

# Effective-One-Body Theory, & Applications to Gravitational-Wave Observations

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# Outline

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- **Lectures I-II: Basics of gravitational waves**
- **Lectures III-IV: Motivations and development of effective-one-body (EOB) theory (two-body dynamics and waveforms)**
- **Lecture V: Using waveform models to infer astrophysical and cosmological information through gravitational-wave observations**
- **Lecture VI: Using waveform models to probe dynamical gravity and extreme matter with gravitational-wave observations**

(NR simulation: Ossokine, AB & SXS @AEI)

(visualization credit: Benger @ Airborne Hydro Mapping Software & Haas @AEI)



## References:

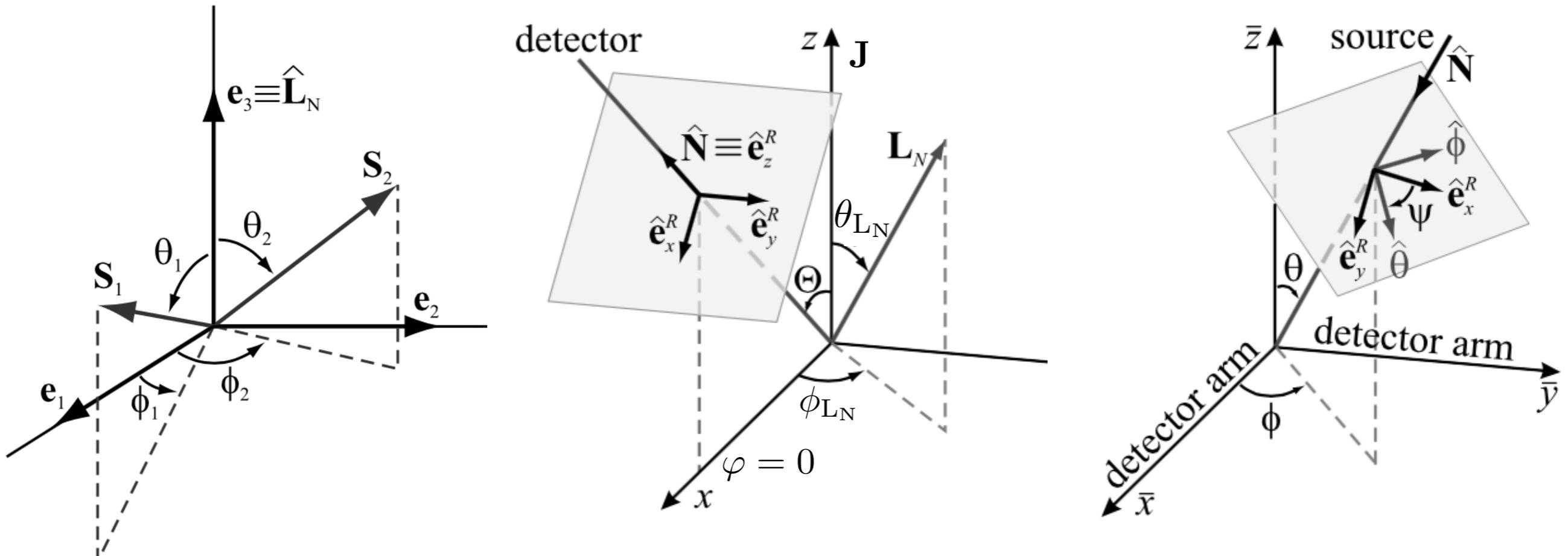
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- **M. Maggiore's books**: “Gravitational Waves Volume I: Theory and Experiments” (2007) & “Gravitational Waves Volume II: Astrophysics and Cosmology” (2018).
- **E. Poisson & C. Will's book**: “Gravity” (2015).
- **E.E. Flanagan & S.A. Hughes' review**: arXiv:0501041.
- **AB's Les Houches School Proceedings**: arXiv:0709.4682.
- **AB & B. Sathyaprakash's review**: arXiv:1410.7832.
- **UMD/AEI graduate course on GW Physics & Astrophysics taught in Winter-Spring 2017**: <http://www.aei.mpg.de/2000472>.

# Parameters in GW from quasi-circular binary systems

| Binary             |   | GW propagation                                    | Detector orientation |
|--------------------|---|---|----------------------|
| $M, \nu, S_1, S_2$ | $\theta_{S_1}, \theta_{S_2}, \phi_{S_1} - \phi_{S_2}$ | $\theta_{LN}, \phi_{LN}, \phi_{S_1} + \phi_{S_2}$ | $\Theta, \varphi$    |
| Basic              | Local   | Directional                                       | $\theta, \phi, \psi$ |

plus distance D & time at coalescence  $t_c$



(AB, Chen & Vallisneri 03)

- **15 parameters** for waveforms from **quasi-circular** binary systems

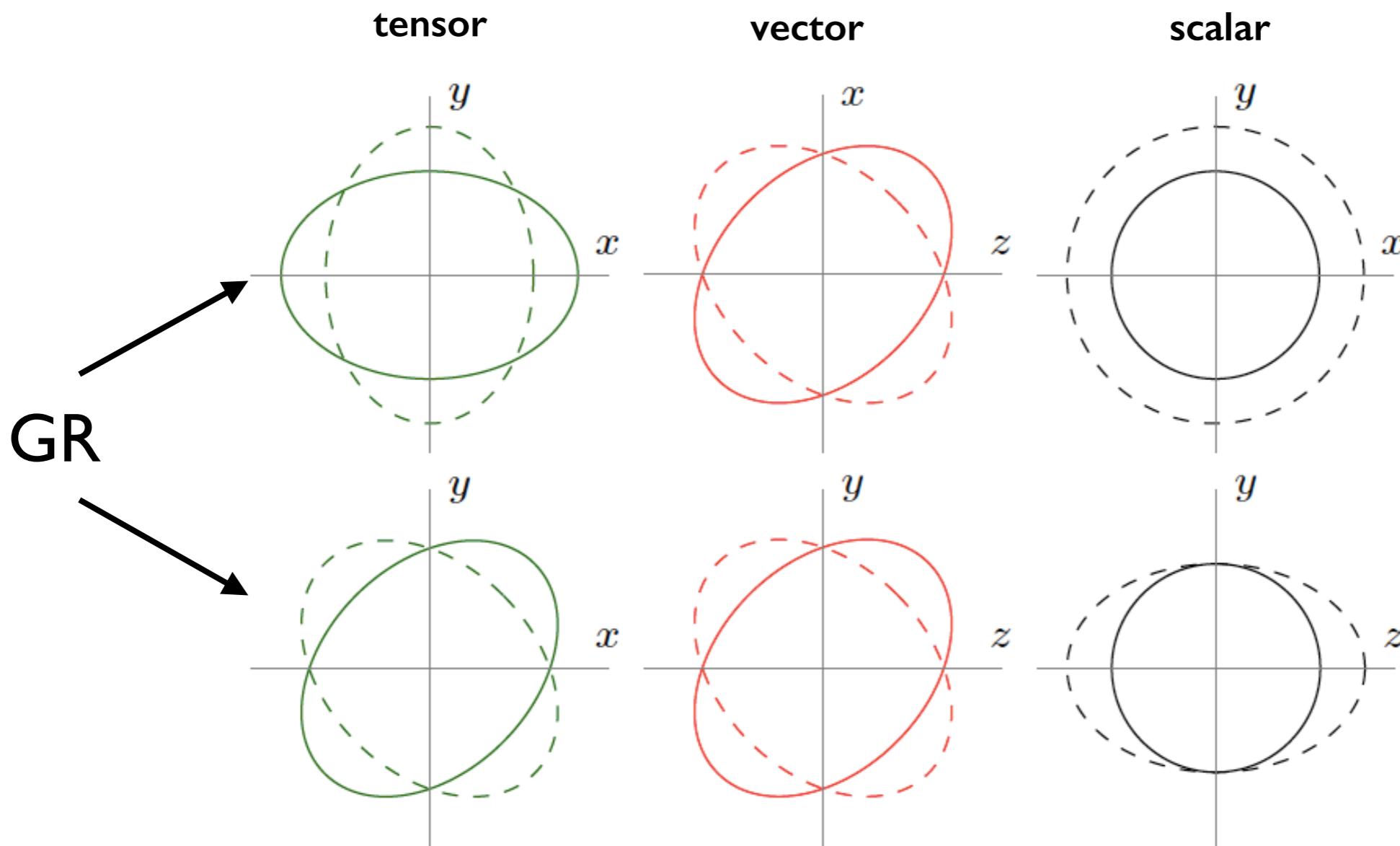
# GW polarizations in gravity

- Generic metric theories of gravity have 6 geometrically distinct polarizations:

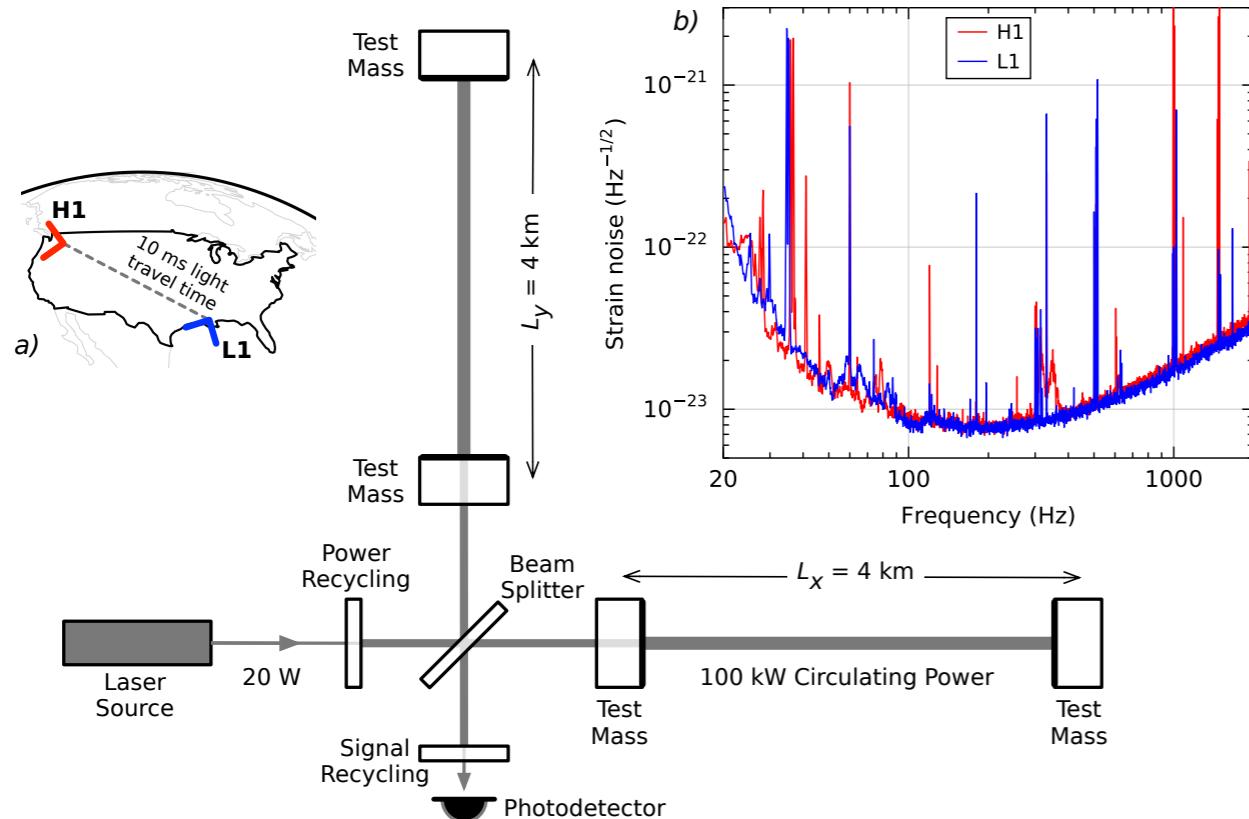
(Will 72)

$$h_{ij}(\mathbf{x}) = \sum_A h_A(\mathbf{x}) e_{ij}^A \quad A = 1, \dots, 6$$

**6 amplitudes**      **6 polarization tensors**



# Detector antenna patterns for different GW polarizations



$$D^{ij} = \frac{1}{2}(d_x^i d_x^j - d_y^i d_y^j)$$

**differential-arm detector**

**detector tensor**

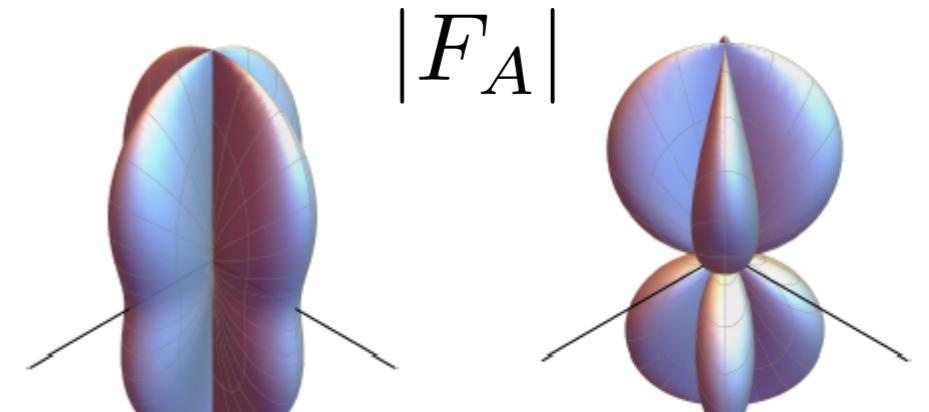
$$h_I(t, \mathbf{x}_I) = D_I^{ij} h_{ij}(t, \mathbf{x}_I)$$

$$= \sum_A h_A(t, \mathbf{x}_I) (D_I^{ij} e_{ij}^A)$$

**depends on  
GW source**

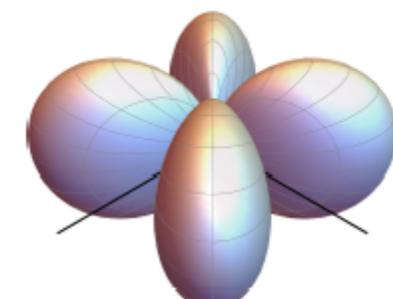
**depends only on  
geometry,  $F_A$**

- Angular responses of a differential-arm detector

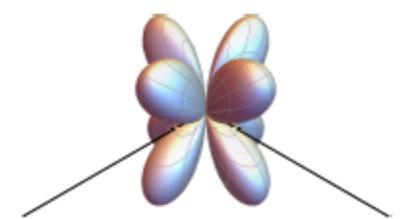


(a) Plus (+)

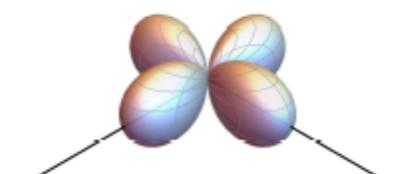
(b) Cross (x)



(c) Vector-x (x)



(d) Vector-y (y)



(e) Scalar (s)

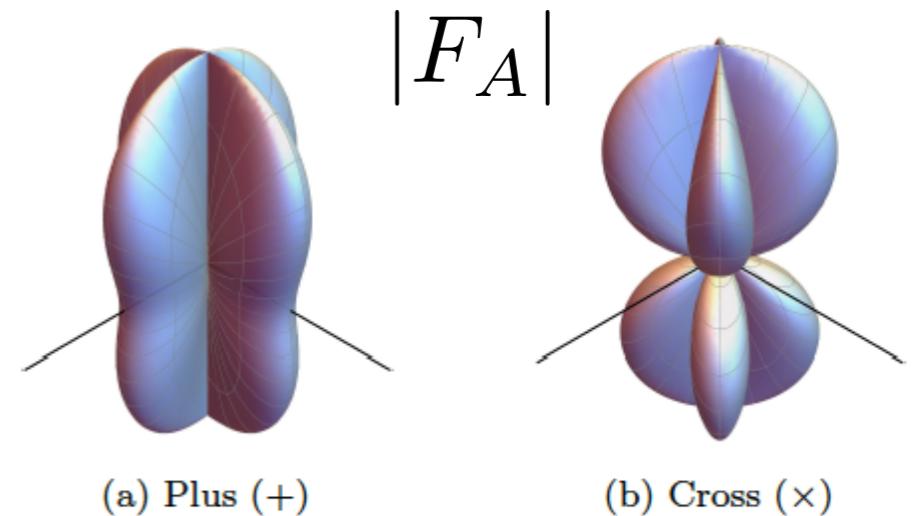
# Testing extra GW polarizations with two LIGOs & Virgo

$$\begin{aligned}
 h_I(t, \mathbf{x}_I) &= D_I^{ij} h_{ij}(t, \mathbf{x}_I) \\
 &= \sum_A h_A(t, \mathbf{x}_I) (D_I^{ij} e_{ij}^A)
 \end{aligned}$$

↓  
 detector tensor  
 ↑  
 depends on  
 GW source

↑ depends only on  
 geometry,  $F_A$

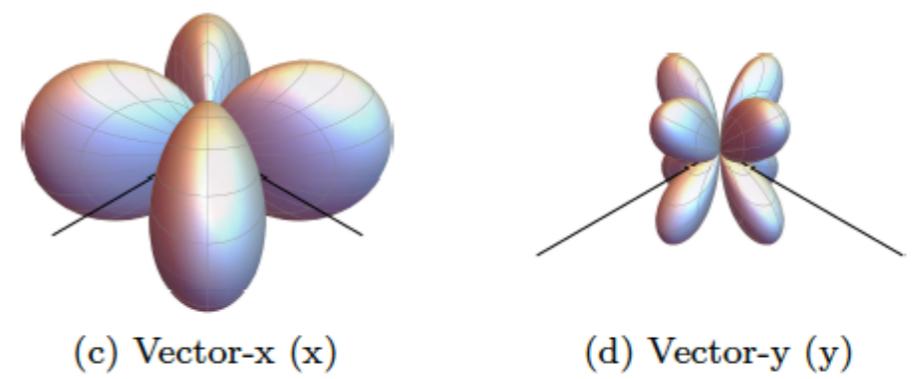
- Angular responses of a differential-arm detector



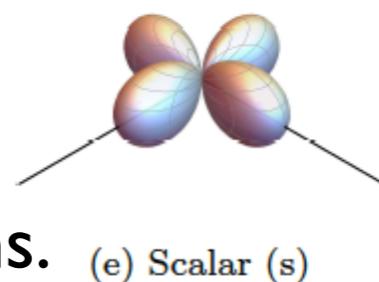
examples of antenna patterns:

$$\begin{aligned}
 F_+(\vartheta, \varphi, \psi) &= -\frac{1}{2} (1 + \cos^2 \vartheta) \cos 2\varphi \cos 2\psi \\
 \text{tensor} &\quad - \cos \vartheta \sin 2\varphi \sin 2\psi ,
 \end{aligned}$$

$$\begin{aligned}
 F_{b/1}(\vartheta, \varphi, \psi) &= \mp \frac{1}{2} \sin^2 \vartheta \cos 2\varphi \\
 \text{scalar} &
 \end{aligned}$$



- Two LIGOs are nearly co-aligned, approximately sensitive to the same linear combination of polarizations.



# GW signal observed by ground-based detectors

- Polarizations for **non-spinning compact objects**:

$$h_+(t) = \frac{2\mathcal{M}}{D} (\mathcal{M}\omega)^{2/3} (1 + \cos^2 \Theta) \cos(2\Phi + 2\Phi_0)$$

$$h_\times(t) = \frac{4\mathcal{M}}{D} (\mathcal{M}\omega)^{2/3} \cos \Theta \sin(2\Phi + 2\Phi_0) \quad (\text{Finn \& Chernoff 93})$$

- Detector response:  $h(t) = h_+(t) F_+ + h_\times(t) F_\times$

$$\tilde{F}_\times = \frac{2 \cos \Theta F_\times}{[(1+\cos^2 \Theta)^2 F_+^2 + 4 \cos^2 \Theta F_\times^2]^{1/2}} \quad \tilde{F}_+ = \frac{(1+\cos^2 \Theta) F_+}{[(1+\cos^2 \Theta)^2 F_+^2 + 4 \cos^2 \Theta F_\times^2]^{1/2}}$$

$$\tilde{F}_+^2 + \tilde{F}_\times^2 = 1 \quad \Rightarrow \quad \cos \alpha \equiv \tilde{F}_+ \quad \sin \alpha \equiv \tilde{F}_\times$$

- Defining:  $\mathcal{A}(\theta, \phi, \psi; \Theta) = [(1 + \cos^2 \Theta)^2 F_+^2 + 4 \cos^2 \Theta F_\times^2]^{1/2}$

$$\tan \alpha(\theta, \phi, \psi; \Theta) = \frac{2 \cos \Theta F_\times}{(1+\cos^2 \Theta) F_+}$$

## GW signal observed by ground-based detectors (contd.)

- Polarizations for **non-spinning compact objects**:

$$h_+(t) = \frac{2\mathcal{M}}{D} (\mathcal{M}\omega)^{2/3} (1 + \cos^2 \Theta) \cos(2\Phi + 2\Phi_0)$$

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- Detector response:  $h(t) = h_+(t) F_+ + h_\times(t) F_\times$

$$h(t) = \frac{2\mathcal{M}}{D} \mathcal{A} [\mathcal{M}\omega(t)]^{2/3} \cos(2\Phi(t) + 2\Phi_0 - \alpha)$$

$\uparrow$  averaging over angles  
 $\overline{\mathcal{A}^2} = 16/25$

$\uparrow$  it can be re-absorbed in initial phase

- Defining:  $\mathcal{A}(\theta, \phi, \psi; \Theta) = [(1 + \cos^2 \Theta)^2 F_+^2 + 4 \cos^2 \Theta F_\times^2]^{1/2}$

$$\tan \alpha(\theta, \phi, \psi; \Theta) = \frac{2 \cos \Theta F_\times}{(1 + \cos^2 \Theta) F_+}$$

## GW signal observed by ground-based detectors (contd.)

- Polarizations for **non-spinning compact objects**:

$$h_+(t) = \frac{2\mathcal{M}}{D} (\mathcal{M}\omega)^{2/3} (1 + \cos^2 \Theta) \cos(2\Phi + 2\Phi_0)$$

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- Detector response:  $h(t) = h_+(t) F_+ + h_\times(t) F_\times$

$$h(t) = \frac{2\mathcal{M}}{D} \mathcal{A} [\mathcal{M}\omega(t)]^{2/3} \cos(2\Phi(t) + 2\Phi_0 - \alpha)$$

$\uparrow$  averaging over angles  
 $\overline{\mathcal{A}^2} = 16/25$

$\uparrow$  it can be re-absorbed in initial phase

- Fourier transform:

$$\tilde{h}(f) = \int_{-\infty}^{+\infty} e^{2\pi ift} h(t) dt = \frac{1}{2} \int_{-\infty}^{+\infty} dt \mathcal{A}(t) [e^{2\pi ift+i\Phi_{\text{GW}}(t)} + e^{2\pi ift-i\Phi_{\text{GW}}(t)}]$$

## PN templates in stationary phase approximation

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$$\tilde{h}(f) = \frac{1}{2} \int_{-\infty}^{+\infty} dt \cancel{A(t)} [e^{2\pi ift + i\Phi_{\text{GW}}(t)} + e^{2\pi ift - i\Phi_{\text{GW}}(t)}]$$

assuming  $f > 0$

- Dominant contribution from the vicinity of the *stationary* points in the phase  
(Sathyaprakash & Dhurandhar 91)

Assuming  $f > 0$  and posing  $\psi(t) \equiv 2\pi f t - \Phi_{\text{GW}}$

Imposing  $\left(\frac{d\psi}{dt}\right)_{t_f} = 0 \Rightarrow \left(\frac{d\Phi_{\text{GW}}}{dt}\right)_{t_f} = 2\pi f = 2\pi F(t_f)$

Expanding the phase:  $\psi(t_f) = 2\pi f t_f - \Phi_{\text{GW}}(t_f) - \pi \dot{F}(t_f) (t - t_f)^2$

$$\tilde{h}_{\text{SPA}}(f) = \frac{1}{2} \frac{A(t_f)}{\sqrt{\dot{F}(t_f)}} e^{i(2\pi f t_f - \Phi_{\text{GW}}(t_f)) - i\pi/4}$$

- How do we compute  $\Phi_{\text{GW}}(t_f)$  and  $\dot{F}(t_f)$ ?

## PN templates in stationary phase approximation (contd.)

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- We need to solve  $v^3 = \dot{\Phi}_{\text{GW}} M/2$  and  $\dot{E}(v) = -F(v)$

$$t(v) = t_c + M \int_v^{v_c} dv \frac{E'(v)}{F(v)} \quad E'(v) \equiv \frac{dE}{dv}$$

$$\Phi_{\text{GW}}(v) = \Phi_c + 2 \int_v^{v_c} dv v^3 \frac{E'(v)}{F(v)} \quad (\text{Sathyaprakash \& Dhurandhar 91})$$

$$\Rightarrow \psi(f) = 2\pi f t_c - \Phi_c - \pi/4 + 2 \int_v^{v_c} (v_c^3 - v^3) \frac{E'(v)}{F(v)} dv$$

$$\Rightarrow \dot{F}(t_f) = \frac{\dot{\omega}}{\pi} = \frac{96}{5} \frac{1}{\pi} \nu M^{5/3} \omega^{11/3}$$

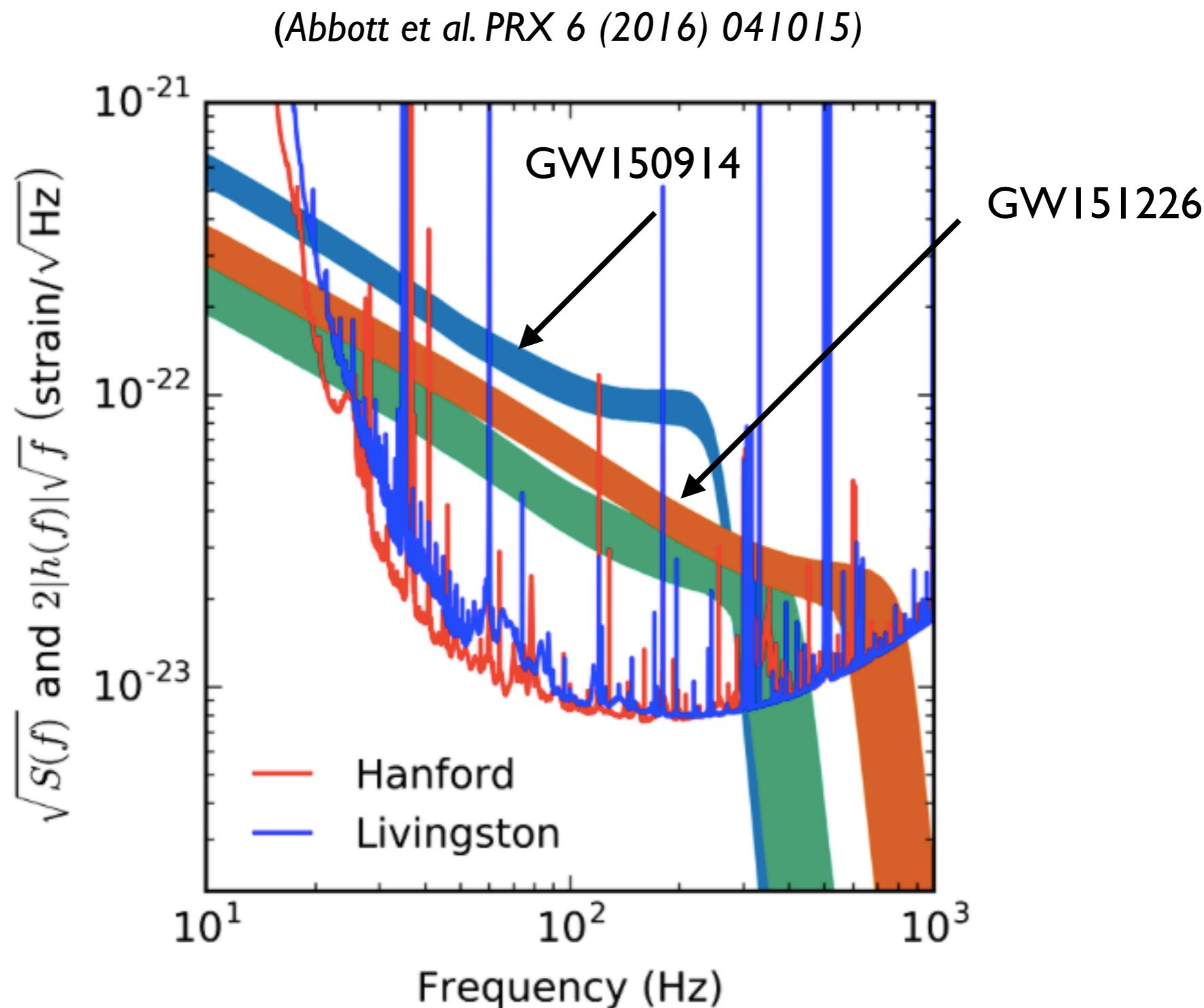
$$\tilde{h}_{\text{SPA}}(f) = \mathcal{A}_{\text{SPA}}(f) e^{i\psi_{\text{SPA}}(f)} \quad \mathcal{A}_{\text{SPA}}(f) = \frac{A}{r} \frac{1}{\pi^{2/3}} \left(\frac{5}{96}\right)^{1/2} \mathcal{M}^{5/6} f^{-7/6}$$

- $\psi_{\text{SPA}}(f) = 2\pi f t_c - \Phi_c - \pi/4 + f^{-5/3} (\psi_0 + \psi_1 f^{2/3} + \psi_{3/2} f + \dots)$

$$\psi_0 = \psi_0(\mathcal{M}), \quad \psi_1 = \psi_1(m_1, m_2), \quad \psi_{3/2} = \psi_{3/2}(m_1, m_2, \mathbf{S} \cdot \mathbf{L}), \dots$$

# GW amplitude in frequency domain during inspiral

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# PN templates in stationary phase approximation: TaylorF2

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$$\tilde{h}(f) = \mathcal{A}_{\text{SPA}}(f) e^{i\psi_{\text{SPA}}(f)}$$

$$\begin{aligned}
 \psi_{\text{SPA}}(f) = & 2\pi f t_c - \Phi_c - \pi/4 + \frac{3}{128} (\pi \mathcal{M} f)^{-5/3} \{ 1 + \\
 & -\frac{5\hat{\alpha}^2}{336\omega_{\text{BD}}} \nu^{2/5} (\pi \mathcal{M} f)^{-2/3} - \frac{128}{3} \frac{\pi^2 D \mathcal{M}}{\lambda_g^2 (1+z)} (\pi \mathcal{M} f)^{2/3} \} \\
 & + \left( \frac{3715}{756} + \frac{55}{9} \nu \right) \nu^{-2/5} (\pi \mathcal{M} f)^{2/3} - 16\pi \nu^{-3/5} (\pi \mathcal{M} f) + 4\beta \nu^{-3/5} (\pi \mathcal{M} f) \\
 & + \left( \frac{15293365}{508032} + \frac{27145}{504} \nu + \frac{3085}{72} \nu^2 \right) \nu^{-4/5} (\pi \mathcal{M} f)^{4/3} - 10\sigma \nu^{-4/5} (\pi \mathcal{M} f)^{4/3} \}
 \end{aligned}$$

dipole radiation →

graviton with non zero mass ↗

spin-orbit ↓

I.5PN

spin-spin ↑

IPN

IPN

I.5PN

I.5PN

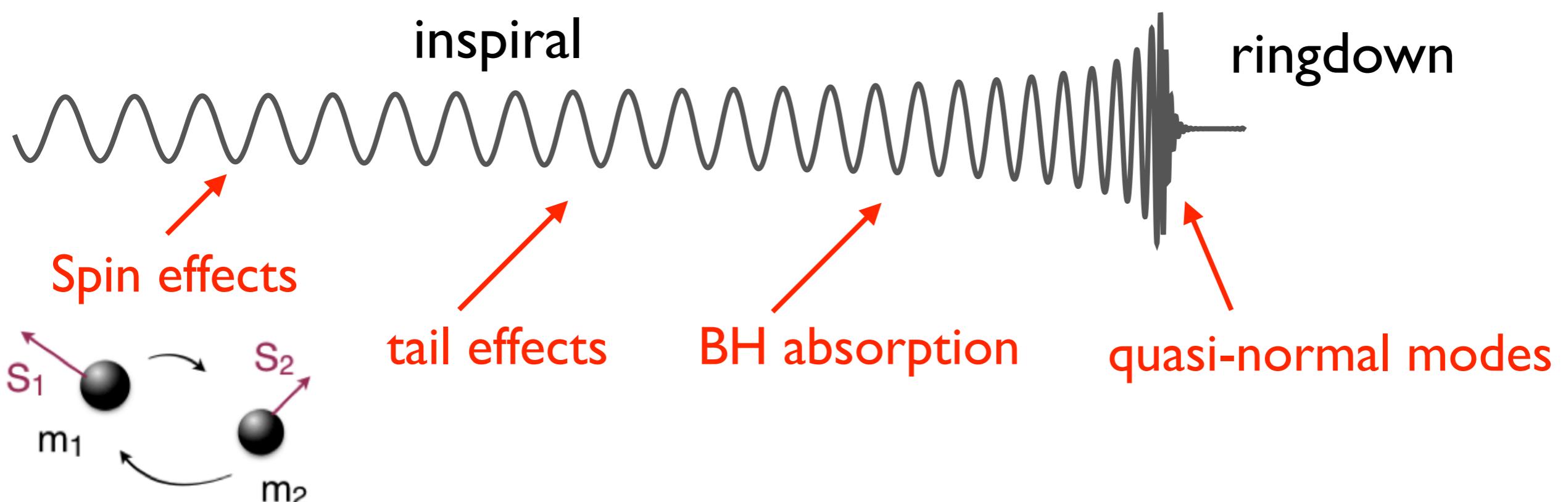
2PN

$$\beta = \frac{1}{12} \sum_{i=1}^2 \chi_i \left[ 113 \frac{m_i^2}{M^2} + 75\nu \right] \hat{\mathbf{L}} \cdot \hat{\mathbf{S}}_i, \quad \sigma = \frac{\nu}{48} \chi_1 \chi_2 \left( -27 \hat{\mathbf{S}}_1 \cdot \hat{\mathbf{S}}_2 + 721 \hat{\mathbf{L}} \cdot \hat{\mathbf{S}}_1 \hat{\mathbf{L}} \cdot \hat{\mathbf{S}}_2 \right)$$

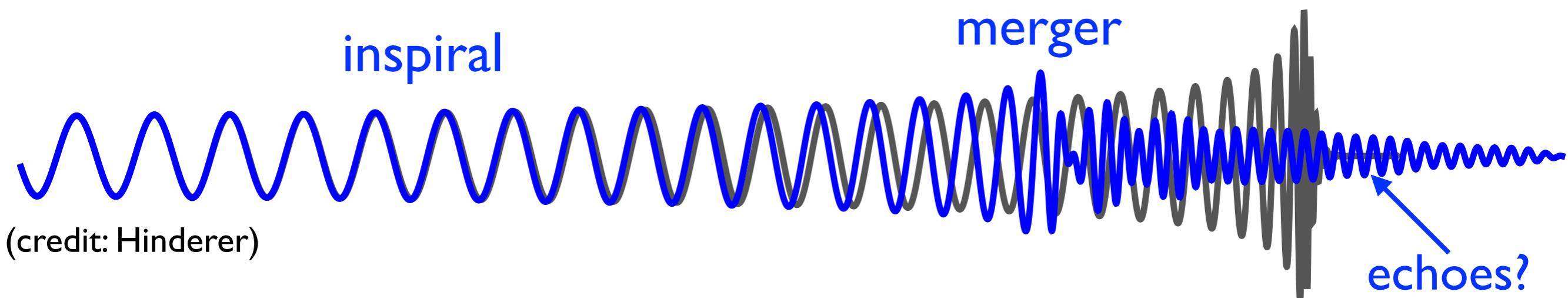
$$\chi_i = \frac{S_i}{m_i^2}$$

# Waveforms encode plethora of physical effects

- Binary black hole



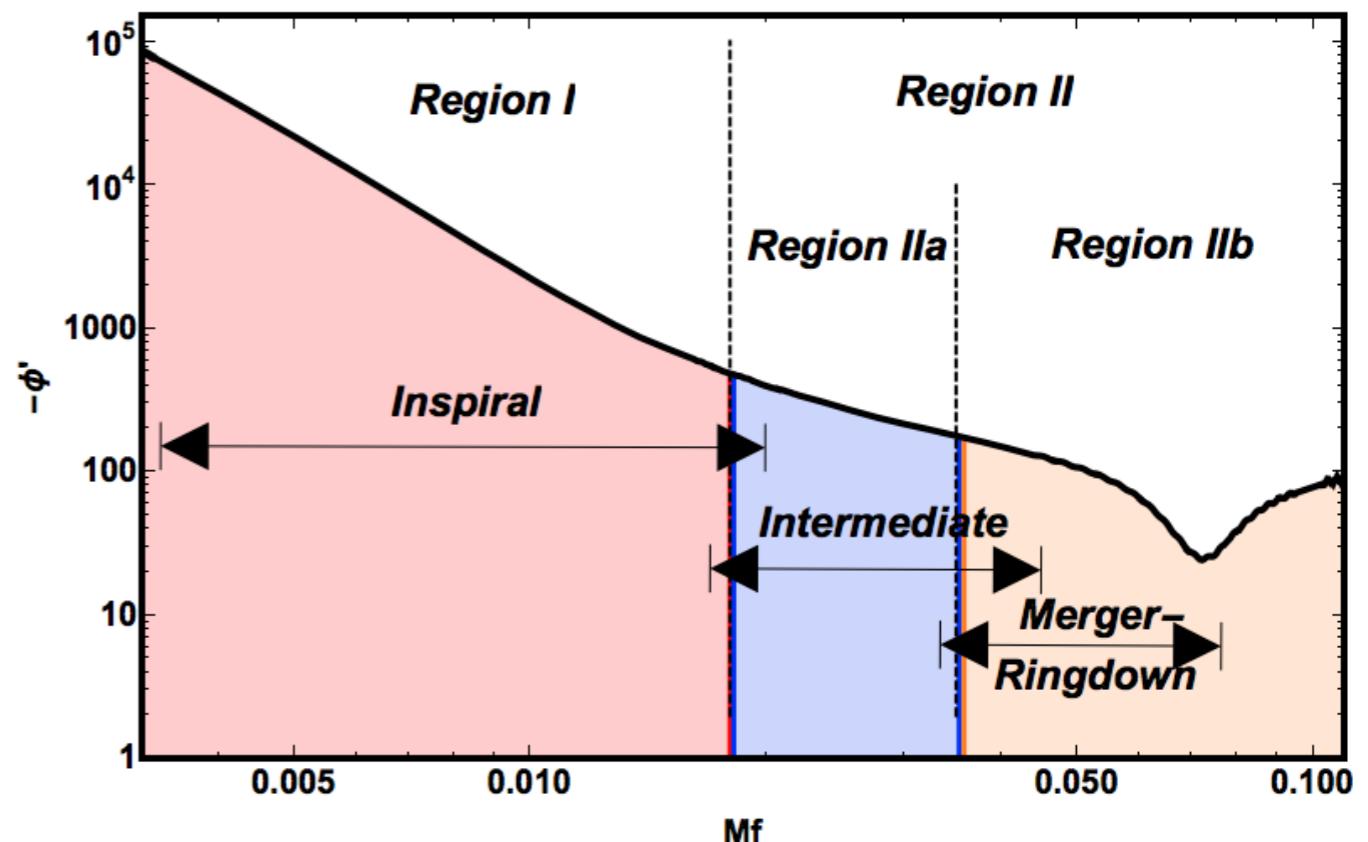
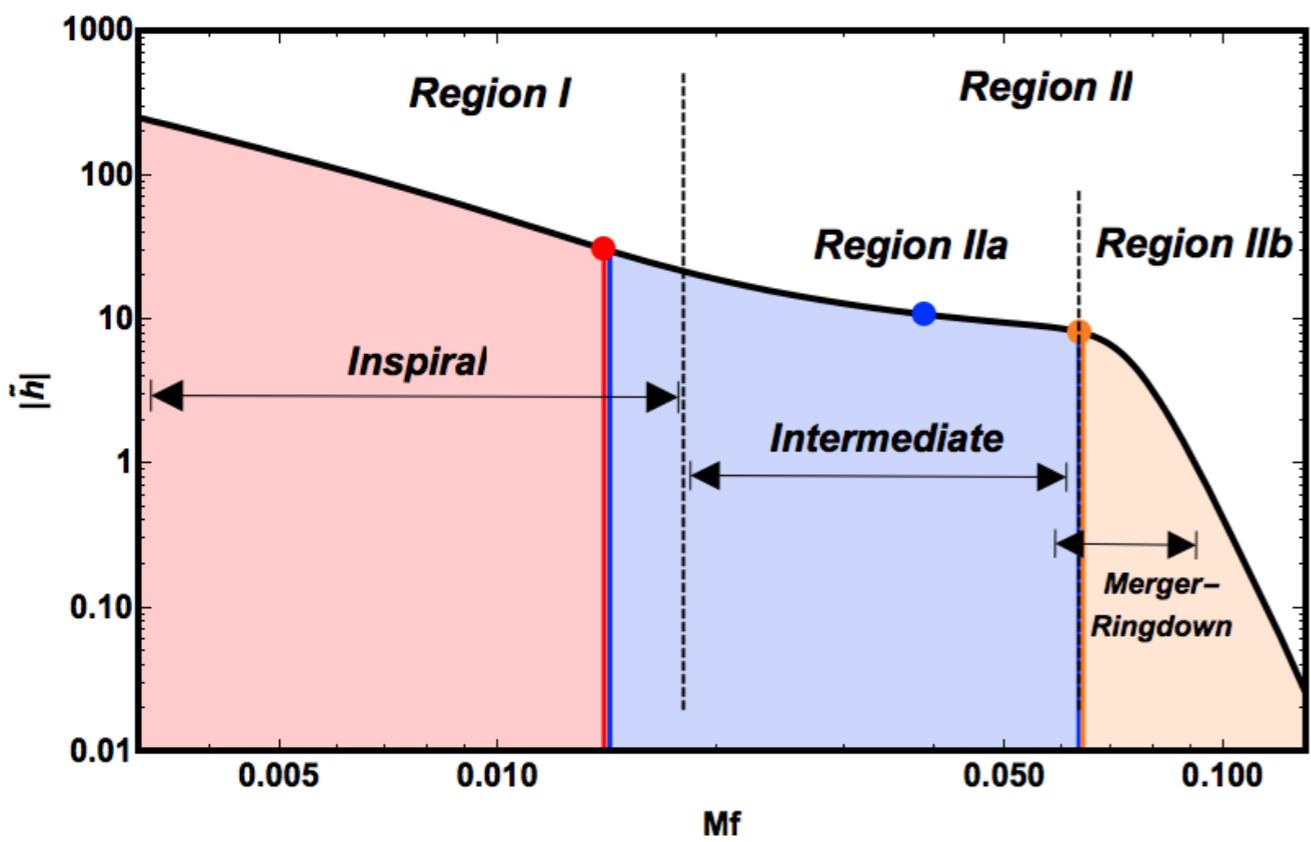
- Compact-object binary with **matter** or in **modified theory** to GR?



# Phenomenological waveforms used in O1 & O2 follow-up analyses

- First works in mid-late 2000 (Pan, AB et al. 07, Ajith et al. 07, Santamaria et al. 10)
- **Fast, frequency-domain** waveform model hybridizing and fitting EOB & NR (Khan et al. 15; Husa et al. 15; Hannam et al. 13)

$$\tilde{h}(f; \lambda_i) = \mathcal{A}(f; \lambda_i) e^{i\phi(f; \lambda_i)}$$



# On phenomenological inspiral-merger-ringdown waveforms

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(Santamaría, Ohme et al. 10)

$$\tilde{h}(f; \lambda_i) = \mathcal{A}(f; \lambda_i) e^{i\phi(f; \lambda_i)}$$

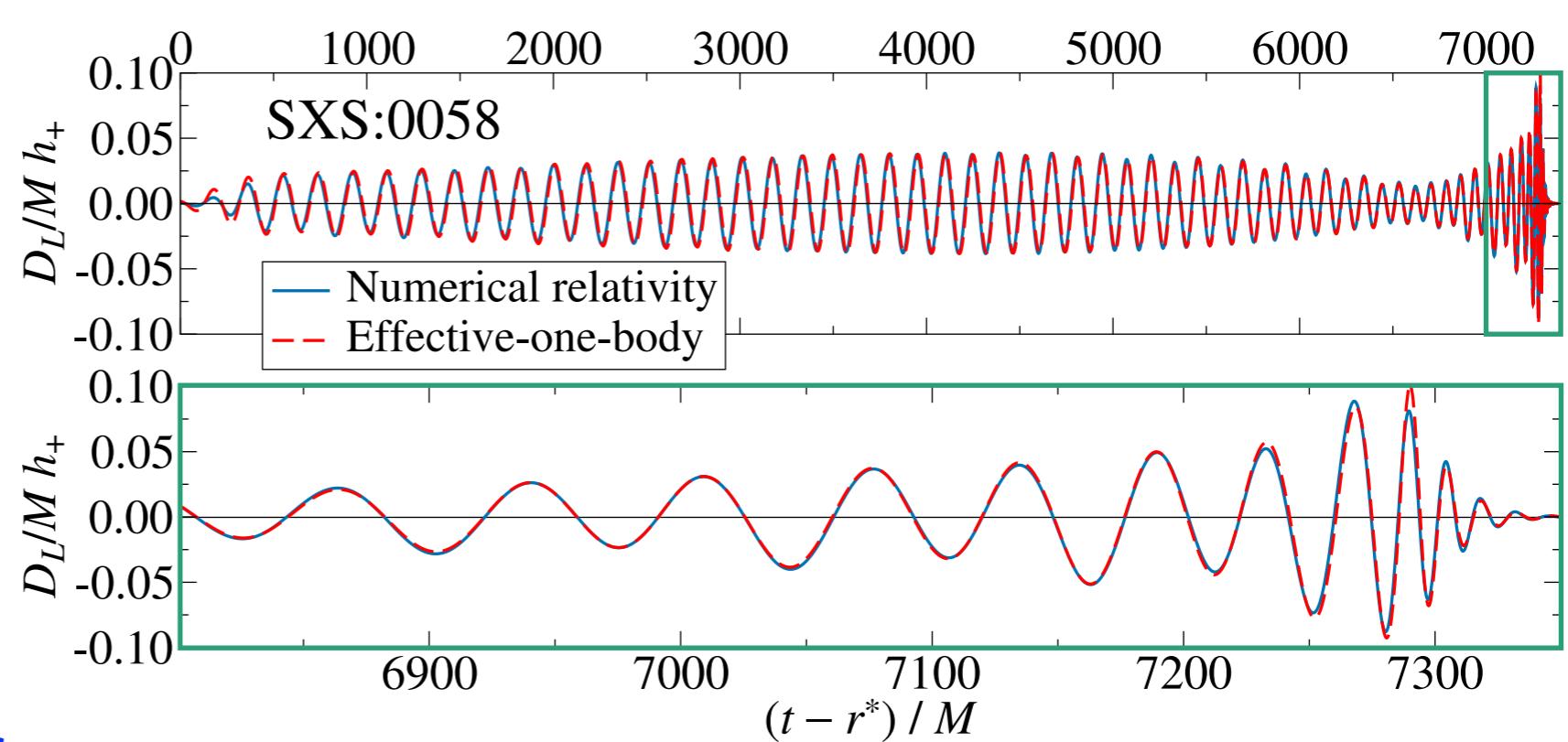
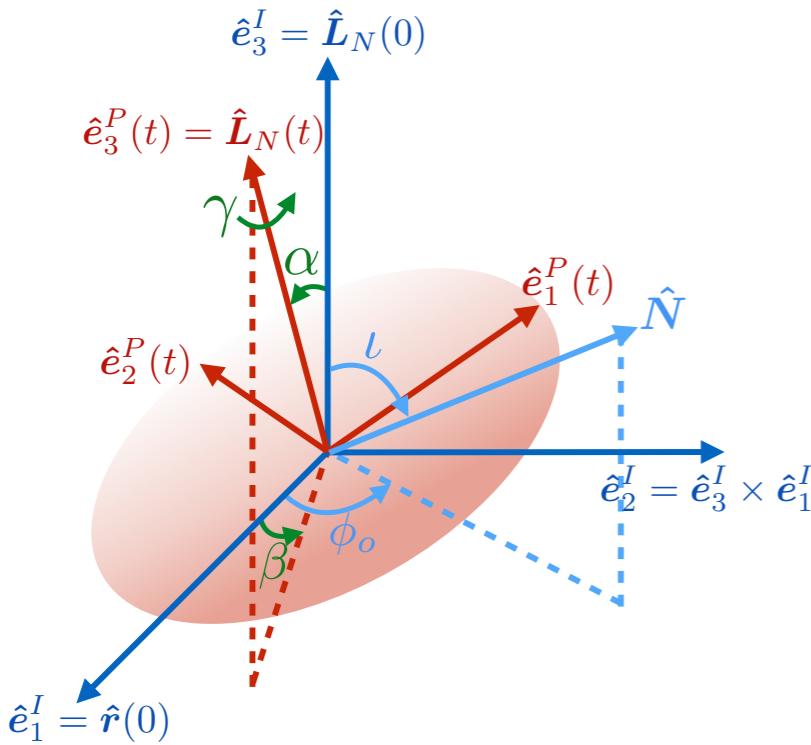
$$\mathcal{A}(f; \lambda_i) \equiv C \begin{cases} \left( \frac{\pi M f}{a_0 \nu^2 + b_0 \nu + c_0} \right)^{-7/6} & \text{if } f < \frac{a_0 \nu^2 + b_0 \nu + c_0}{\pi M} \\ \left( \frac{\pi M f}{a_0 \nu^2 + b_0 \nu + c_0} \right)^{-2/3} & \text{if } \frac{a_0 \nu^2 + b_0 \nu + c_0}{\pi M} \leq f < \frac{a_1 \nu^2 + b_1 \nu + c_1}{\pi M} \\ w \mathcal{L} \left( f, \frac{a_1 \nu^2 + b_1 \nu + c_1}{\pi M}, \frac{a_2 \nu^2 + b_2 \nu + c_2}{\pi M} \right) & \text{if } \frac{a_1 \nu^2 + b_1 \nu + c_1}{\pi M} \leq f < \frac{a_3 \nu^2 + b_3 \nu + c_3}{\pi M} \end{cases}$$

$$\phi(f; \lambda_i) = 2\pi f t_0 + \varphi_0 + \frac{1}{\nu} \sum_{k=0}^7 (x_k \nu^2 + y_k \nu + z_k) (\pi M f)^{(k-5)/3}$$

- Extended to **spin effects**, including **effective precession**.

(Khan et al. 15; Husa et al. 15; Hannam et al. 13)

# Spinning precessing waveform models



## Precessing (co-rotating) frame

(AB, Chen & Vallisneri 03, Boyle et al. 11,  
Schmidt et al. 11, O'Shaugheessy et al. 11 )

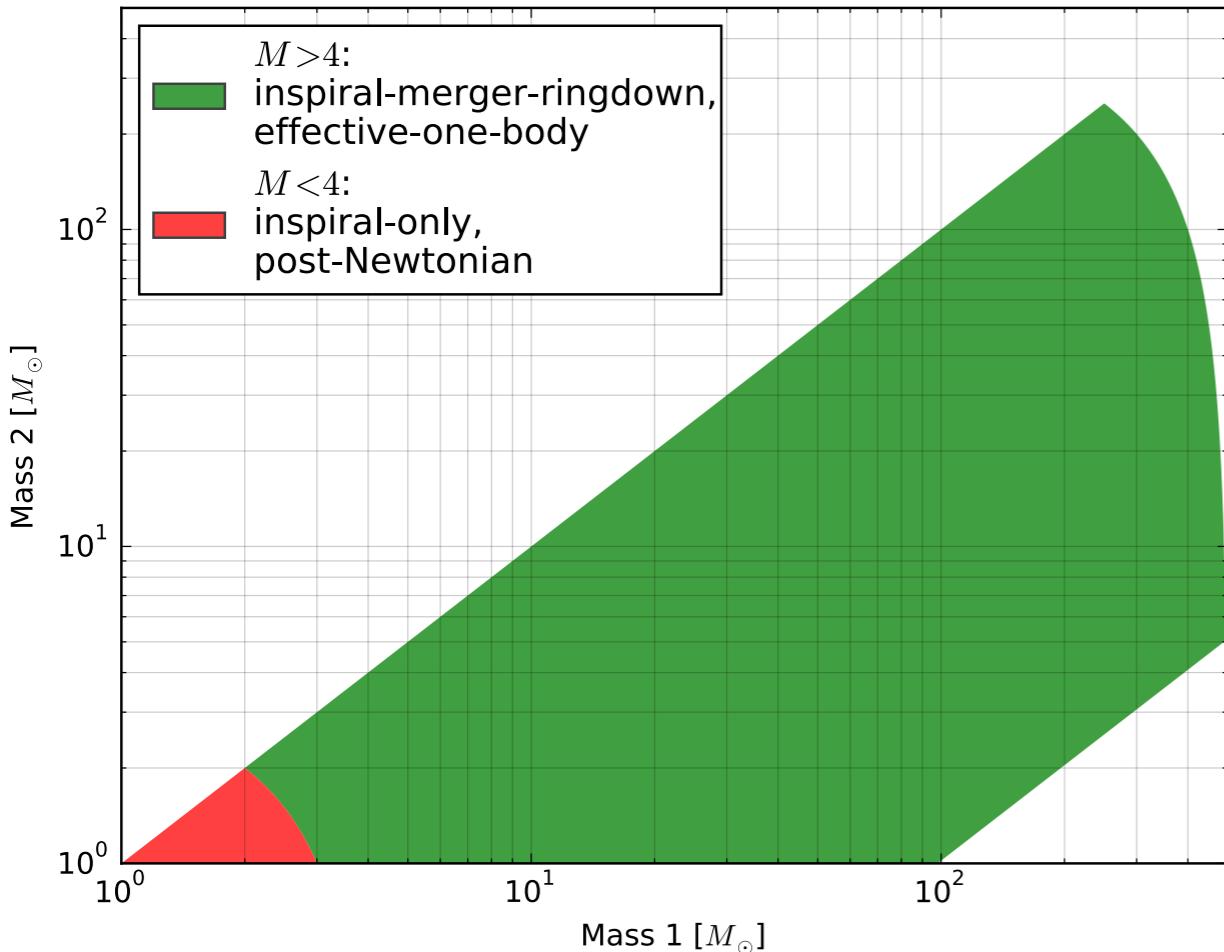
(Pan et al. 14, Babak et al. 16)

- **Single effective-spin precessing** waveform model in **frequency domain** (IMR phenomenological, 13-independent parameters). (Schmidt et al. 12, Hannam et al. 14)
- **Double-spin precessing** waveform model in **time domain** (EOBNR, 15-independent parameters). (Pan et al. 14, Babak et al. 16)

# Detection confidence with modeled search in O2

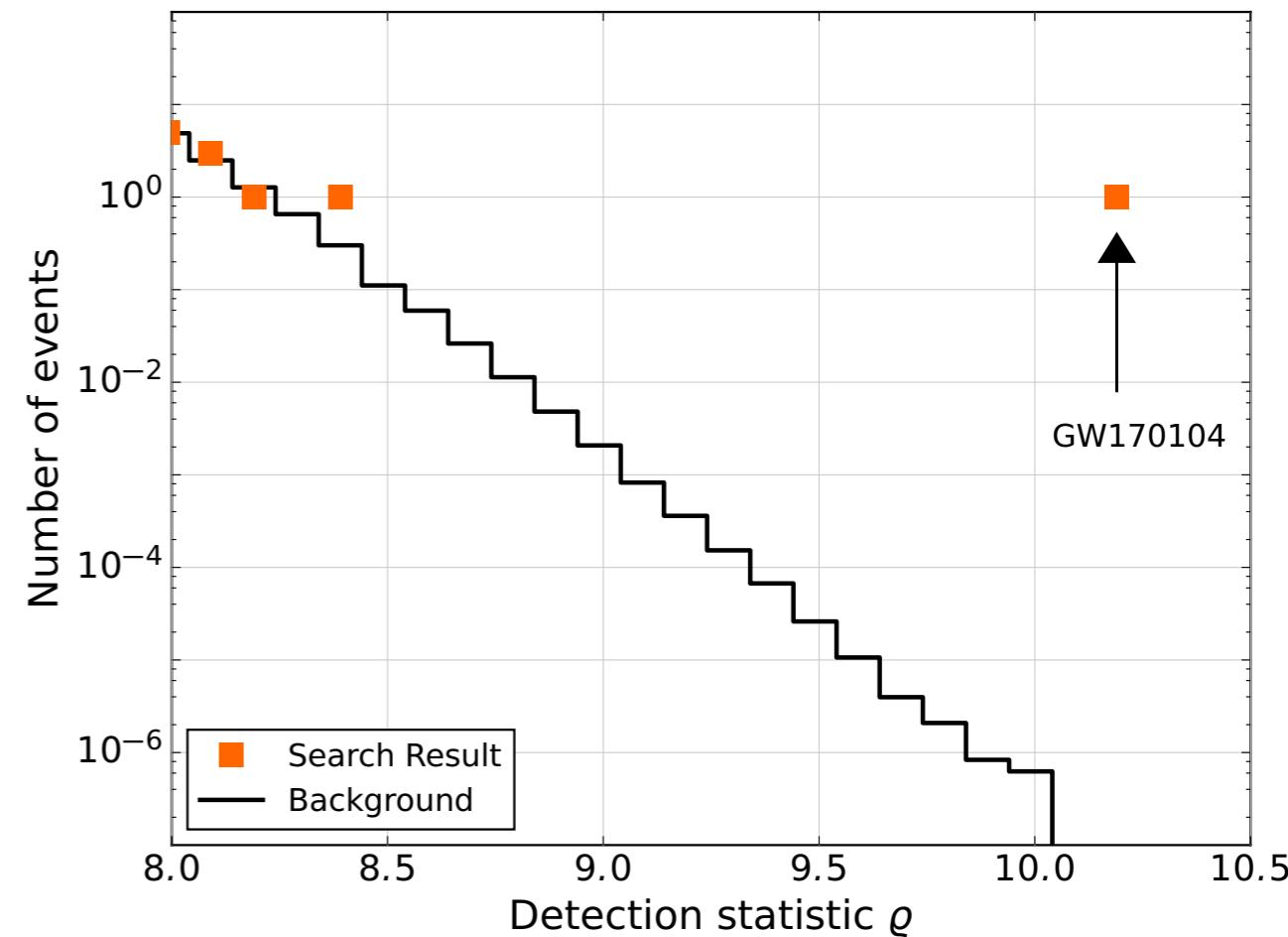
- Matched filtering employed

(*Dal Canton & Harry 17*)



only (2,2) & (2,-2) modes use in templates

(*Abbott et al. PRL 118 (2017) 221101*)



- Modeled-search: at detection statistic value of **GW170104**, **GW170814**, **GW170608**, false alarm rate is less than 1 in 70000, 40000, 3000 years of coincident observing time.
- Modeled-search: false alarm rate is less than 1 in 80,000 years for **GW170817**.

# Detection confidence with modeled search in O2 (contd.)

- Matched filtering employed

$$\hat{d} = \int_{-\infty}^{\infty} dt K(t) d(t)$$

↑ filter

↓ data      ↓ noise      ↓ signal

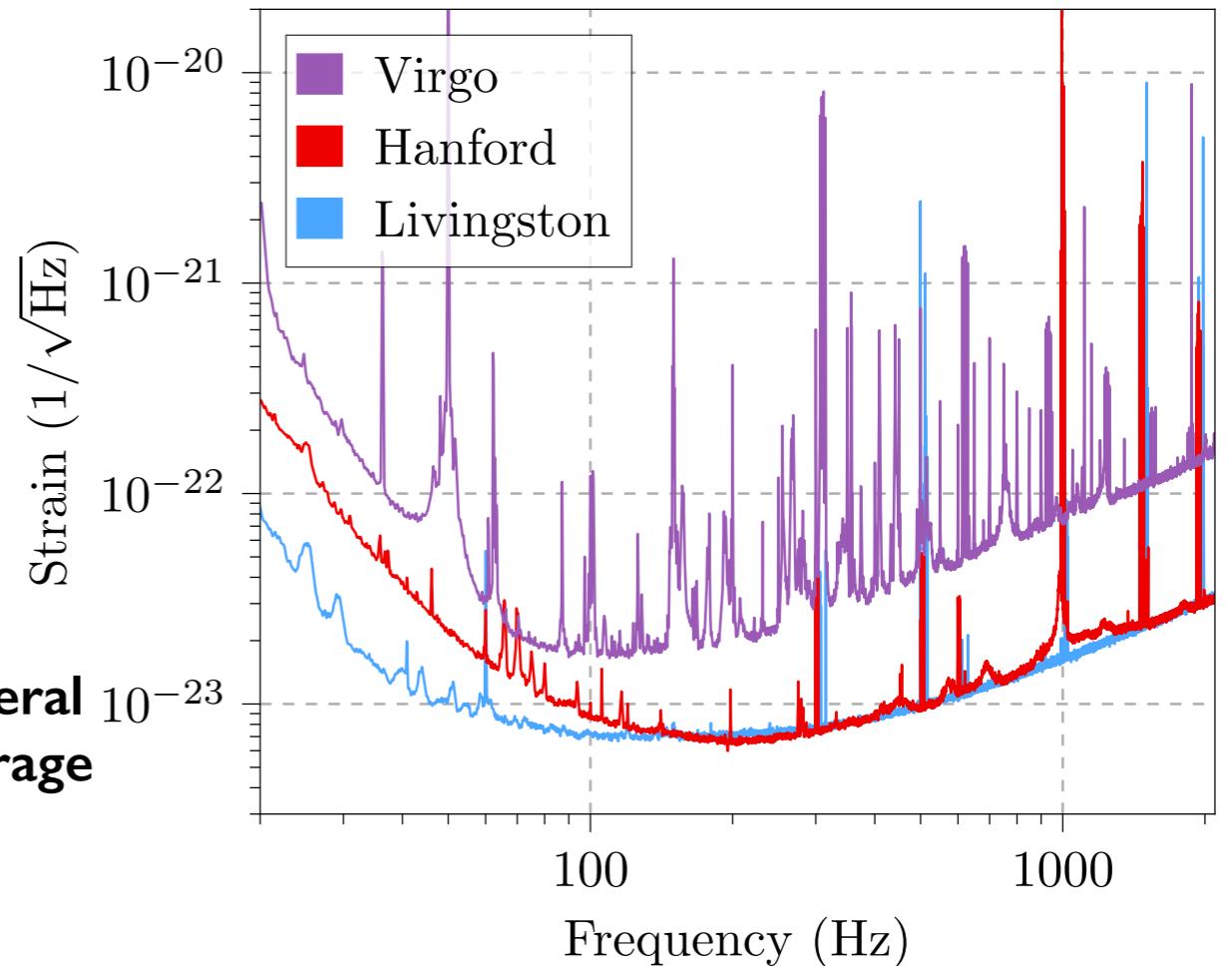
$$d(t) = n(t) + h(t)$$

$$S = \langle \hat{d} \rangle \quad \leftarrow \text{expectation value over several noise realizations/time average}$$

$$N = \sqrt{\langle \hat{d}^2 \rangle - \langle \hat{d} \rangle^2} \Big|_{h=0} \quad \leftarrow \text{variance when signal is absent}$$

• Optimal SNR:  $\left( \frac{S}{N} \right) = \sqrt{(h|h)}$

$$(h_1|h_2) = 4\text{Re} \int_0^{\infty} df \frac{\tilde{h}_1^*(f) \tilde{h}_2(f)}{S_n(f)}$$



• Optimizing SNR:  $\tilde{K}(f) \propto \frac{\tilde{h}(f)}{S_n(f)}$

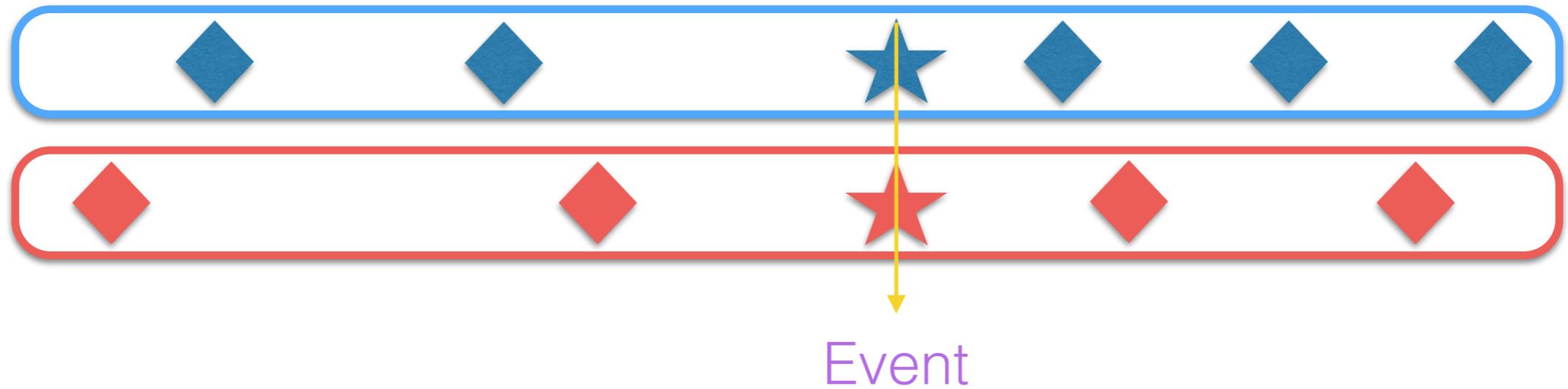
• Statistics:  $\rho = \frac{\hat{d}}{N}$

# Assessing confidence in LIGO detection

Could the **signal** be due to **noise fluctuations**?

(animation by Nuttall)

**Zero-lag**

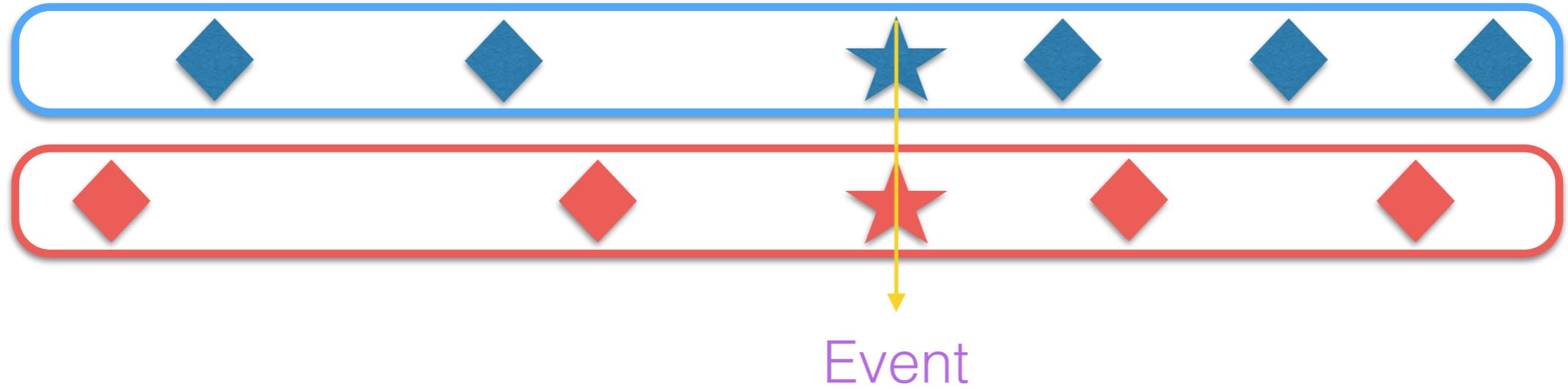


# Assessing confidence in LIGO detection

Building larger background by time-shifting the data

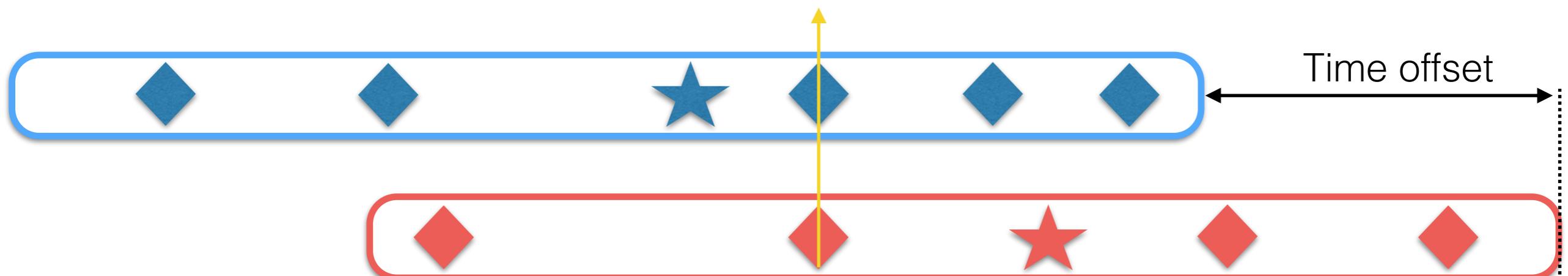
(animation by Nuttall)

## Zero-lag



## Time slide

Background event

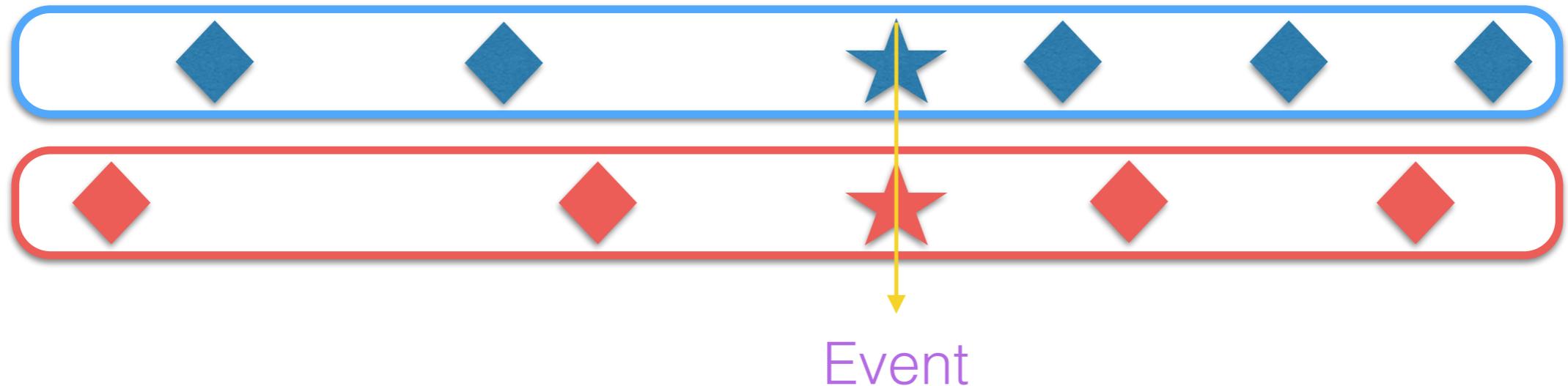


# Assessing confidence in LIGO detection

Building larger background by time-shifting the data

(animation by Nuttall)

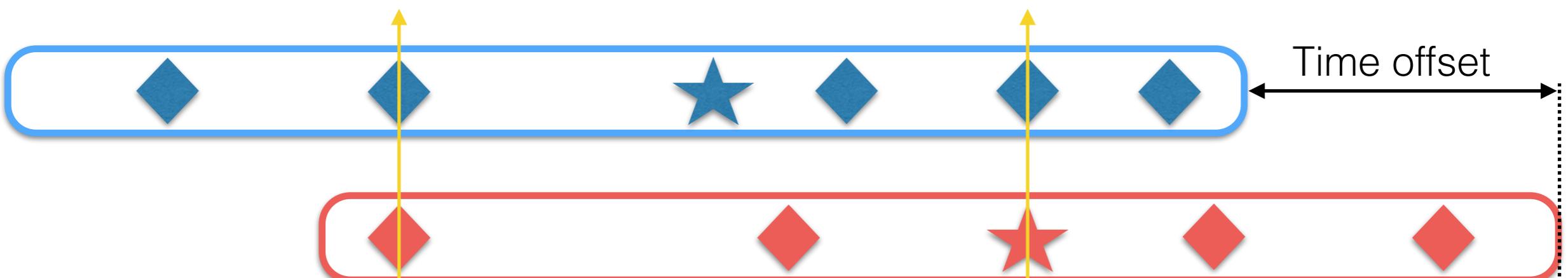
## Zero-lag



## Time slide

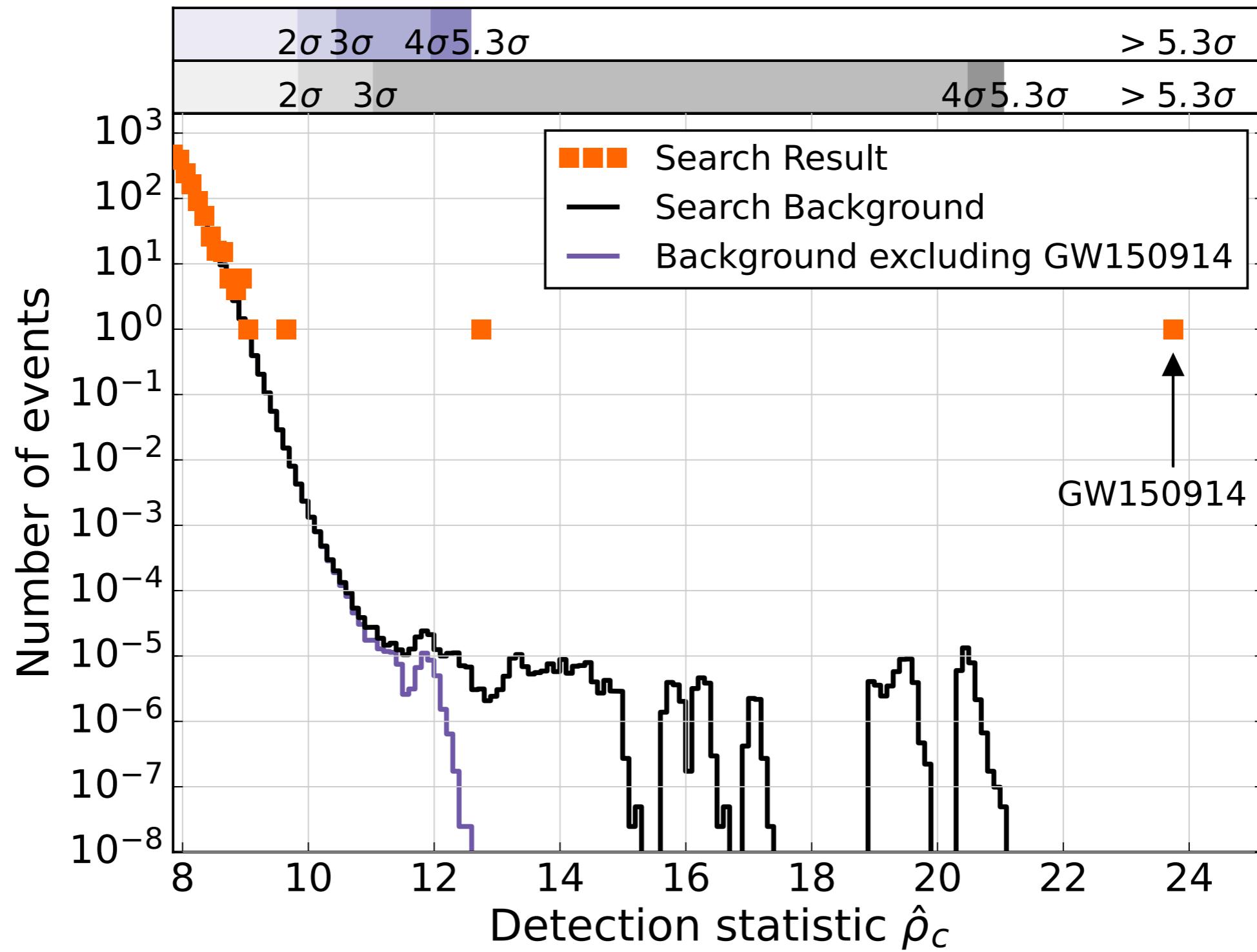
Background event

Event + background



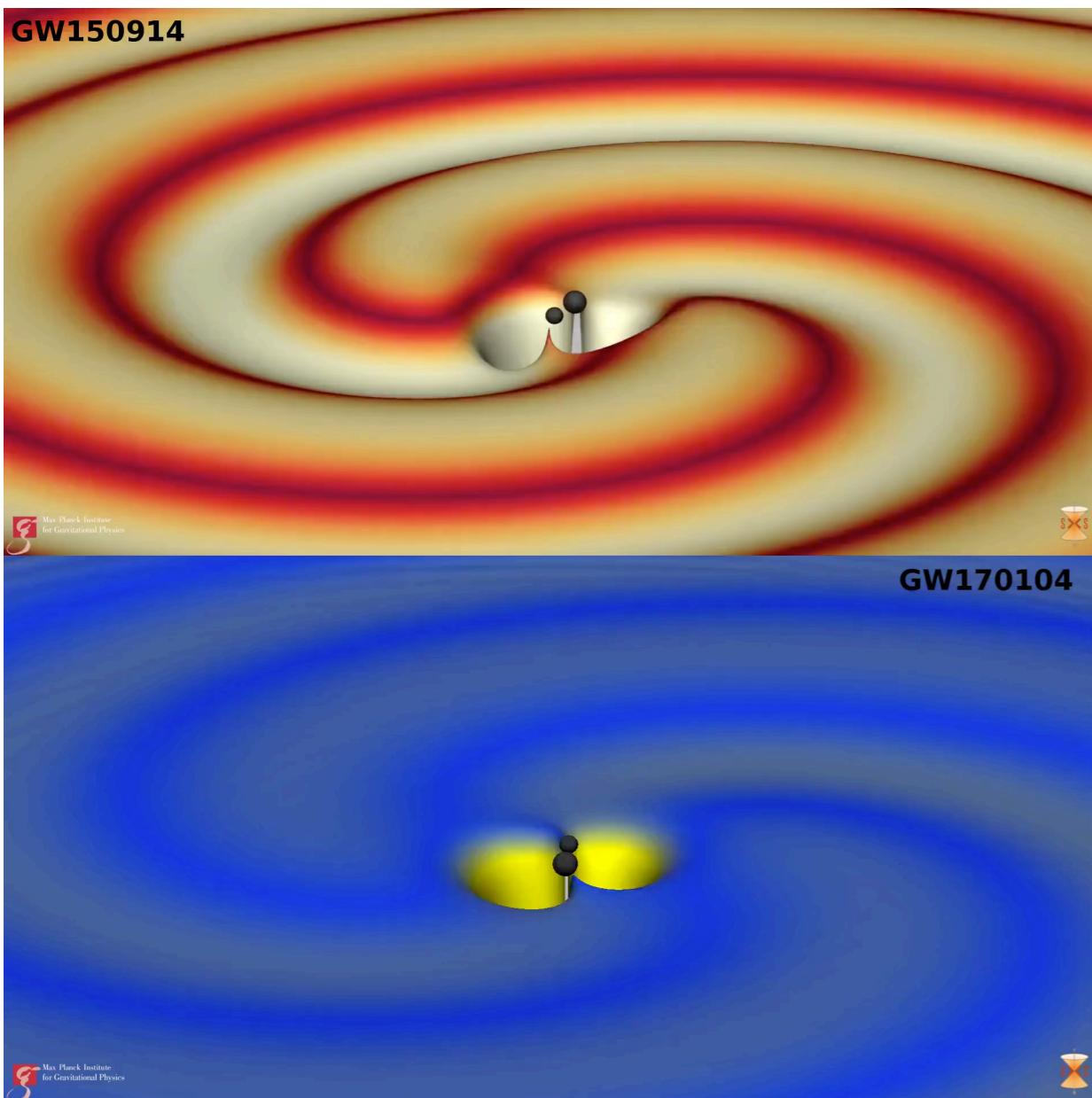
# Assessing confidence in LIGO detection for GW150914

(Abbott et al. Phys.Rev.X6 (2016) no.4, 041015)



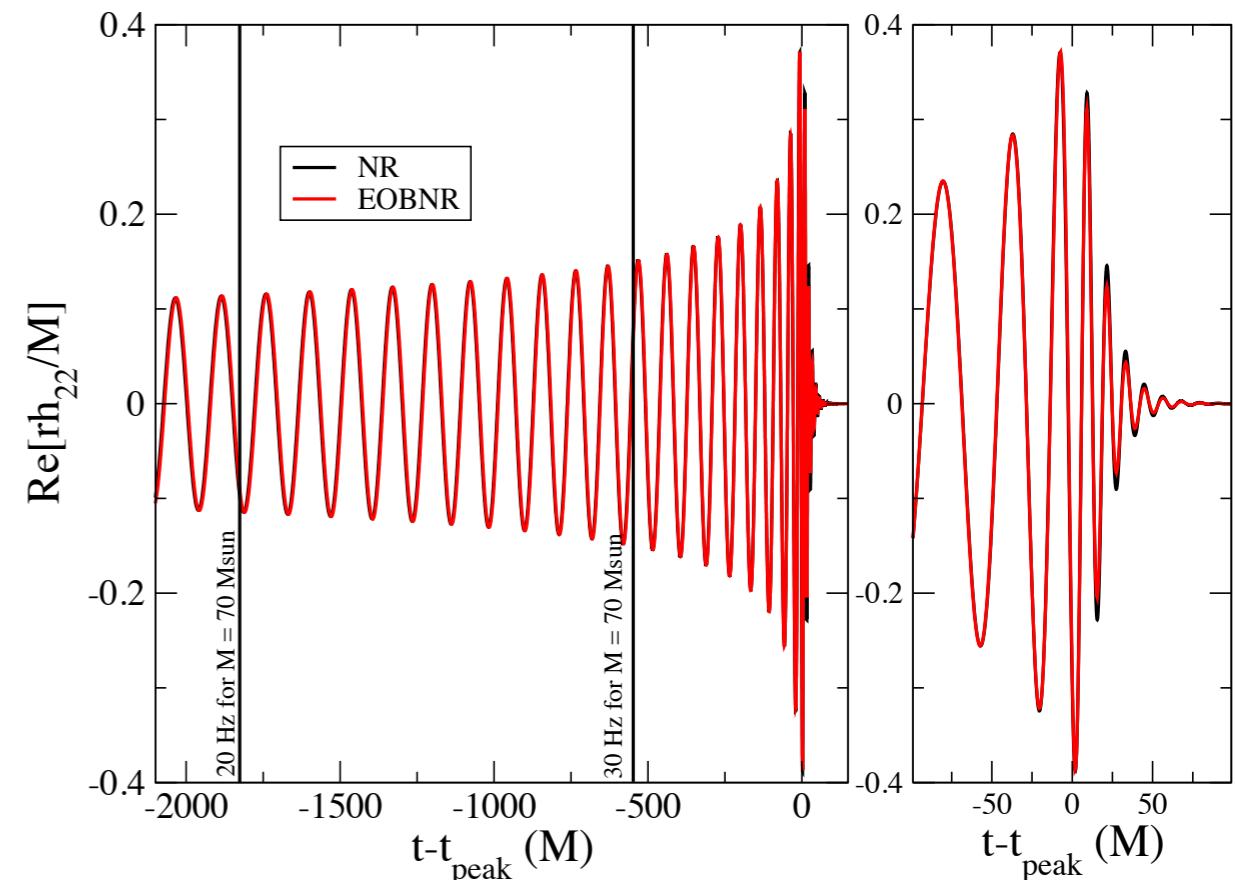
# Assessing possible modeling systematics of GW events

(visualization credit: Dietrich, Haas @AEI)



(Ossokine, AB & SXS project)

$$\odot M_{\odot} - 9 \nabla = W^{\odot} M_{\odot} - 09 = W$$



- **Systematics due to modeling are smaller than statistical errors.**

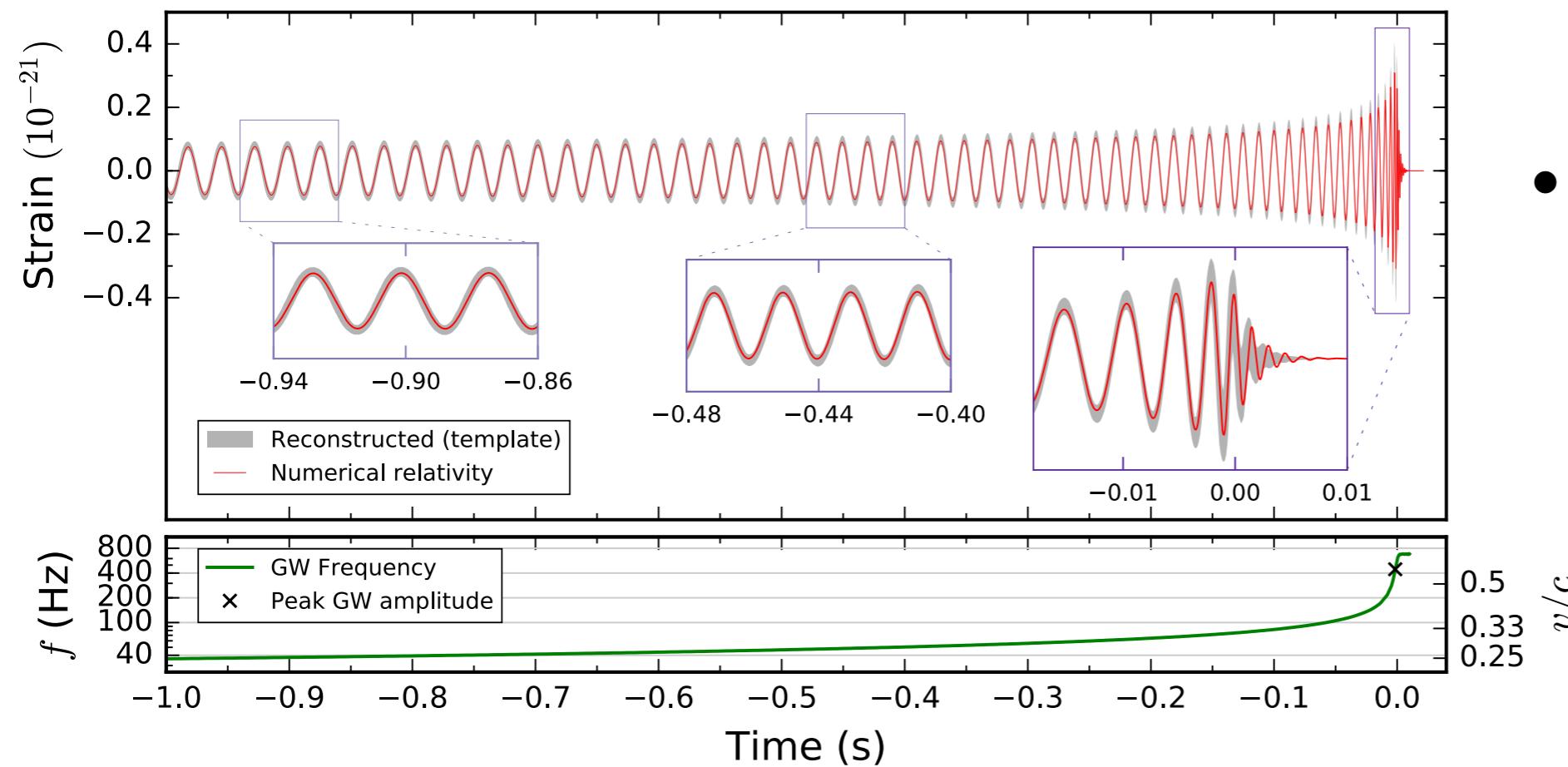
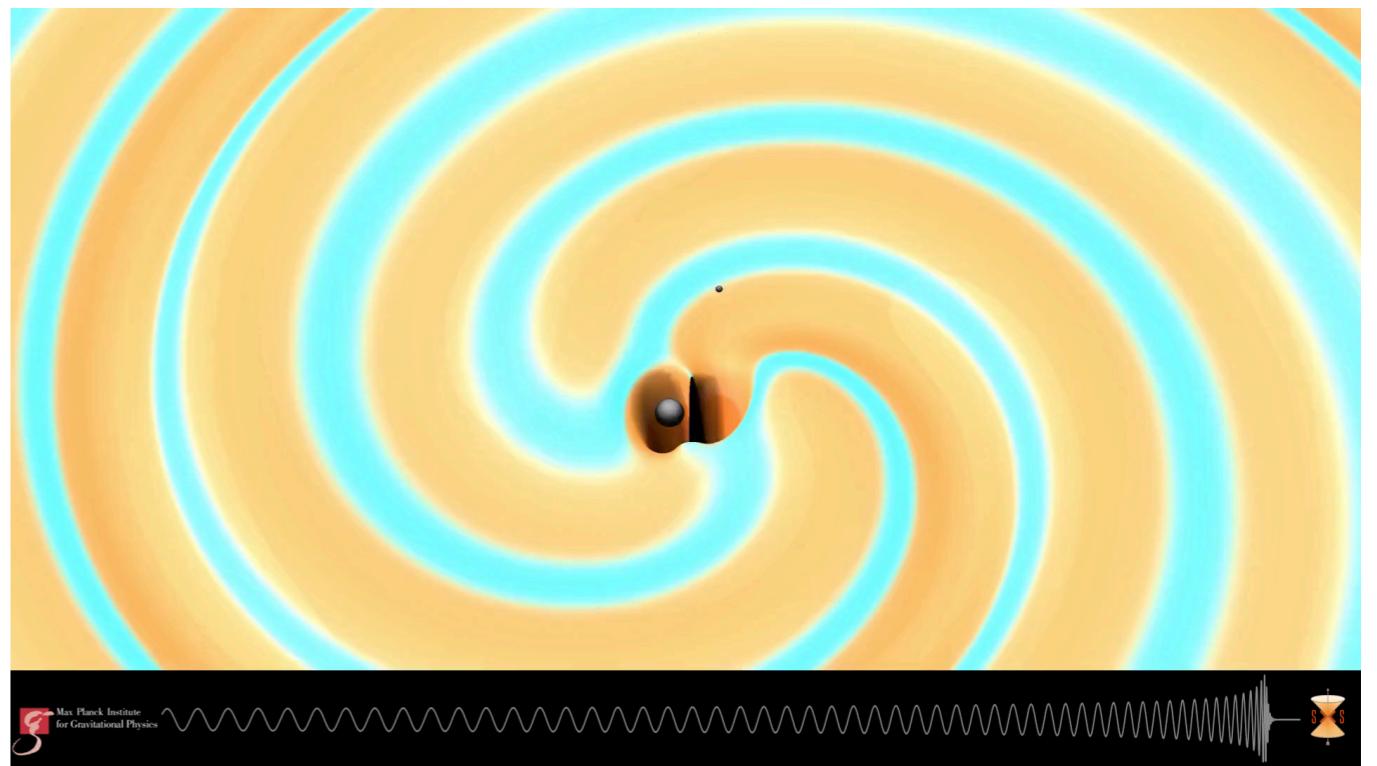
(see also Abbott et al.  
CQG 34 (2017) 104002 )

# Assessing possible modeling systematics of GW events

(visualization credit: Dietrich, Haas @AEI)

(Ossokine, AB & SXS project)

(Abbott et al. PRL 116 (2016) 241103)



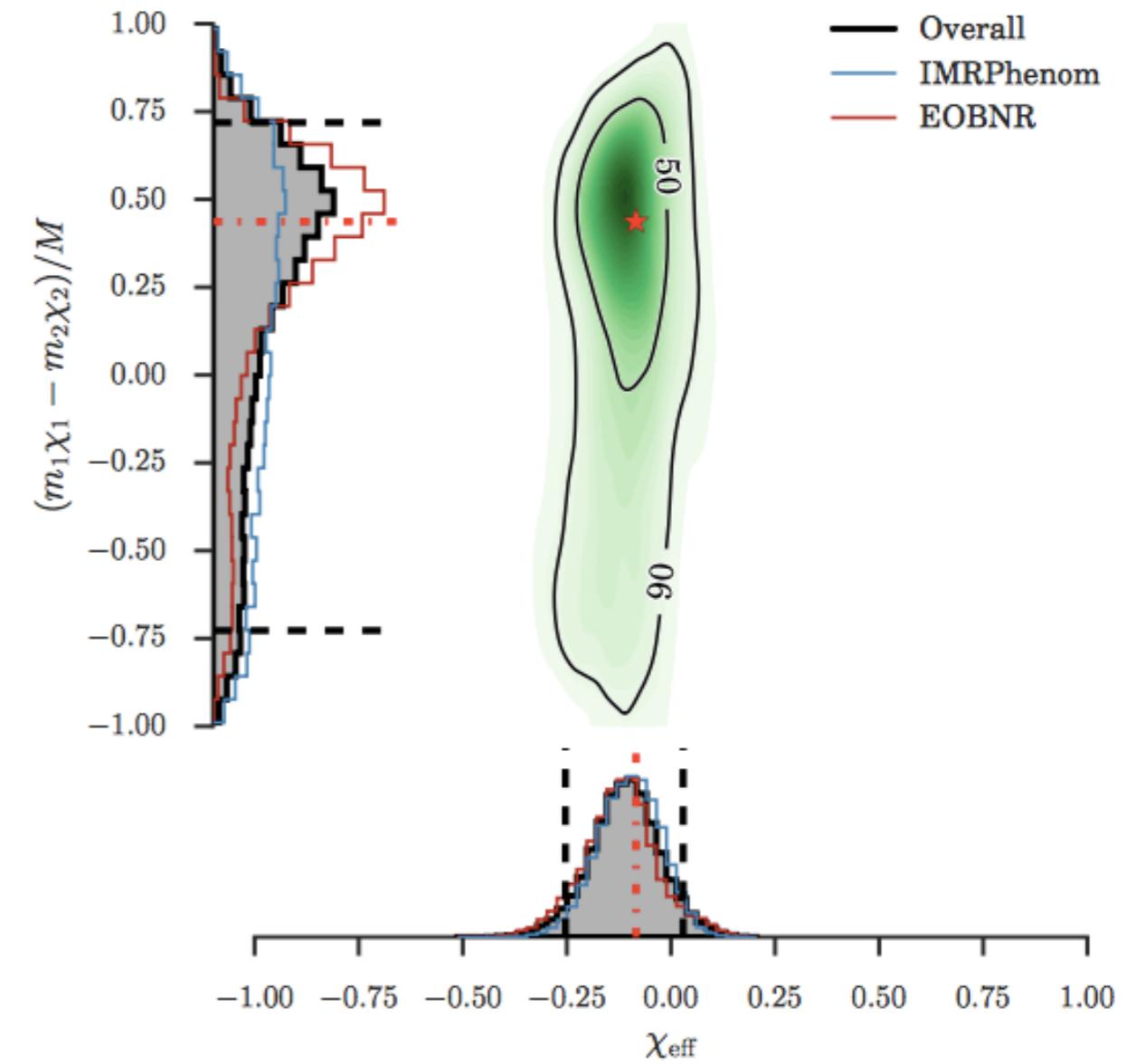
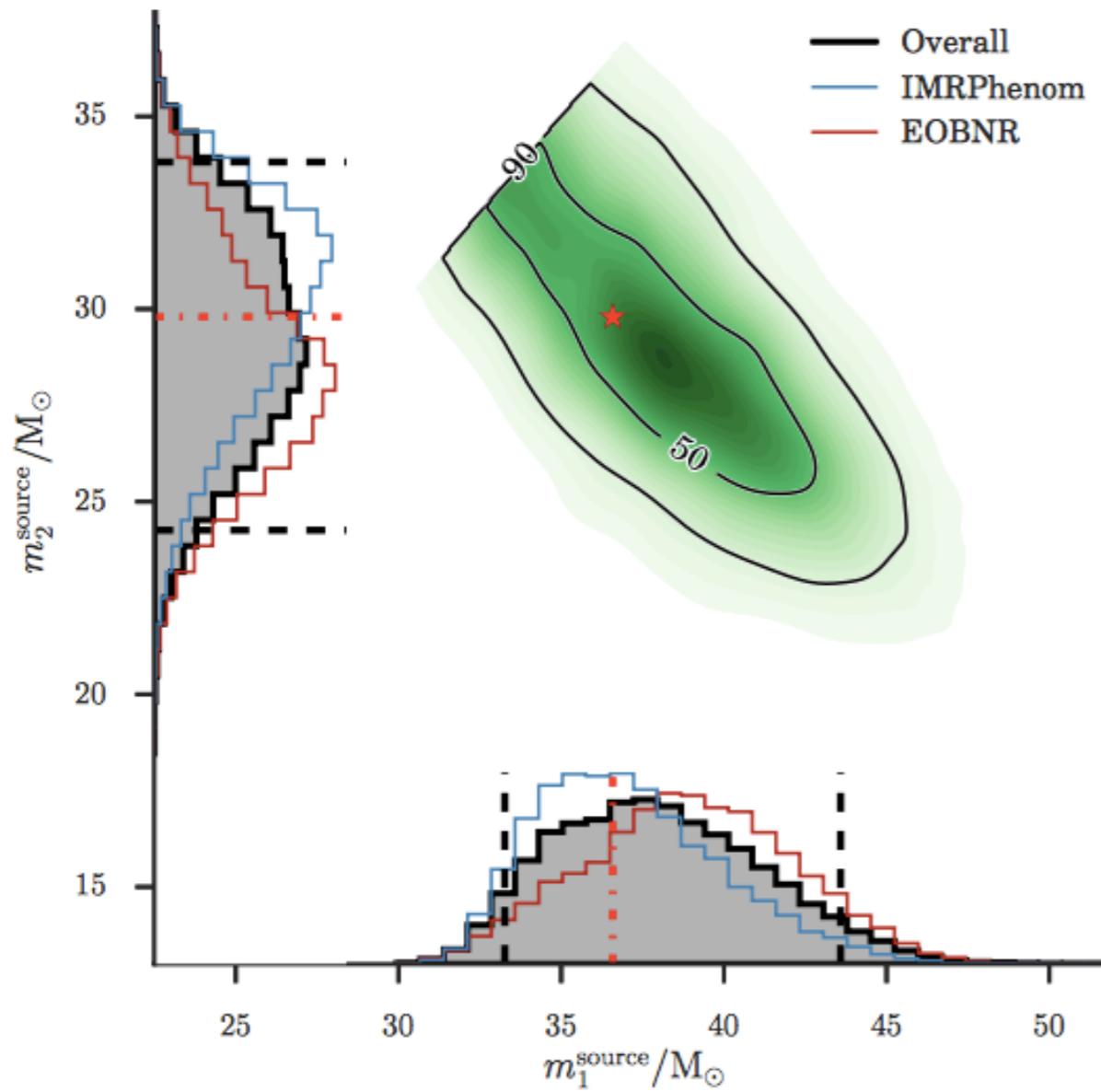
- **Systematics due to modeling are smaller than statistical errors.**

(see also Abbott et al.  
CQG 34 (2017) 104002 )

# On systematics due to modeling comparing to NR waveform

- Mock signal from NR simulation with parameters close to GW150914.

(Abbott et al. CQG 34 (2017) 104002 )

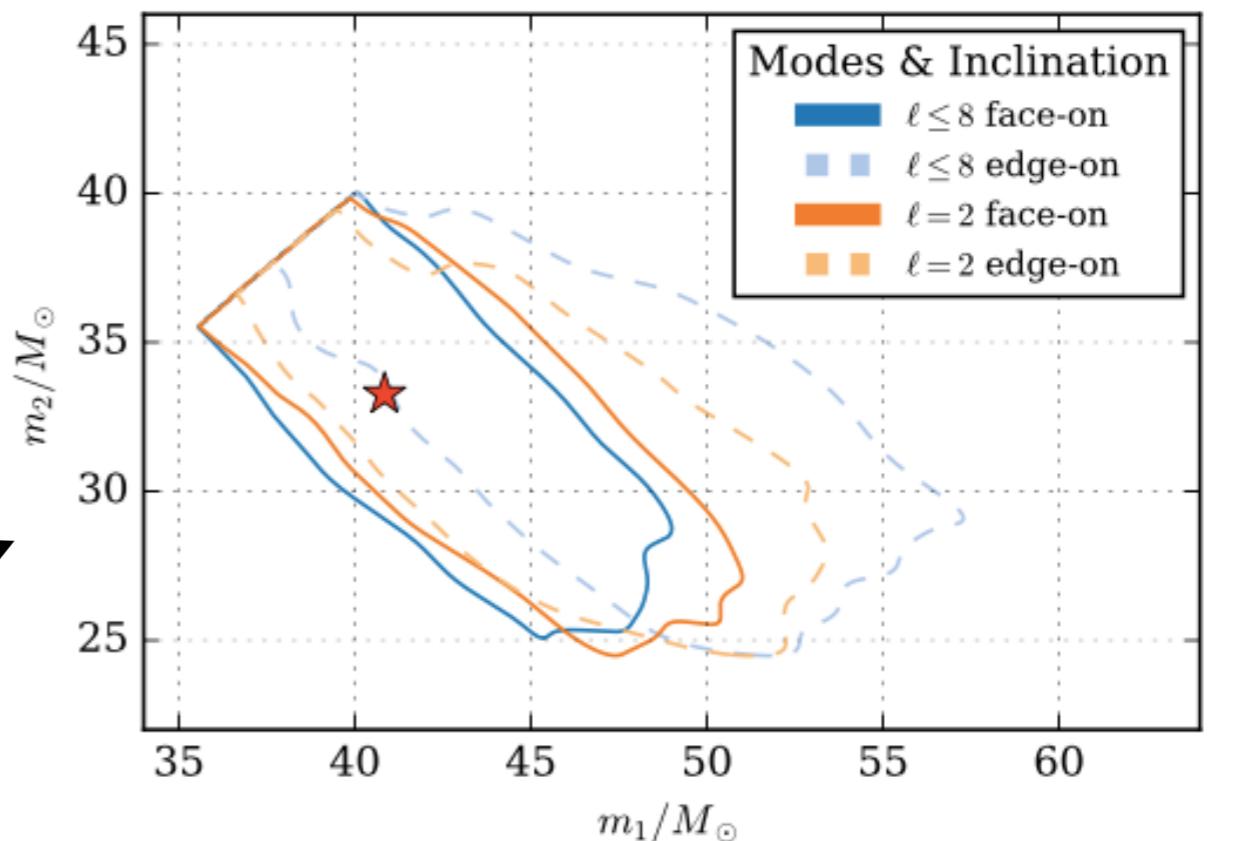
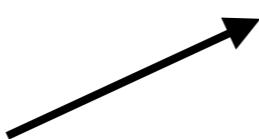


- Overall, no evidence for systematic bias relative to the statistical error of original parameter recovery of GW150914.

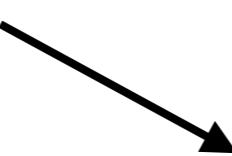
# On systematics due to modeling comparing to NR waveform (contd.)

(Abbott et al. CQG 34 (2017) 104002 )

- Parameter biases are found to occur for some configurations disfavored by data of GW150914.

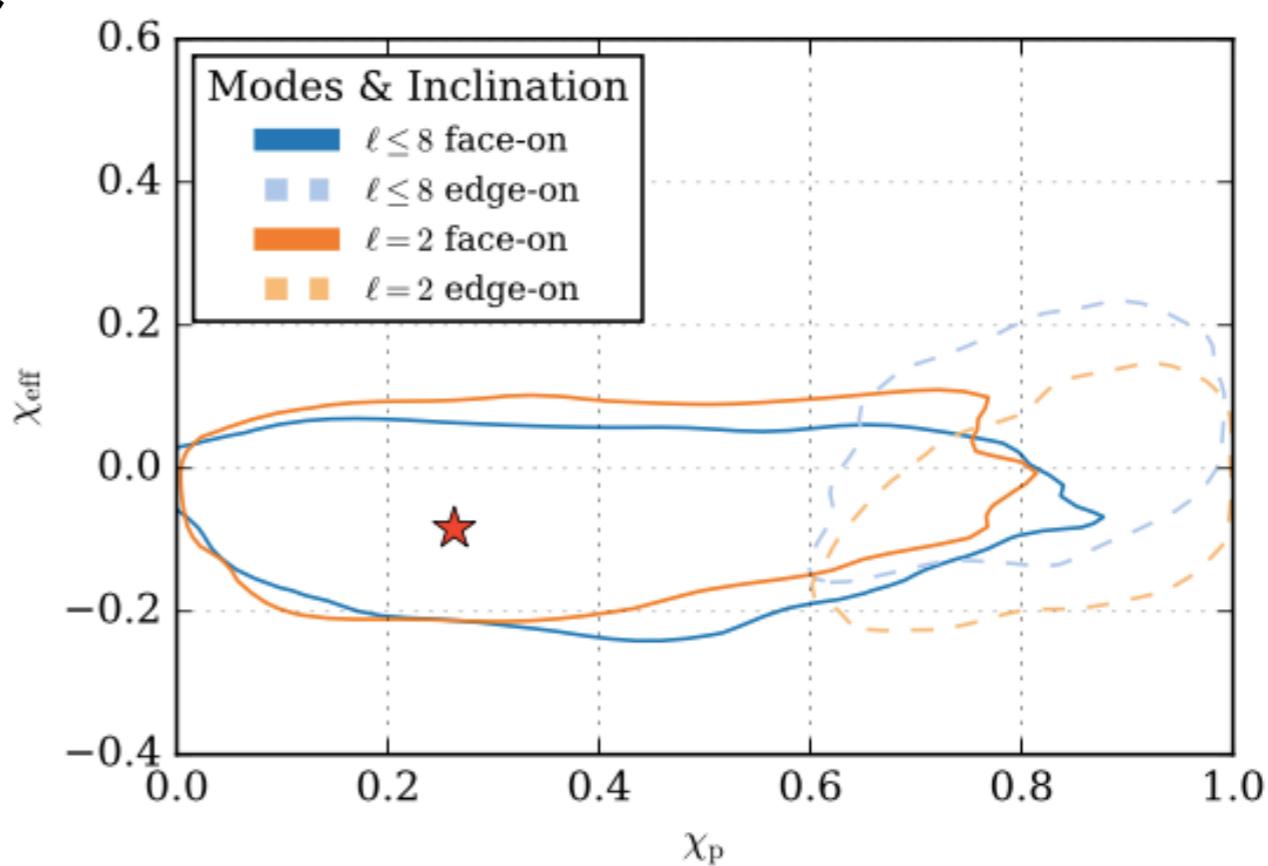


- E.g., biases are present for binaries inclined edge-on to the detector over a small range of choices of polarization angles.



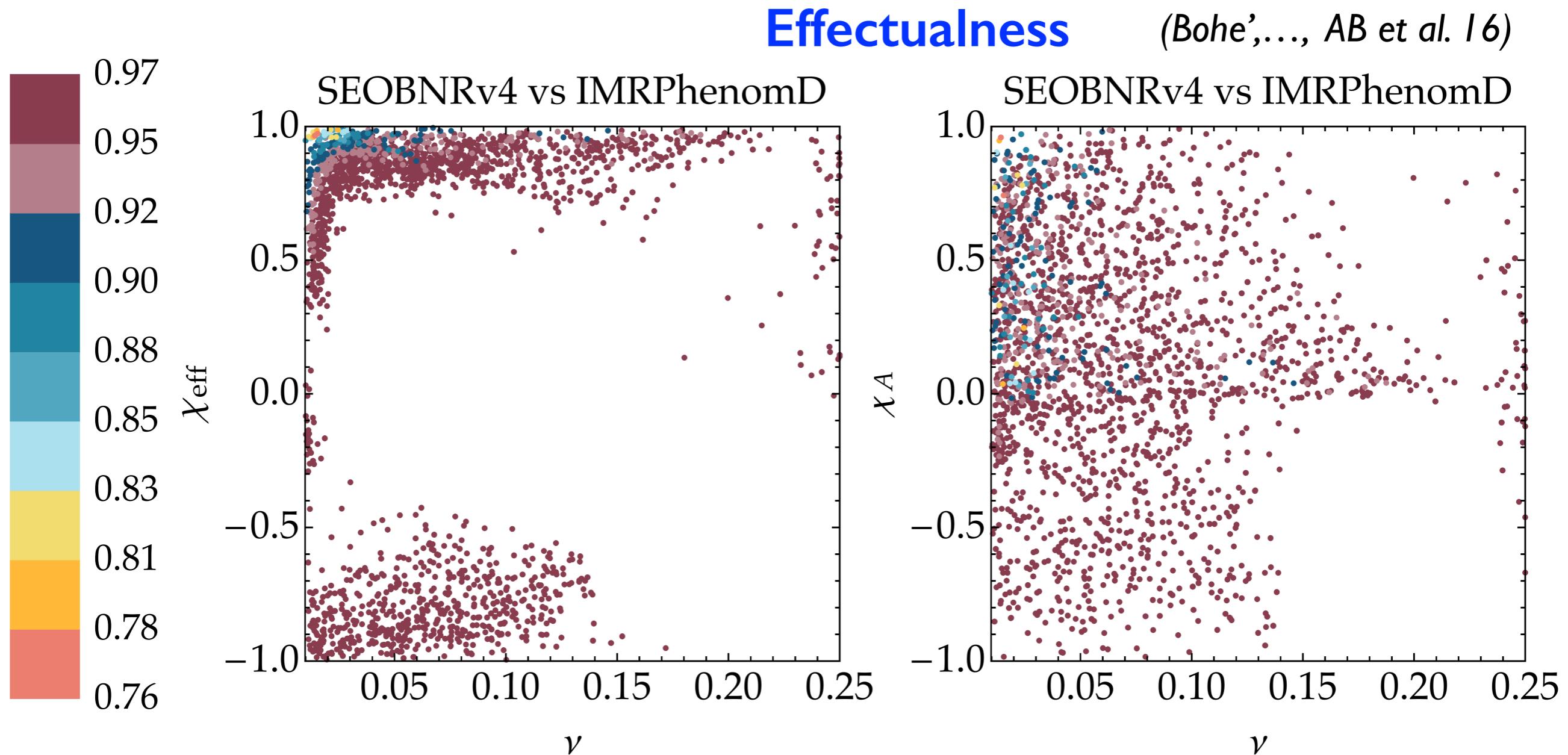
(see also Williamson et al. 2017)

- Biases can be present for binaries with eccentricity  $> 0.05$ .



# Comparing EOBNR & IMRPhenom models: detection

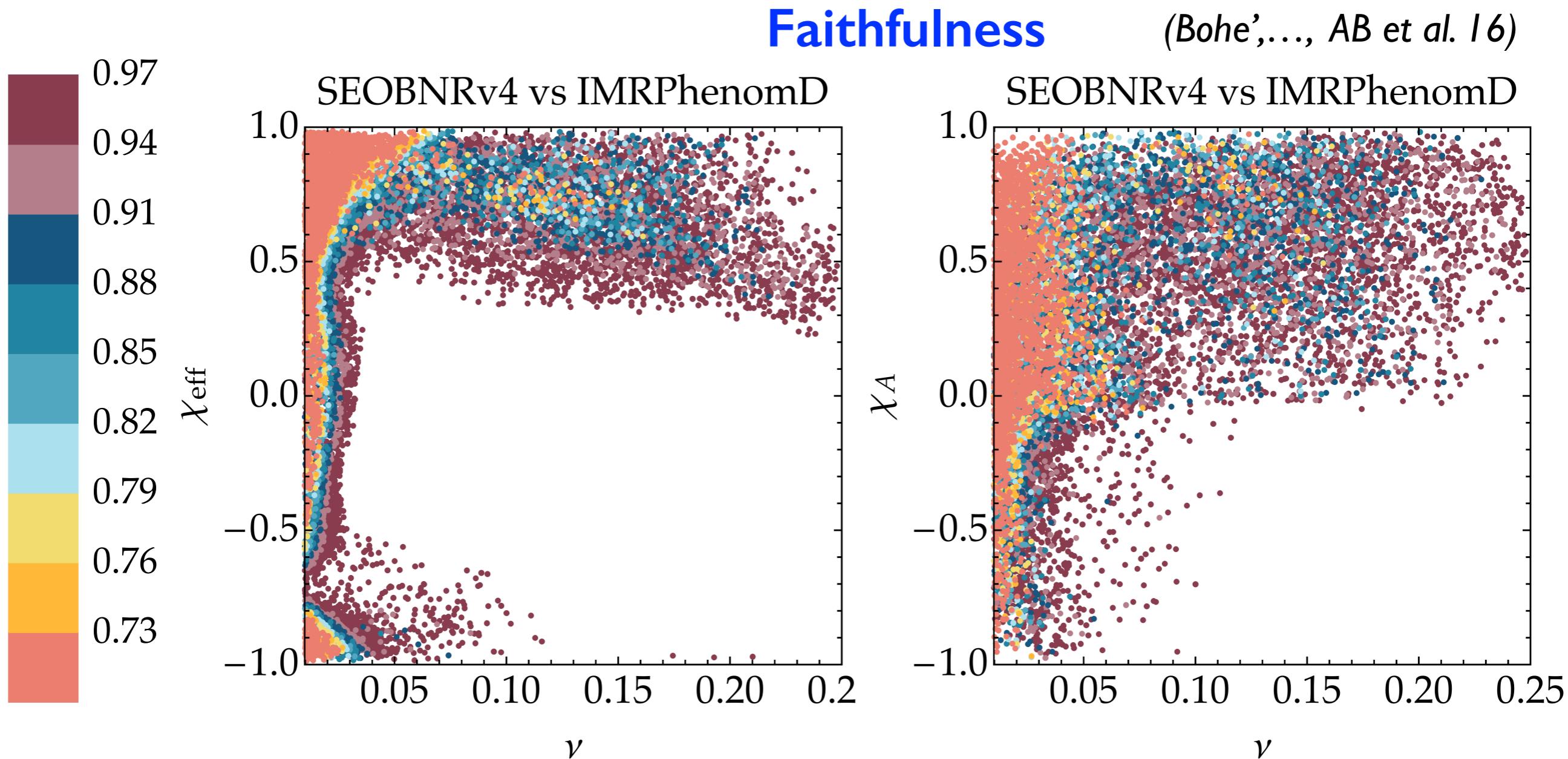
- Aligned/anti-aligned waveform models. Only dominant (2,2) mode.



[Note that only 2.1% of 100,000 points have matches < 97%.]

# Comparing EOBNR & IMRPhenom models: inferring parameters

- Aligned/anti-aligned waveform models. Only dominant (2,2) mode.
- Differences for large mass ratios ( $> 4$ ) and large spins ( $> 0.8$ ).

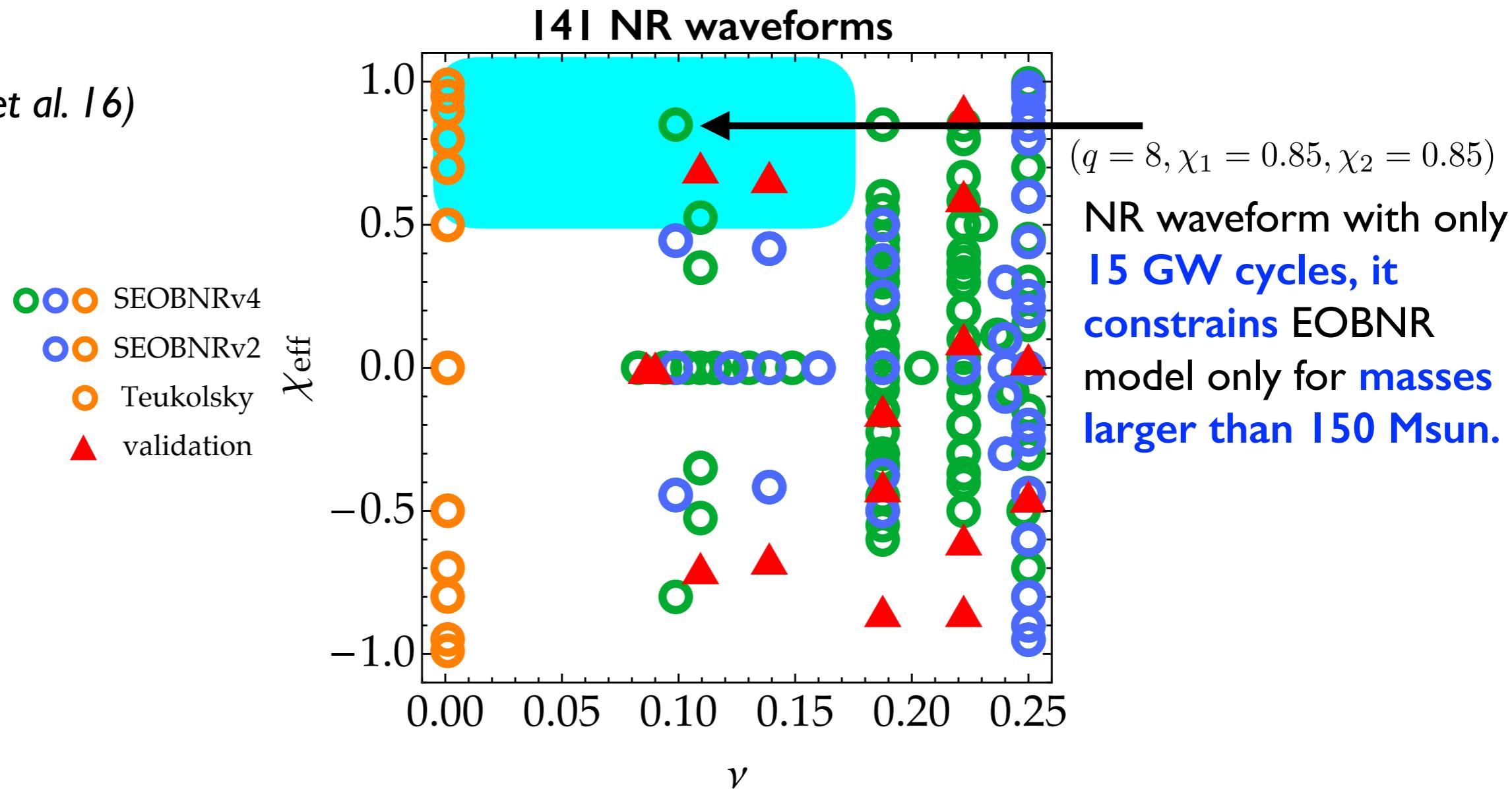


[Note that only 7% of 200,000 points have matches < 97%.]

# Extending waveform model in all BBH parameter space

- Difficult to run **NR simulations** for **large mass ratios** ( $> 4$ ) and **large spins** ( $> 0.8$ ), with **large number** of GW **cycles** ( $> 50$ ).

(Bohe',..., AB et al. 16)



- For large mass ratios ( $> 4$ ) combine **PN & GSF** results in **EOB** framework.  
*(Damour 09; Barausse et al. 12, Le Tiec et al. 12, Bini et al. 12-16, Antonelli et al. in progress)*
- Inclusion of **GSF** also important for **EMRIs** (LISA) and **IMRIs** (3G detectors).

# Inferring sources' properties/model selection upon detection

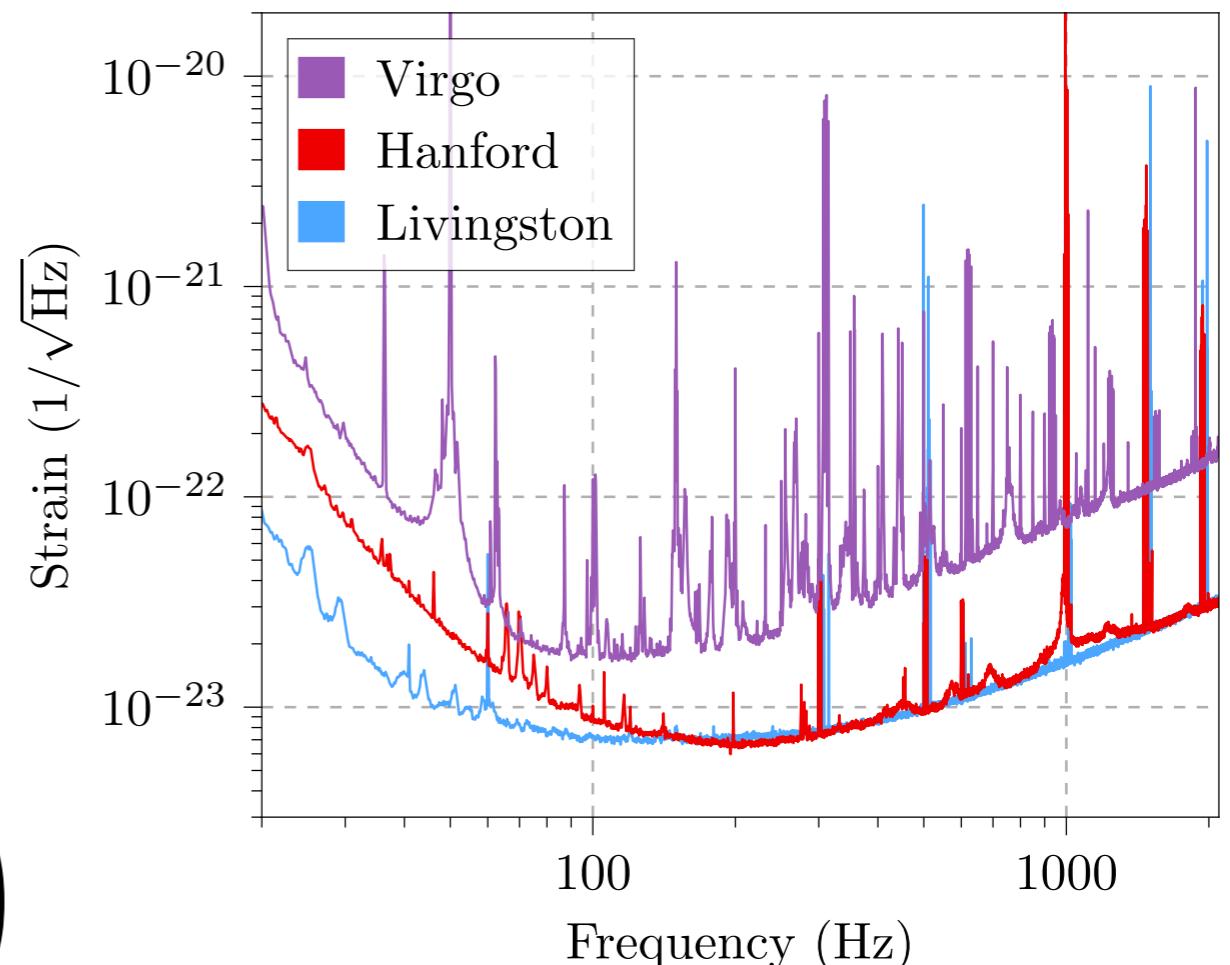
- Bayes theorem:

$$\mathcal{P}(\theta|d, \mathcal{H}) \propto \text{Likelihood function} \downarrow \times \text{prior probability} \downarrow$$
$$\mathcal{P}(\theta|d, \mathcal{H}) \propto \Lambda(d|\theta, \mathcal{H}) \times \mathcal{P}(\theta, \mathcal{H})$$

↑  
posterior probability  
distribution

- Likelihood function for observed data  $d(t) = n(t) + h(t)$ , given hypothesis that there is GW signal with parameters  $\theta$ :

$$\Lambda(d|\theta, \mathcal{H}) \propto \exp \left( -2 \sum_i \frac{|\tilde{d}(f_i) - \tilde{h}(f_i; \theta)|^2}{S_n(f_i)} \right)$$



# Inferring sources' properties/model selection upon detection

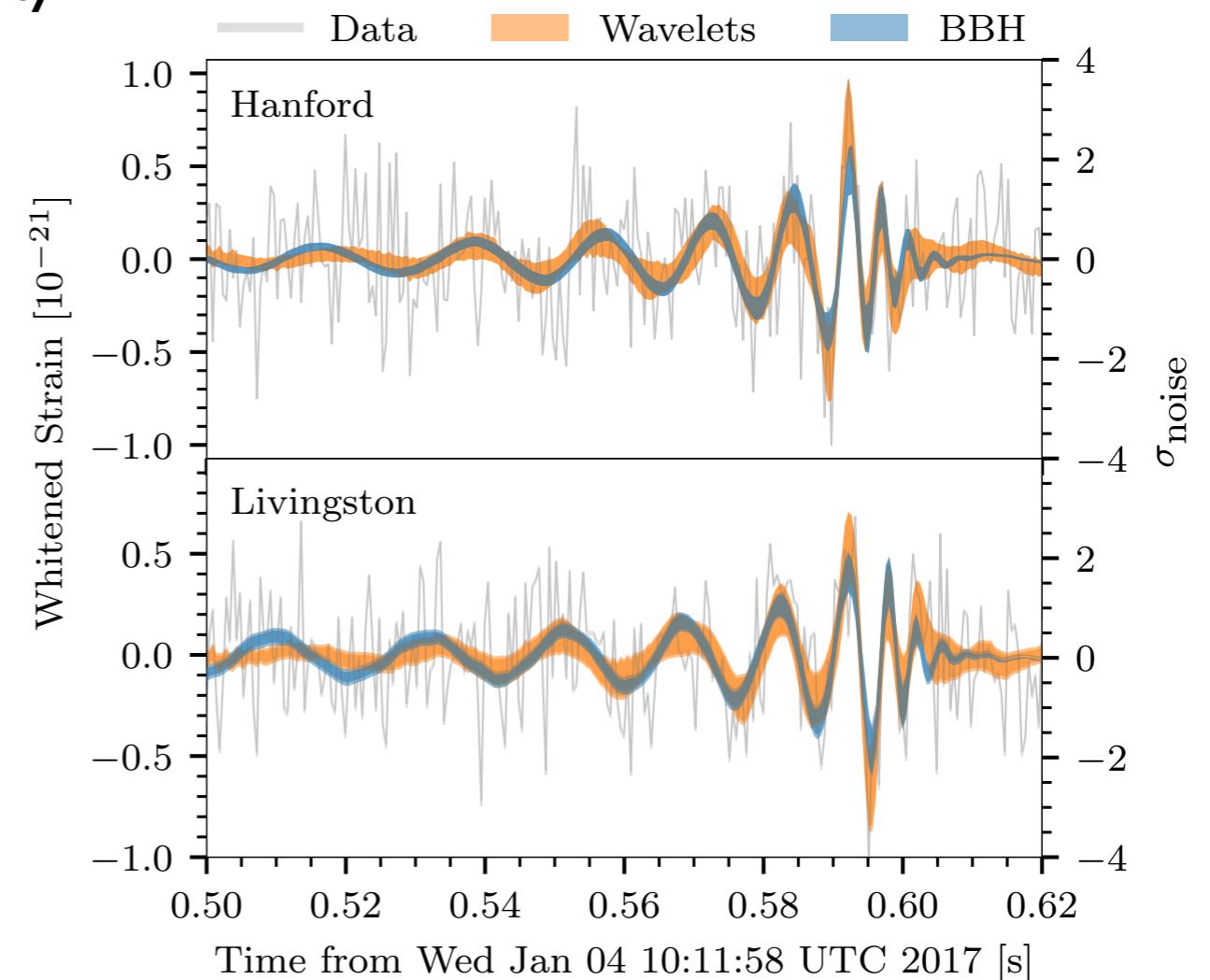
- Bayes theorem:

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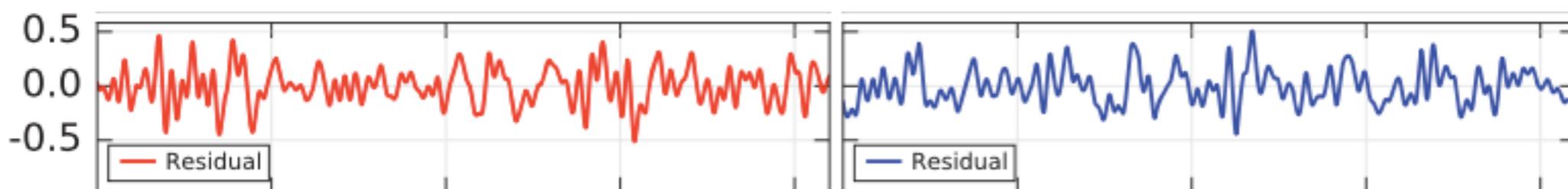
↑  
posterior probability  
distribution

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$$\Lambda(d|\theta, \mathcal{H}) \propto \exp \left( -2 \sum_i \frac{|\tilde{d}(f_i) - \tilde{h}(f_i; \theta)|^2}{S_n(f_i)} \right)$$

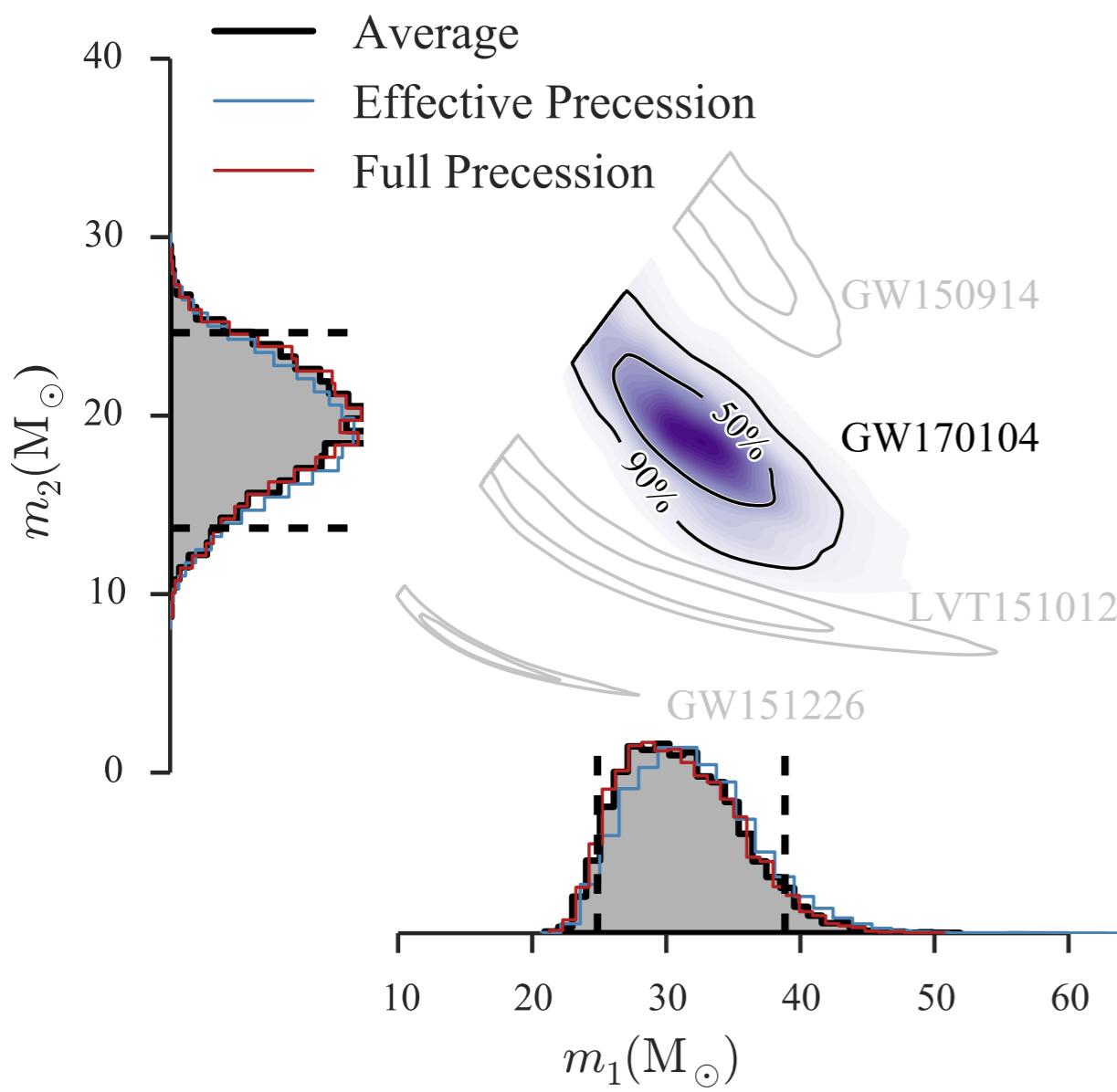


- Subtracting best-fit GR waveform model (MaP) from data

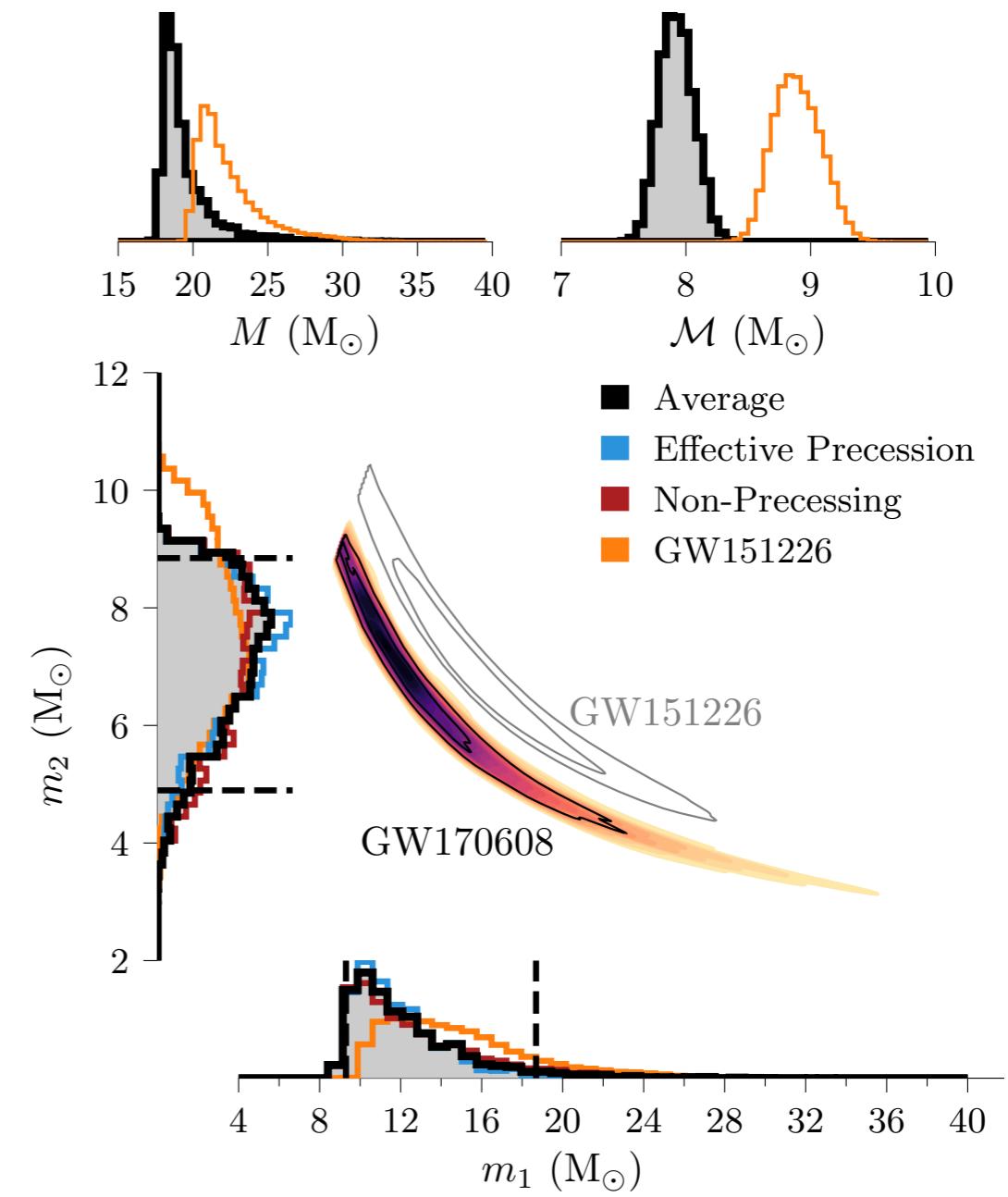


# Unveiling binary black-hole properties: masses

(Abbott et al. PRL 118 (2017) 221101)

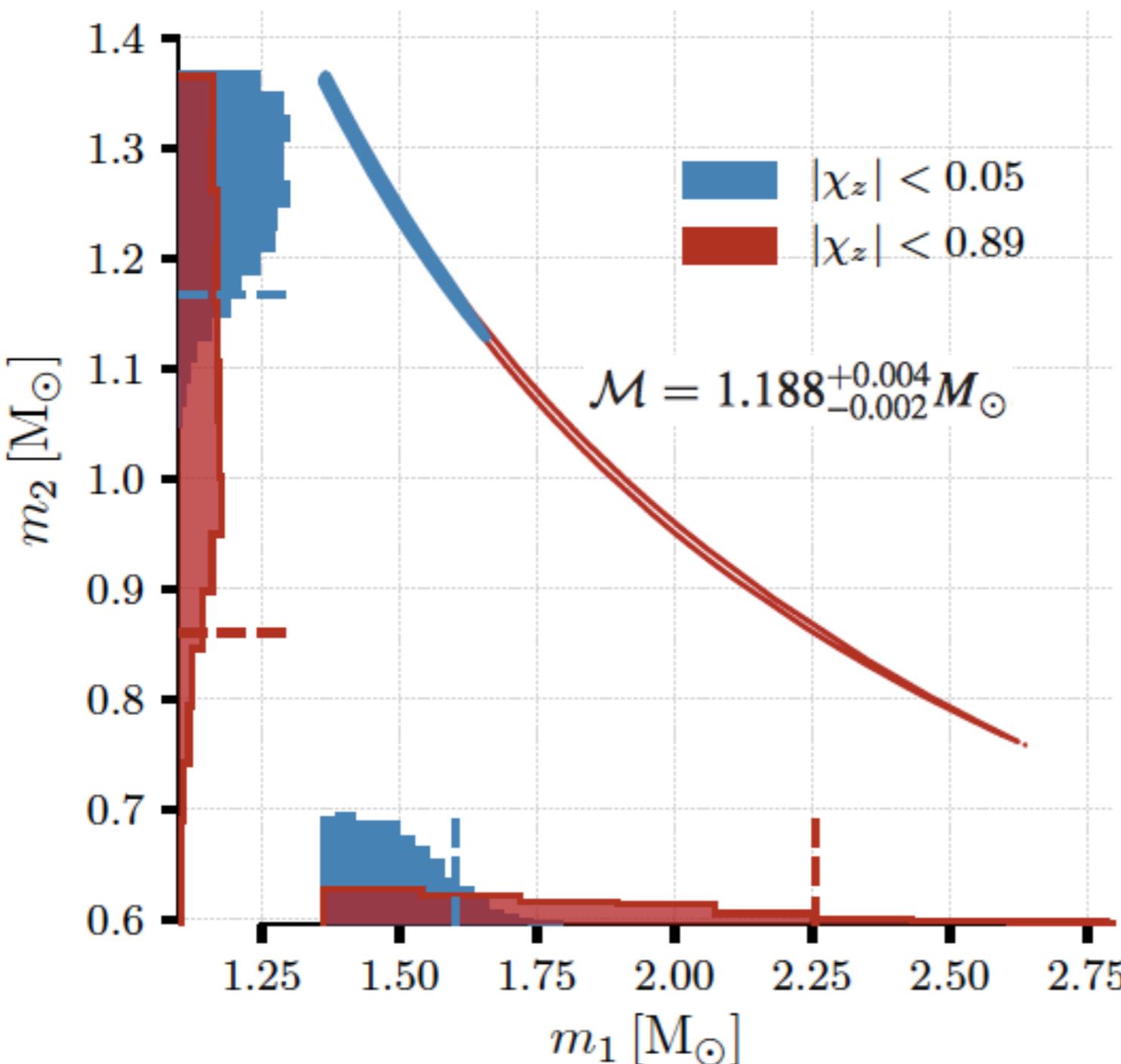


(Abbott et al. ApJ 851 (2017) L35)



- Chirp mass is best measured. Individual masses can be better measured if merger is observed, because total mass is measured at merger.

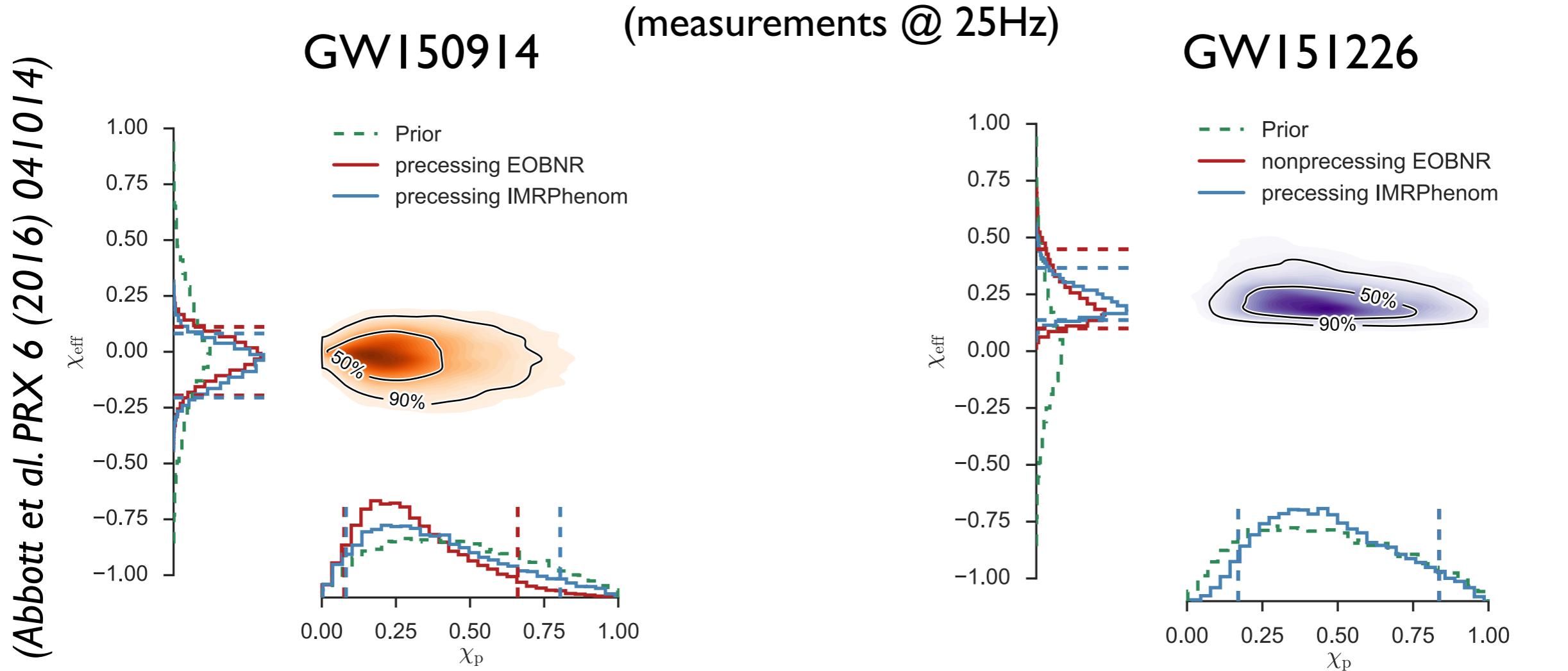
# Unveiling binary neutron star properties: masses



- **Degeneracy** between masses and spins.
- **Fastest-spinning** neutron star has (dimensionless) **spin  $\sim 0.4$** .
- Observation of **binary pulsars** in our galaxy indicate spins are **not larger than  $\sim 0.04$** .

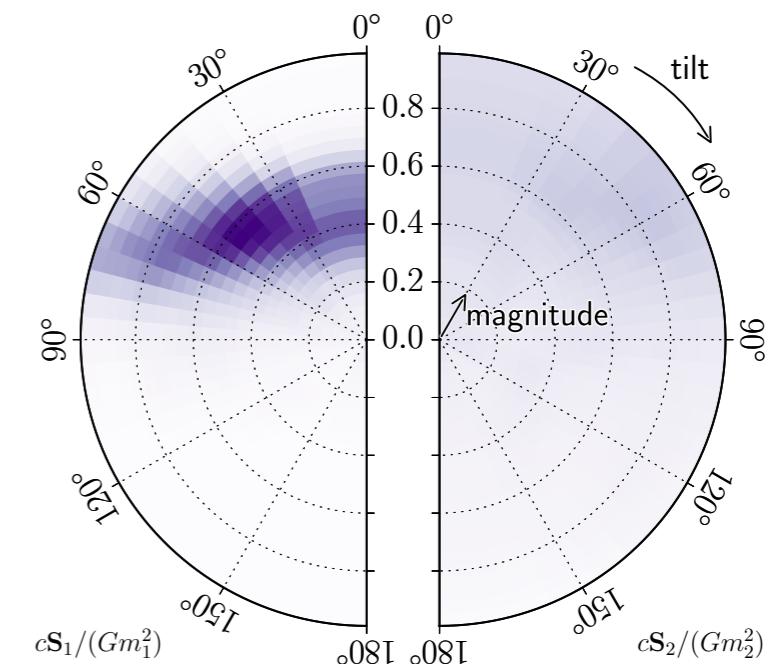
# Unveiling binary black-hole properties: spins

(Abbott et al. PRL 116 (2016) 241103)



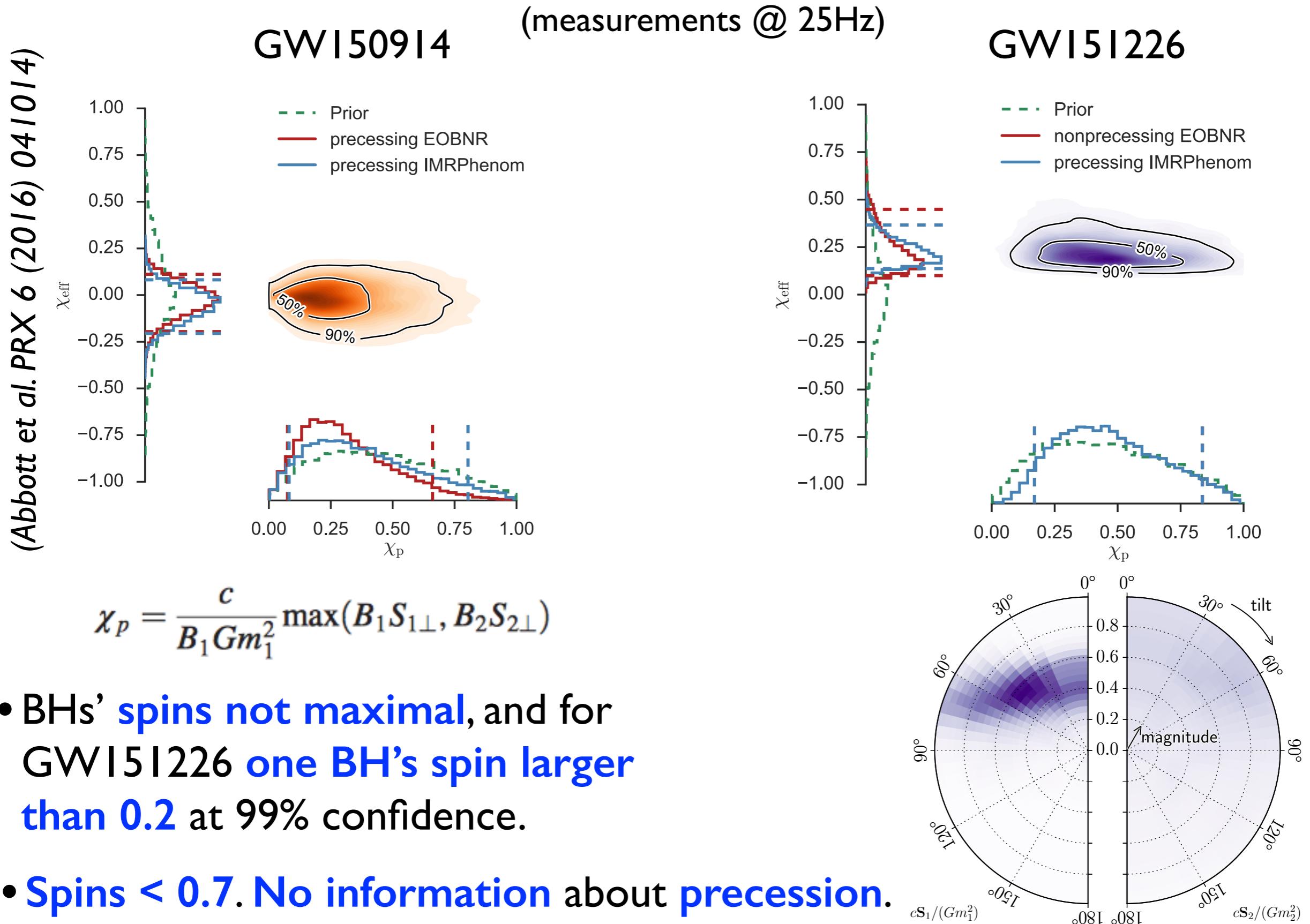
$$\chi_{\text{eff}} = \frac{c}{GM} \left( \frac{S_1}{m_1} + \frac{S_2}{m_2} \right) \cdot \frac{\mathbf{L}}{|\mathbf{L}|} \quad (\text{constant through 2PN order})$$

- BHs' **spins not maximal**, and for GW151226 **one BH's spin larger than 0.2** at 99% confidence.
- Spins < 0.7. No information about precession.

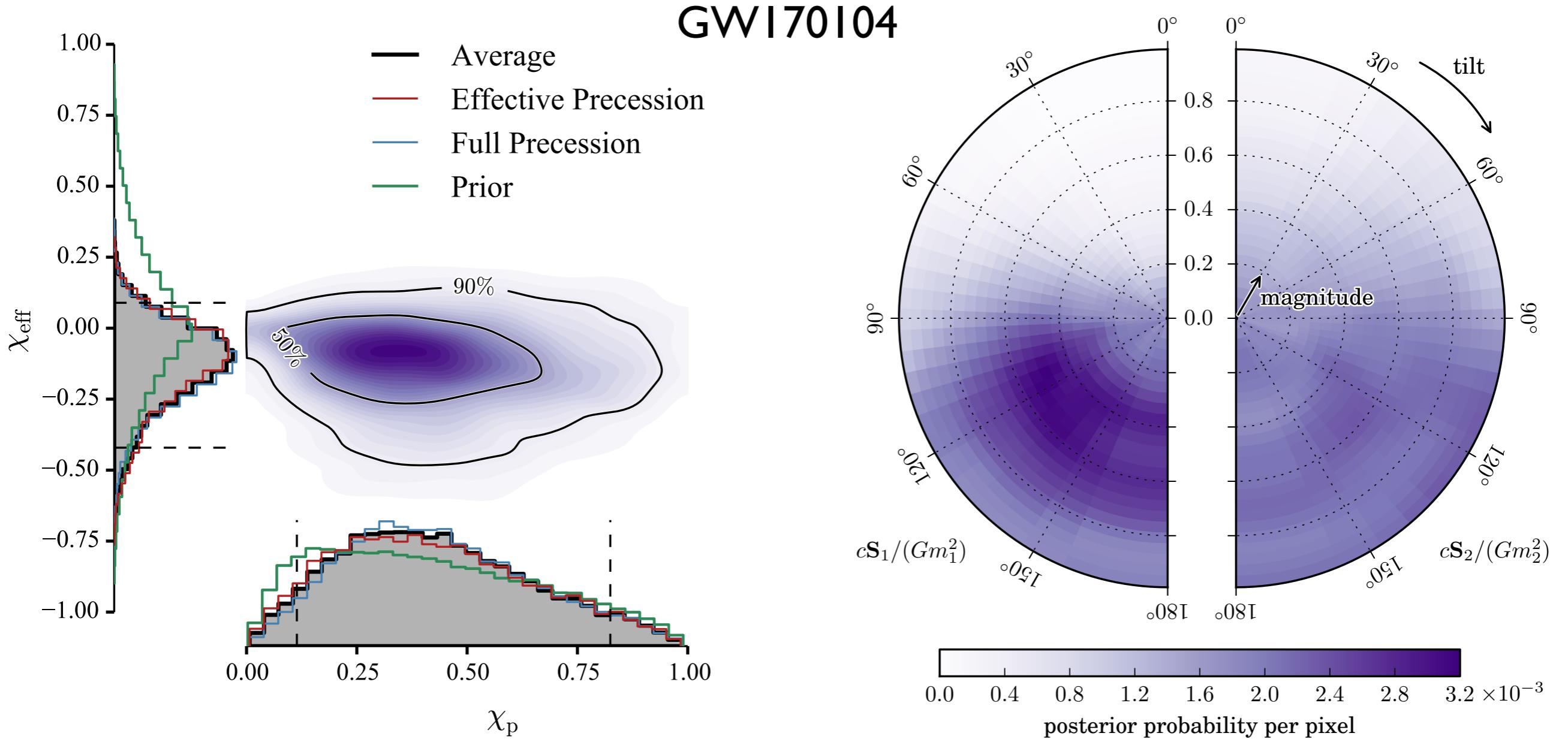


# Unveiling binary black-hole properties: spins

(Abbott et al. PRL 116 (2016) 241103)



# Unveiling binary black holes properties: spins



- Probability that  $\chi_{\text{eff}} < 0$  is 82%

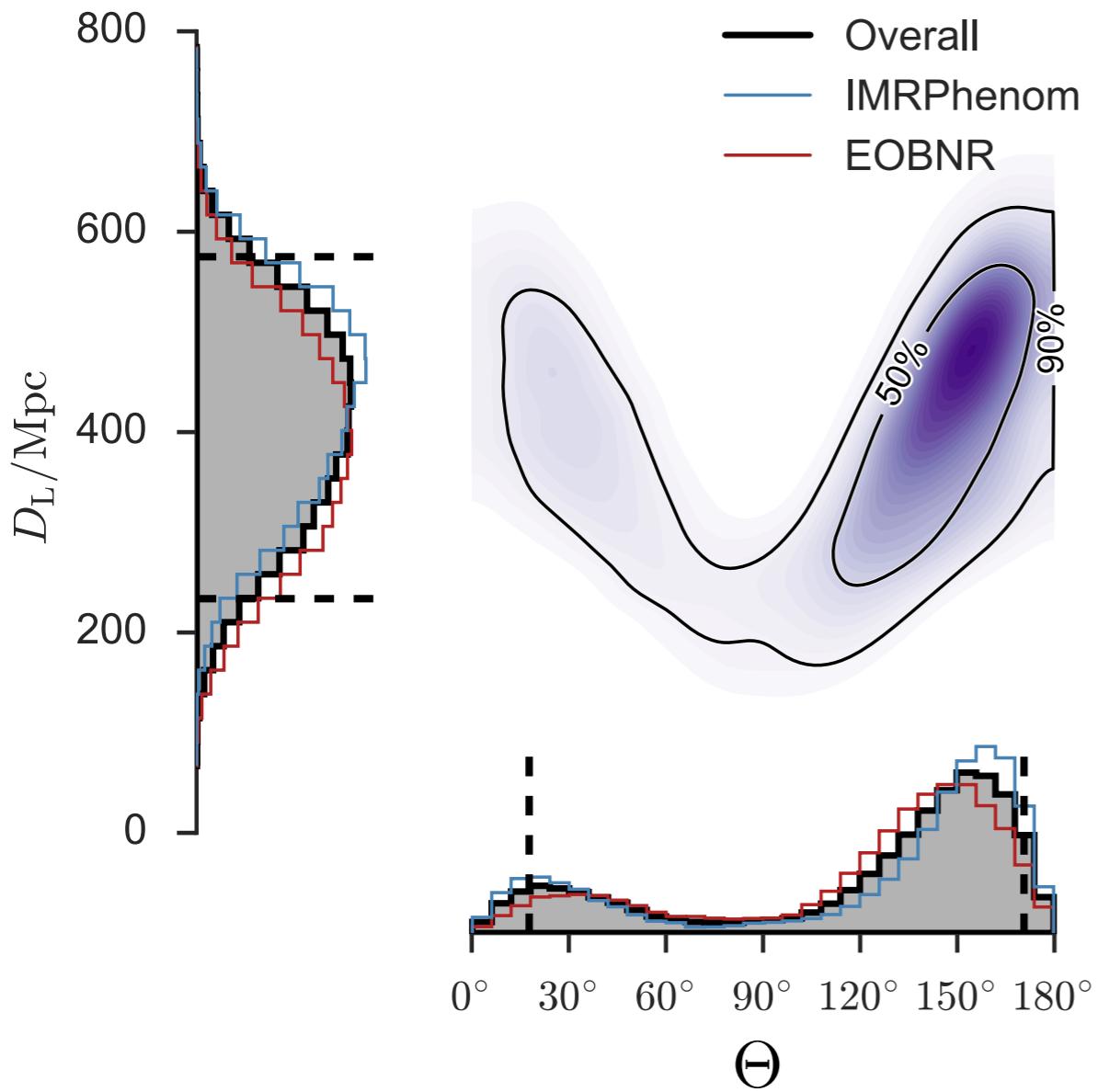
(Abbott et al. PRL 118 (2017) 221101)

- Measurements disfavor large total spin positively aligned, but do not exclude zero spins.

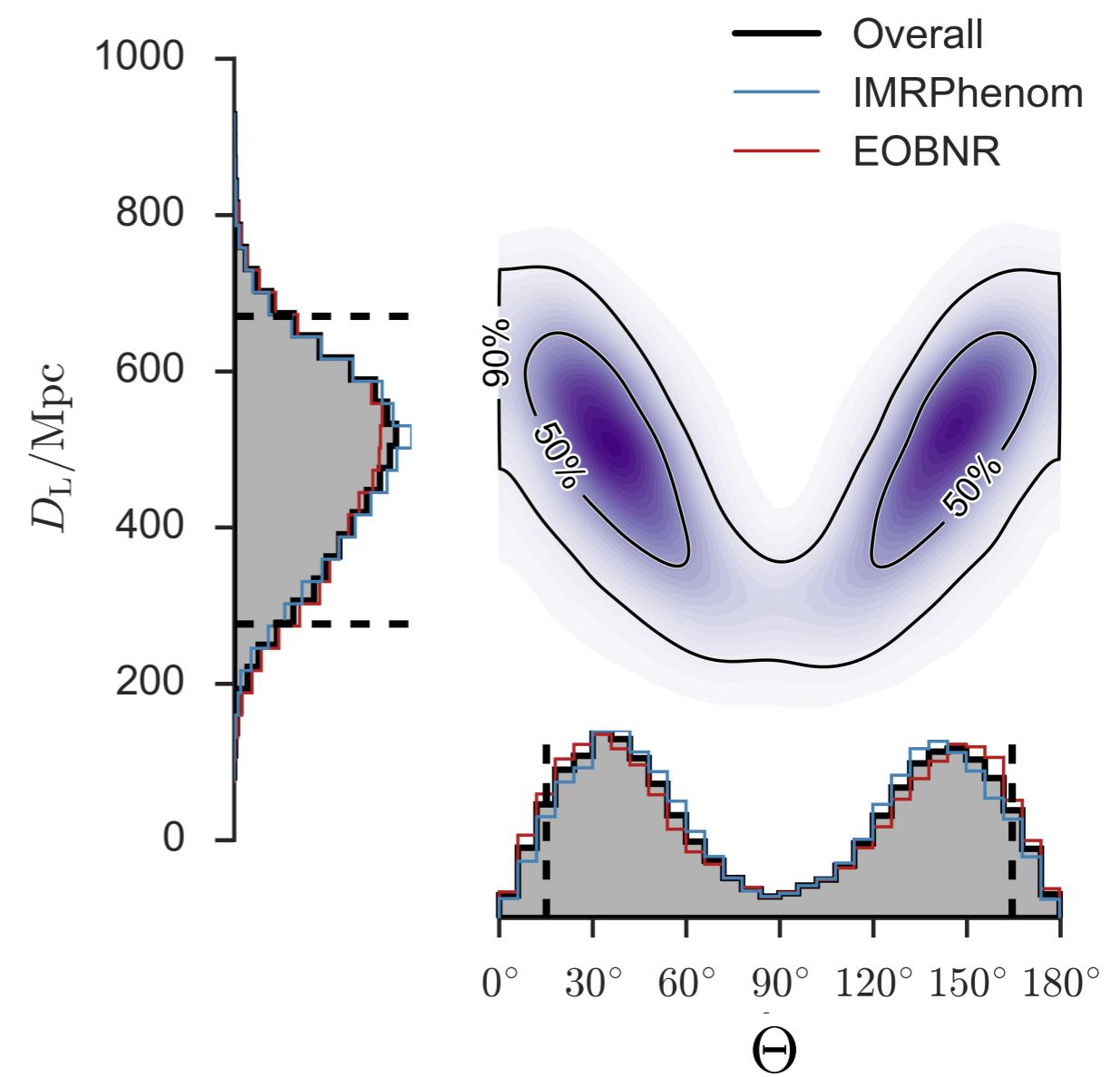
# Binaries' distance and inclination

(Abbott et al. PRX 6 (2016) 041015)

GW150914



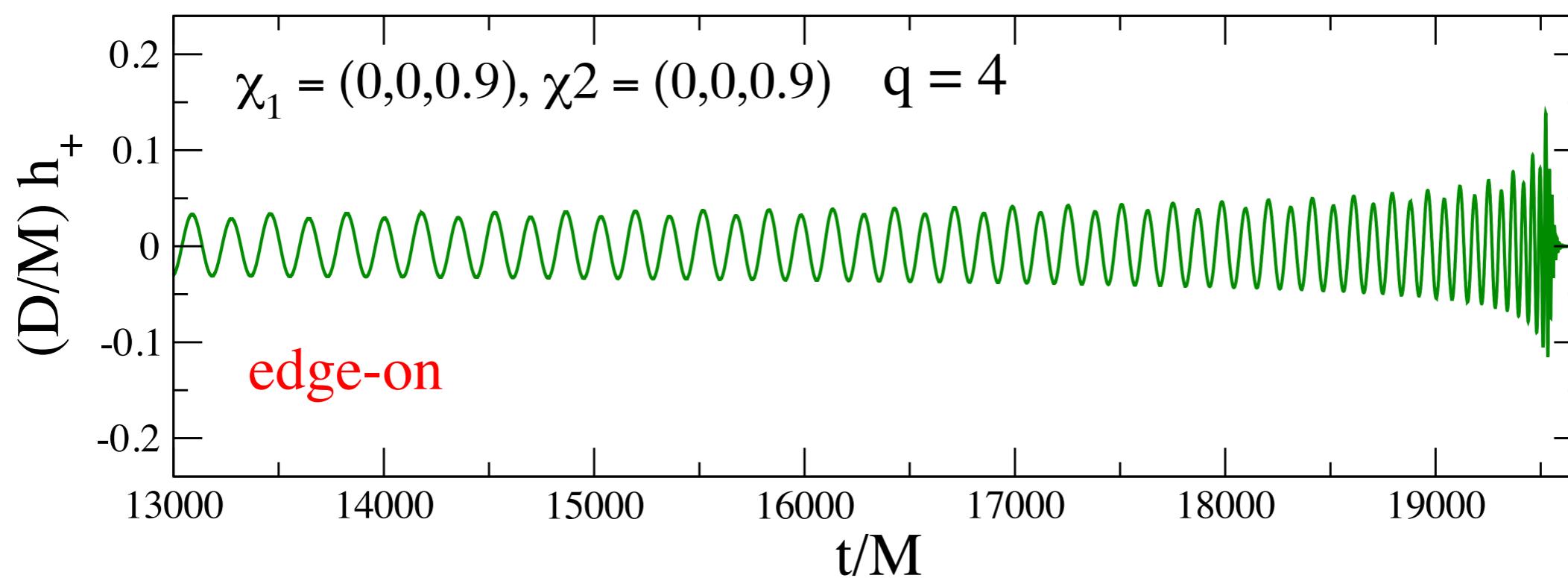
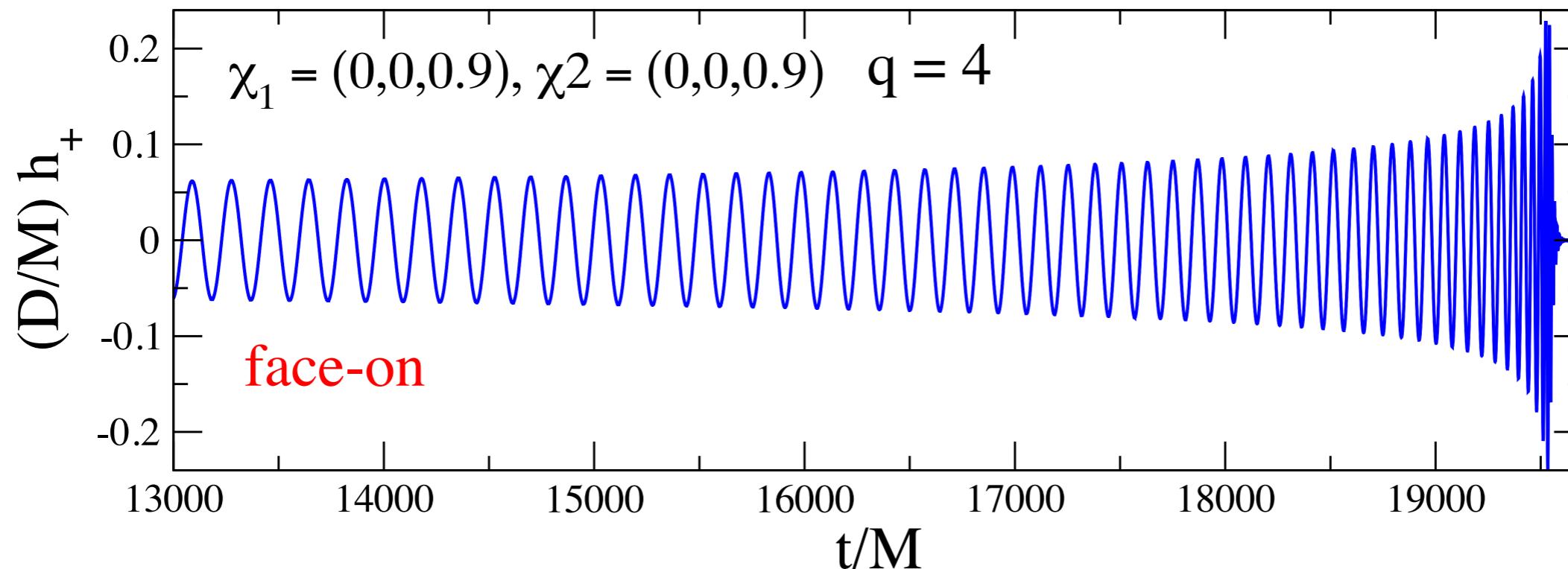
GW151226



# Effect of orientation of binary's orbital plane

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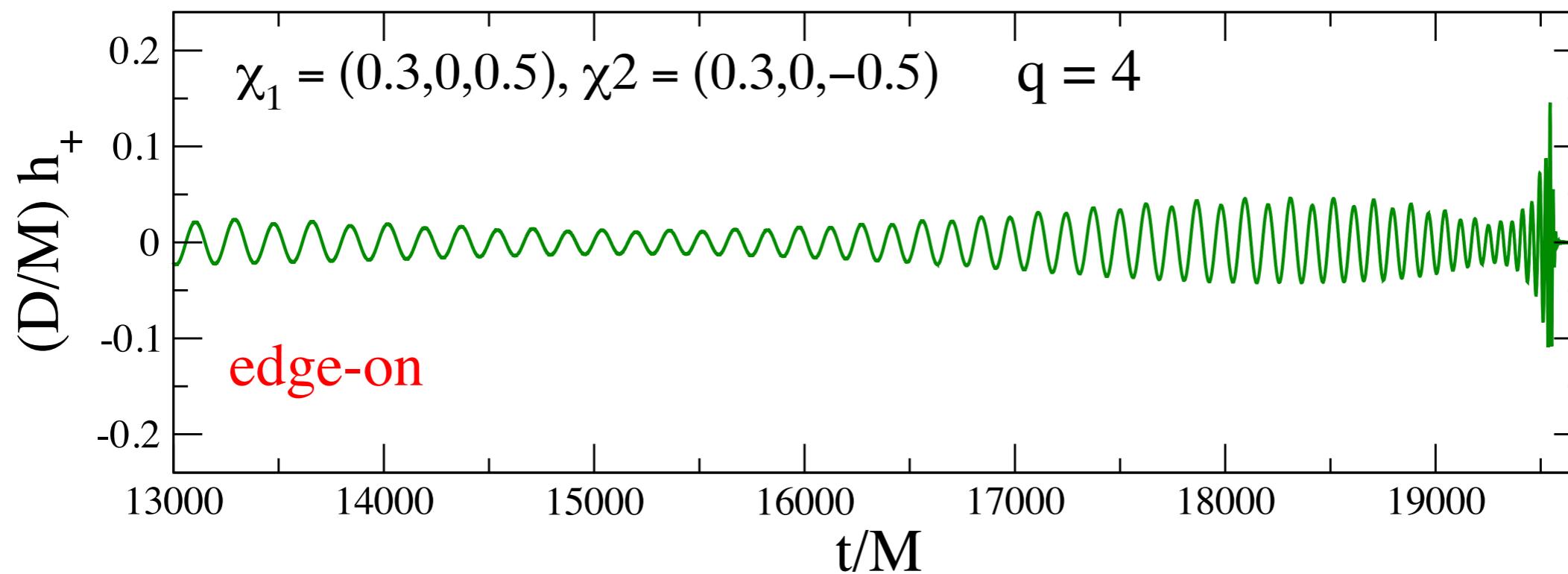
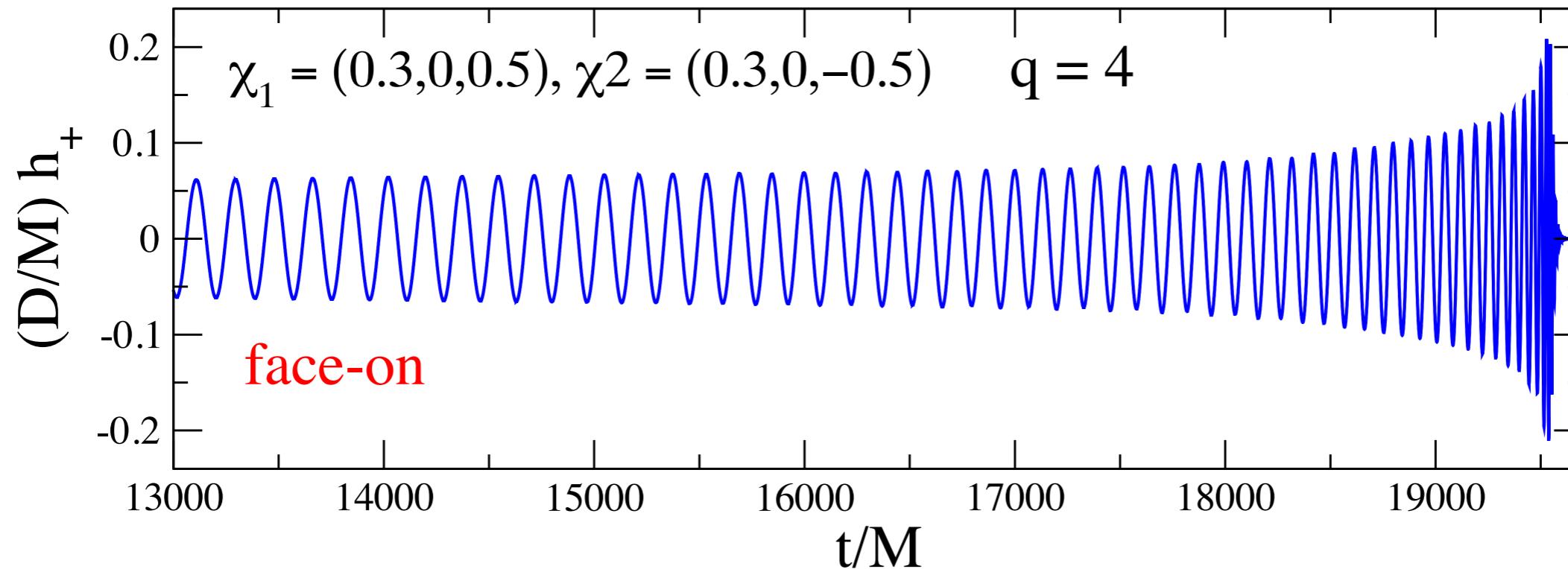
spin nonprecessing binary



# Effect of orientation of binary's orbital plane

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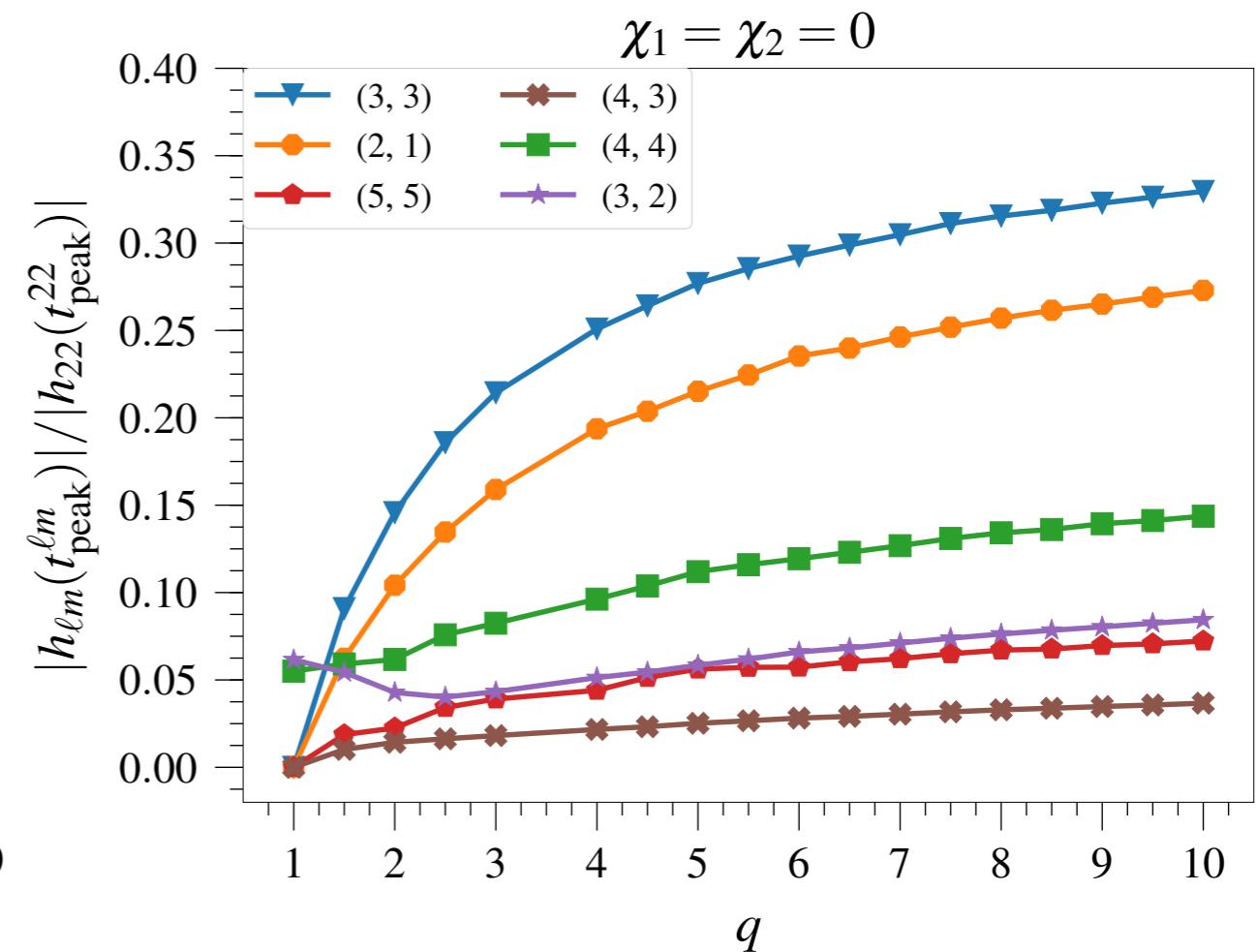
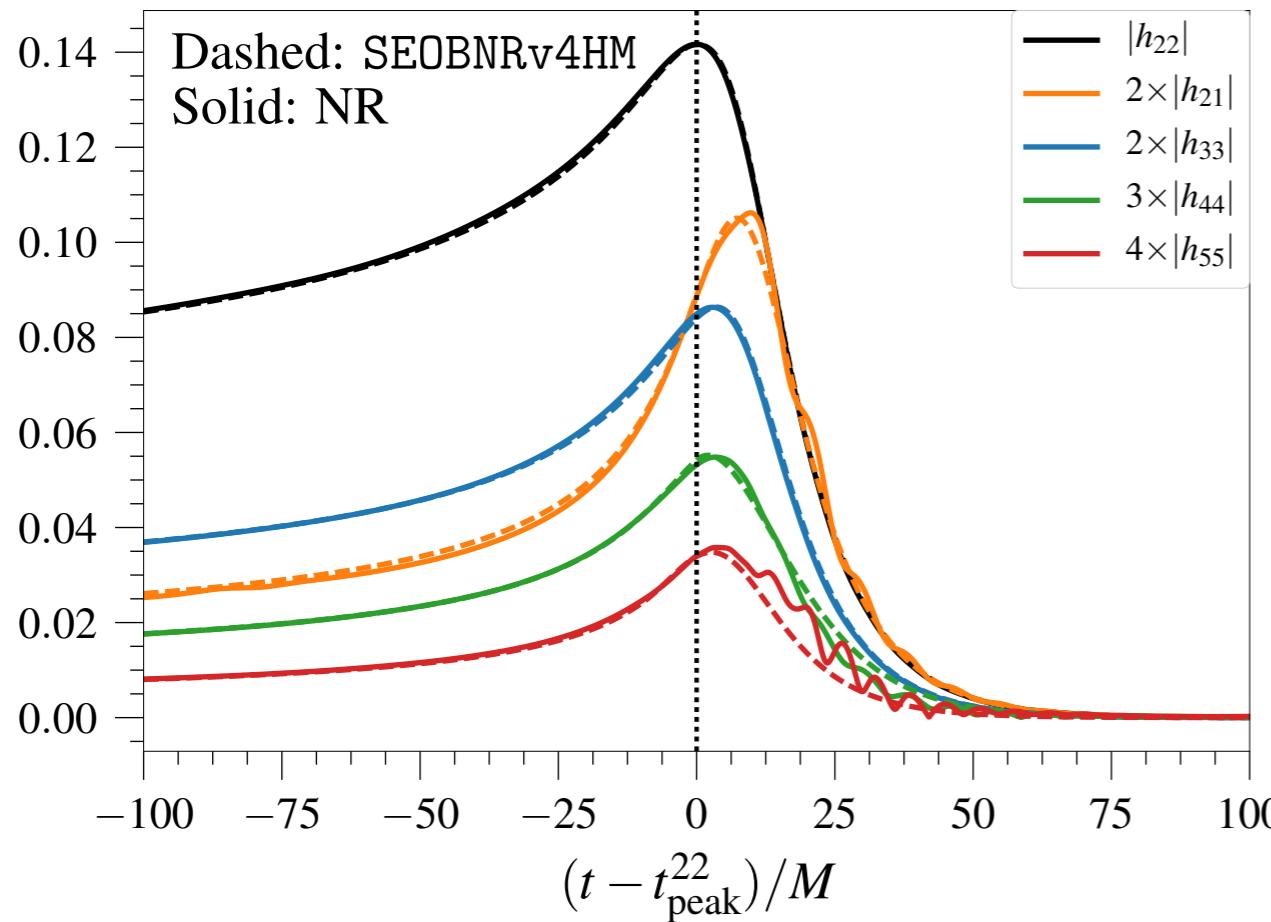
spin precessing binary



# Importance of higher harmonics: varying mass ratio

$$h_+(t; \Theta, \varphi) - i h_\times(t; \Theta, \varphi) = \sum_{\ell=2}^{\infty} \sum_{m=-\ell}^{\ell} {}_{-2}Y_{\ell m}(\Theta, \varphi) h_{\ell m}(t)$$

(Cotesta, AB et al. 18)



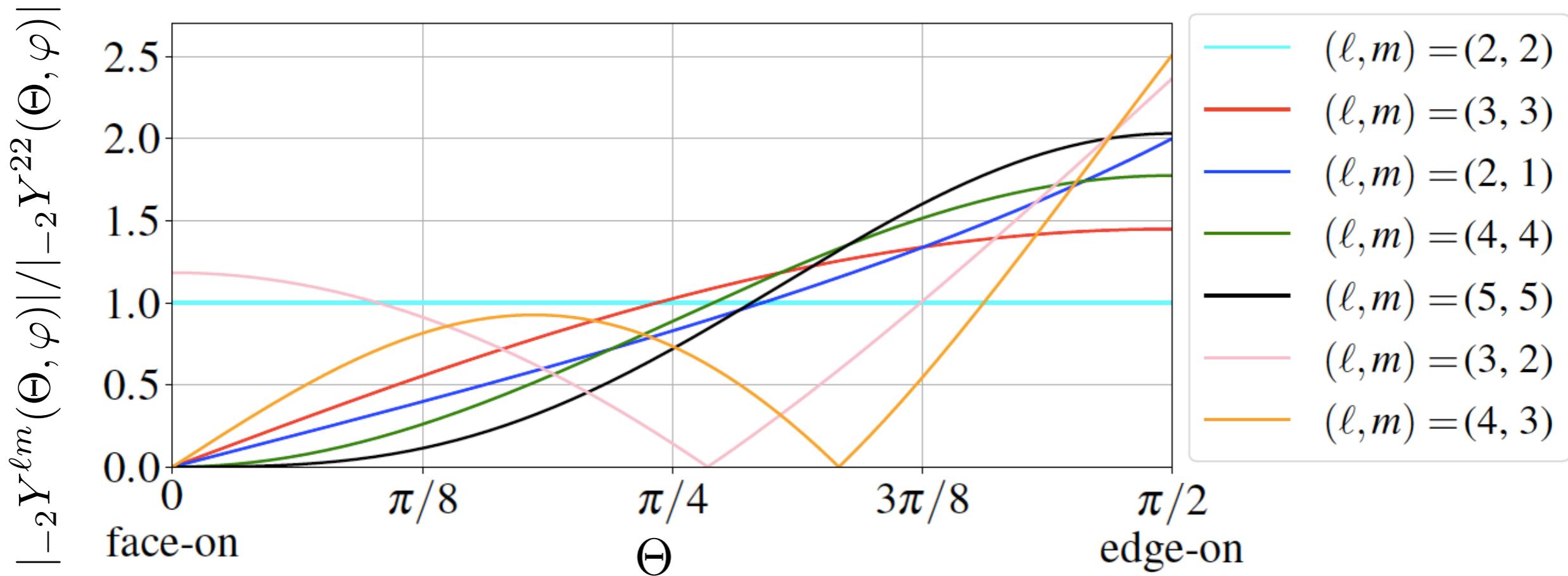
- Merger-ringdown EOBNR model **reproduces time & phase shifts between NR modes' at peak.**

# Importance of higher harmonics also depends on geometric factor

$$h_+(t; \Theta, \varphi) - i h_\times(t; \Theta, \varphi) = \sum_{\ell=2}^{\infty} \sum_{m=-\ell}^{\ell} {}_{-2}Y_{\ell m}(\Theta, \varphi) h_{\ell m}(t)$$

(Cotesta, AB et al. 18)

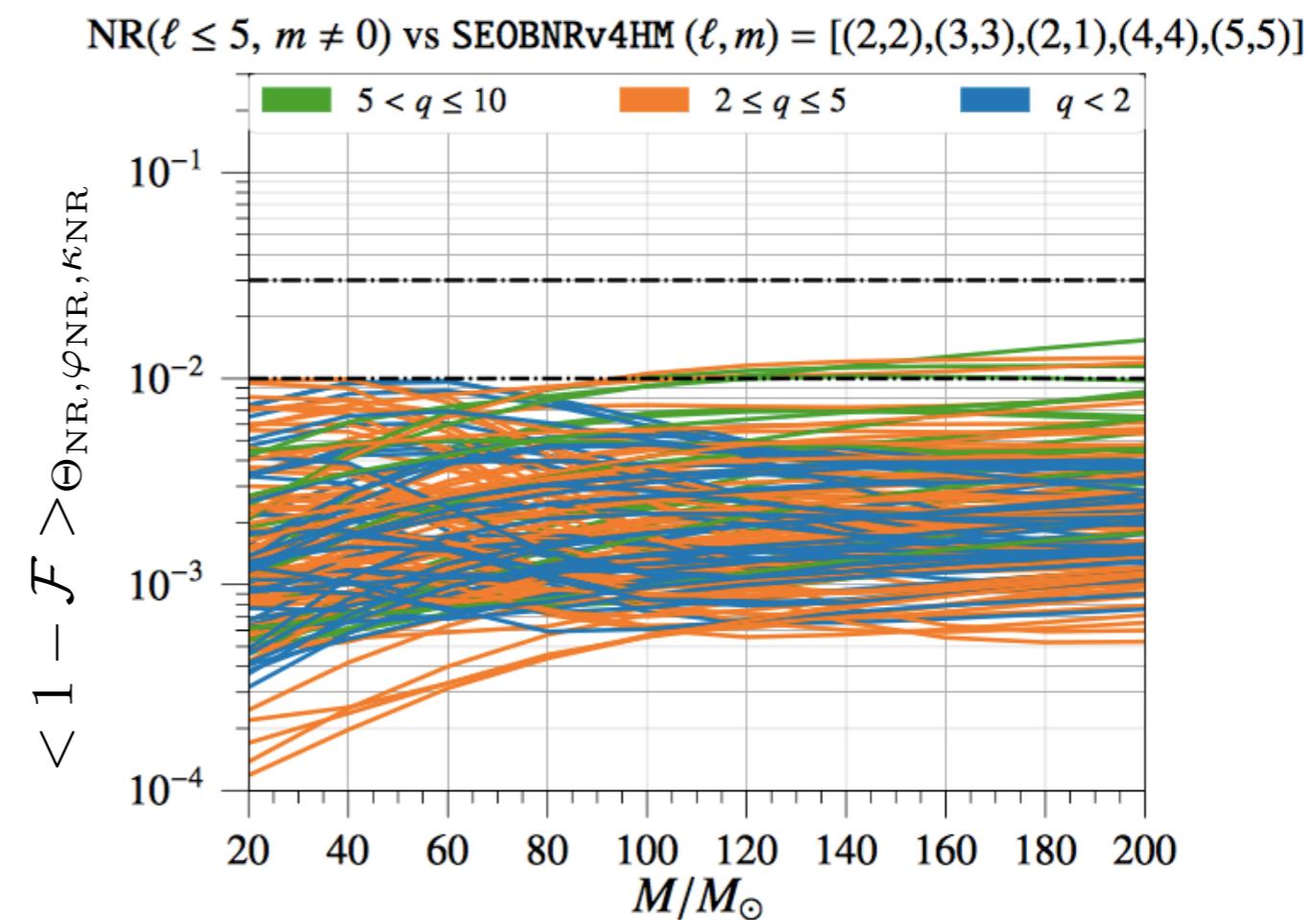
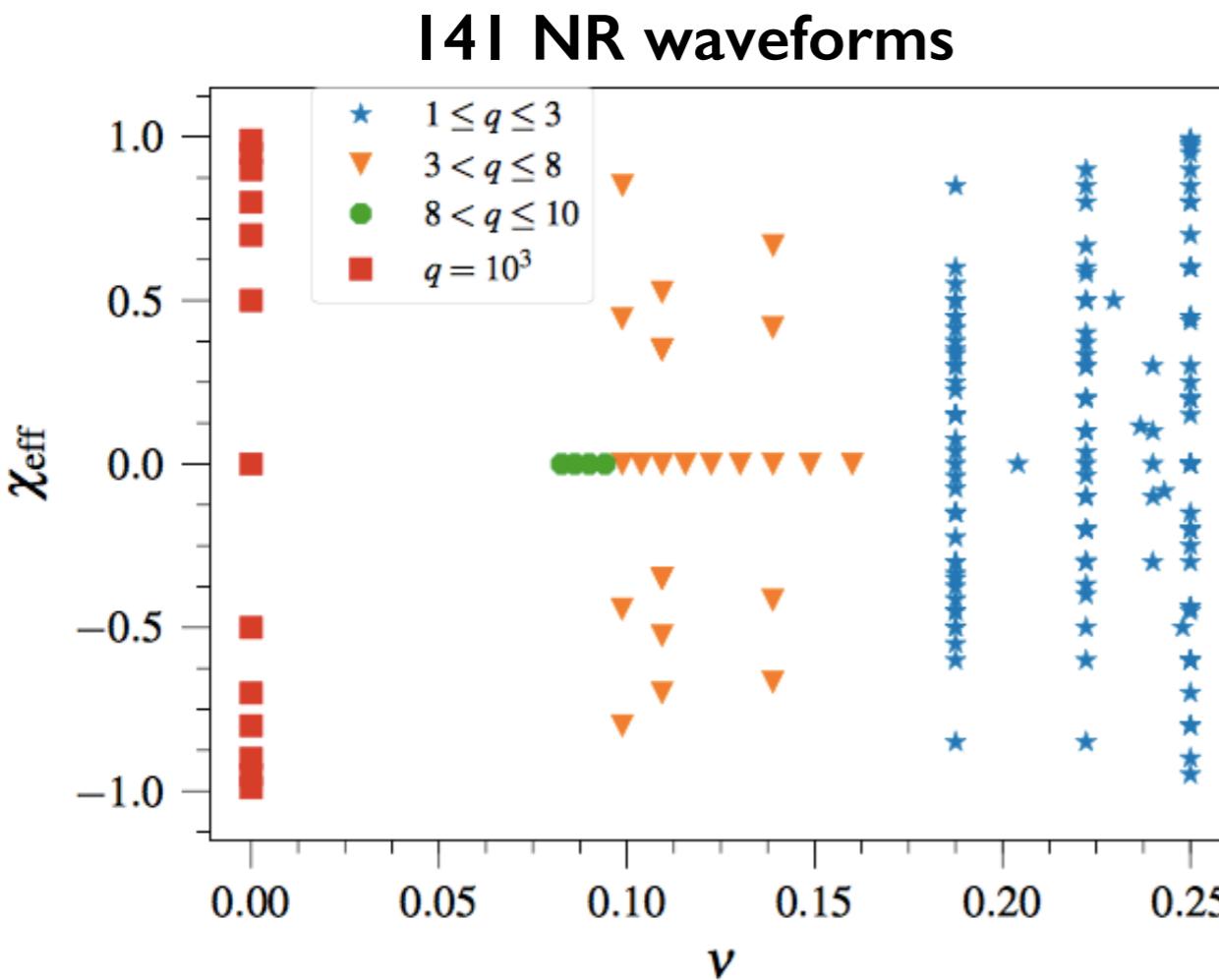
↑ **geometric factor** is important  
to determine **strength** of  
**higher harmonics**



# Accuracy of multipolar EOBNR model against NR

- Non-precessing spin EOBNR waveform model with (2,1), (3,3), (4,4) & (5,5) modes.

(Cotesta, AB et al. 18)

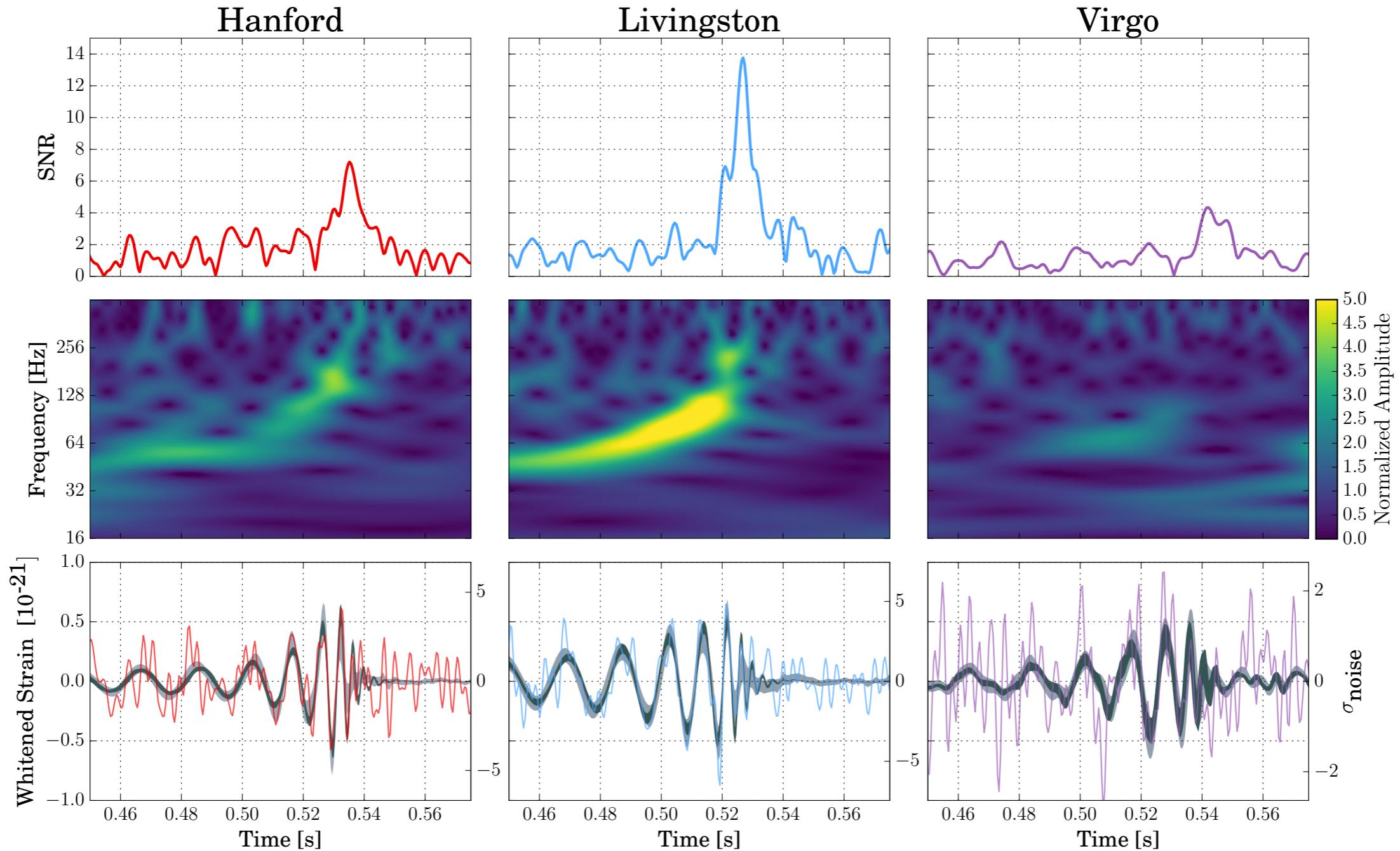


- Extending analysis to other modes requires producing accurate NR waveforms for those modes.

(for modeling see also Mehta et al. 17, London et al. 17; for searches see Capano, ..., AB 16, Harry et al. 18)

# Three-detector observation during O2: GW170814

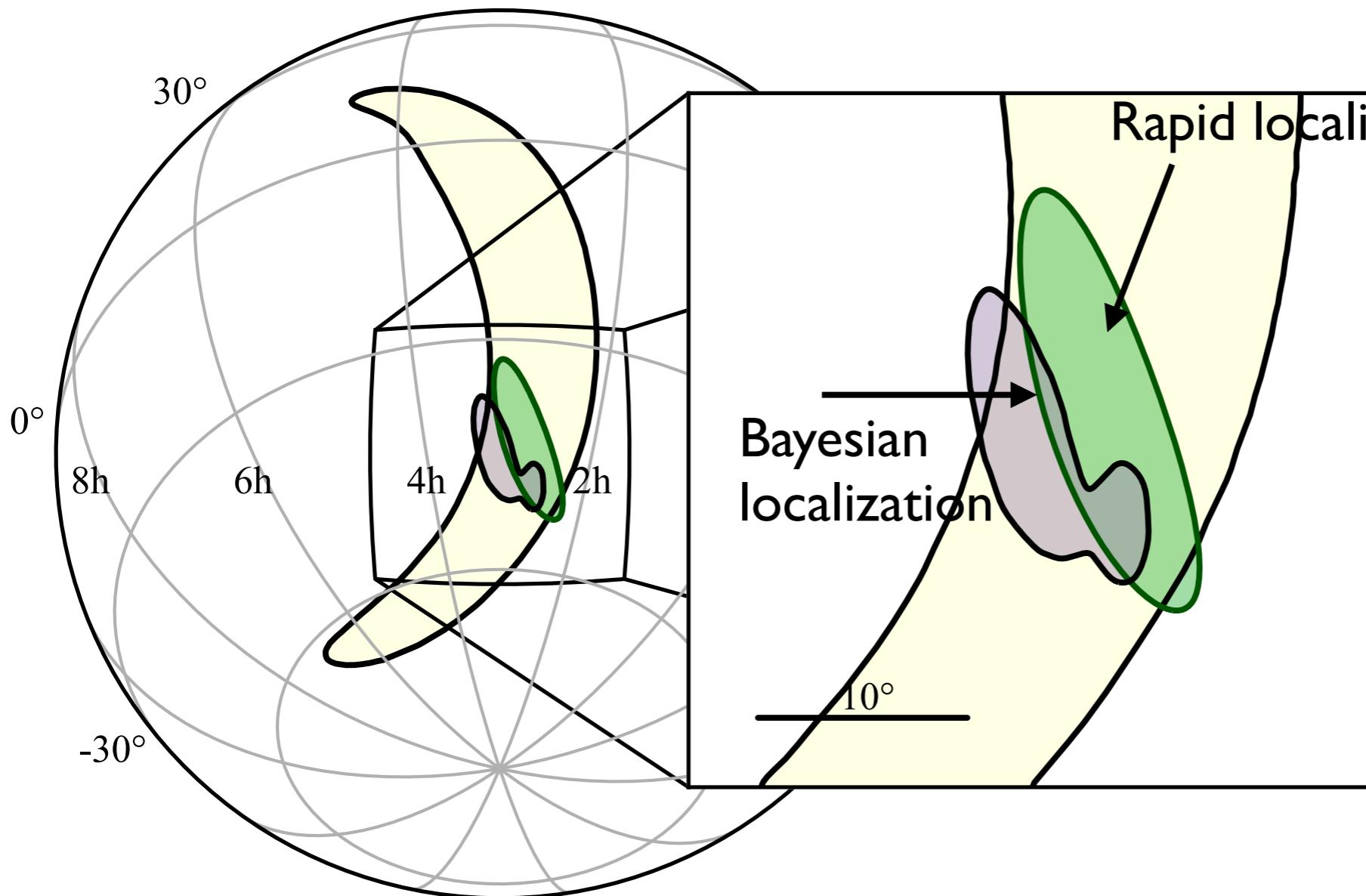
(Abbott et al. PRL 119 (2017) 141101)



- **GW170814:** SNR=7.3 (Hanford), 13.7 (Livingston), 4.4 (Virgo)

# Three-detector observation improves source sky-localization

(Abbott et al. PRL 119 (2017) 141101)

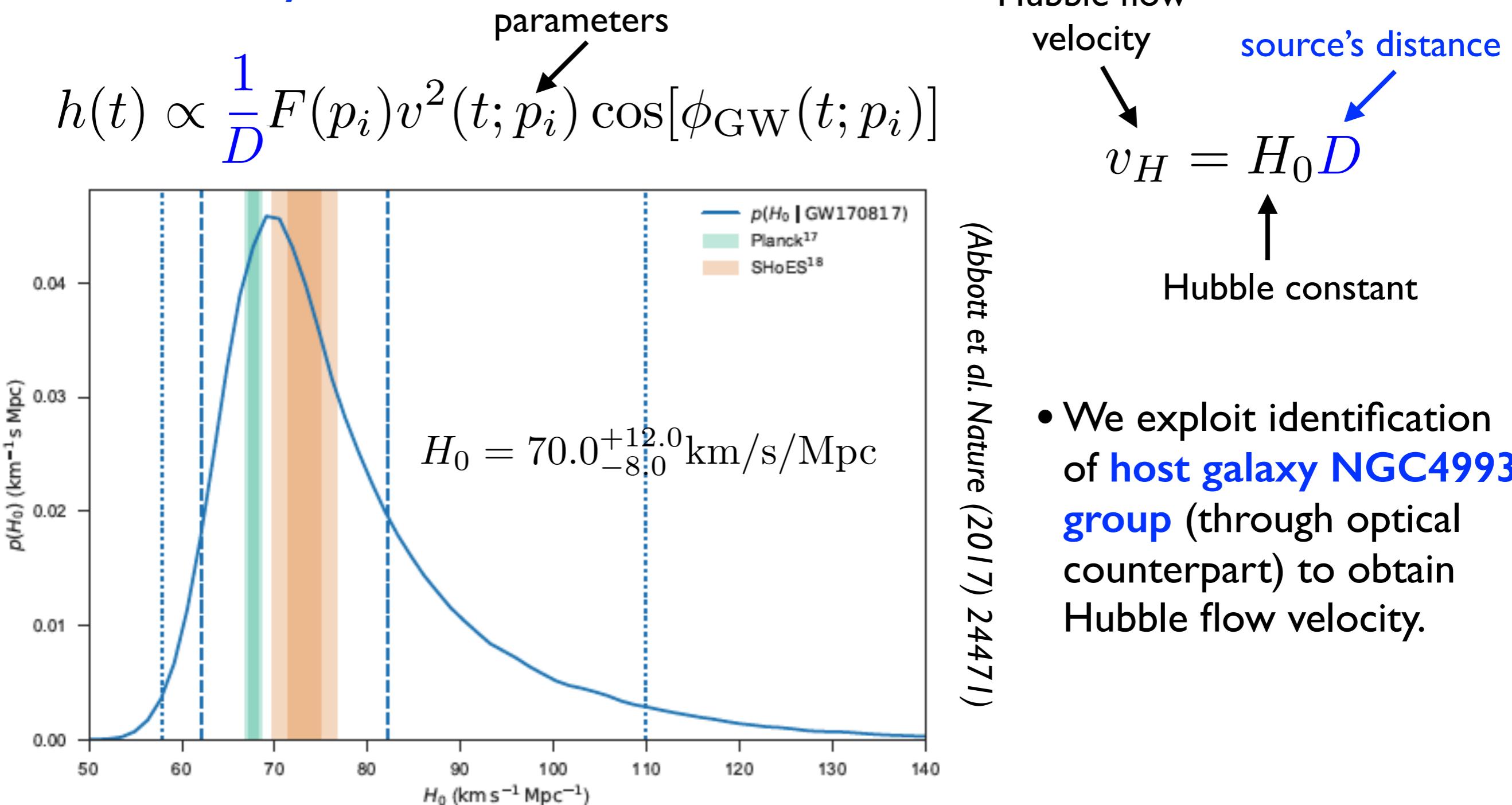


- GW **amplitude & phase information at different detectors** improve localization.

- **GW170814**: with Virgo, **sky-localization reduced** from 1160 to **60 square degrees** (90% credible regions). **Distance** measurement also **improved**.

# Cosmography: Inference of Hubble constant with GW170817

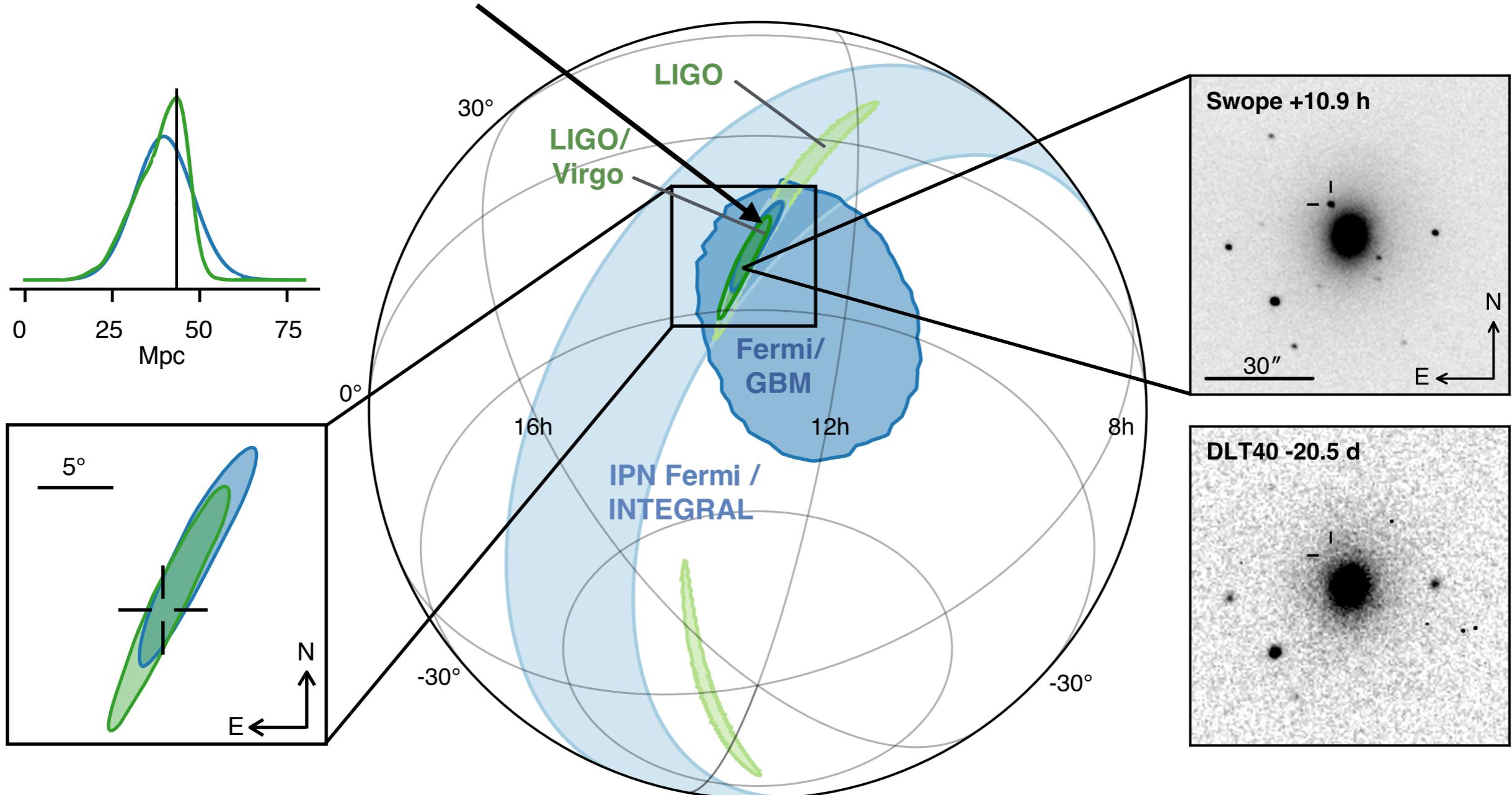
- Compact-object **binaries are standard candles** (sirens). (Schutz 1986)
- **Standard candles are sources whose distance from Earth can be inferred from their luminosity.**



# Sky localization and rapid EM follow up

• 28 square degrees

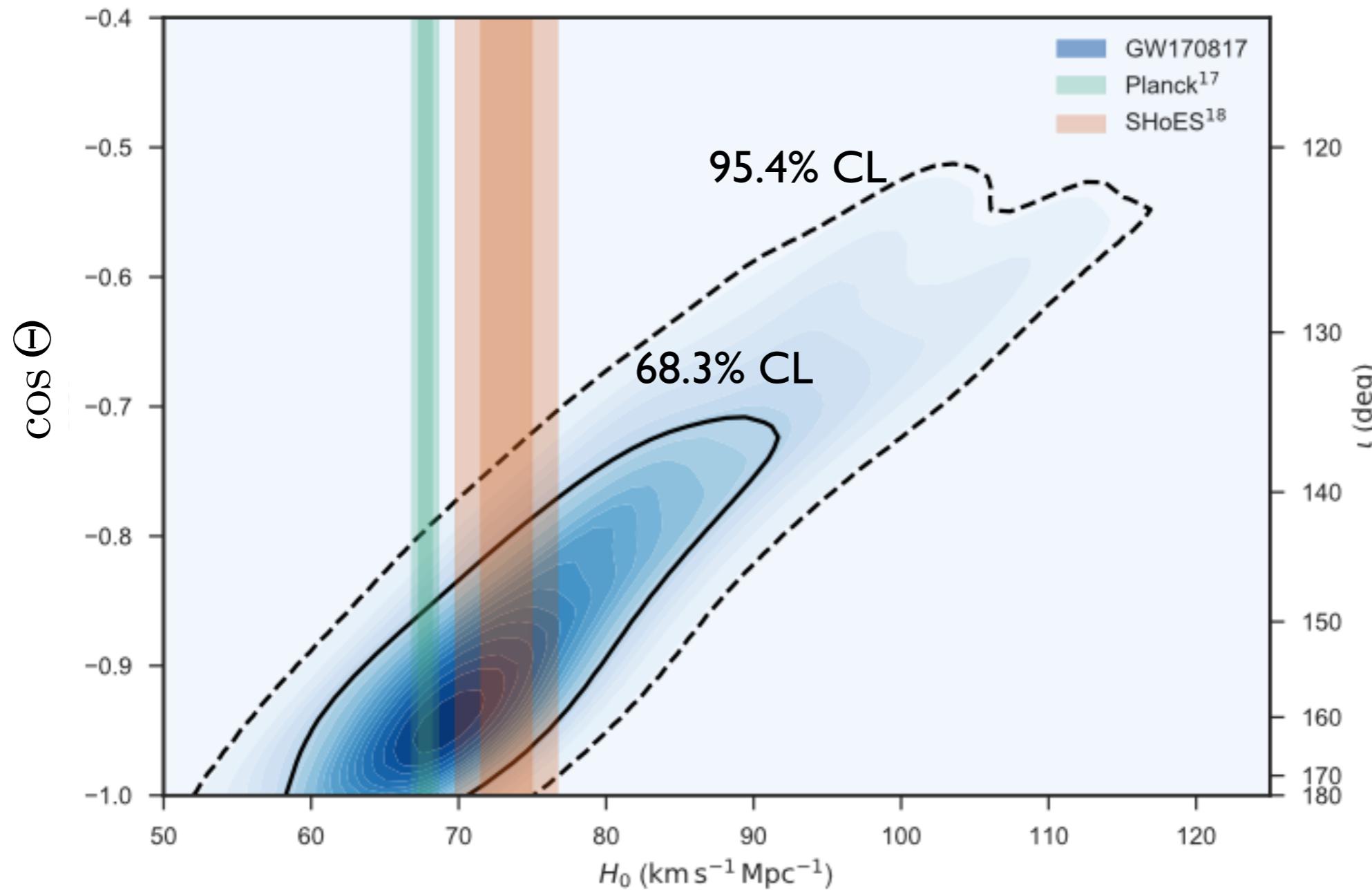
03:05:23 August 18, 2017



(Abbott et al. APJ 848 (2017) L12)

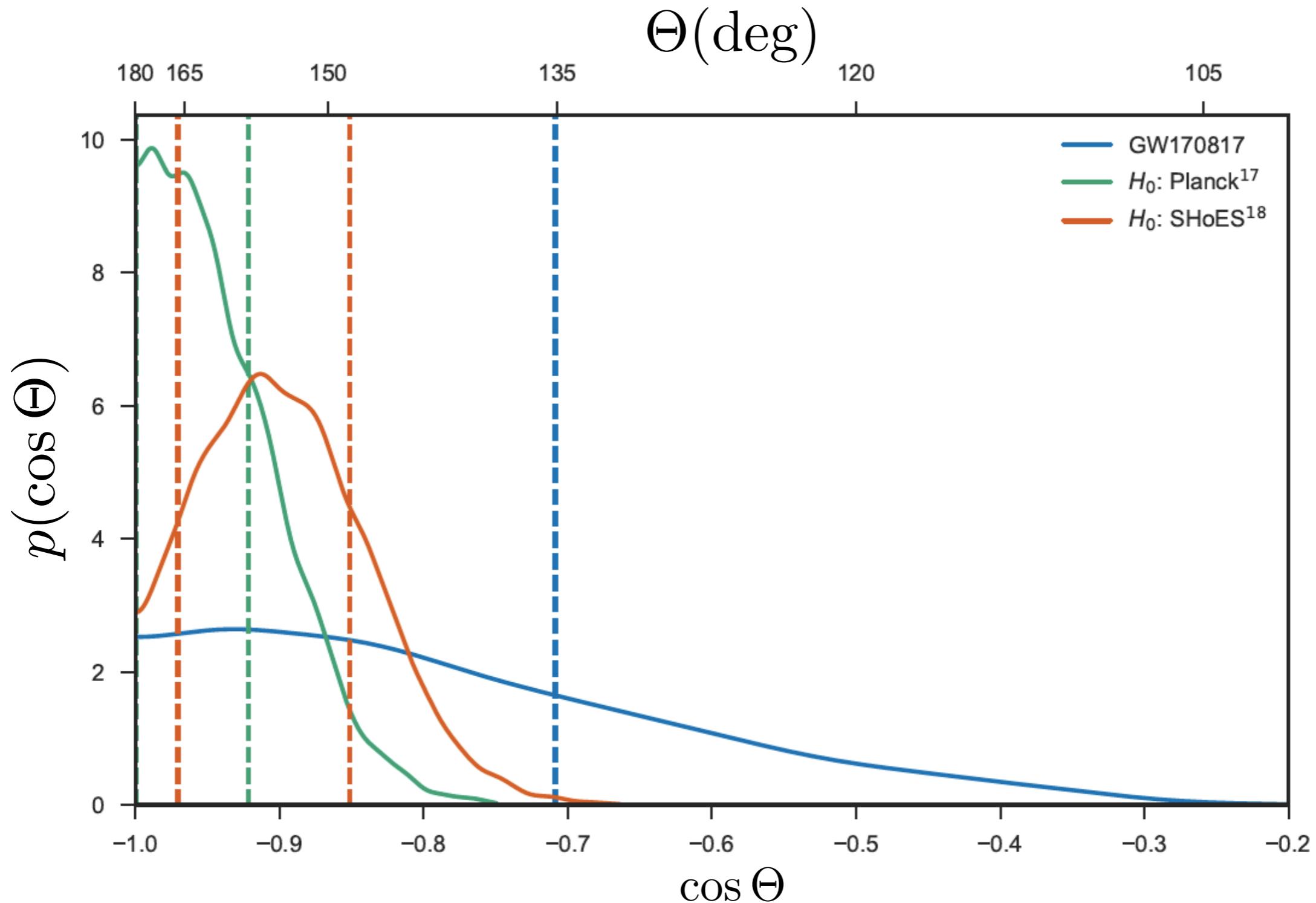
# Hubble parameter correlates with binary's inclination

$$h(t) = \frac{2\mathcal{M}}{D} \mathcal{A}(\theta, \phi, \psi; \Theta) [\mathcal{M}\omega(t)]^{2/3} \cos(2\Phi(t) + 2\Phi_0 - \alpha)$$



(Abbott et al. Nature (2017) 24471)

# Constraints on binary's inclination angle using Planck/SHoES



(Abbott et al. Nature (2017) 24471)