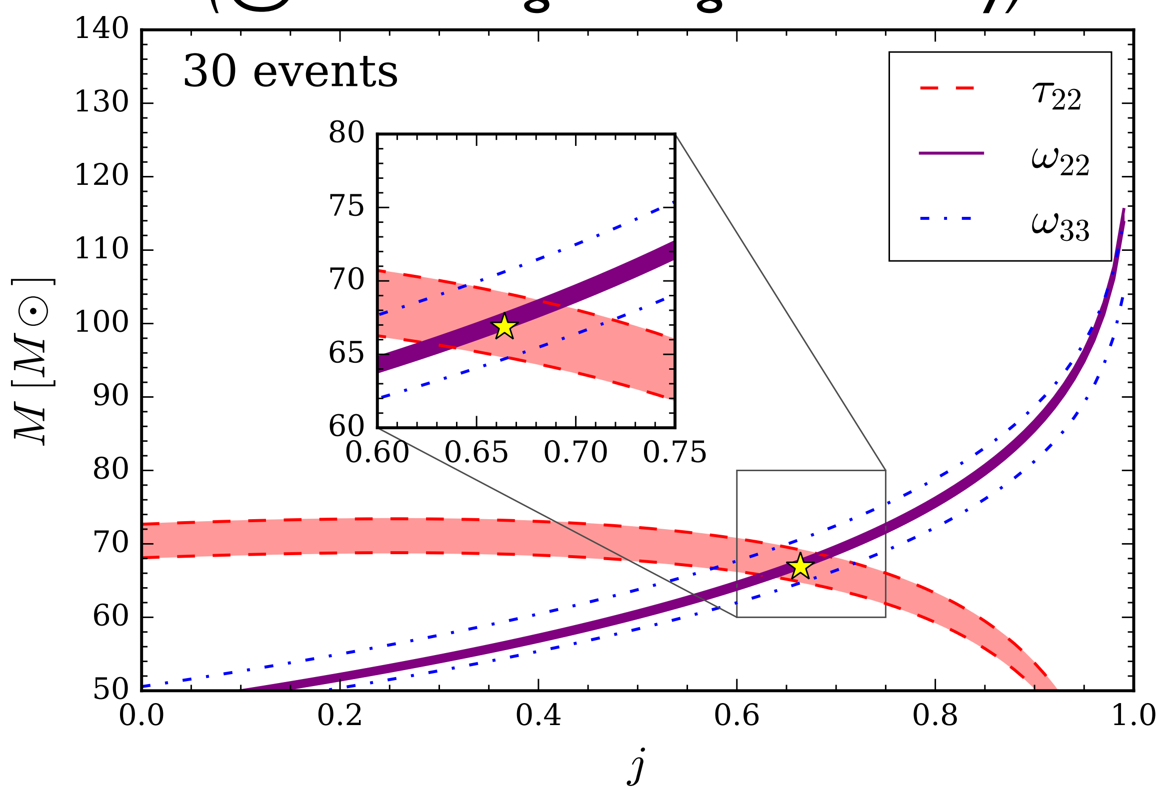


(@ LIGO/Virgo design sensitivity)



(Brito, AB & Raymond 18)

(@ LIGO/Virgo design sensitivity)

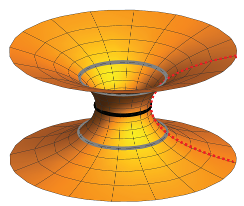


using pEOBNR

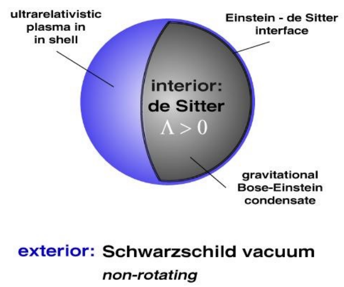
Remnant: black hole or exotic compact object (ECO)?

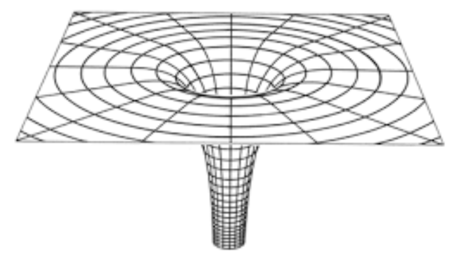
horizonless objects

wormhole



Gravastar





black hole

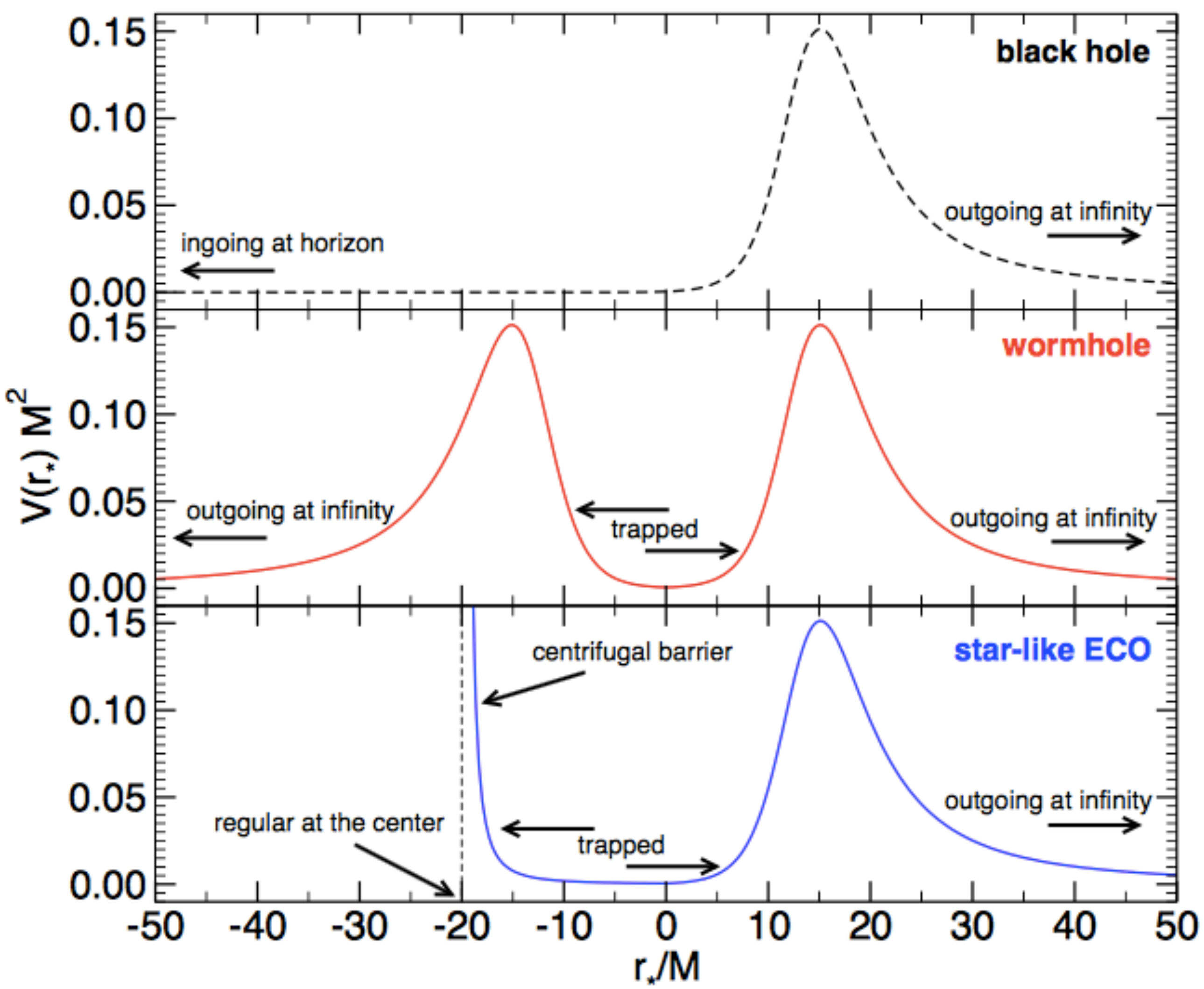
boson stars, fermion stars, etc.

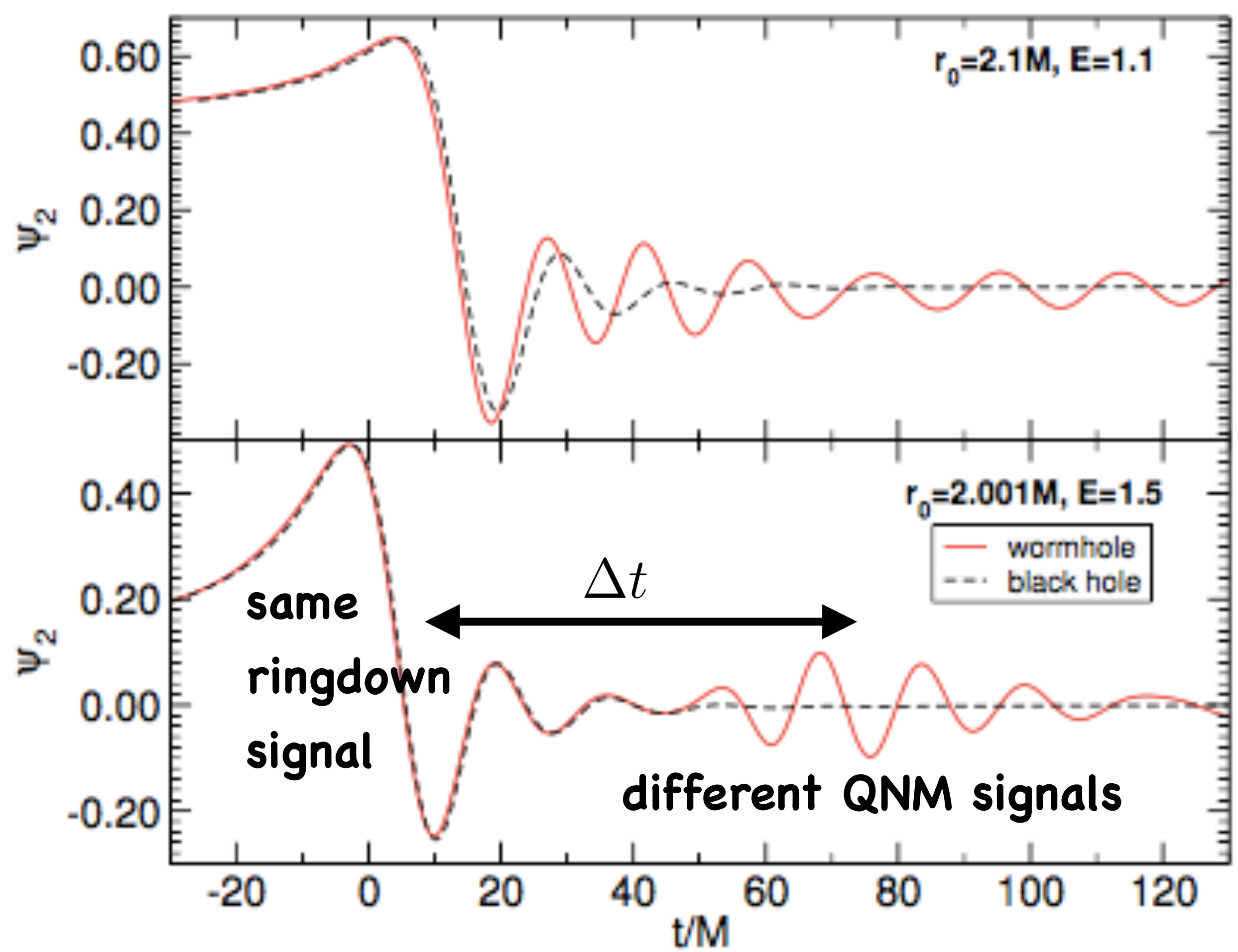
(e.g., Giudice et al. 16)

- If **remnant** is **horizonless,** and/or **horizon is replaced** by **"surface"**, new modes in the spectrum, and ringdown signal is modified: **echoes signals emitted after merger.**

(Damour & Solodukhin 07, Cardoso, Franzin & Pani 16)

(Cardoso et al. 16)





GW polarizations in gravity

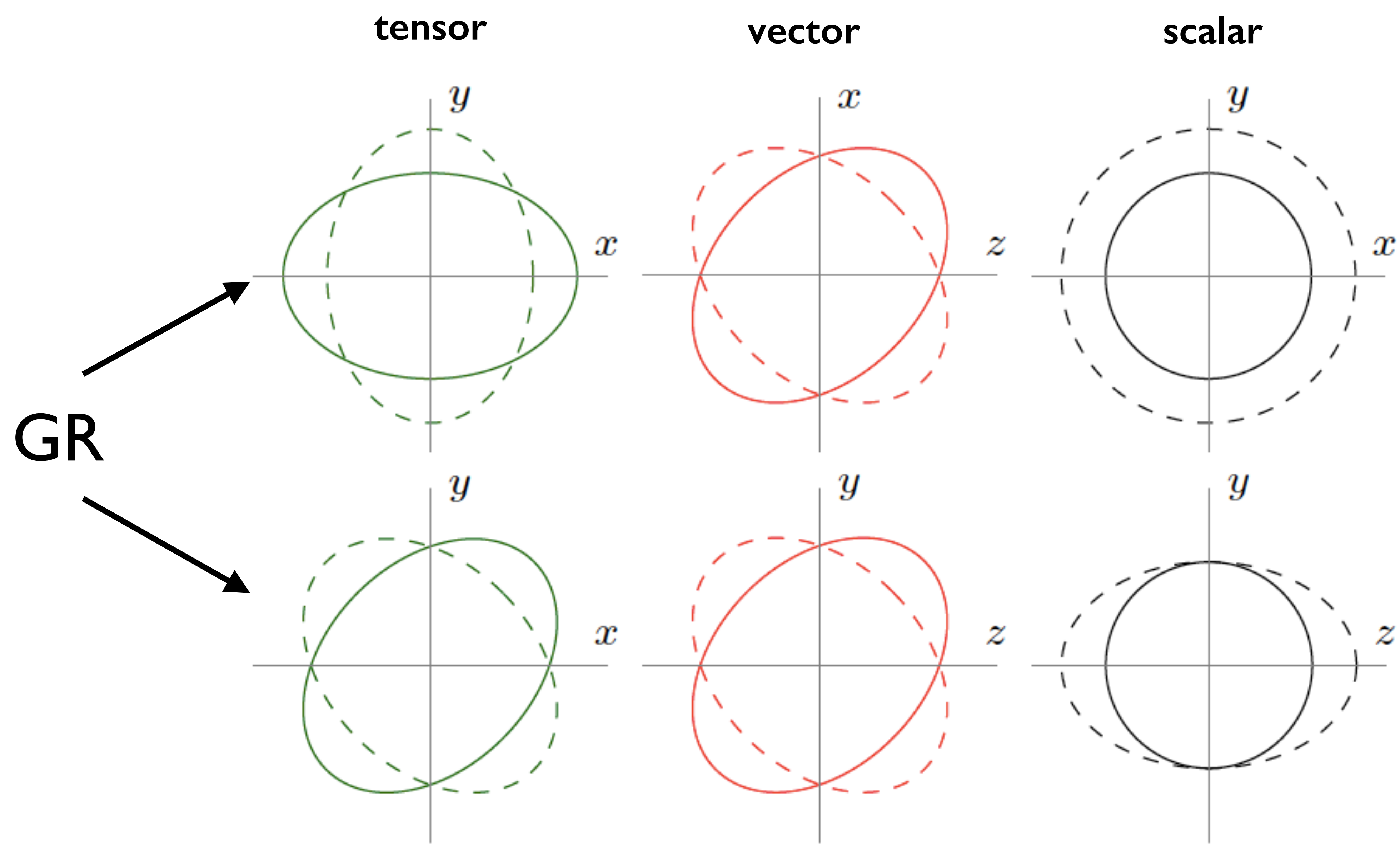
- **Generic metric theories of gravity** have **6** geometrically distinct **polarizations:**

(Will 72)

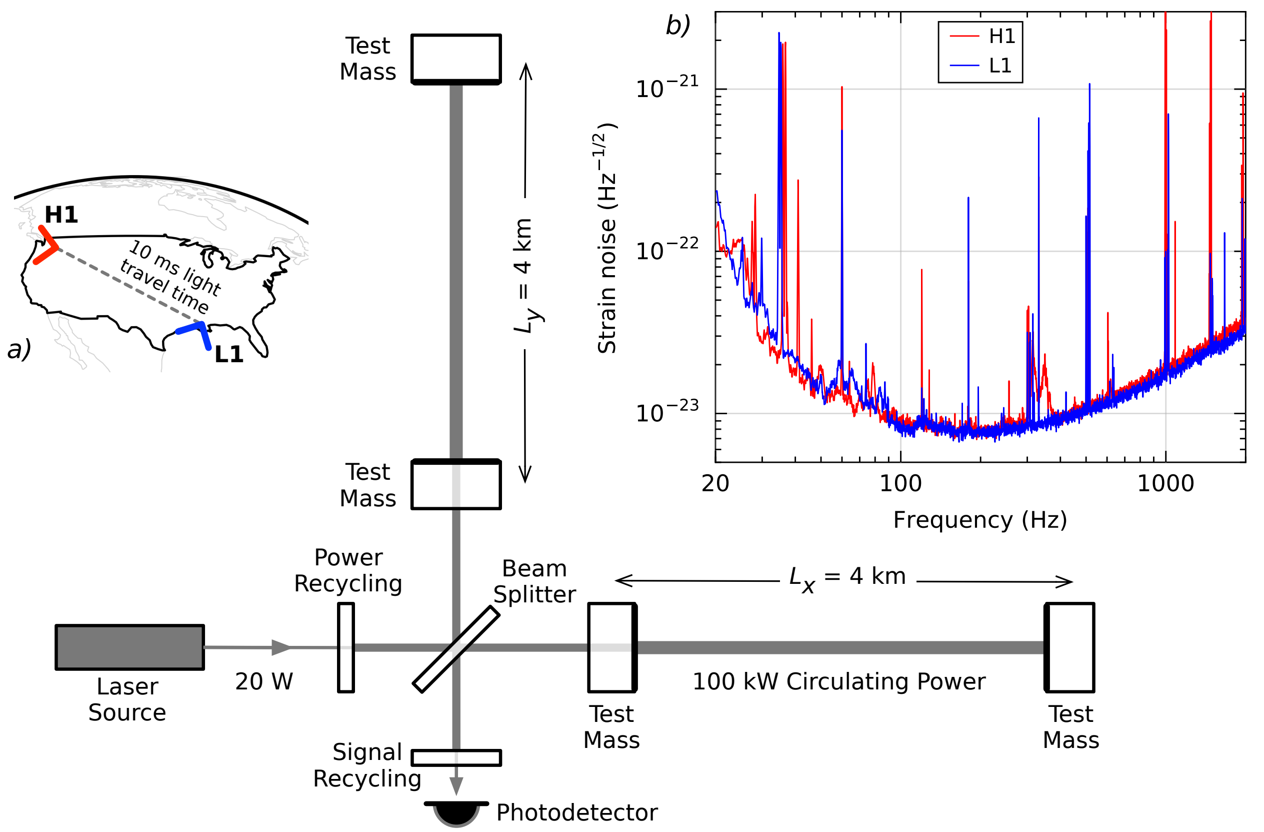
$$h_{ij}(\mathbf{x}) = \sum_A h_A(\mathbf{x})\, e^A_{ij} \qquad A = 1, \cdots, 6$$

6 amplitudes

6 polarization tensors



Detector antenna patterns for different GW polarizations



$$D^{ij} = \frac{1}{2}(d_x^i\, d_x^j - d_y^i\, d_y^j)$$

differential-arm detector

detector tensor

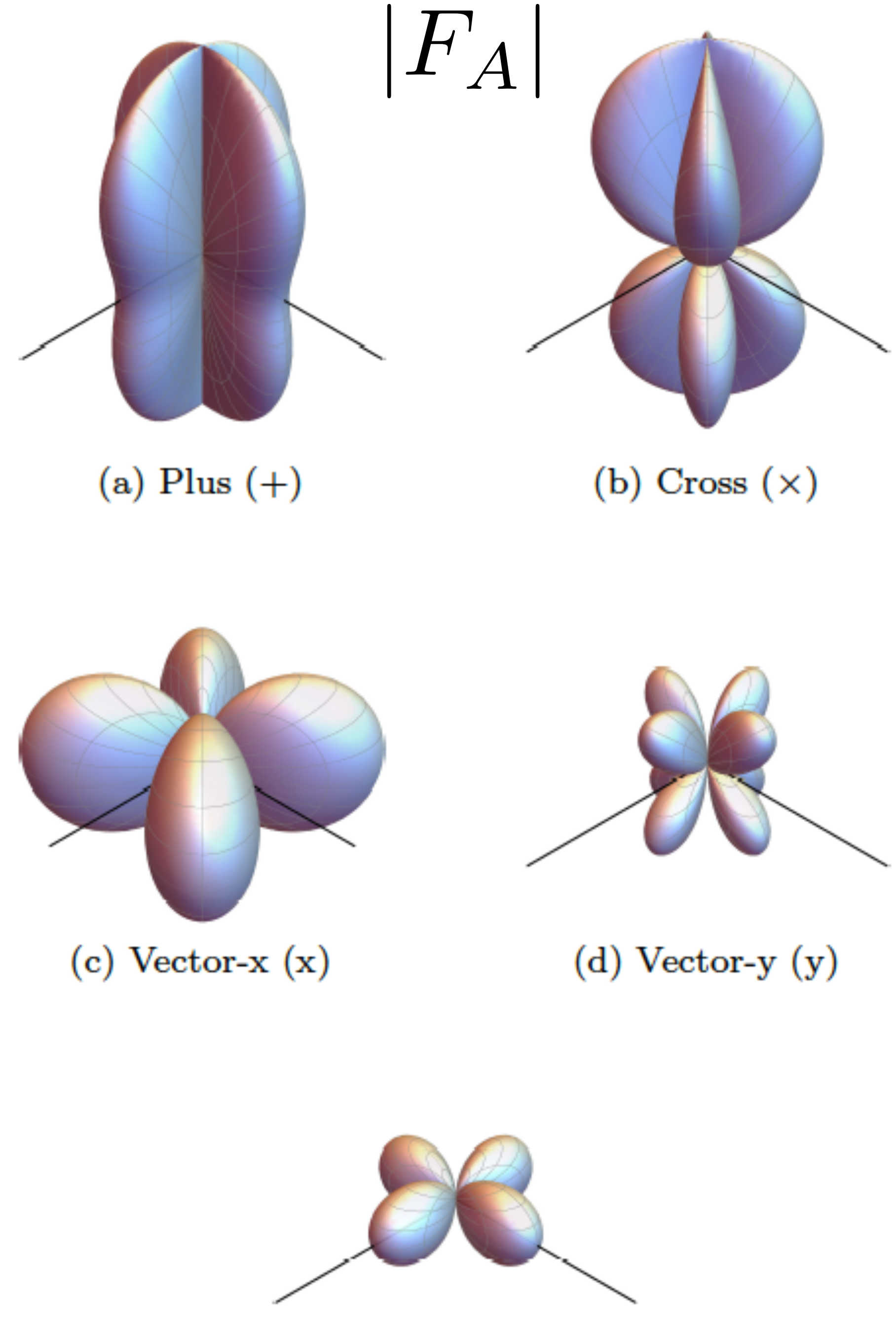
$$h_{\rm I}(t, \mathbf{x}_I) = D_I^{ij}\, h_{ij}(t, \mathbf{x}_I)$$
$$= \sum_A h_A(t, \mathbf{x}_I)\, (D_I^{ij}\, e_{ij}^A)$$

depends on GW source

depends only on geometry, F_A

- **Angular responses of a differential-arm detector**

$|F_A|$



(a) Plus (+) (b) Cross (×)

(c) Vector-x (x) (d) Vector-y (y)

(e) Scalar (s)

Testing extra GW polarizations with two LIGOs & Virgo

$$h_I(t,\mathbf{x}_I) = \underbrace{D_I^{ij}}_{\text{detector tensor}}\, h_{ij}(t,\mathbf{x}_I)$$

$$= \sum_A \underbrace{h_A(t,\mathbf{x}_I)}_{\text{depends on GW source}}\, \underbrace{(D_I^{ij}\, e_{ij}^A)}_{\text{depends only on geometry, } F_A}$$

examples of antenna patterns:

tensor

$$F_+(\vartheta,\varphi,\psi) = -\frac{1}{2}\,(1+\cos^2\vartheta)\cos 2\varphi\cos 2\psi - \cos\vartheta\sin 2\varphi\sin 2\psi\ ,$$

scalar

$$F_{\mathrm{b}/1}(\vartheta,\varphi,\psi) = \mp\frac{1}{2}\sin^2\vartheta\cos 2\varphi$$

- **Two LIGOs** are **nearly co-aligned,** approximately sensitive to the **same linear combination** of polarizations.
- **Five (differential-arm) detectors** would allow to extract all the **5 polarizations (2 scalar polarization are degenerate).**
- Observation of **GW170814 with two LIGOs plus Virgo** allowed to test **pure tensor polarizations against pure scalar (vector)**, finding **Bayes factors 1000 (200).** (Abbott et al. PRL 119 (2017) 141101)

Tests of Lorentz Invariance/Bounding Graviton Mass

- Phenomenological approach: **modified dispersion relation.** GWs travel at speed different from speed of light. (Will 94, Mirshekari, Yunes & Will 12)

$$E^2 = p^2c^2 + Ap^\alpha c^\alpha$$

$$\alpha \geq 0$$

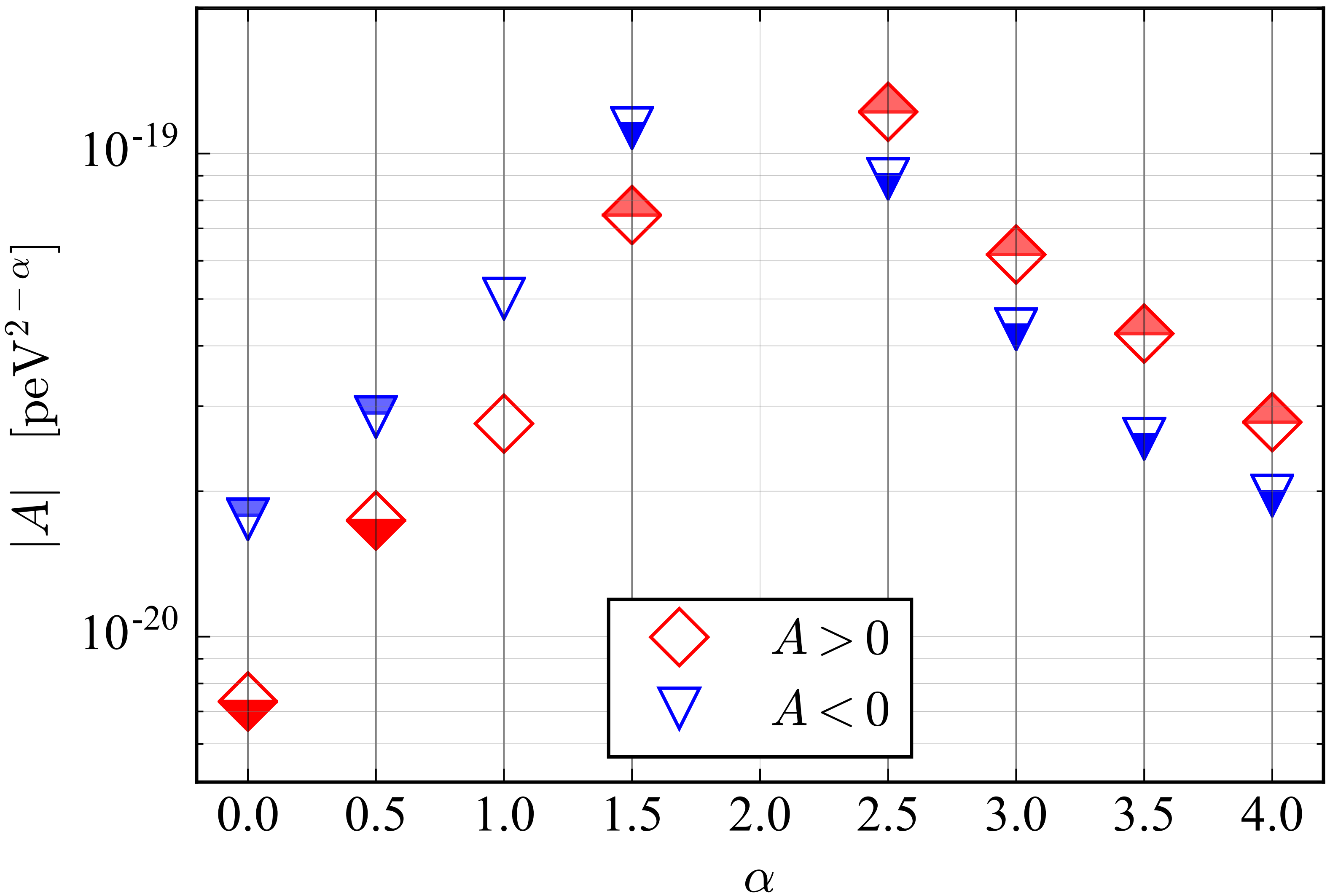
$$\frac{v_g}{c} = 1 + (\alpha - 1)\frac{A}{2}E^{\alpha-2}$$

$$m_g \leq 7.7 \times 10^{-23} \mathrm{eV/c^2}$$

$$\alpha = 0, A > 0$$

$$\lambda_g = \frac{h}{m_g c}$$

(Abbott et al. PRL118 (2017))



Constraints on speed of GWs & test of equivalence principle

- **Combining GW** and **GRB** observations: *(Abbott et al. APJ 848 (2017) L12)*

$$\frac{\Delta c}{c} \simeq c\,\frac{\Delta t}{D}$$

$$\Delta t = t_{\rm EM} - t_{\rm GW}$$

$$\Delta c = c_{\rm GW} - c$$

$$-4 \times 10^{-15} \leq \frac{\Delta c}{c} \leq 7 \times 10^{-16}$$

assuming GRB is emitted 10 s after GW signal

assuming observed time delay is entirely due to different speed $\Delta t \simeq 1.7 s$

- **Strong constraints** on **scalar-tensor** and **vector-tensor theories of gravity**.

(Creminelli et al. 17, Ezquiaga et al. 17, Sakstein et al. 17, Baker et al. 17)

- **EM waves** & **GWs follow same geodesic. Metric perturbations** (e.g., due to potential between source and Earth) **affect** their **propagation** in **same way.**

$$\delta t_{\rm S} = -\frac{1+\gamma}{c^3}\int_{r_{\rm e}}^{r_{\rm o}} U(\boldsymbol{r}(l))dl$$

gravitational potential of Milky Way outside sphere of 100 kpc

(Shapiro 1964)

$$-2.6 \times 10^{-7} \leqslant \gamma_{\rm GW} - \gamma_{\rm EM} \leqslant 1.2 \times 10^{-6}$$

(Abbott et al. APJ 848 (2017) L12)

Solving two-body problem in General Relativity (including radiation)

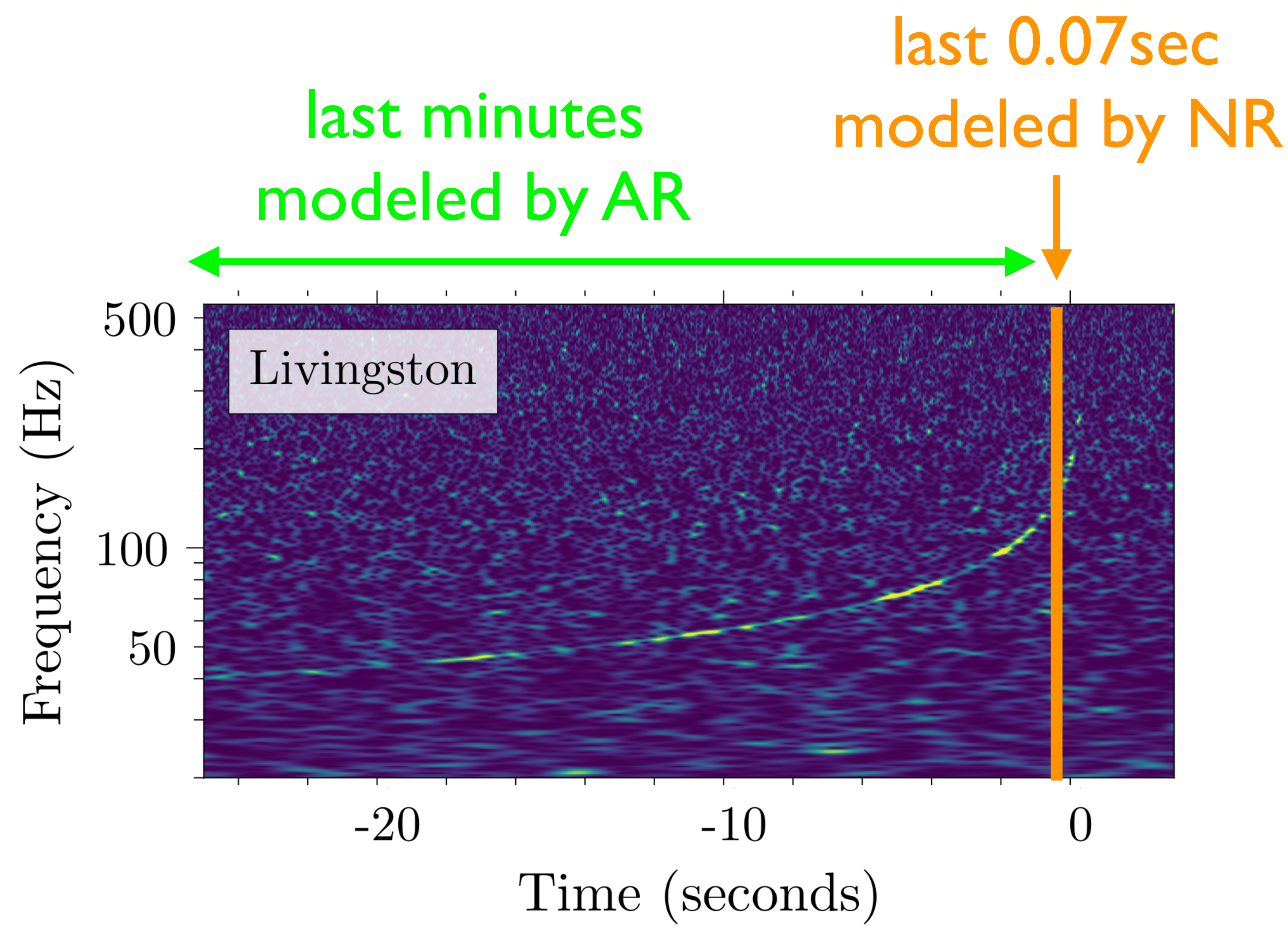
$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}$$

- **GR** is **non-linear theory**. Complexity similar to QCD.

- Einstein's field equations can be solved:
  - **approximately,** but **analytically** (**fast** way)
  - **exactly**, but **numerically** on supercomputers (**slow** way)

(Abbott et al. PRL 119 (2017) 161101)

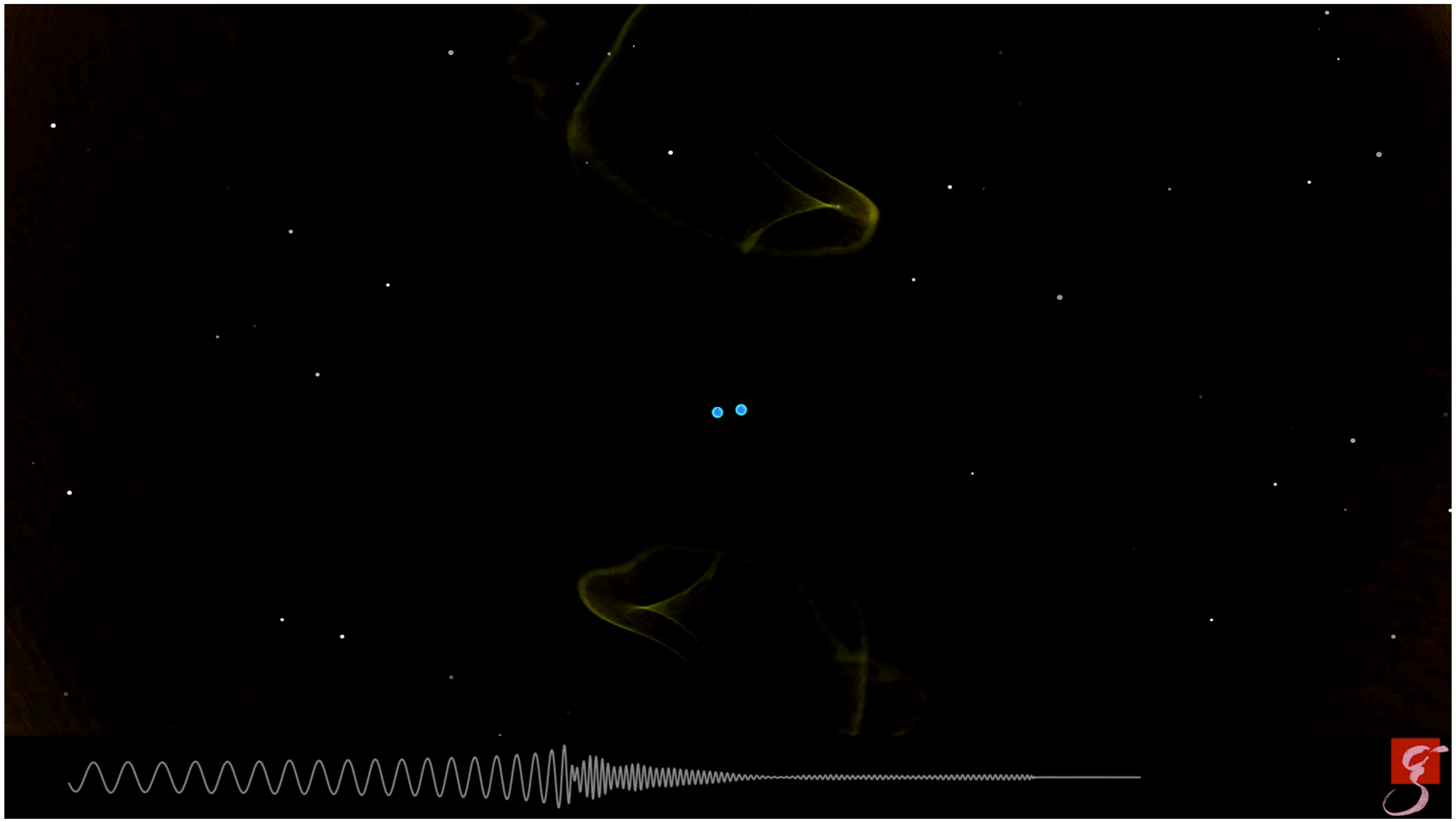
- **GW170817**: SNR=32 (strong), 3000 cycles (from 30 Hz), one **minute.**



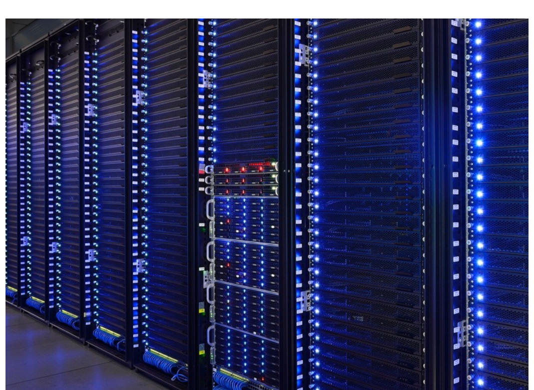
- **Synergy** between **analytical** and **numerical relativity** is **crucial.**

Numerical-relativity simulation of GW170817

(numerical simulation: Dietrich @ AEI and BAM collaboration)

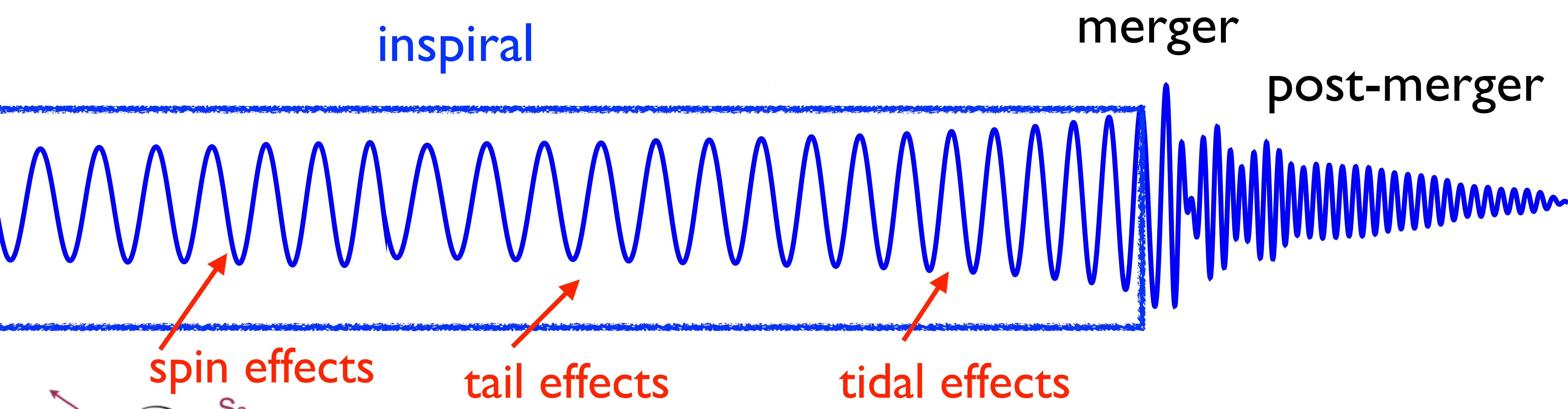


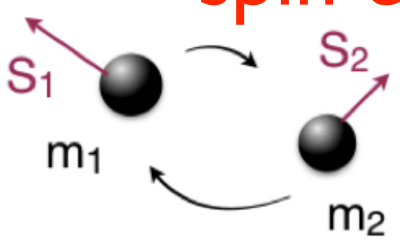
(visualization: Dietrich, Ossokine, Pfeiffer & Buonanno @ AEI)



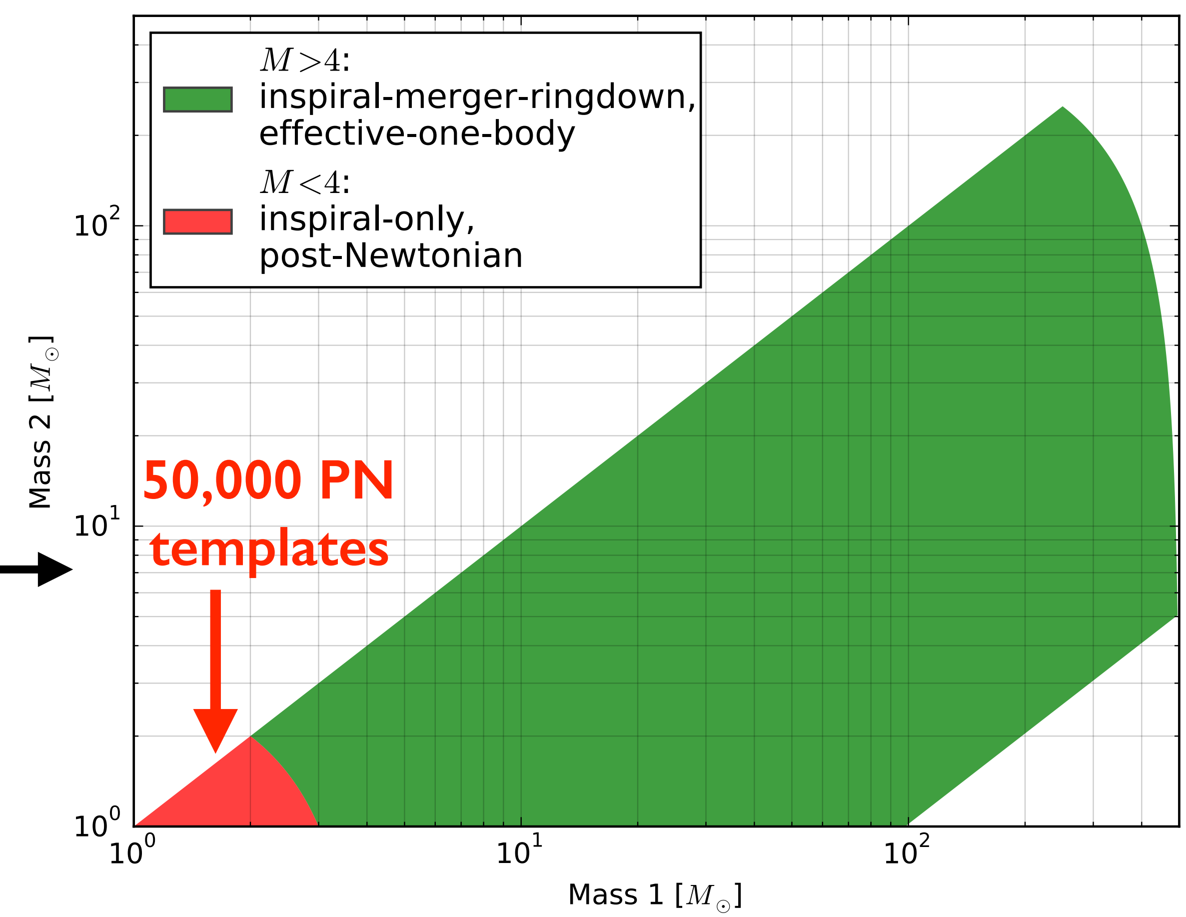
Minerva:
High-Performance Computer Cluster
@ AEI Potsdam (~10,000 cores)

Analytical waveform modeling for GW170817





- **PN waveform model** was used for:
  - **template bank:** to observe GW170817
  - **Bayesian analyses:** to infer astrophysical, fundamental physics information of GW170817

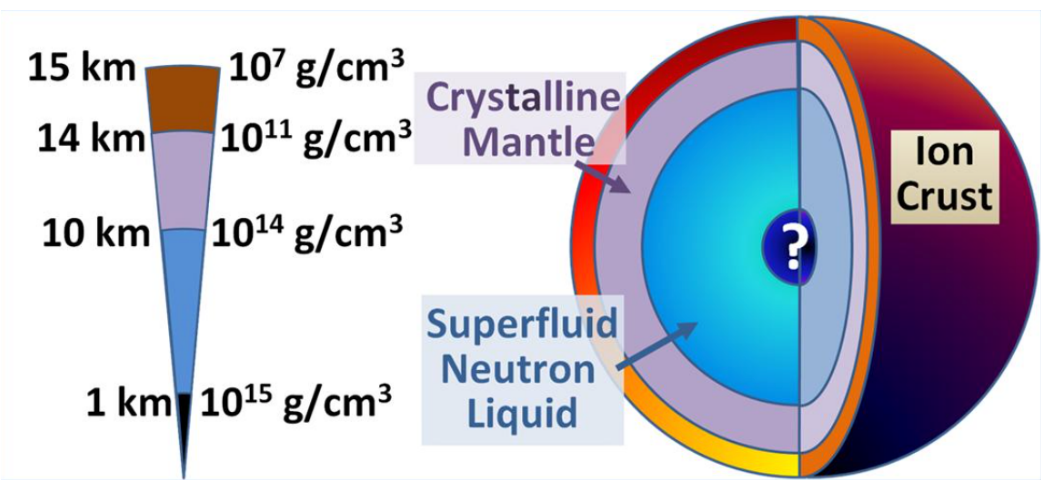


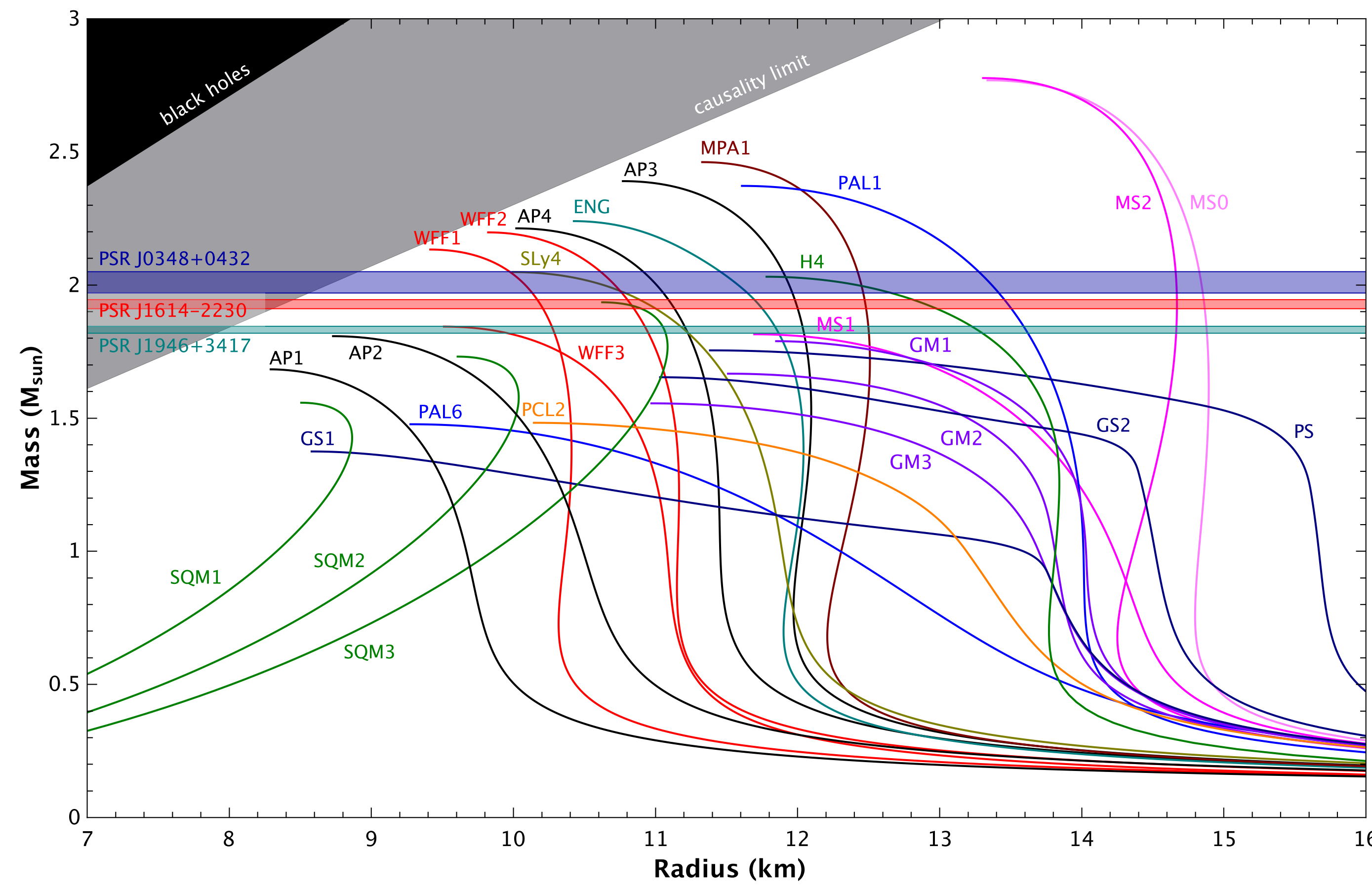
(Dal Canton & Harry 16)

Probing equation of state of neutron stars

Neutron Star:

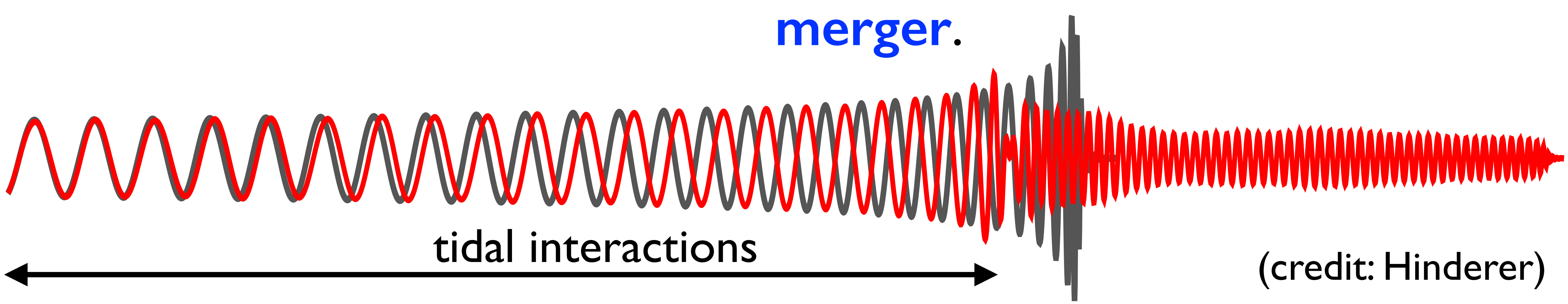
- mass: 1-3 Msun
- radius: 9-15 km
- core density > 10^{14}g/cm^3





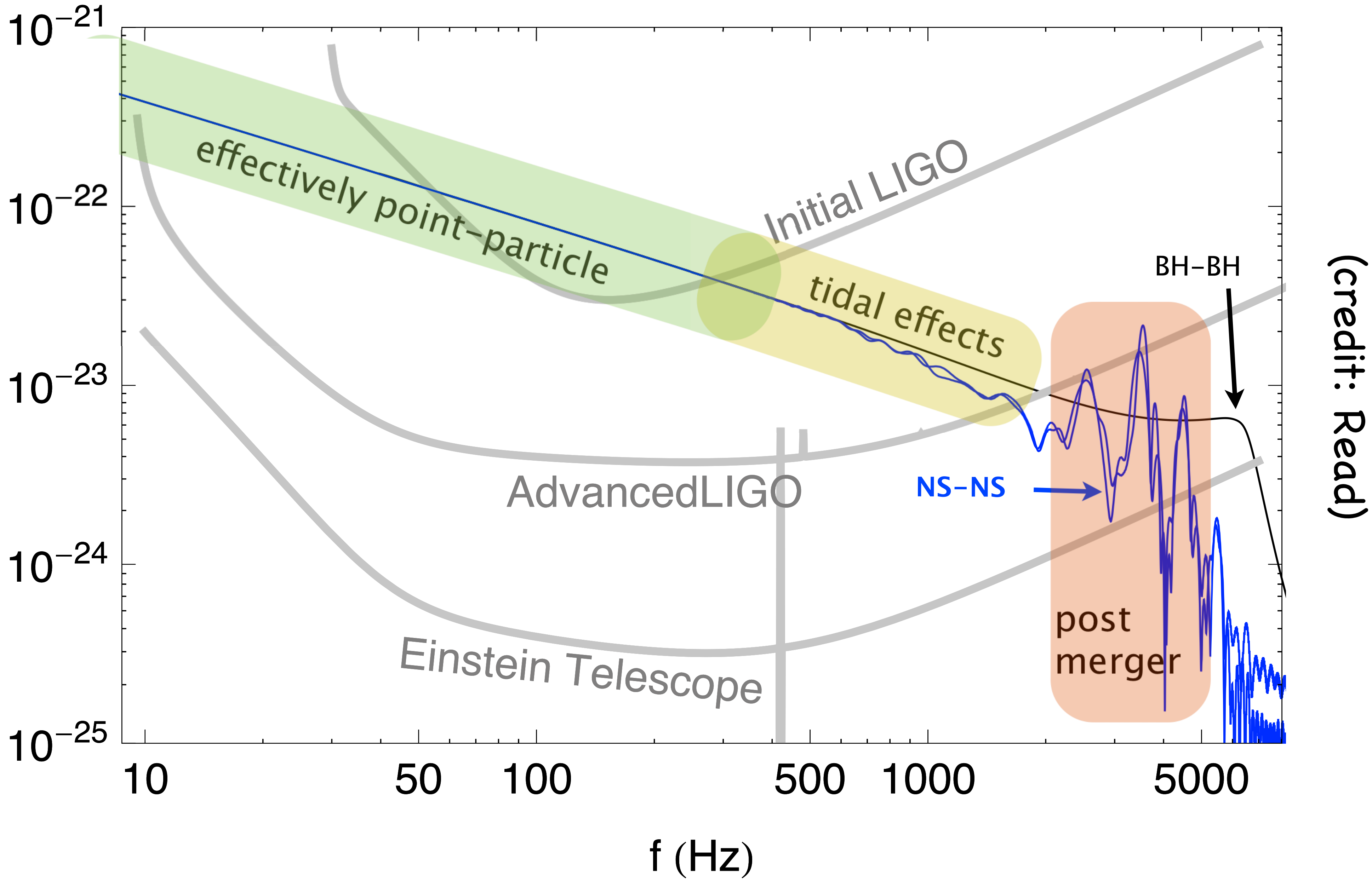
(Antoniadis et al. 2016)

- **NS equation of state** (EOS) affects gravitational **waveform** during **late inspiral, merger** and **post-merger**.



(credit: Hinderer)

Probing equation of state of neutron stars



- **Tidal effects imprinted** on gravitational waveform during inspiral through **parameter** λ.
- λ **measures** star's **quadrupole deformation** in response to companion **perturbing tidal field:**

$$Q_{ij} = -\lambda\, \mathcal{E}_{ij}$$

NS deformation in external tidal field

- In presence of **external potential**, (non-rotating) **NS acquires** a **deformation**: self-gravitating fluid is perturbed from equilibrium configuration

- **Quadrupolar tidal field:** $\mathcal{E}_{ij} = \partial_i \partial_j U_{\rm ext}$

Newtonian tidal deformations

- **Gravitational potential** generated by **perturbed NS**

$$U(t, \mathbf{x}) = -G \int d^3x' \, \frac{\rho(t, \mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|}$$

equilibrium configuration

$$\rho(t, \mathbf{x}') = \overline{\rho}(r') + \delta\rho(t, \mathbf{x}')$$

perturbations

- **Multipole expansion around CM:**

outside NS

$$\frac{1}{|\mathbf{x} - \mathbf{x}'|} = \frac{1}{r} + \frac{\mathbf{x} \cdot \mathbf{x}'}{r^3} + \frac{(3\, n_i\, n_j - \delta_{ij})}{2r^3}\, x'_i\, x'_j + \ldots \qquad n_i = \frac{x_i}{r} \qquad r > r'$$

$$U(t, \mathbf{x}) = -\frac{G\, m_{\rm NS}}{r} - \frac{G(3n_i\, n_j - \delta_{ij})}{2r^3}\, Q_{ij} + \ldots$$

$$Q_{ij} = \int d^3x' \rho(t, \mathbf{x}')\, (x'_i\, x'_j - \frac{1}{3} r'^2\, \delta_{ij})$$

NS deformation in external tidal field (contd.)

- **Total gravitational potential** outside NS:

$$U(t,\mathbf{x}) = -\frac{G m_{\rm NS}}{r} - \frac{3G}{2r^3}\, n_i\, n_j\, Q_{ij} + \mathcal{O}\left(\frac{1}{r^4}\right) + \frac{1}{2}\, x_i\, x_j\, \mathcal{E}_{ij} + \mathcal{O}(x^3)$$

- Considering **quasi-static perturbations** (tidal force frequency much smaller than NS's eigenfrequency of normal mode of oscillation, i.e., f modes):

$$Q_{ij} = -\lambda\, \mathcal{E}_{ij} \qquad\qquad k_2 = \frac{3}{2}\, \frac{G\lambda}{R_{\rm NS}^5}$$

$$U(t,\mathbf{x}) = -\frac{G m_{\rm NS}}{r} + \frac{1}{2}\, \mathcal{E}_{ij}\, x_i\, x_j \left[1 + 2k_2 \left(\frac{R_{\rm NS}}{r}\right)^5\right] + \mathcal{O}\left(\frac{1}{r^4}\right) + \mathcal{O}(x^3)$$

$$g_{00} = 1 - \frac{2G m_{\rm NS}}{r} + \mathcal{E}_{ij}\, x_i\, x_j \left[1 + 2k_2 \left(\frac{R_{\rm NS}}{r}\right)^5\right] + \mathcal{O}\left(\frac{1}{r^4}\right) + \mathcal{O}(x^3)$$

PN templates in stationary phase approximation: TaylorF2

$$\tilde{h}(f) = \mathcal{A}_{\rm SPA}(f)\, e^{i\psi_{\rm SPA}(f)}$$

0PN

graviton with non zero mass

1PN

dipole radiation

-1PN

spin-orbit

1PN

1.5PN

1.5PN

2PN

2PN

$$\begin{aligned}
\psi_{\rm SPA}(f) \;=\; & 2\pi f t_c - \Phi_c - \pi/4 + \frac{3}{128}(\pi\mathcal{M}f)^{-5/3}\,\Big\{1+ \\
& -\frac{5\hat{\alpha}^2}{336\omega_{\rm BD}}\nu^{2/5}(\pi\mathcal{M}f)^{-2/3} - \frac{128}{3}\frac{\pi^2 D\,\mathcal{M}}{\lambda_g^2(1+z)}(\pi\mathcal{M}f)^{2/3} \\
+ & \left(\frac{3715}{756}+\frac{55}{9}\nu\right)\nu^{-2/5}(\pi\mathcal{M}f)^{2/3} - 16\pi\,\nu^{-3/5}(\pi\mathcal{M}f) + 4\beta\,\nu^{-3/5}(\pi\mathcal{M}f) \\
+ & \left(\frac{15293365}{508032}+\frac{27145}{504}\nu+\frac{3085}{72}\nu^2\right)\nu^{-4/5}(\pi\mathcal{M}f)^{4/3} - 10\sigma\,\nu^{-4/5}(\pi\mathcal{M}f)^{4/3} \\
& \cdots - \frac{39}{2}\nu^{-2}\,\tilde{\Lambda}\,(\pi\mathcal{M}f)^{10/3}\Big\}
\end{aligned}$$

spin-spin

5PN

tidal

Depends on EOS & compactness

it can be large

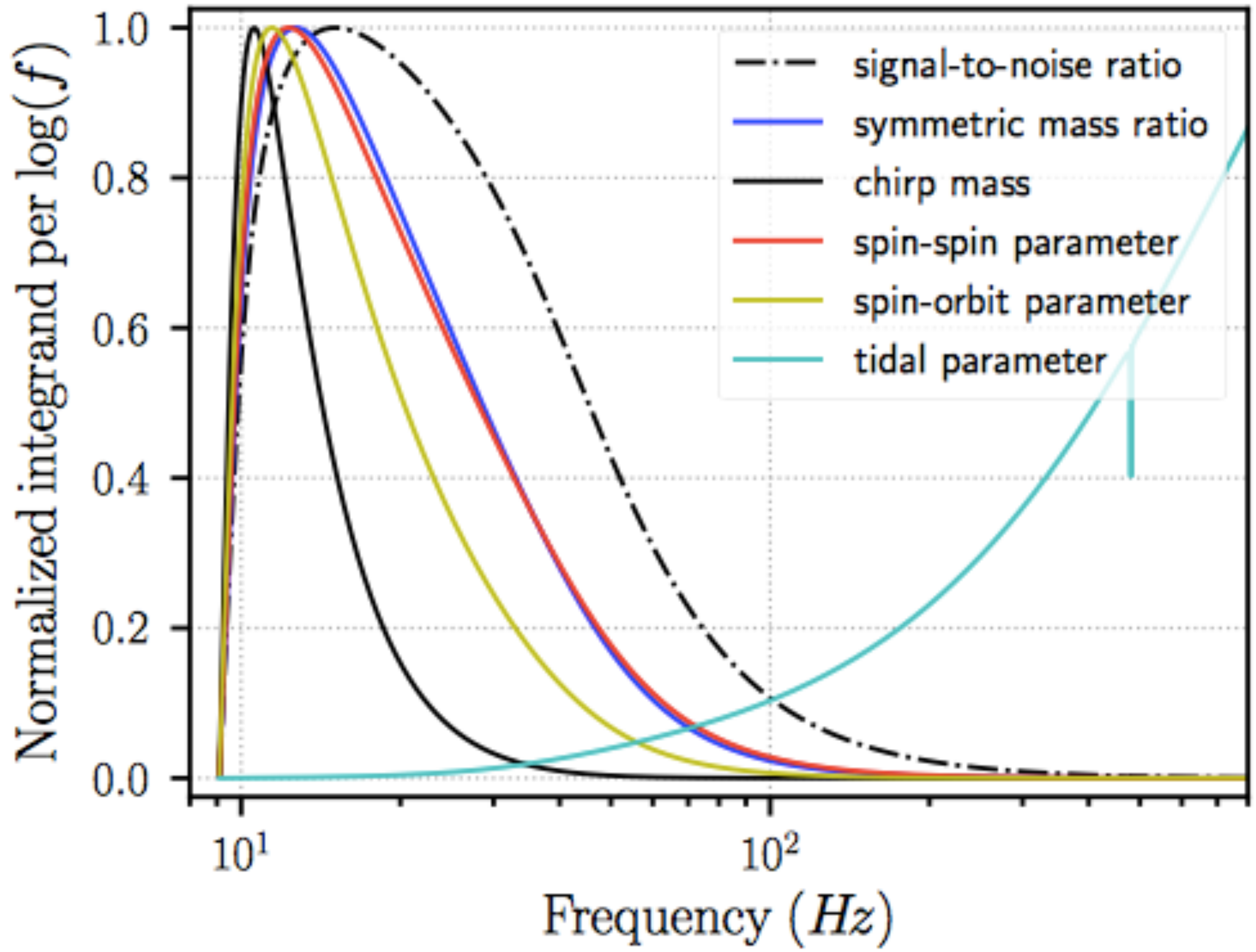
$$\tilde{\Lambda} = \frac{16}{13}\frac{(m_1+12m_2)m_1^4\Lambda_1 + (m_2+12m_1)m_2^4\Lambda_2}{(m_1+m_2)^5}$$

$$\Lambda = \frac{\lambda}{m_{\rm NS}^5} = \frac{2}{3}k_2\left(\frac{R_{\rm NS}c^2}{Gm_{\rm NS}}\right)^5$$

Probing equation of state of neutron stars

- **Where in frequency** the information about (intrinsic) **binary parameters predominantly comes from**.

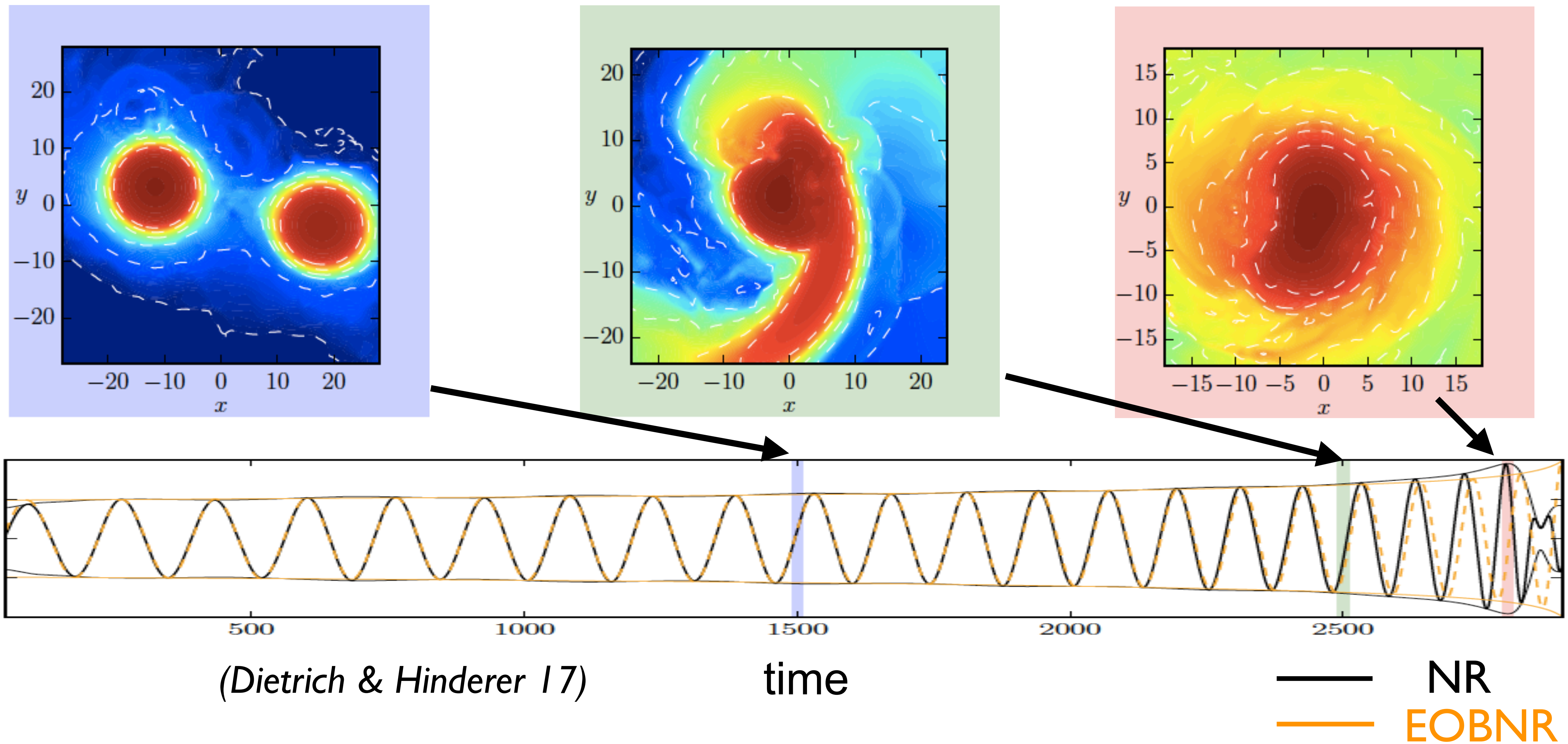
(Harry & Hinderer 17, see also Damour et al . 12)



- **Tidal effects** typically **change** overall **number of GW cycles** from 30 Hz (about 3000) **by one single cycle!**

State-of-art waveform models for binary neutron stars

- Synergy between **analytical** and **numerical work** is **crucial.**

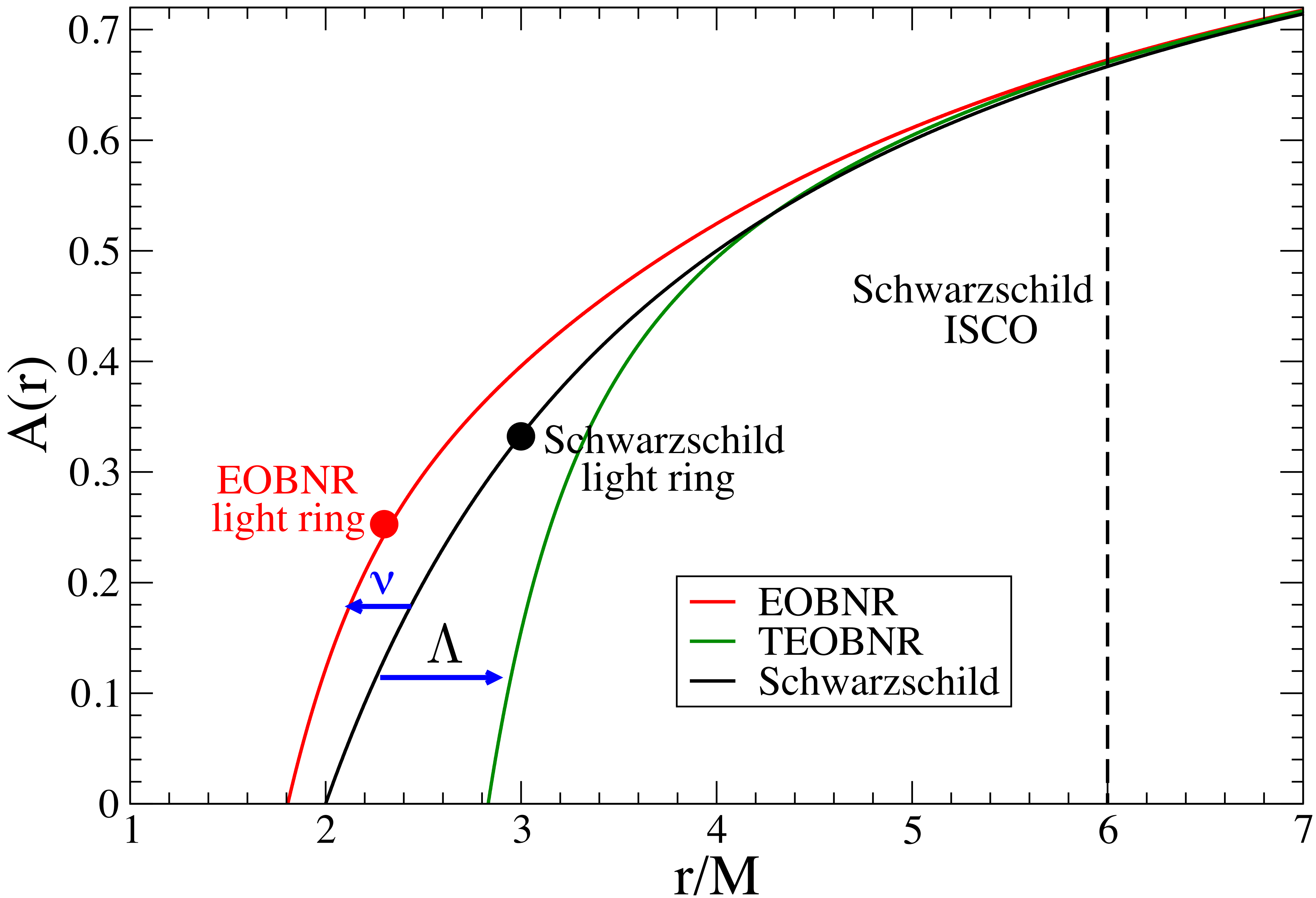


(Dietrich & Hinderer 17)

(Damour 1983, Flanagan & Hinderer 08, Binnington & Poisson 09, Vines et al. 11, Damour & Nagar 09, 12, Bernuzzi et al. 15, Hinderer et al. 16, Steinhoff et al. 16, Dietrich et al. 17-18, Nagar et al. 18)

Strong-field effects in presence of matter in EOB theory

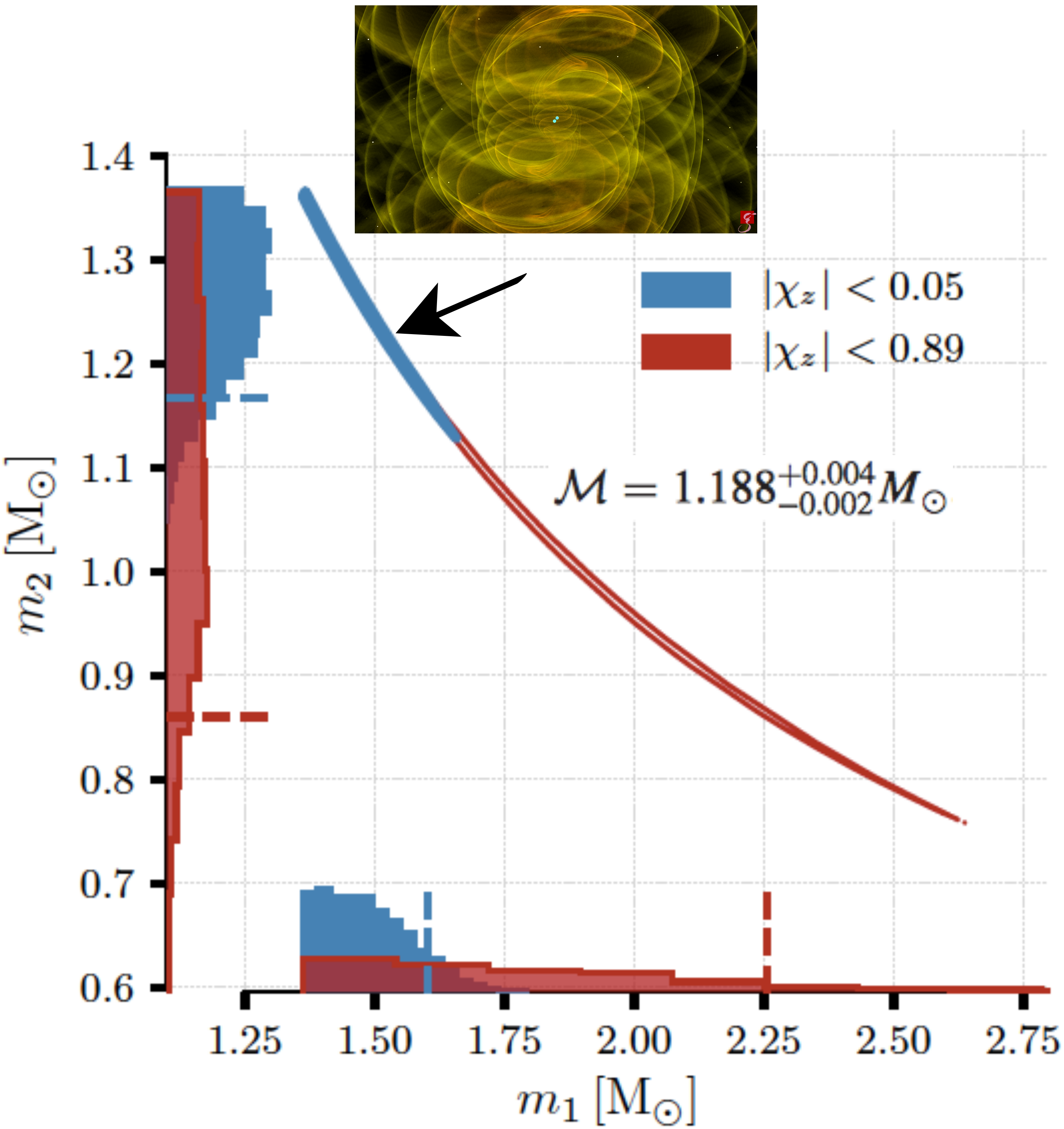
$$A(r) = A_\nu(r) + A_{\rm tides}(r)$$



(Hinderer et al. 2016, Steinhoff et al. 2016, see also Bernuzzi et al. 15)

Tides make **gravitational** interaction **more attractive**

Unveiling binary neutron star properties: masses

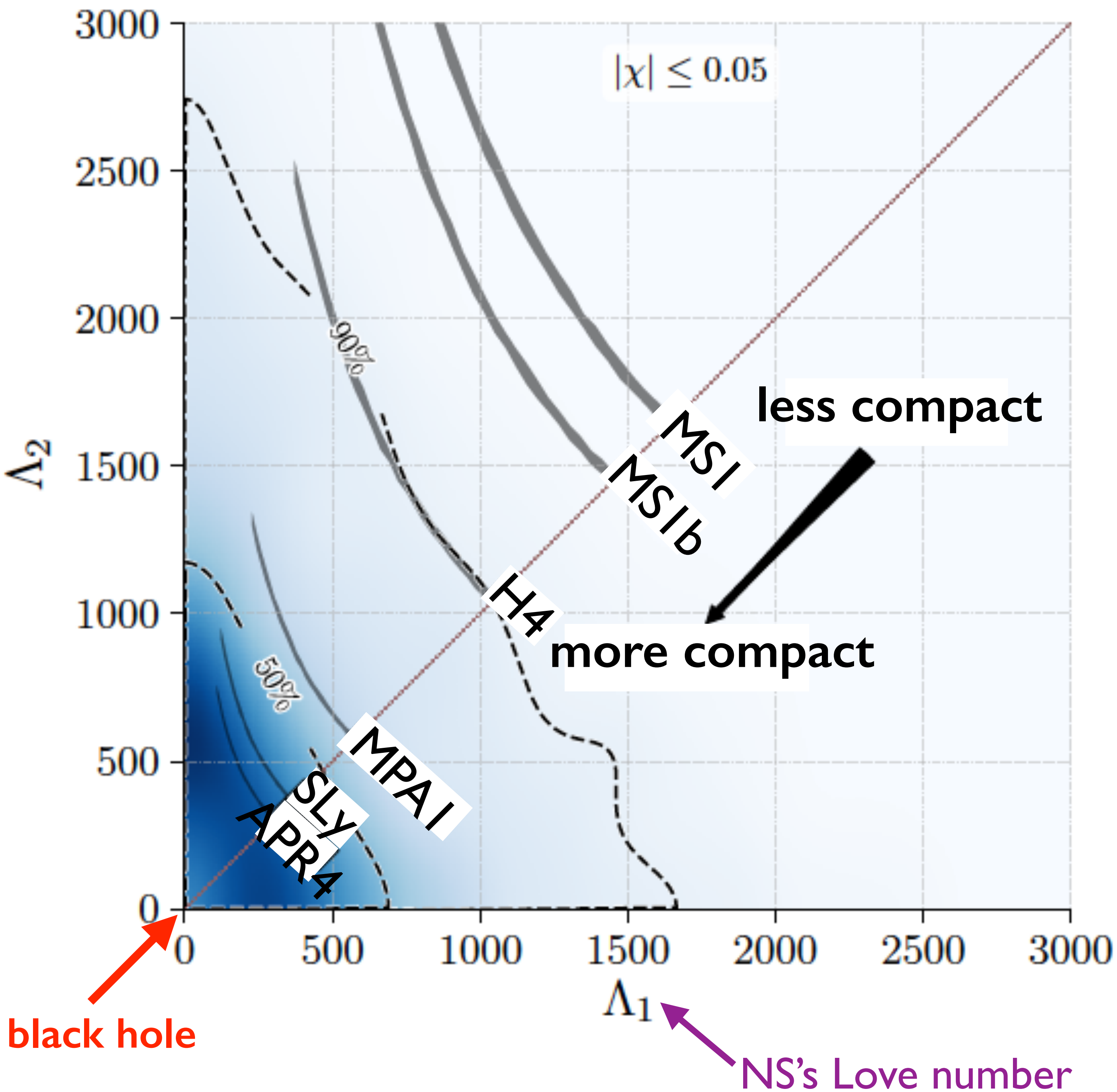


(Abbott et al. PRL 119 (2017) 161101)

- **Degeneracy** between masses and spins.
- **Fastest-spinning** neutron star has (dimensionless) **spin ~0.4.**
- Observation of **binary pulsars** in our galaxy indicate spins are **not larger than ~0.04**.

Constraining Love numbers with GW170817

(Abbott et al. PRL 119 (2017) 161101)



Depends on EOS & compactness

$$\Lambda = \frac{\lambda}{m_{NS}^5} = \frac{2}{3} k_2 \left(\frac{R_{NS} c^2}{G m_{NS}} \right)^5$$

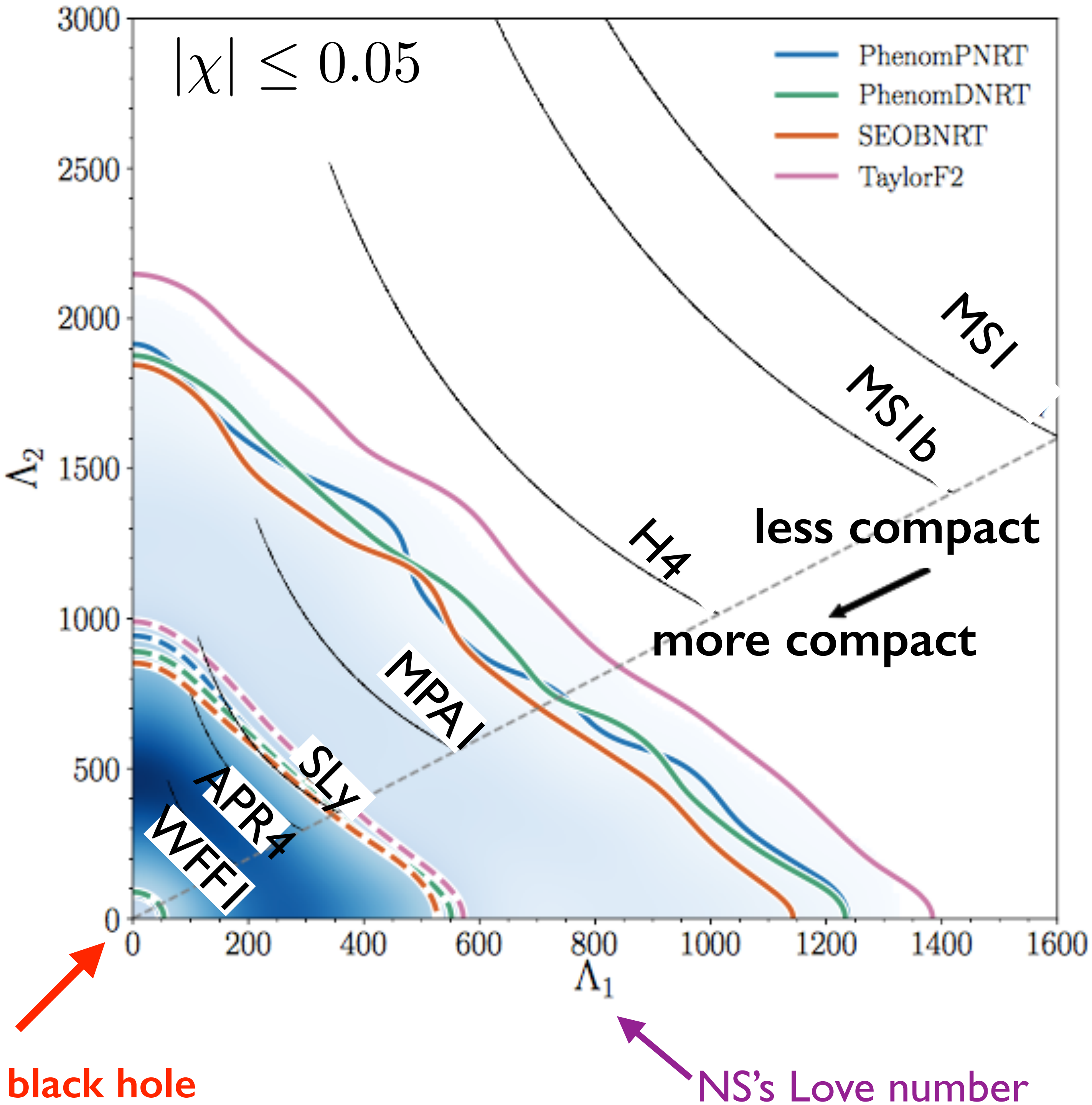
- **Effective tidal deformability** enters **GW phase at 5PN order:**

$$\tilde{\Lambda} = \frac{16}{13} \frac{(m_1 + 12m_2)m_1^4 \Lambda_1 + (m_2 + 12m_1)m_2^4 \Lambda_2}{(m_1 + m_2)^5}$$

- With **state-of-art waveform models, tides are reduced by ~20%.** New analyses are out.

Constraining Love numbers with GW170817: new analysis

(Abbott et al. arXiv:1805.11579)



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- **Effective tidal deformability** enters **GW phase at 5PN order:**

$$\tilde{\Lambda} = \frac{16}{13} \frac{(m_1 + 12 m_2) m_1^4 \Lambda_1 + (m_2 + 12 m_1) m_2^4 \Lambda_2}{(m_1 + m_2)^5}$$

Including dynamical tidal effects in EOB model

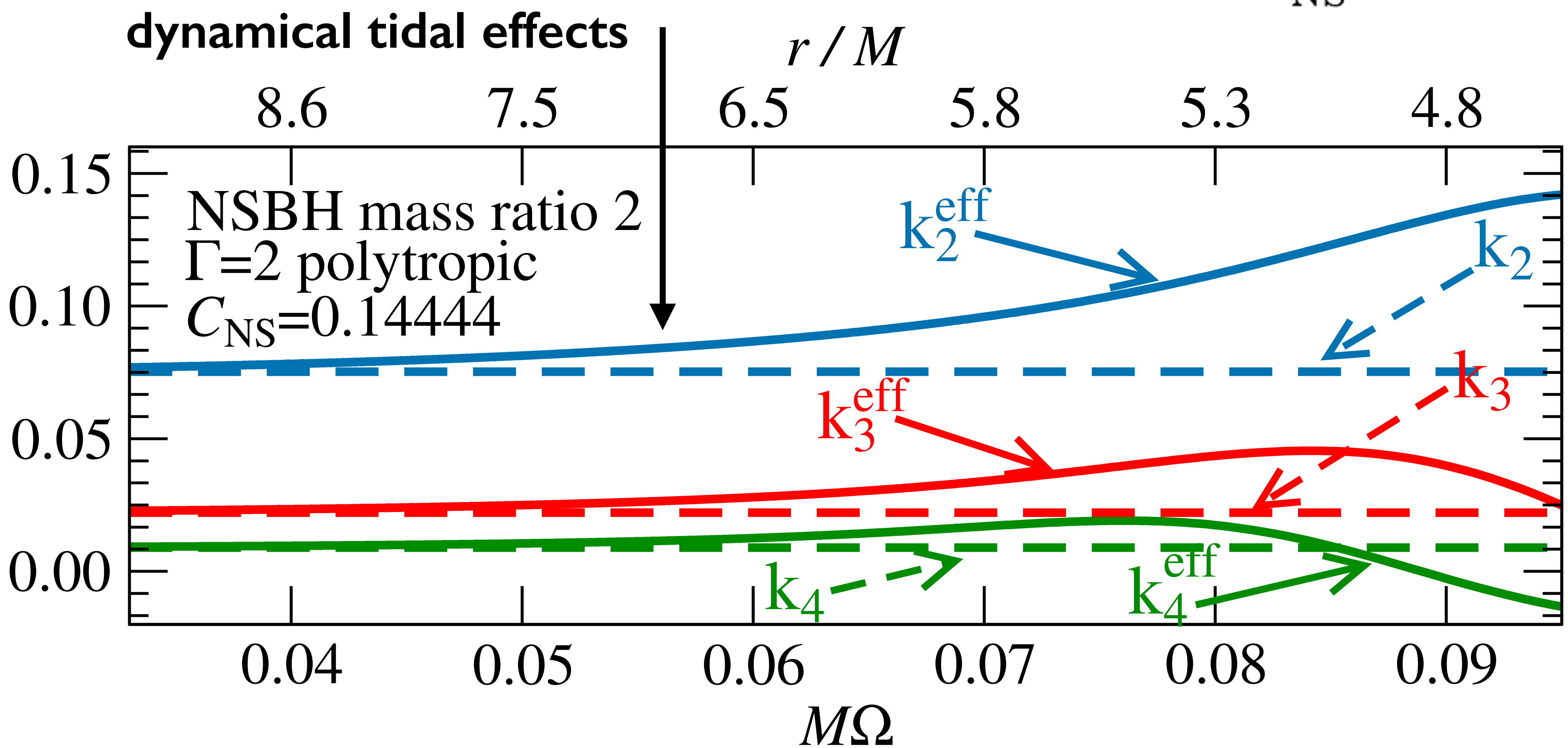
- **Dynamical tides**: NS's f-modes can be excited toward merger.
- **Tidal force frequency approaches** eigenfrequency of **NS's normal modes of oscillation**, resulting in an enhanced, more complex tidal response.

(Kokkotas et al. 1995, Flanagan et al. 08, Hinderer, … AB et al. 16, Steinhoff, … AB et al. 16)

(Hinderer, …, AB et al. 16)

$$k_2 = \frac{3}{2}\frac{\lambda}{R_{\rm NS}^5}$$

NS's effective response to dynamical tidal effects



Systematics versus statistical errors for GW170817

$|\chi| \leq 0.05$

EOB with dynamical tides

(Bohe', … AB et al. 15, Hinderer, … AB et al. 16, Steinhoff, … AB et al. 16)

EOB with adiabatic tides

(Damour et al. 14, Bernuzzi et al. 15, Nagar et al. 18)

Low-spin prior, $\chi_i \leq 0.05$	SEOBNRv4T	TEOBResumS	PhenomDNRT
Detector frame chirp mass $\mathcal{M}^{\rm det}$	$1.1975^{+0.0001}_{-0.0001} M_\odot$	$1.1975^{+0.0001}_{-0.0001} M_\odot$	$1.1975^{+0.0001}_{-0.0001} M_\odot$
Chirp mass $\mathcal{M}$	$1.186^{+0.001}_{-0.001} M_\odot$	$1.186^{+0.001}_{-0.001} M_\odot$	$1.186^{+0.001}_{-0.001}$
Primary mass m_1	$(1.36,\ 1.56)\ M_\odot$	$(1.36,\ 1.53)\ M_\odot$	$(1.36,\ 1.57)\ M_\odot$
Secondary mass m_2	$(1.19,\ 1.36)\ M_\odot$	$(1.22,\ 1.36)\ M_\odot$	$(1.19,\ 1.36)\ M_\odot$
Total mass m	$2.73^{+0.04}_{-0.01} M_\odot$	$2.73^{+0.03}_{-0.01} M_\odot$	$2.73^{+0.04}_{-0.01} M_\odot$
Mass ratio q	$(0.76,\ 1.00)$	$(0.79,\ 1.00)$	$(0.76,\ 1.00)$
Effective spin $\chi_{\rm eff}$	$0.00^{+0.02}_{-0.01}$	$0.00^{+0.01}_{-0.01}$	$0.00^{+0.02}_{-0.01}$
Primary dimensionless spin χ_1	$(0.00,\ 0.03)$	$(0.00,\ 0.02)$	$(0.00,\ 0.03)$
Secondary dimensionless spin χ_2	$(0.00,\ 0.03)$	$(0.00,\ 0.03)$	$(0.00,\ 0.03)$
Tidal deformability $\tilde{\Lambda}$ with flat prior (symmetric/HPD)	$280^{+430}_{-220}/280^{+280}_{-280}$	$340^{+520}_{-260}/\ 340^{+350}_{-330}$	$310^{+510}_{-240}/\ 310^{+380}_{-290}$

(Khan et al 14, Dietrich et al. 18)

Boson stars as black-hole/neutron-star mimickers

$$S = \int d^4x\sqrt{-g}\left[\frac{R}{16\pi} - \nabla^\alpha\Phi\nabla_\alpha\Phi^* - V(|\Phi|^2)\right]$$

- **Boson stars** are self-gravitating configurations of a complex scalar field

	$V(\|\Phi\|^2)$	$M_{\rm max}$	Compactness
Mini BS	$\mu^2\Phi^2$	$\left(\frac{85\text{peV}}{\mu}\right) M_\odot$	0.08
Massive BS	$\mu^2\Phi^2 + \frac{\lambda}{2}\|\Phi\|^4$	$\sqrt{\lambda}\left(\frac{270\text{MeV}}{\mu}\right)^2 M_\odot$	0.158
Neutron star		$2-4\,M_\odot$	0.3
Solitonic BS	$\mu^2\Phi^2\left(1-\frac{2\|\Phi\|^2}{\sigma_0^2}\right)^2$	$\left(\frac{\mu}{\sigma_0}\right)^2\left(\frac{700\text{TeV}}{\mu}\right)^3 M_\odot$	0.349
Black hole		∞	0.5

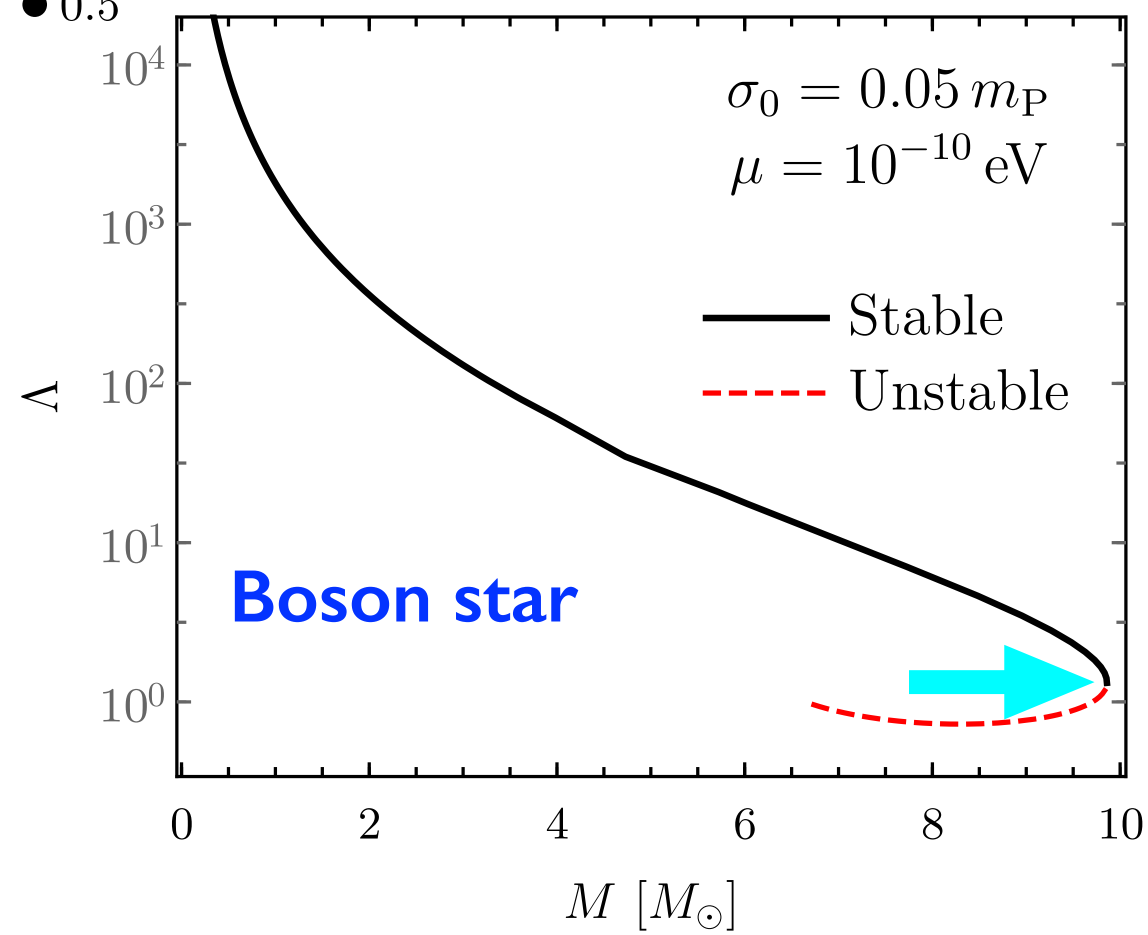
(credit: Sennett)

$$\mathcal{C} = \frac{GM}{Rc^2}$$

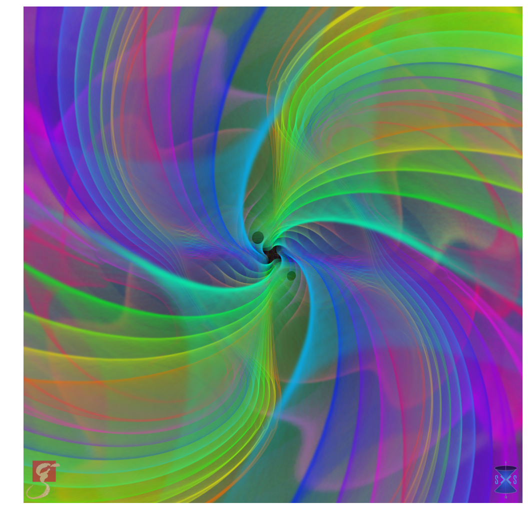
(Sennett…AB et al. 17)
(see also Cardoso et al. 17, Johnson-Mcdaniel 18)

$$\Lambda = \lambda/M^5$$

- **Black holes:** $\Lambda = 0$
- **Neutron stars:** $\Lambda_{\rm min} \sim 10$
- **Boson stars:** $\Lambda_{\rm min} \sim 1$

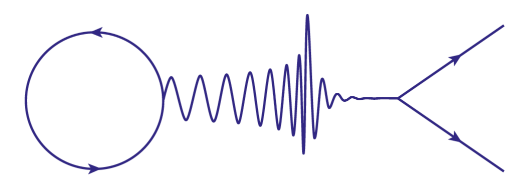


The new era of precision gravitational-wave astrophysics



- Theoretical groundwork in **analytical and numerical relativity** has allowed us to build **faithful waveform models** to **search** for signals, **infer properties** and **test GR**.

- EOB theory **combines AR (PN, PM, GSF) & NR results** in suitable manner to **improve** waveform **accuracy**.



- We can now **learn about gravity** in the genuinely **highly dynamical, strong field** regime.

- We can now **unveil properties of neutron stars** unaccessible in other ways.

- **To take full advantage of discovery potential** in next years and decades **we need** to continue to make **precise theoretical predictions.**

- **Analytical relativity** work **still needed** to cover the **entire parameter space.** New **opportunities** for **theoretical physicists to contribute!**