Astrophysics and Data Analysis
Lecture 2: Sources

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Gravitational Wave Sources

- This lecture will focus on gravitational wave sources. For each source type, we will discuss several aspects
  - *Source Astrophysics.* Current evidence for and knowledge of progenitors, estimates of event rates.
  - *Detection.* Which detectors, recent results from LIGO (where relevant).
  - *Science.* What can we hope to learn about astrophysics from observations of these sources?
- Will also mention appropriate modelling techniques (without details) and relevant data analysis methods. The latter will be discussed in more detail in Lecture 3.

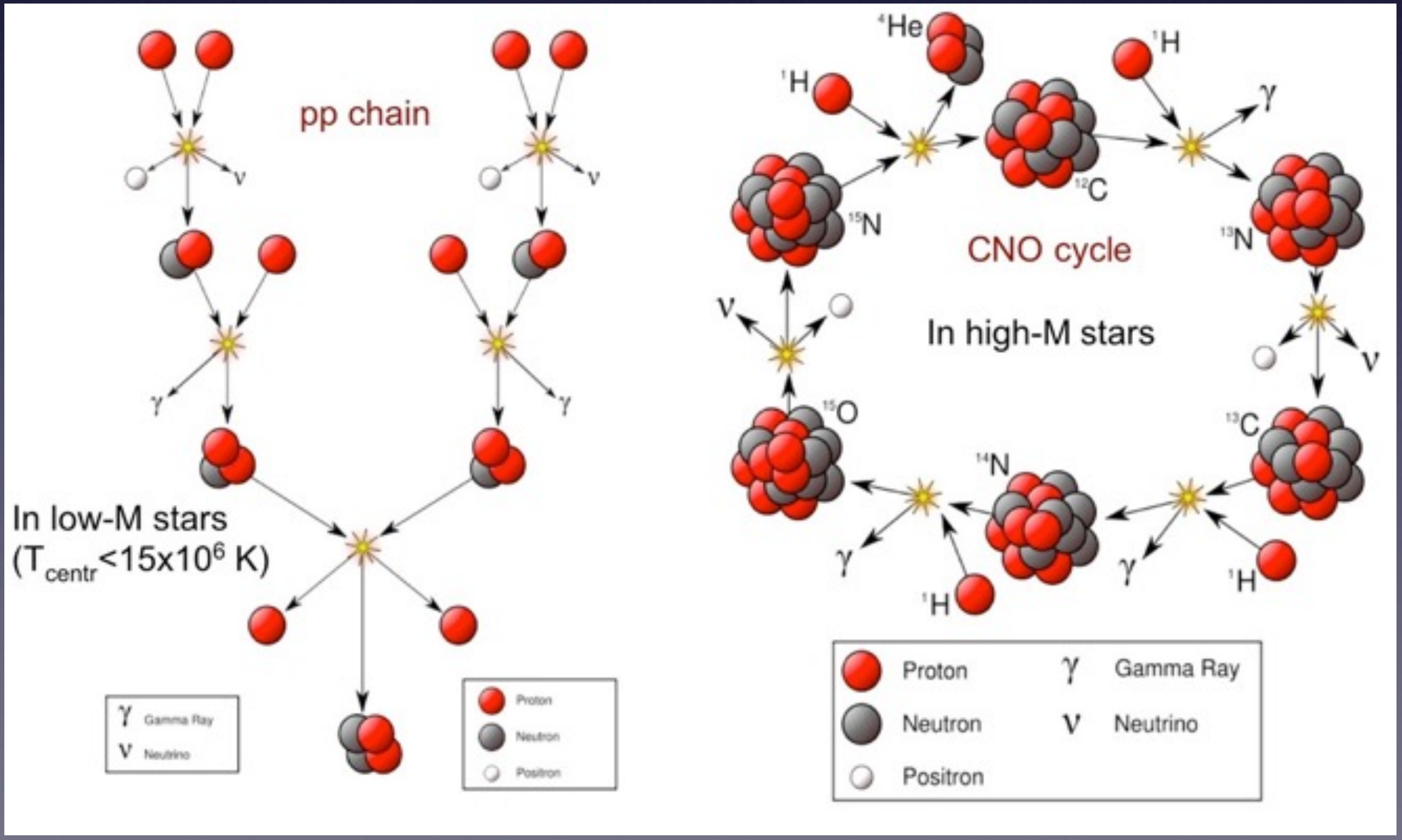
Gravitational Wave Sources: Compact Binary Coalescence

Comparable Mass Compact Binaries

- The inspirals and mergers of compact binaries are some of the most important gravitational wave sources for both ground and space based detectors.
- Can categorize compact binaries in terms of their mass ratios
  - ➡ *Comparable mass compact binaries (CMCB).* Two components are of similar mass. Divide into *stellar mass comparable mass binaries* for LIGO and *intermediate mass/supermassive comparable mass binaries* for LISA.
  - ➡ *Intermediate mass ratio inspirals.* Mass ratios of 100-1000:1 for both LIGO (1:100) and LISA (1,000:1,000,000).
  - ➡ *Extreme mass ratio inspirals.* Inspiral of a stellar mass object into a supermassive object, with mass ratio >10,000:1.

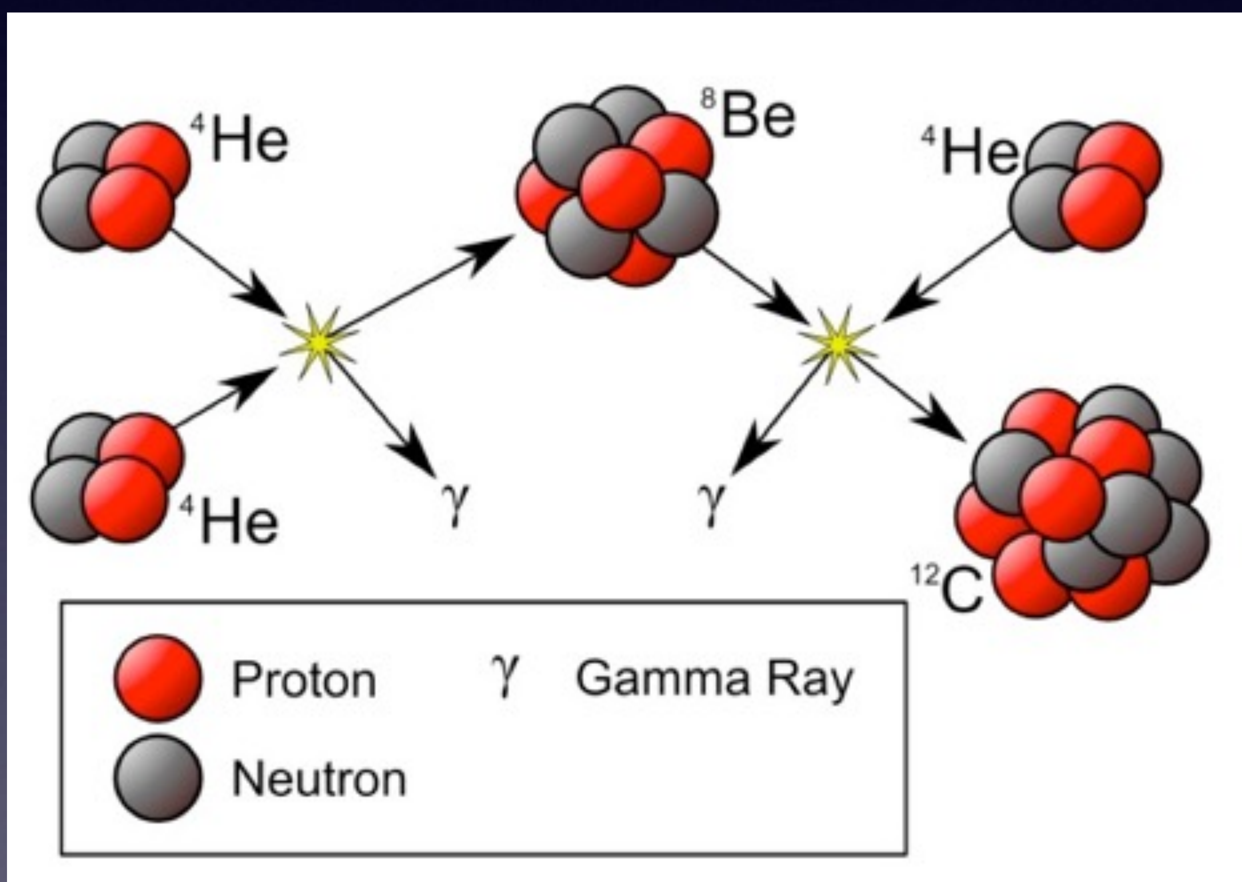
CMCB Inspirals for LIGO

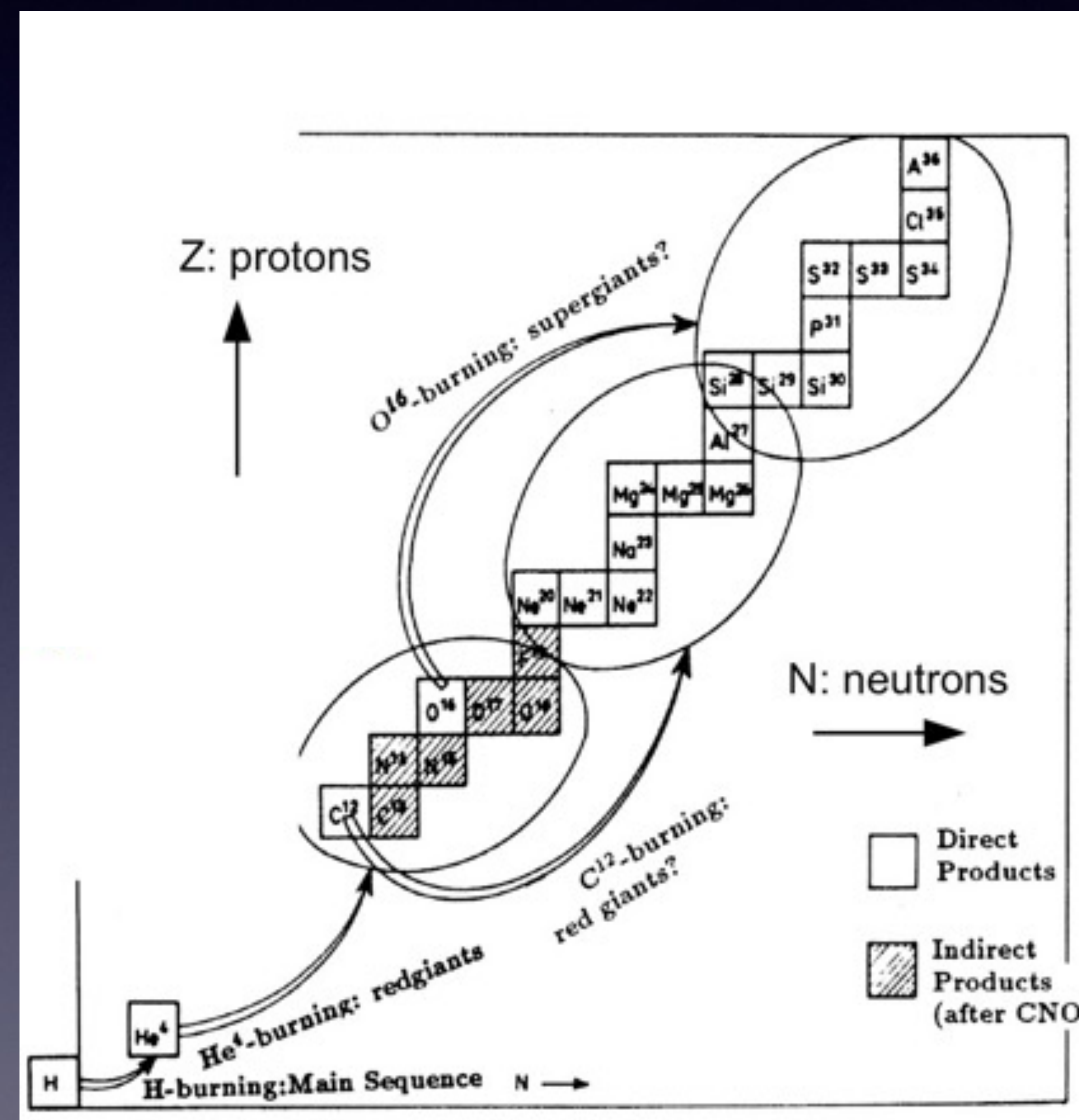
- Mergers between compact remnants formed as endpoint of stellar evolution are an important source for LIGO.
- Stars are supported by nuclear burning for most of their lifetime - pp chain for low mass stars or CNO cycle for high mass.



Stellar Evolution

- As fuel runs out at a radius, the star collapses and heats up and then a new nuclear reaction takes over.

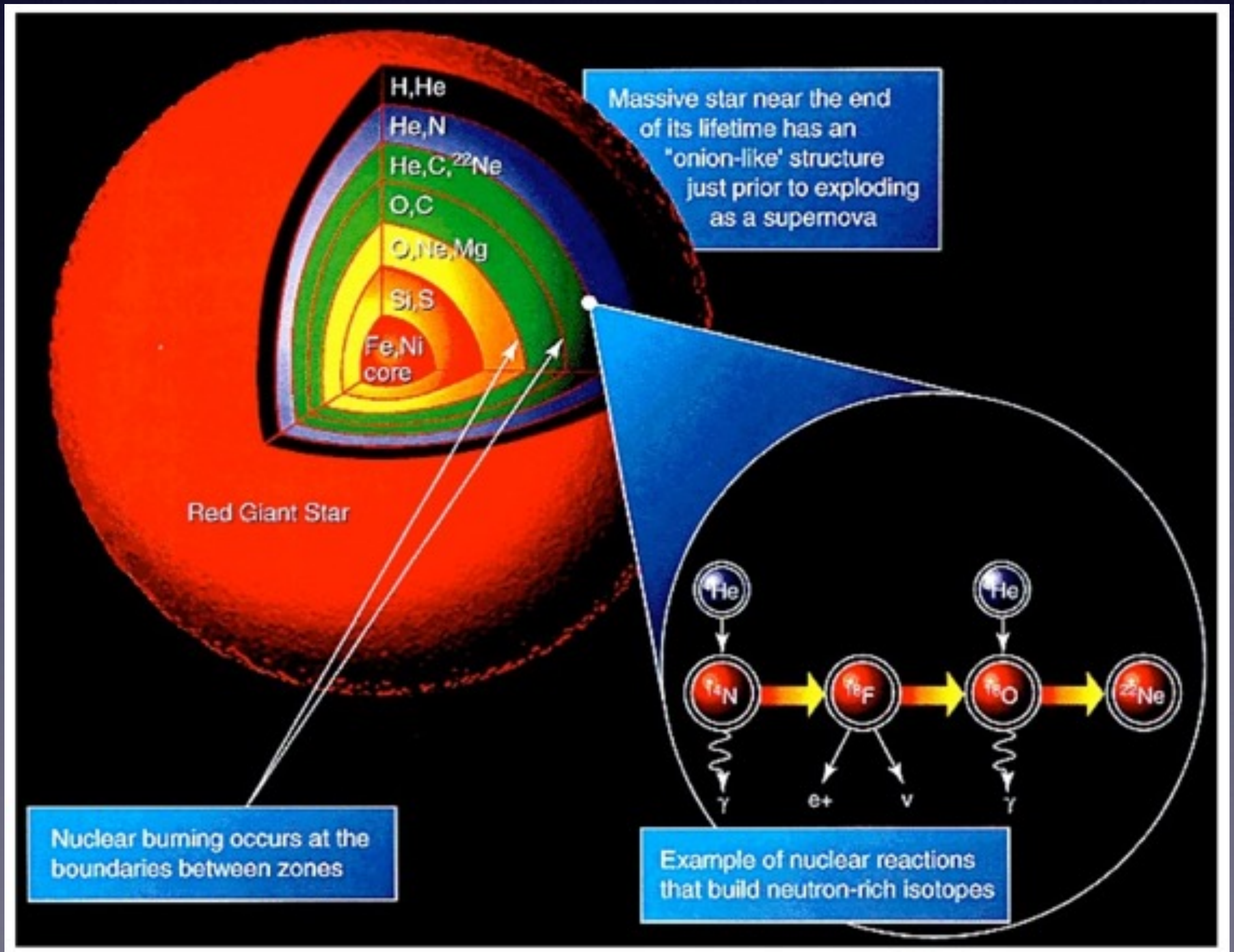




- Not energetically favourable for fusion after the Iron/ Nickel peak.

Stellar Evolution

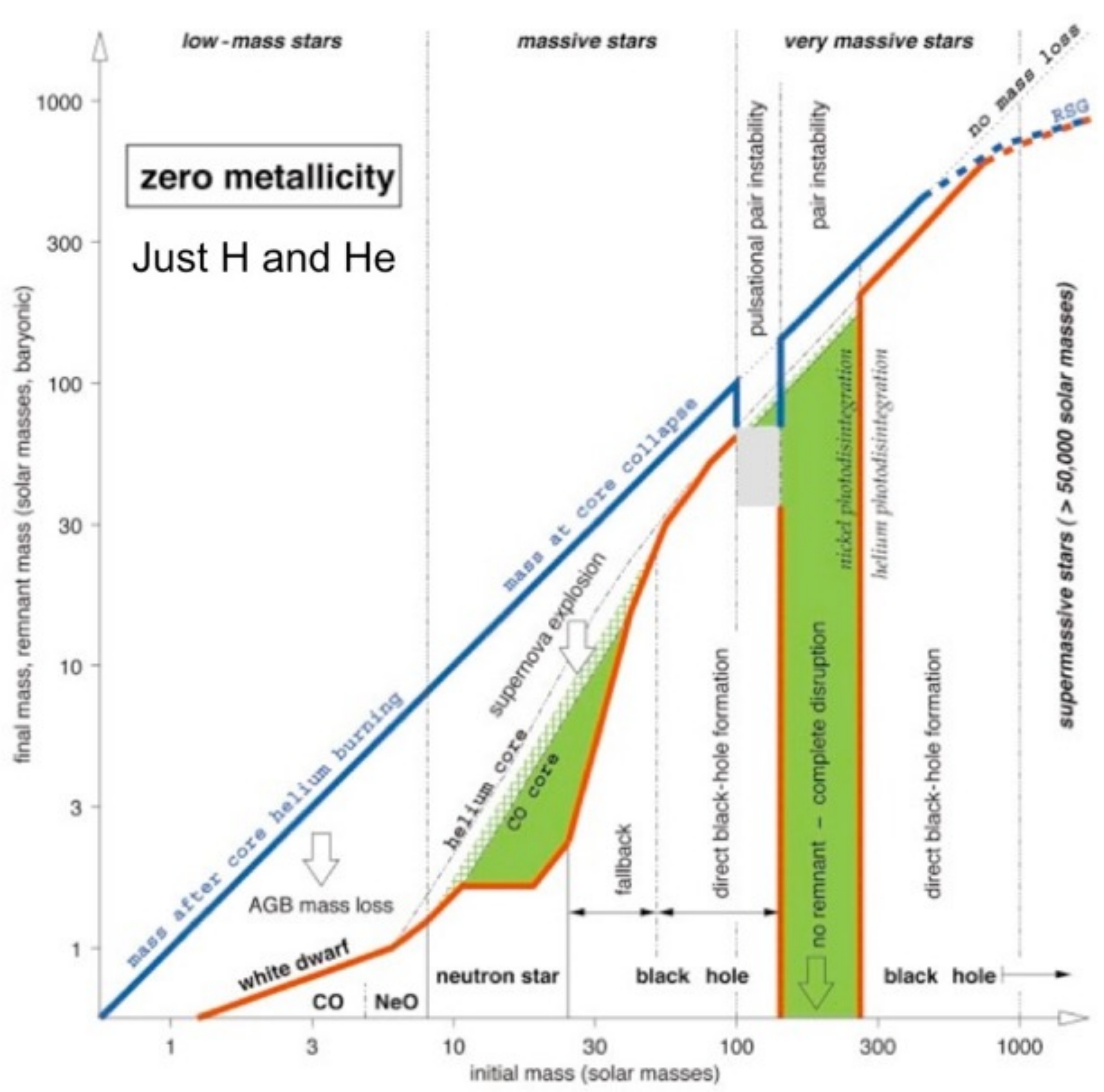
- For low-mass stars, degeneracy pressure halts stellar collapse - helium or carbon/oxygen white dwarf.
- Heavier mass stars end up with an iron core and "onion" layer structure.



Stellar Evolution

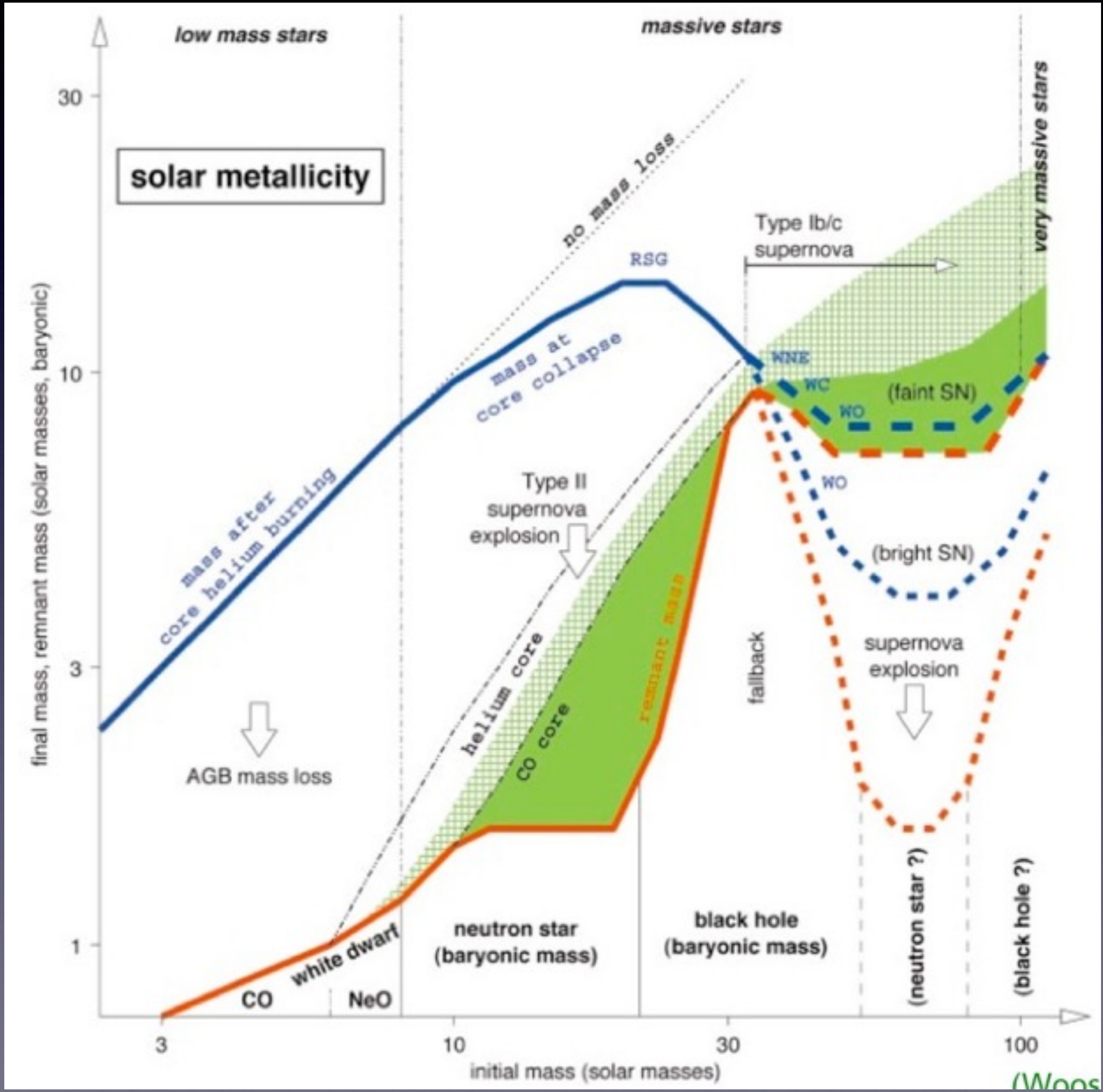
- End state of evolution depends on mass
  - ➡ $M \lesssim 8M_\odot$ *forms white dwarf*. Degenerate core cools for eternity. Radius approximately 1% of solar radius. Maximum mass of a white dwarf is 1.4 solar masses (electrons become relativistic).
  - ➡ $8M_\odot \lesssim M \lesssim 25M_\odot$ *forms neutron star.* Formed if core passes Chandrasekhar limit while still relatively cool or when high density leads to electron-capture reaction. Core collapses to nuclear density. Electrons and protons recombine to form neutrons, and neutrinos emitted. Often accompanied by supernova. Neutron star supported by neutron degeneracy pressure, typical radius is 10-15km. Mass $M \sim 1.4 - 2M_\odot$.
  - ➡ $M \gtrsim 25 - 40M_\odot$ *forms black hole*. Formed when core suffers catastrophic loss of pressure (electron capture at high density, photo dissociation at high temperature) and NS exceeds mass limit. Can form by direct formation or fallback in supernova.

Endstate of Stellar Evolution



Endstate of stellar evolution at zero metallicity (Woosley and Heger 2002)

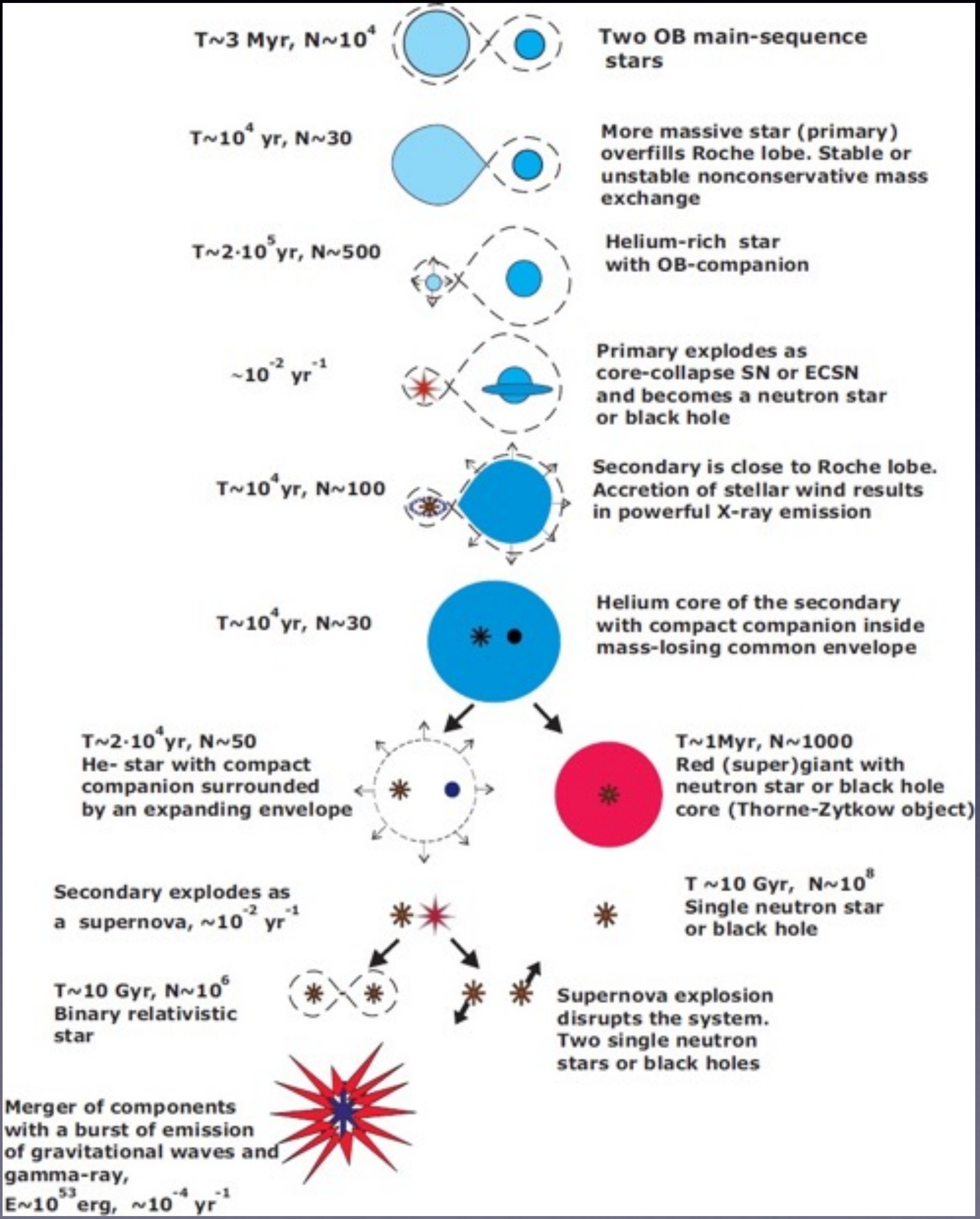
Endstate of Stellar Evolution



Endstate of stellar evolution at zero metallicity (Woosley and Heger 2002)

Forming Comparable Mass Compact Binaries

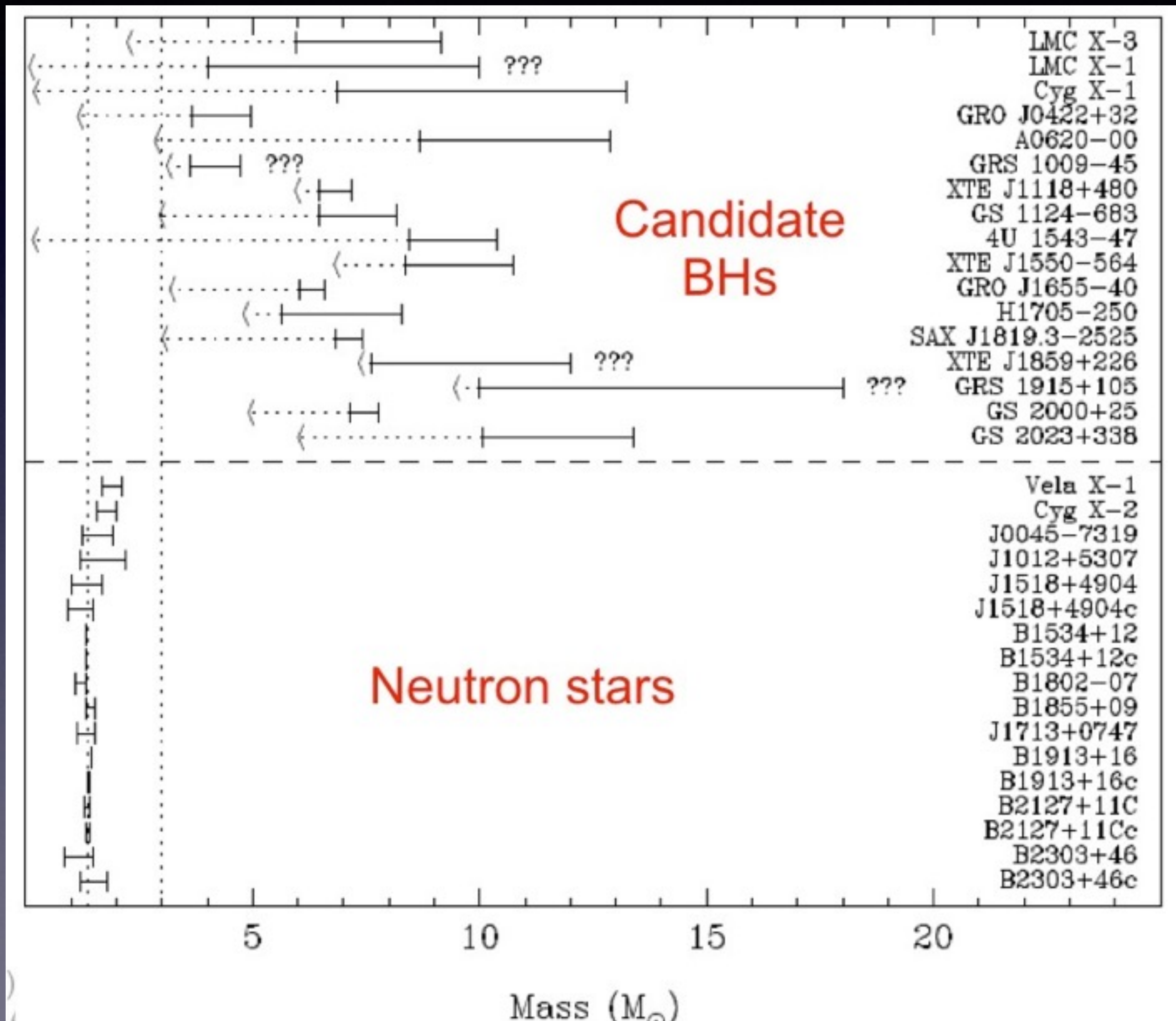
- Need both components of a binary to form a BH or NS, and neither to be ejected in a supernova.
- Some physics poorly understood
  - Common envelope evolution
  - Kick imparted during a supernova explosion.
- Expect binaries to have circularized before entering LIGO band.



Astrophysical Evidence

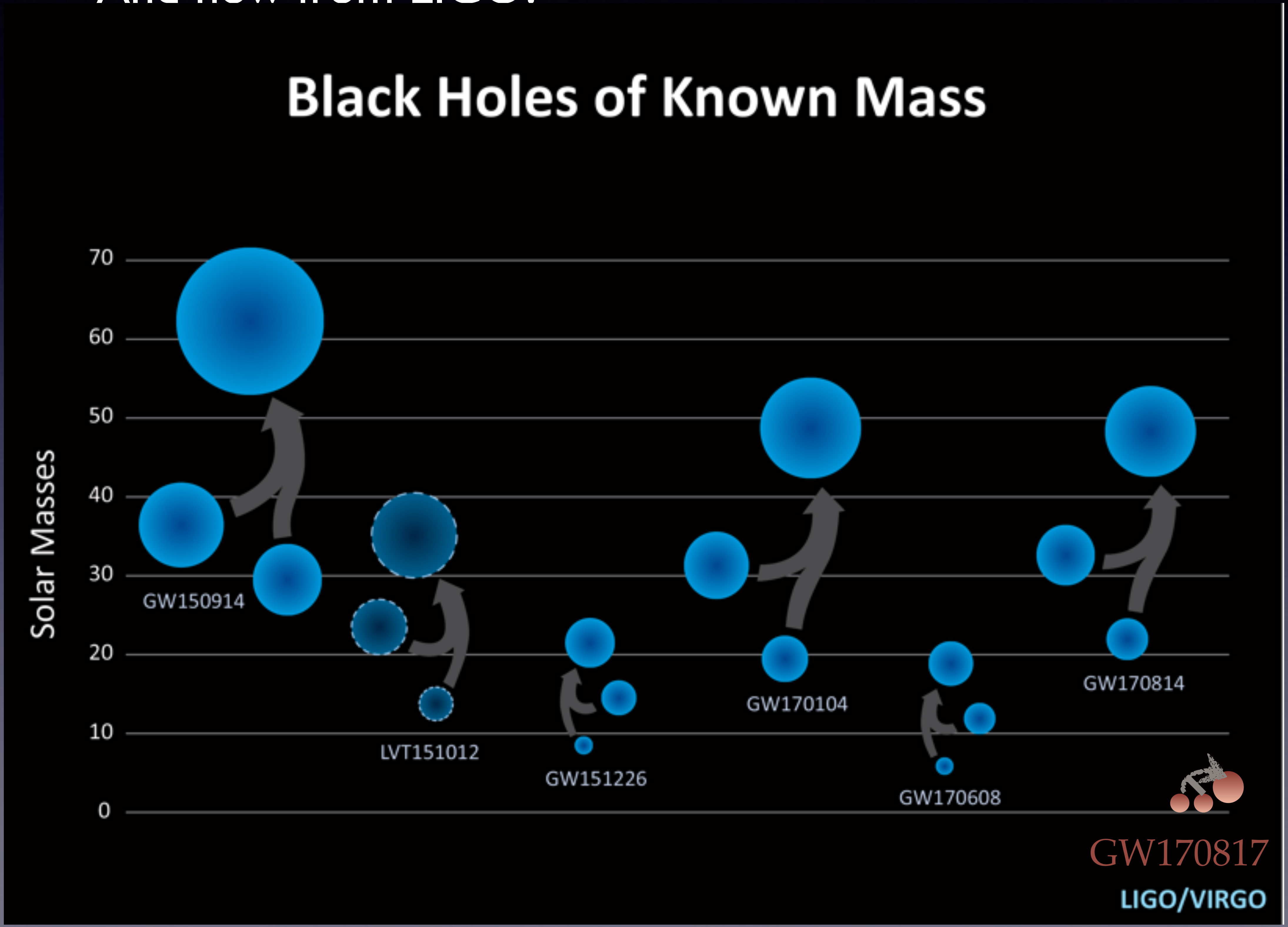
- There is good observational evidence for all three types of remnant.
- White Dwarfs
  - ➡ *Isolated white dwarfs*. Identified from spectral characteristics.
  - ➡ *Cataclysmic Variables*. White dwarf accreting from main sequence companion.
- Neutron Stars and Black Holes
  - ➡ *Pulsars*. Rotating Neutron Stars. Many detected, including some in binaries (and four NS-NS binaries).
  - ➡ *X-ray binaries*. See X-ray emission from some binary systems. Caused by accretion onto very compact object (radius less than 100km). Can estimate masses in some circumstances. Inferred mass of accretor exceeds NS limit in ~20 cases.

Astrophysical Evidence



Astrophysical Evidence

- And now from LIGO!

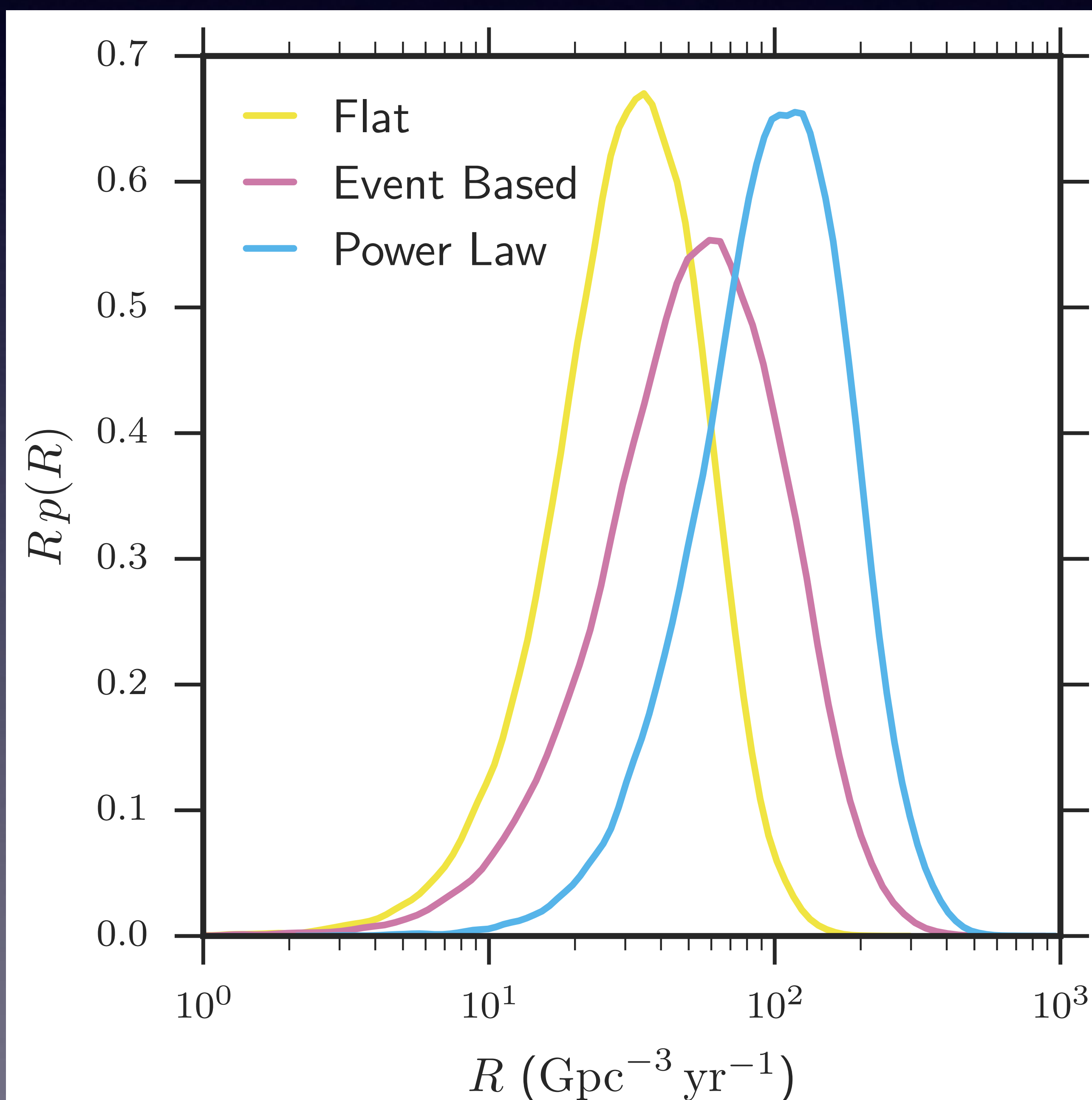


Detection

- These are sources for ground-based interferometers, i.e., LIGO/Virgo.
- Detection methods include *matched filtering* and *unmodelled searches.*
- Waveform modelling
  - Post-Newtonian expansion for inspiral.
  - Numerical relativity for merger/ringdown.
  - Combined in IMR Phenom and EOB models.

Merger Rates

- LIGO observations have provided a measurement of the merger rate.



Mass distribution	$R/(\mathrm{Gpc}^{-3}\mathrm{yr}^{-1})$		
	PyCBC	GstLAL	Combined
	Event based		
GW150914	$3.2^{+8.3}_{-2.7}$	$3.6^{+9.1}_{-3.0}$	$3.4^{+8.6}_{-2.8}$
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All	53^{+100}_{-40}	56^{+105}_{-42}	55^{+99}_{-41}
	Astrophysical		
Flat in log mass	31^{+43}_{-21}	30^{+43}_{-21}	30^{+43}_{-21}
Power Law (−2.35)	100^{+136}_{-69}	95^{+138}_{-67}	99^{+138}_{-70}

Overall: ~12-213 Gpc^{-3}yr^{-1} (one merger per galaxy every 50,000 to one million years.

LIGO CMCB Inspiral Rates

- Predict tens to hundreds of events per year.

Epoch			2015 – 2016	2016 – 2017	2018 – 2019	2020+	2024+
Planned run duration			4 months	9 months	12 months	(per year)	(per year)
Expected burst range/Mpc		LIGO	40 – 60	60 – 75	75 – 90	105	105
		Virgo	—	20 – 40	40 – 50	40 – 70	80
		KAGRA	—	—	—	—	100
Expected BNS range/Mpc		LIGO	40 – 80	80 – 120	120 – 170	190	190
		Virgo	—	20 – 65	65 – 85	65 – 115	125
		KAGRA	—	—	—	—	140
Achieved BNS range/Mpc		LIGO	60 – 80	60 – 100	—	—	—
		Virgo	—	25 – 30	—	—	—
		KAGRA	—	—	—	—	—
Estimated BNS detections			0.05 – 1	0.2 – 4.5	1 – 50	4 – 80	11 – 180
Actual BNS detections			0	1	—	—	—
90% CR	% within	5 deg^2	< 1	1 – 5	1 – 4	3 – 7	23 – 30
		20 deg^2	< 1	7 – 14	12 – 21	14 – 22	65 – 73
	median/deg^2		460 – 530	230 – 320	120 – 180	110 – 180	9 – 12
Searched area	% within	5 deg^2	4 – 6	15 – 21	20 – 26	23 – 29	62 – 67
		20 deg^2	14 – 17	33 – 41	42 – 50	44 – 52	87 – 90

LVC, Liv. Rev. Rel. **19** 1 (2016)

LIGO CMCB Inspiral Rates

- Compare to indirect predictions pre-GW150914.

TABLE IV: Compact binary coalescence rates per Mpc^3 per Myr.[a]

Source	R_{low}	R_{re}	R_{high}	R_{max}
NS-NS (Mpc^{-3} Myr^{-1})	0.01 [1]	1 [1]	10 [1]	50 [16]
NS-BH (Mpc^{-3} Myr^{-1})	6×10^{-4} [18]	0.03 [18]	1 [18]	
BH-BH (Mpc^{-3} Myr^{-1})	1×10^{-4} [14]	0.005 [14]	0.3 [14]	

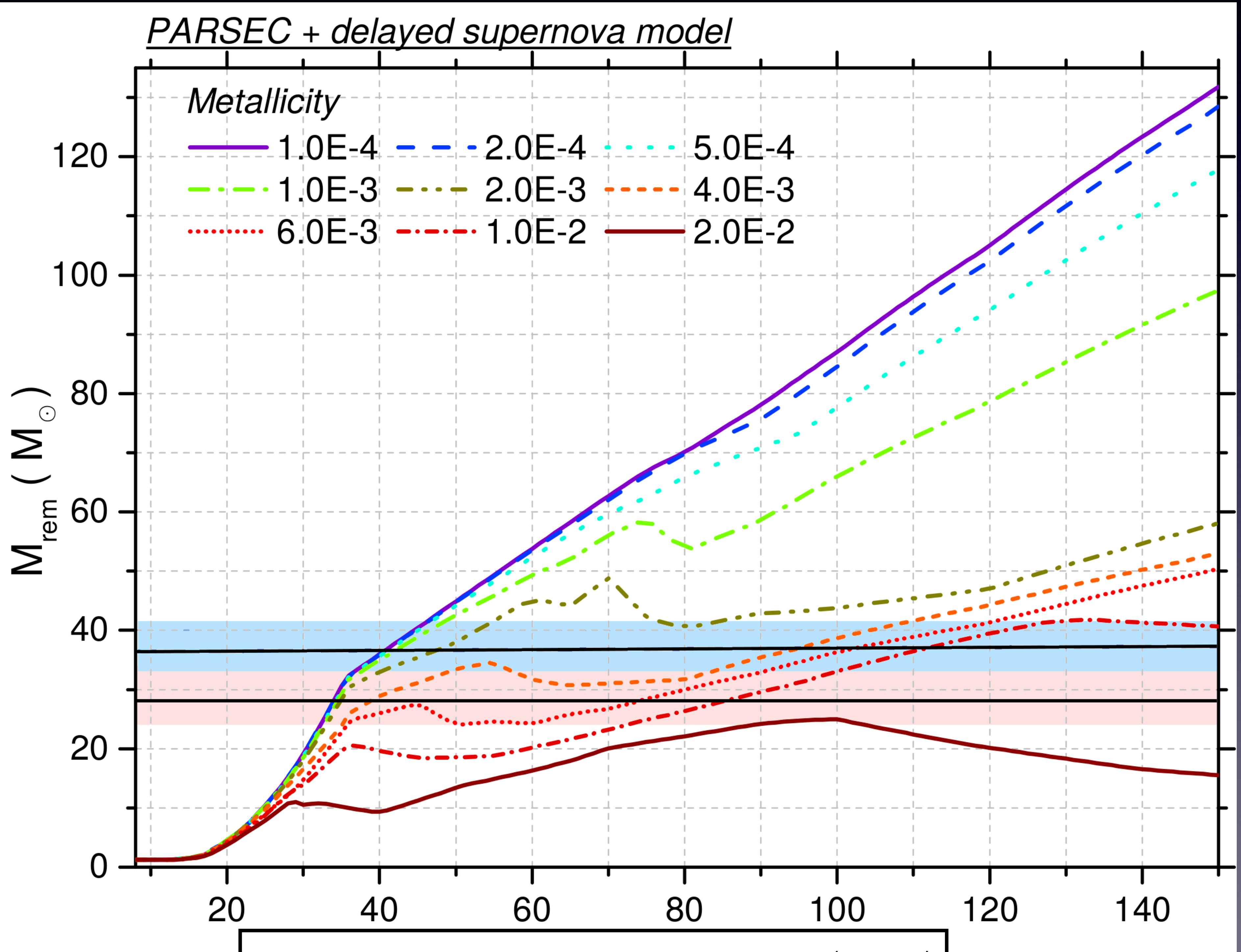
TABLE V: Detection rates for compact binary coalescence sources.

IFO	Source[a]	$\dot{N}_{\mathrm{low}}$ yr^{-1}	$\dot{N}_{\mathrm{re}}$ yr^{-1}	$\dot{N}_{\mathrm{high}}$ yr^{-1}	$\dot{N}_{\mathrm{max}}$ yr^{-1}
	NS-NS	2×10^{-4}	0.02	0.2	0.6
	NS-BH	7×10^{-5}	0.004	0.1	
Initial	BH-BH	2×10^{-4}	0.007	0.5	
	IMRI into IMBH			$< 0.001^{b}$	0.01^{c}
	IMBH-IMBH			$10^{-4\,d}$	$10^{-3\,e}$
	NS-NS	0.4	40	400	1000
	NS-BH	0.2	10	300	
Advanced	BH-BH	0.4	20	1000	
	IMRI into IMBH			10^{b}	300^{c}
	IMBH-IMBH			0.1^{d}	1^{e}

Abadie et al. (LVC), arxiv:1003.2480

Some Recent Results from LIGO

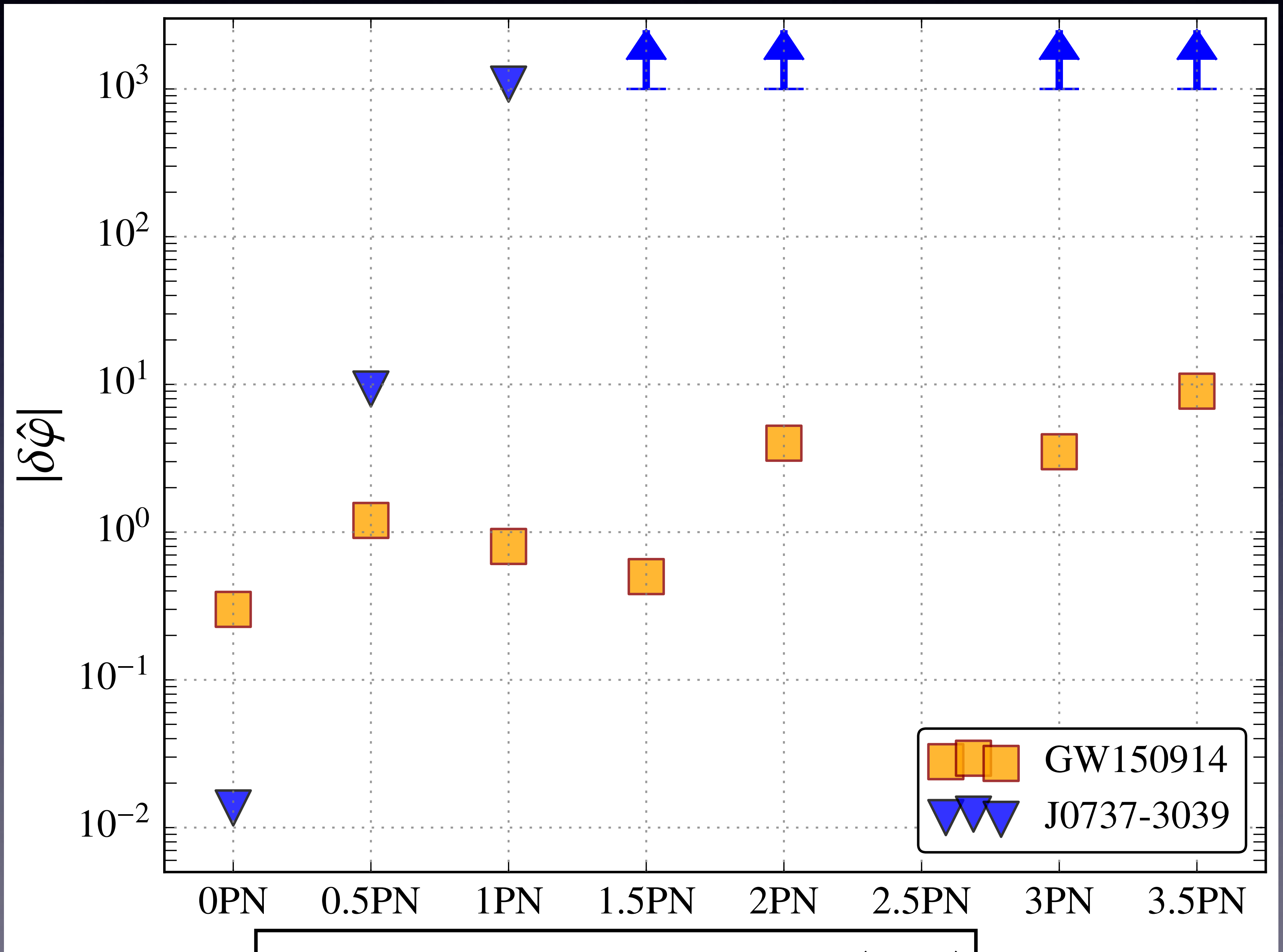
- Black hole formation - masses are surprisingly large.



LVC, *Astrophys. J. Lett.* **818** L22 (2016)

Some Recent Results from LIGO

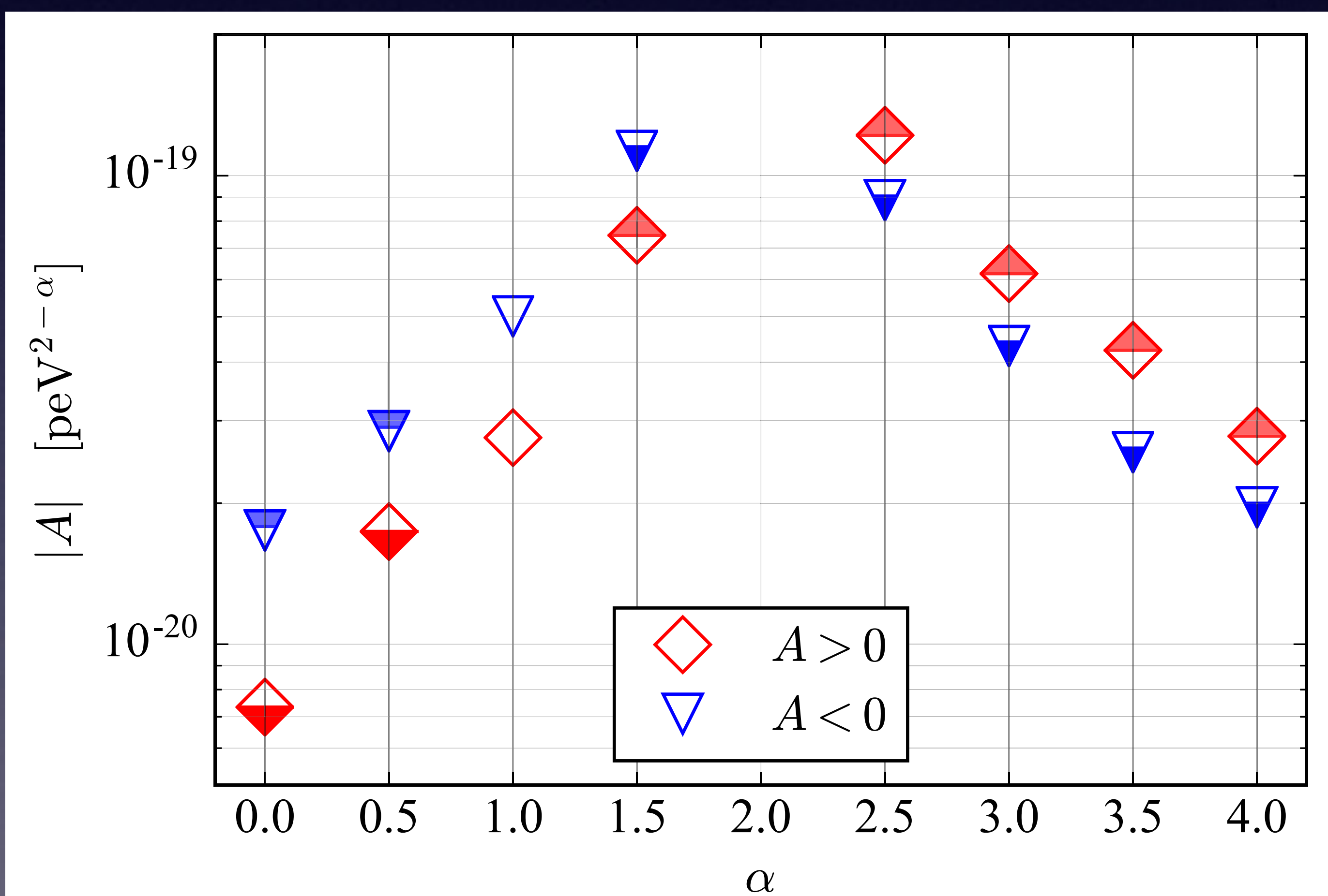
- General relativity is still correct - most stringent tests to date.



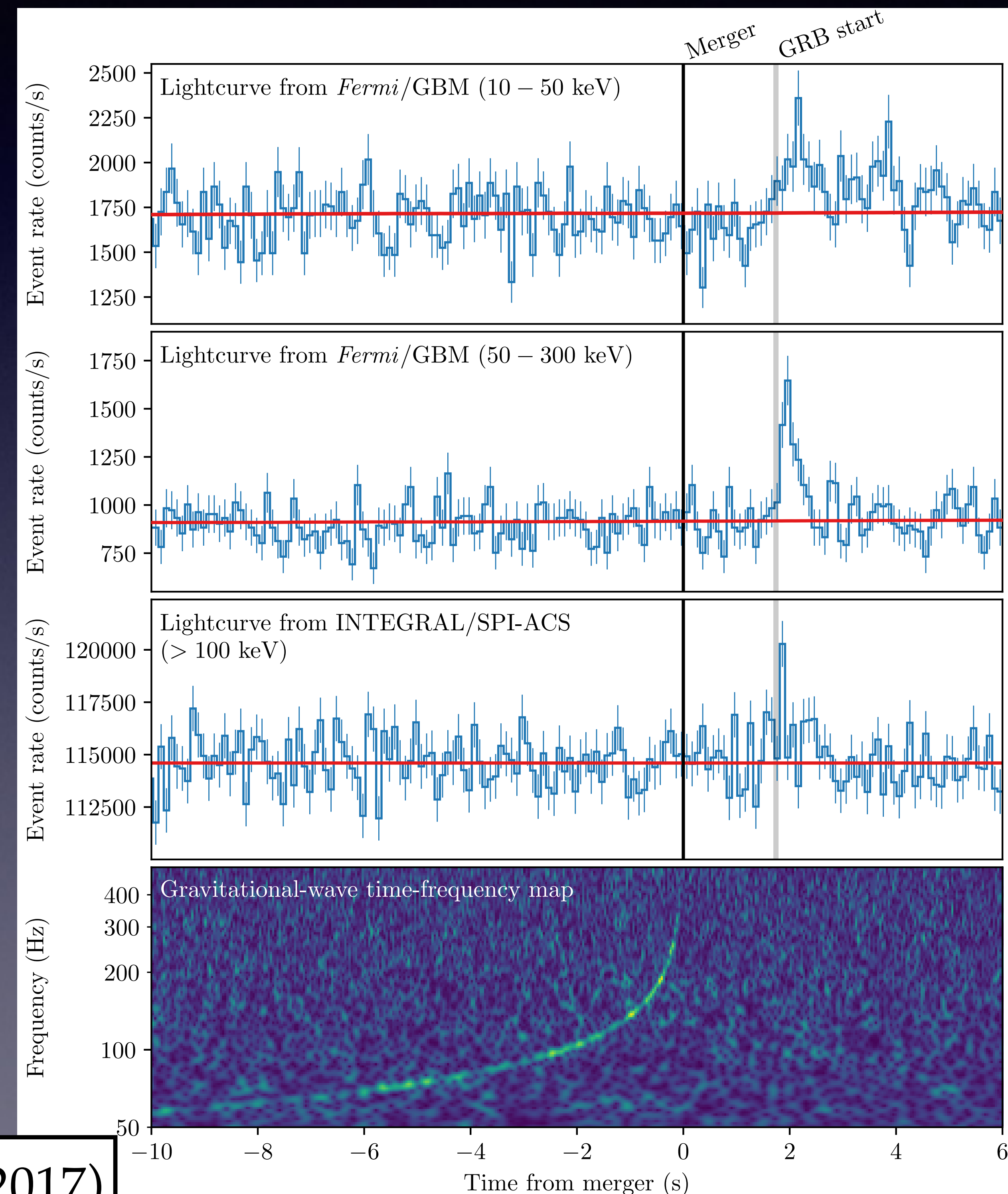
LVC, *Phys. Rev. Lett.* **116** 221101 (2017)

Some Recent Results from LIGO

- General relativity is correct - gravitational waves travel at the speed of light.



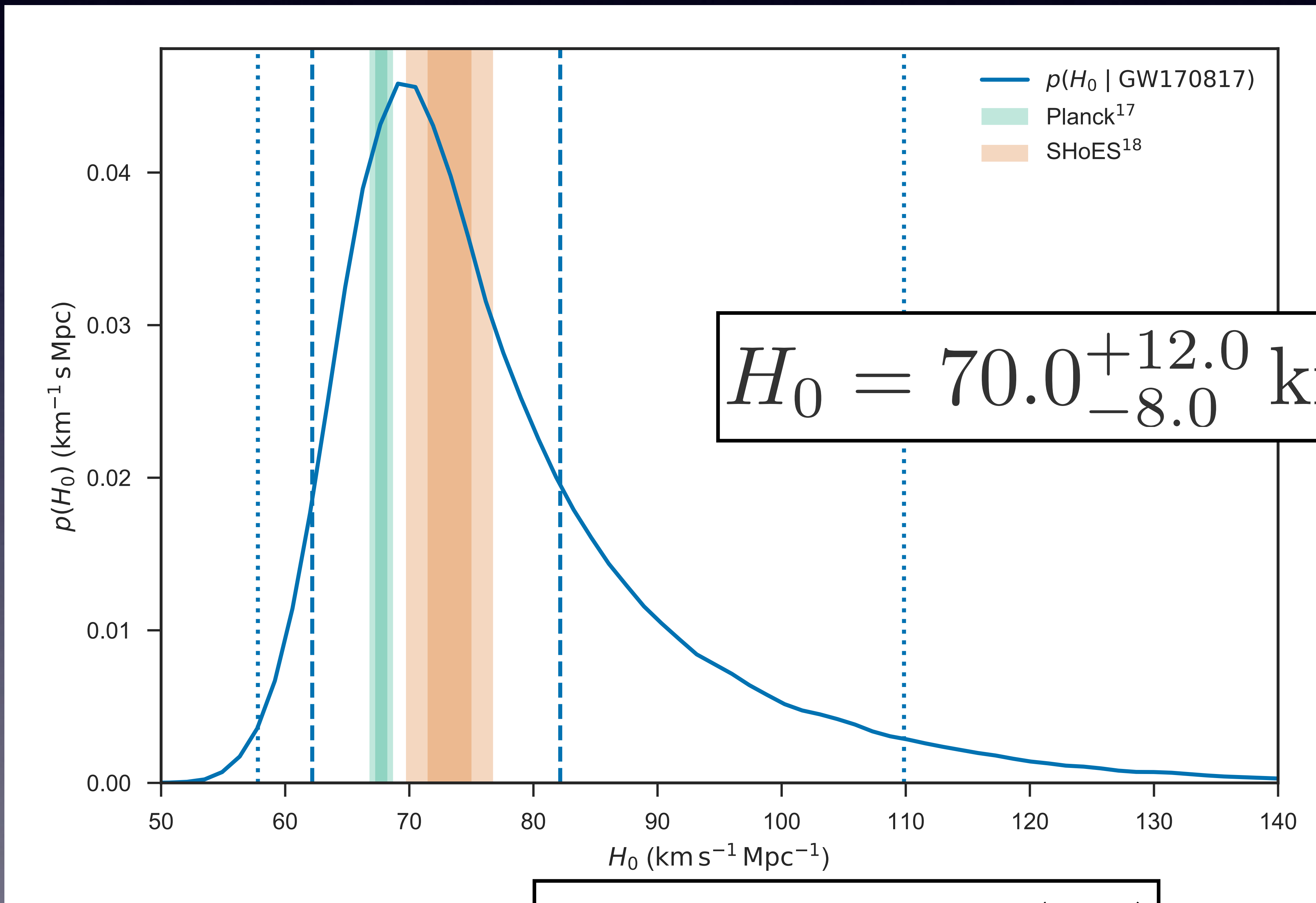
LVC+, *Phys. Rev. Lett.* **118** 221101 (2017)



LVC+, *Astrophys. J. Lett.* **848** L13 (2017)

Some Recent Results from LIGO

- First measurement of the expansion rate of the Universe using gravitational waves.

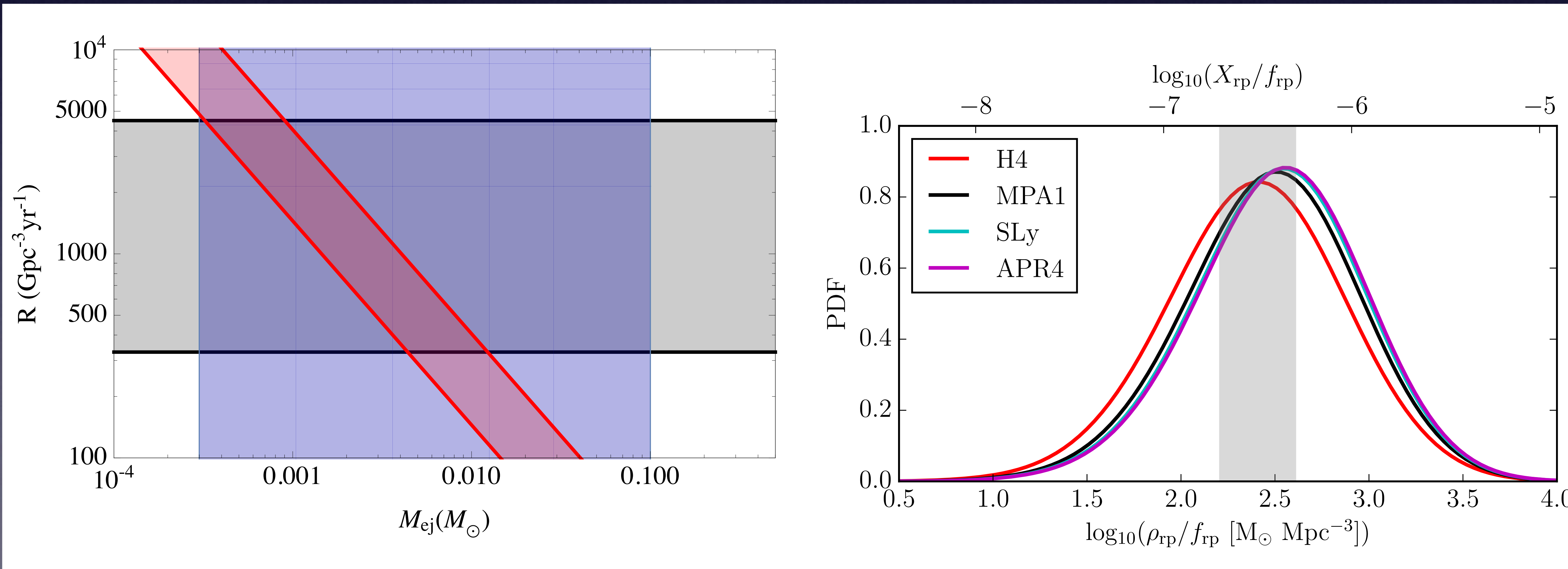


$$H_0 = 70.0^{+12.0}_{-8.0}\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$$

LVC+, *Nature Lett.* **551** 85 (2017)

Some Recent Results from LIGO

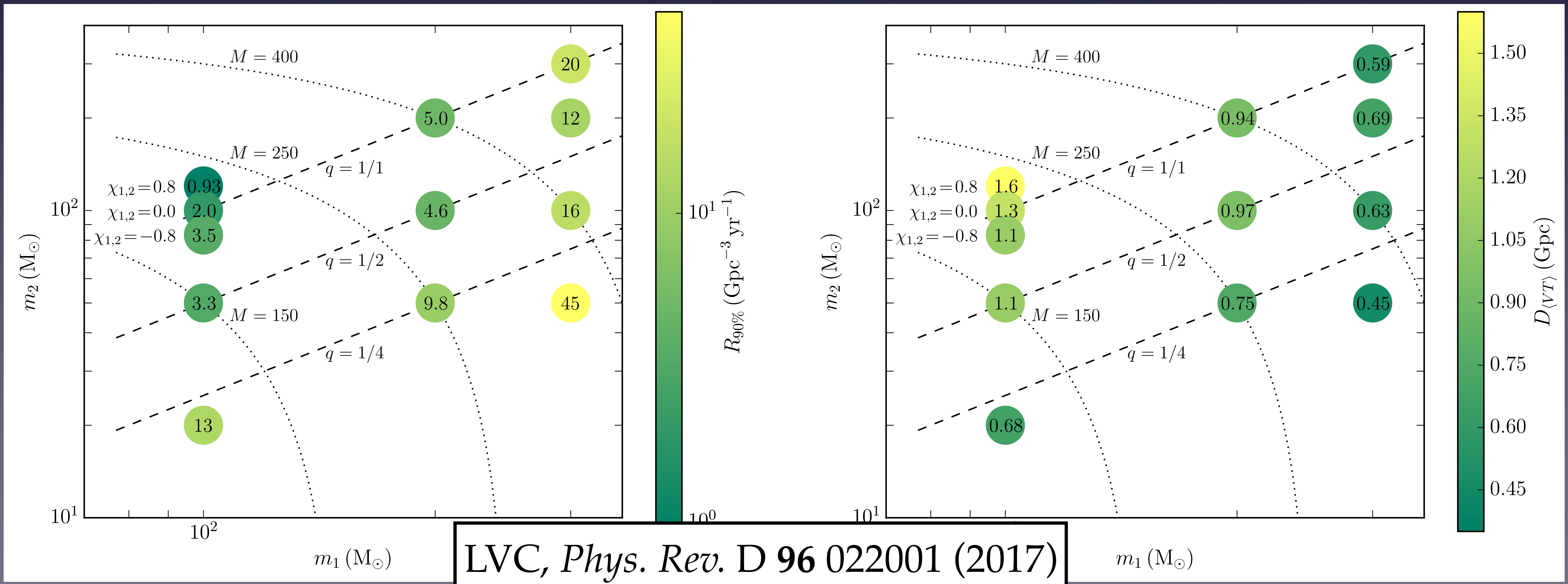
- Remarkably from GW170817 alone we now know that all the Gold in the Universe (and other, less interesting r-process elements) can be made in events like GW170817.



LVC, *Astrophys. J. Lett.* **850** 39 (2017)

Some Recent Results from LIGO

- Black holes of intermediate mass (IMBH) may exist.
  - Formation in early Universe, e.g., pop III stars
  - Dynamical formation in clusters, but mechanism not fully understood
  - Some observational evidence, e.g., ULXs M82 X-1, NGC 1313 X-2
  - LIGO can constrain the rate of IMBH binary mergers

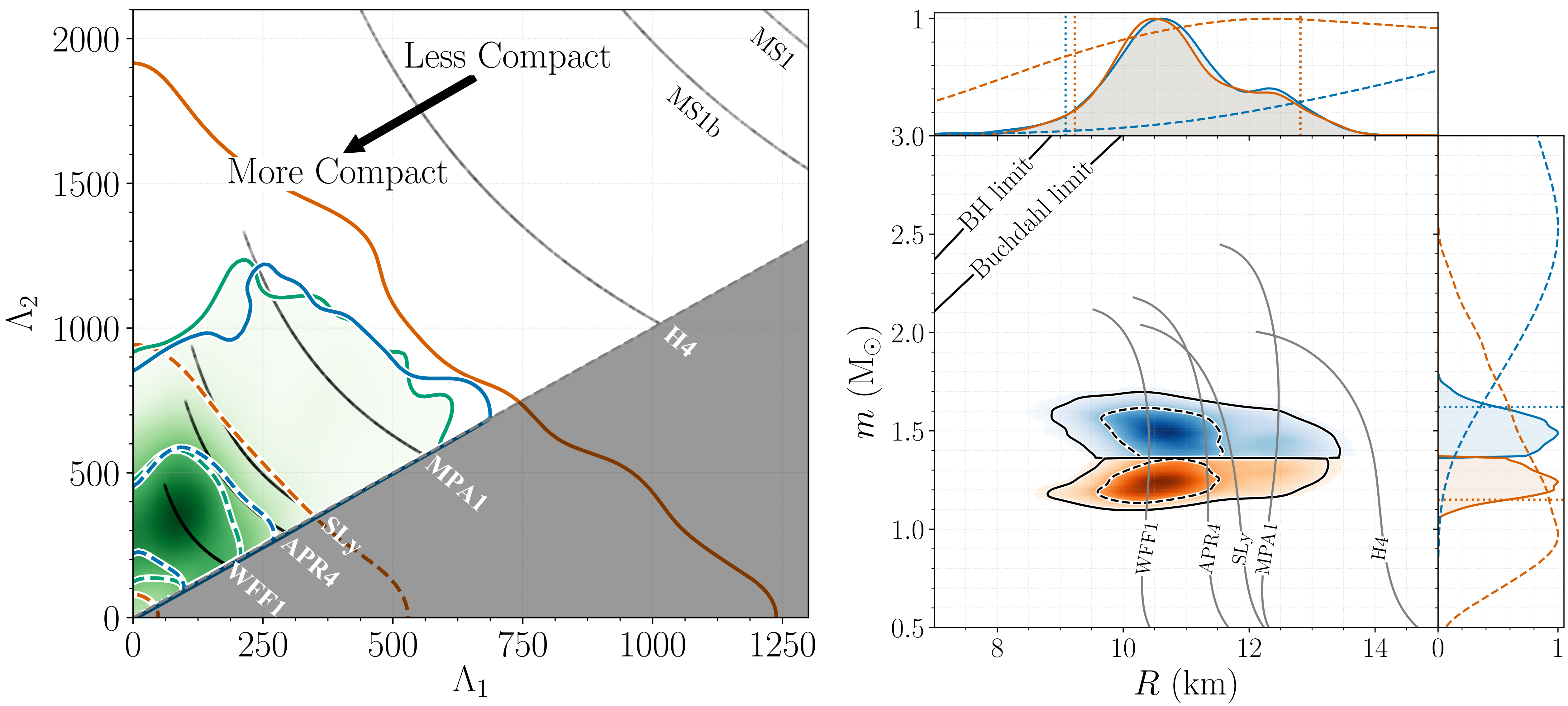


LVC, *Phys. Rev.* D **96** 022001 (2017)

Science with CMCBs

- Science applications of LIGO CMCB inspiral observations are severalfold
    - ➡ *Astrophysics of compact objects*. Observations of event rate and parameter distributions will probe poorly understood physics - initial mass function, binary evolution and endstate of stellar evolution.
    - ➡ *Neutron Star equation of state*. In a NS-BH inspiral, the neutron star will be tidally disrupted at the end of the inspiral phase. The radius/frequency at which this occurs is a gravitational wave observable, and depends on the NS mass-radius relation, which is very uncertain.
    - ➡ *Strong field test of general relativity*. Merger waveforms depend on the full non-linear equations of relativity. Comparison of waveform structure to predictions of numerical relativity will be a very poweful test of the theory of relativity.

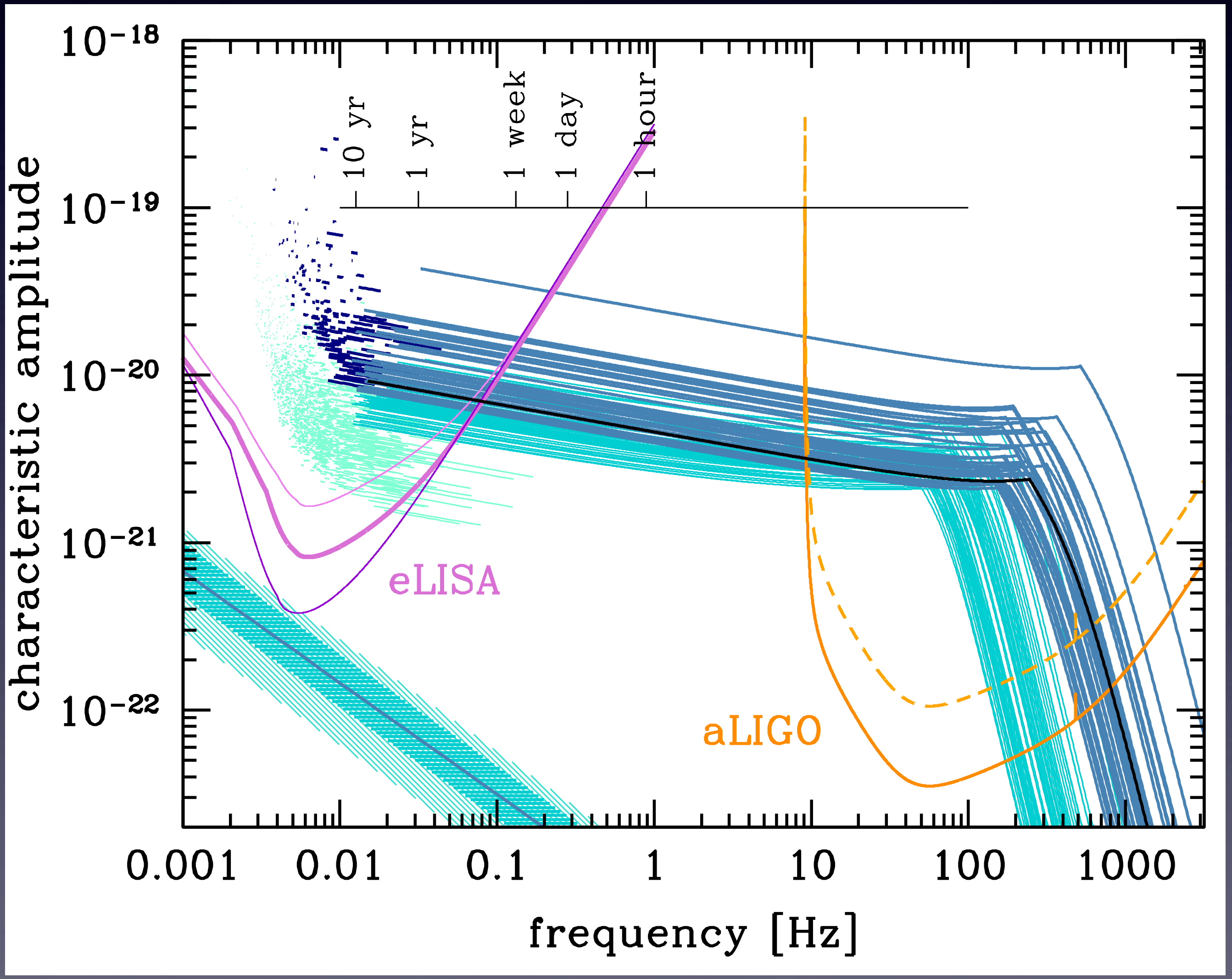
Science with CMCBs



Neutron Star equation of state and radius constraints (LVC, arXiv:1805.11581)

LIGO sources for LISA

- GW150914 would have been observable by LISA ~5 years before being observed by LIGO, with S/N~10 in a 5yr observation. (Sesana 2016)
- LISA provides sky location to ~0.few square degrees and time of coalescence to ~few s.
- Number of events could be high (several hundred) but there are significant uncertainties.
- Formation of massive BHs above pair instability band could be crucial.



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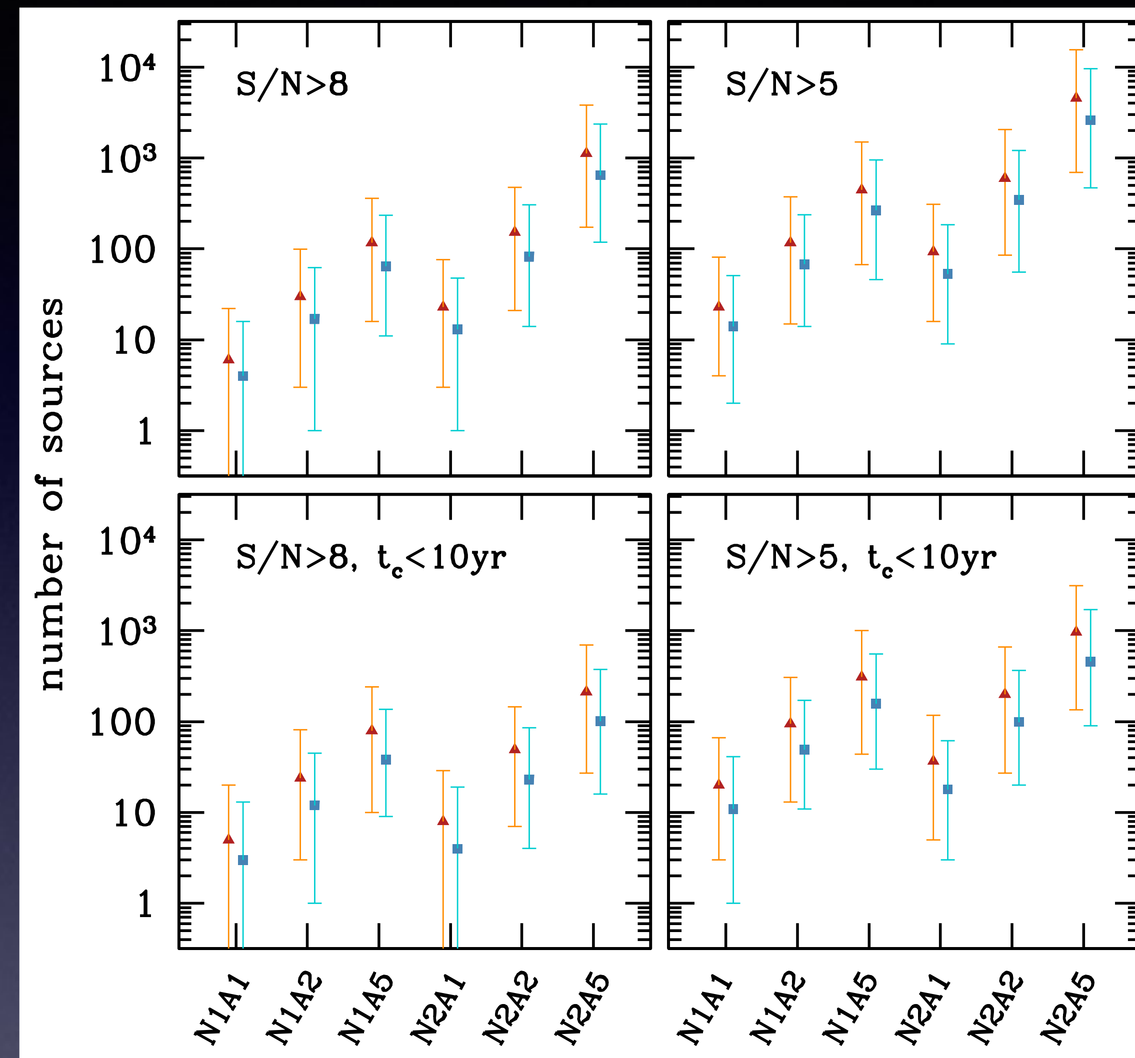

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CMCB Inspirals for LISA

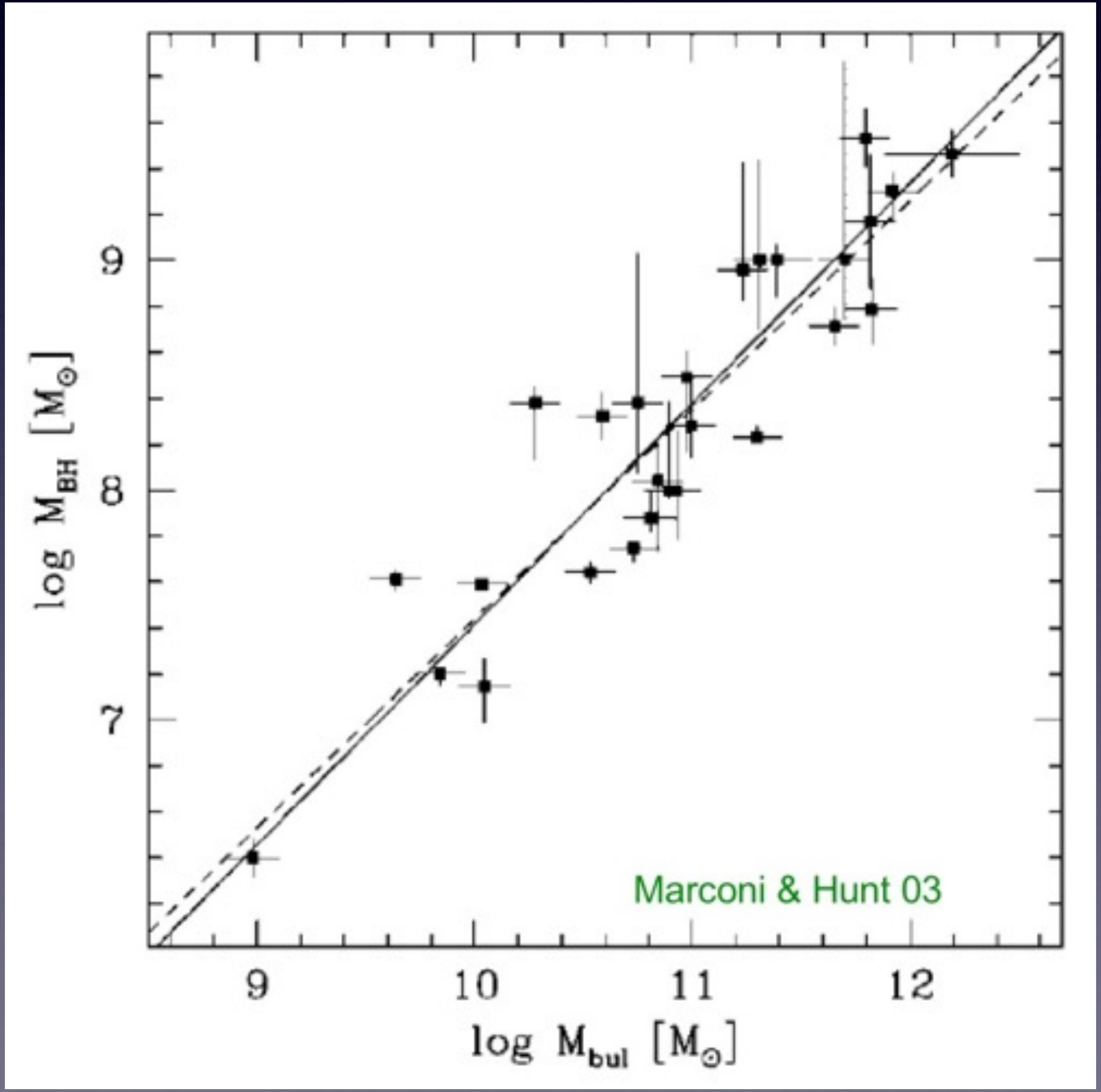
- Merging supermassive black holes with mass

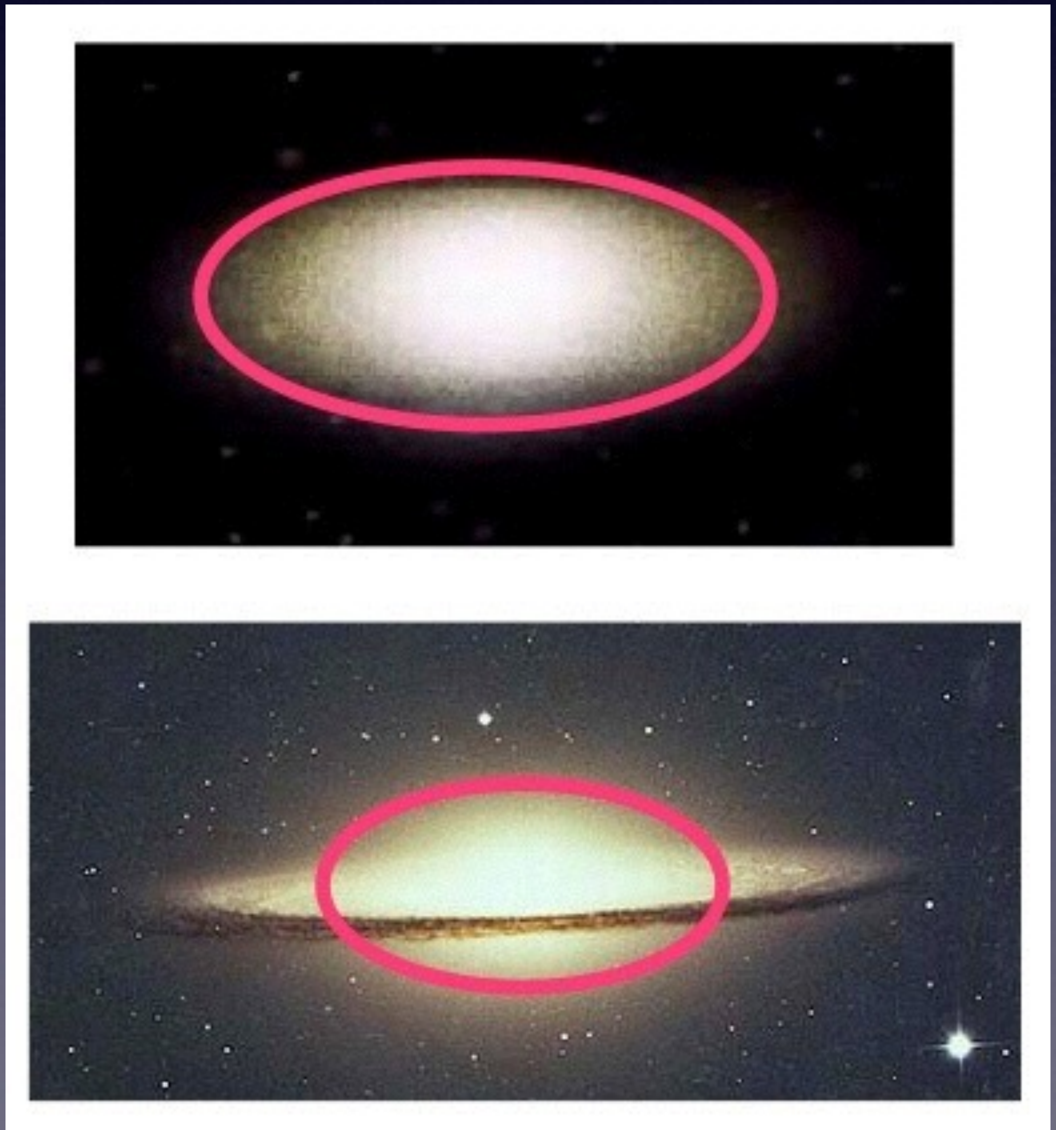
$$M \sim 10^5 M_\odot - 10^7 M_\odot$$

- will be sources for LISA.
- There is very good observational evidence that the centres of most galaxies contain very massive black holes - *Active Galactic Nuclei, Jets, dynamics of galactic central clusters (e.g., Sgr A*).*

CMCB Inspirals for LISA

- Black holes show a very tight correlation between BH mass and both bulge mass and spheroid velocity dispersion, although data is scarce in the LISA range.





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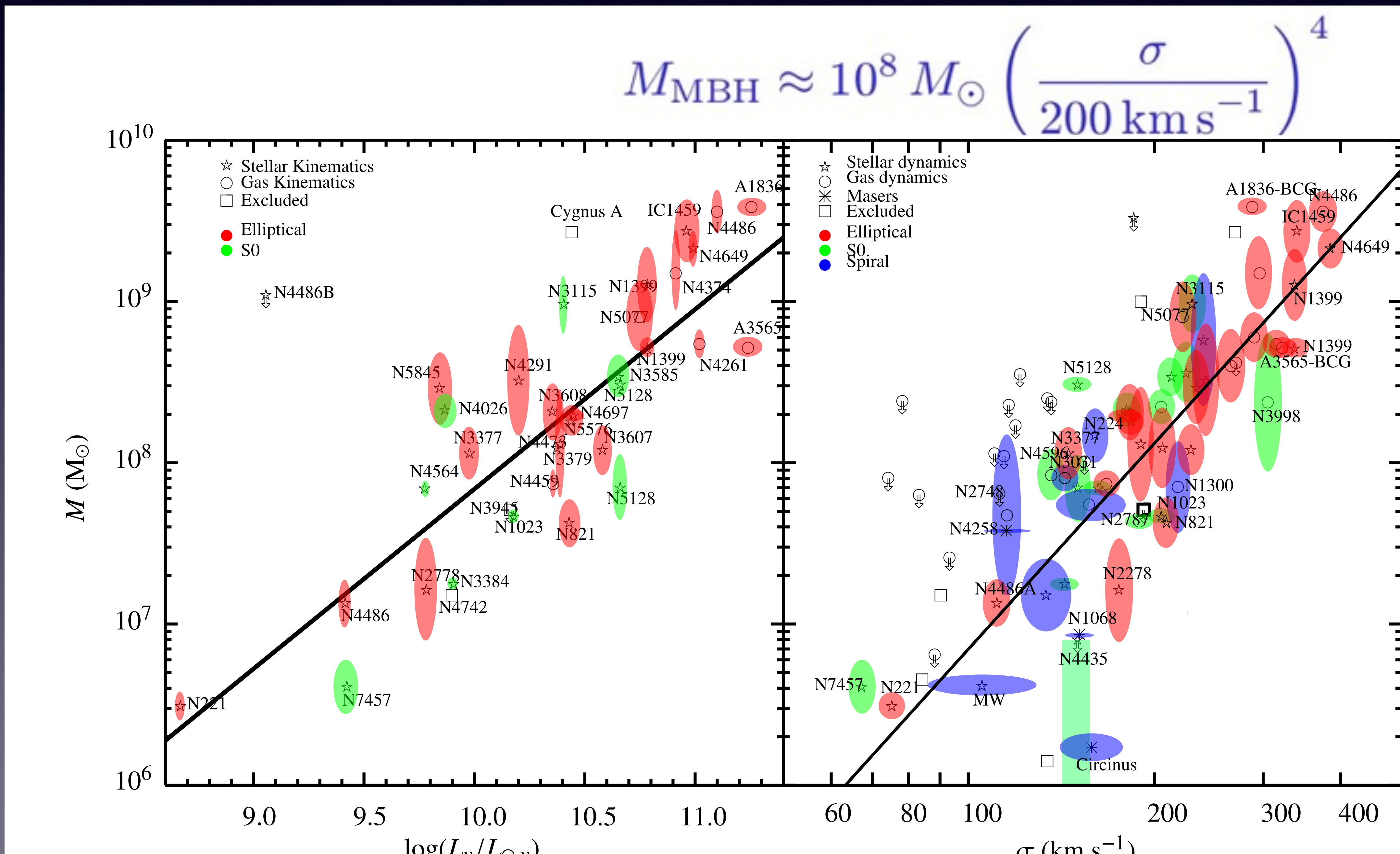
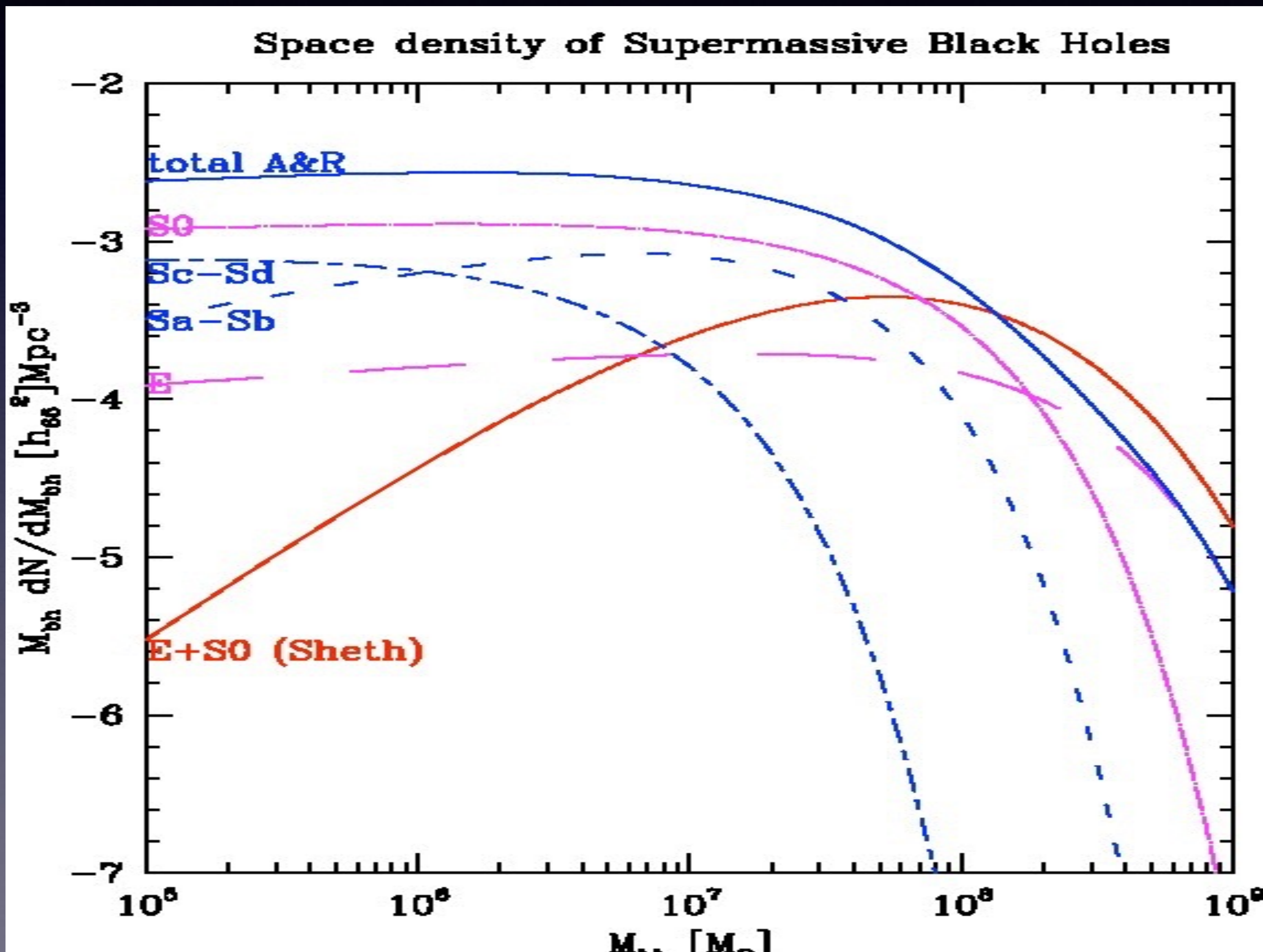


CMCB Inspirals for LISA

- Number of galaxies in LISA mass range highly uncertain.



CMCB Inspirals for LISA

- Expect/observe galaxies to merge.
- Black holes may merge, stall or be ejected. Merging is most likely, although physics somewhat uncertain, e.g., *last parsec problem, natal kicks.*

Galaxy Merger Rate

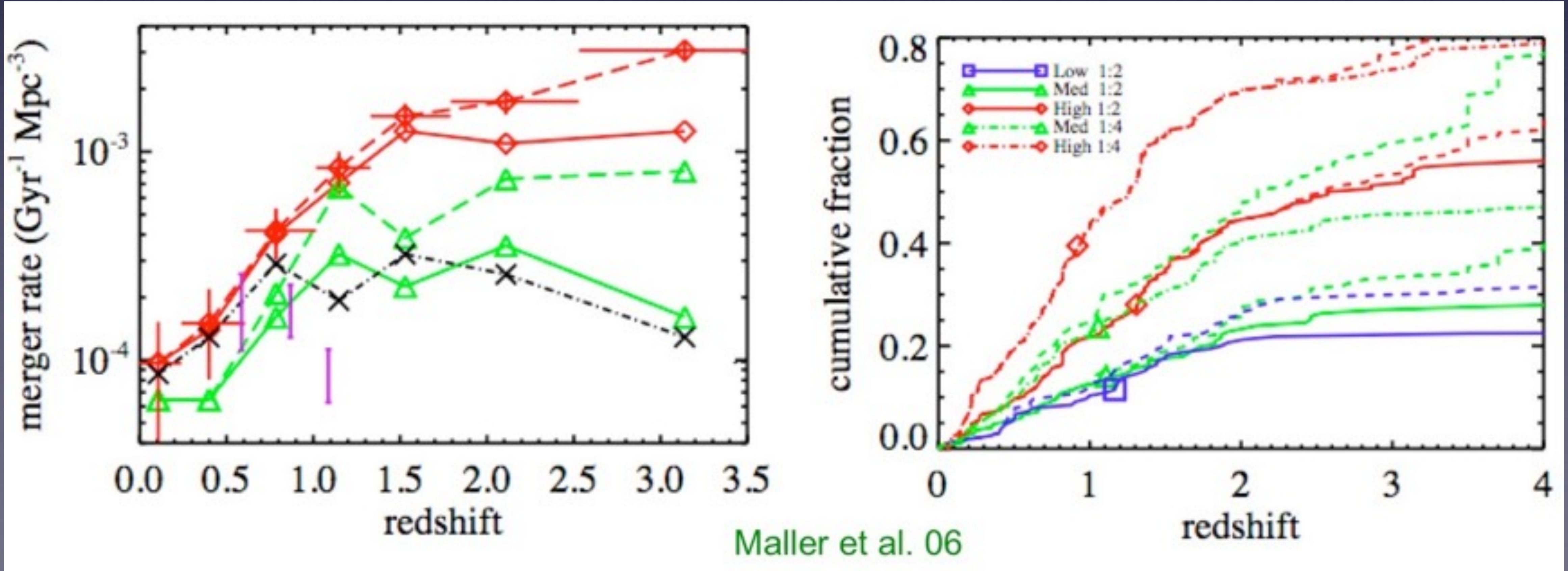
- Observed time between mergers for a galaxy today is

$$\Delta t_{\rm merge} \approx 25 {\rm Gyr}$$

- which gives a merger rate today of

$$\dot{n}_{\rm merg} \approx 2 \times 10^{-4} {\rm Mpc}^{-3} {\rm Gyr}^{-1}$$

- Merger rate was much higher in the past, when galactic black holes were smaller - ideal for LISA!

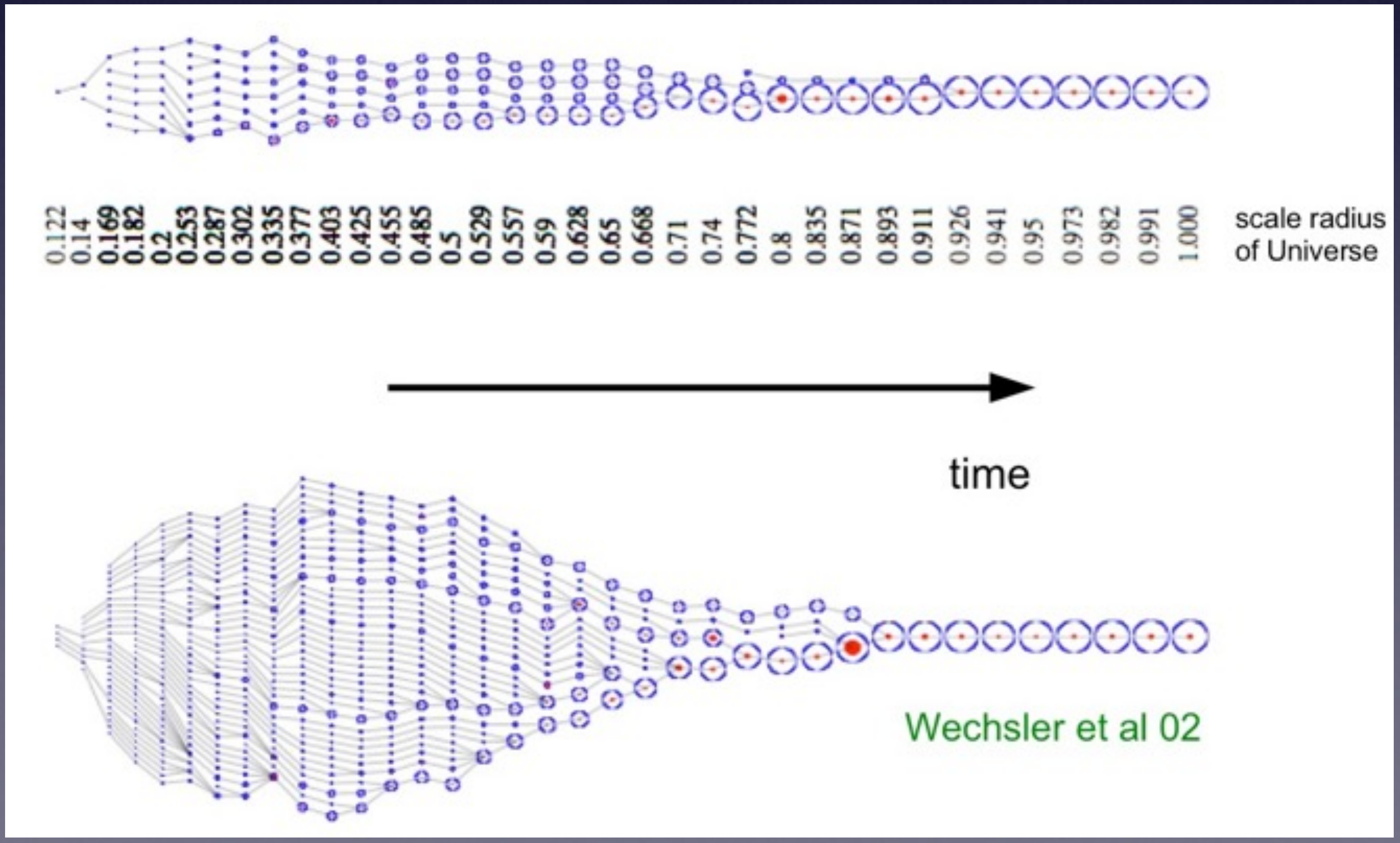


Maller et al. 06

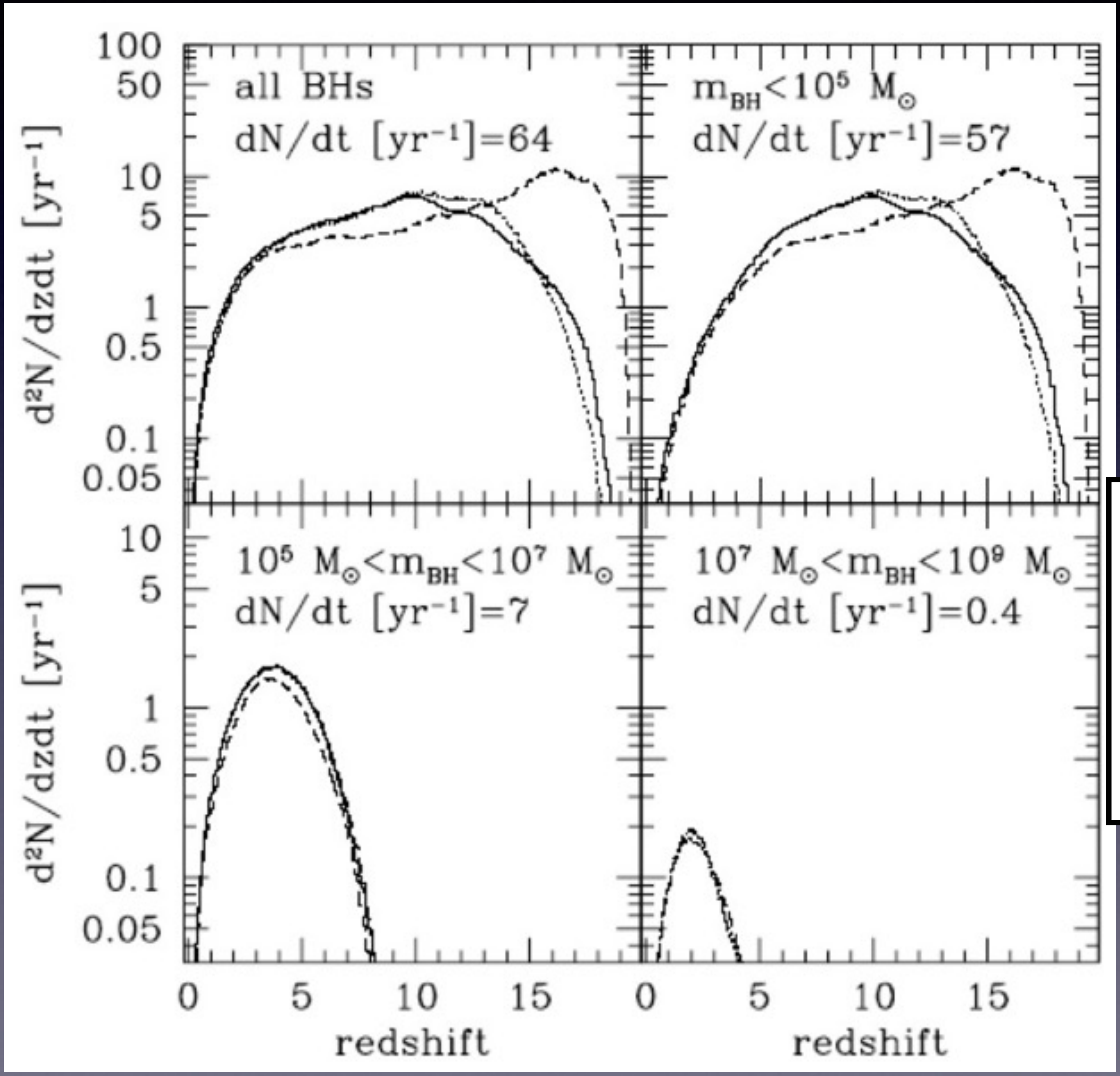
LISA Event Rate

- LISA can detect suitable SMBH mergers out to redshift z=10 or higher. Rate is contaminated by these higher redshift mergers.

- Estimate rate using galaxy merger trees, starting with seed black holes of a certain mass.

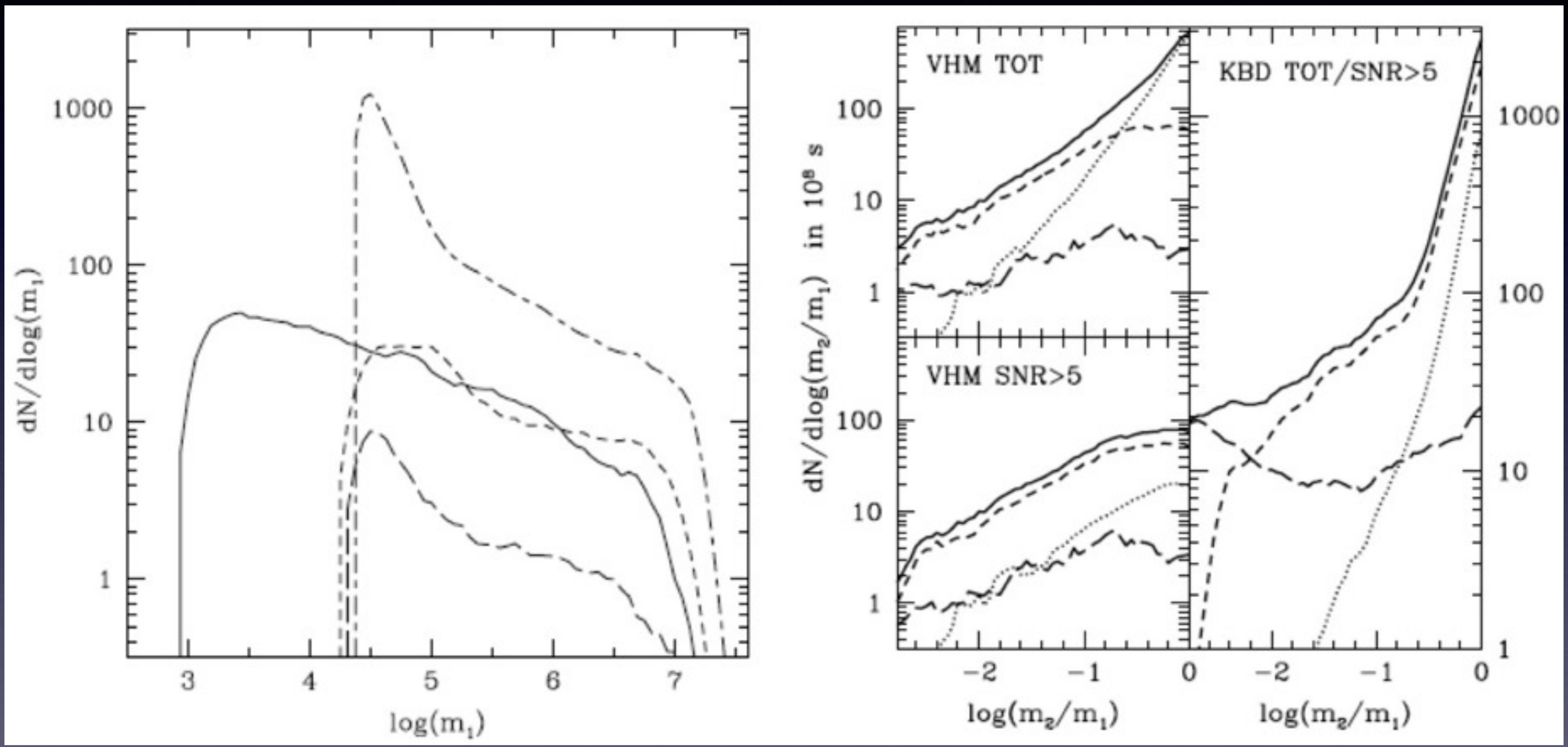


LISA Event Rate



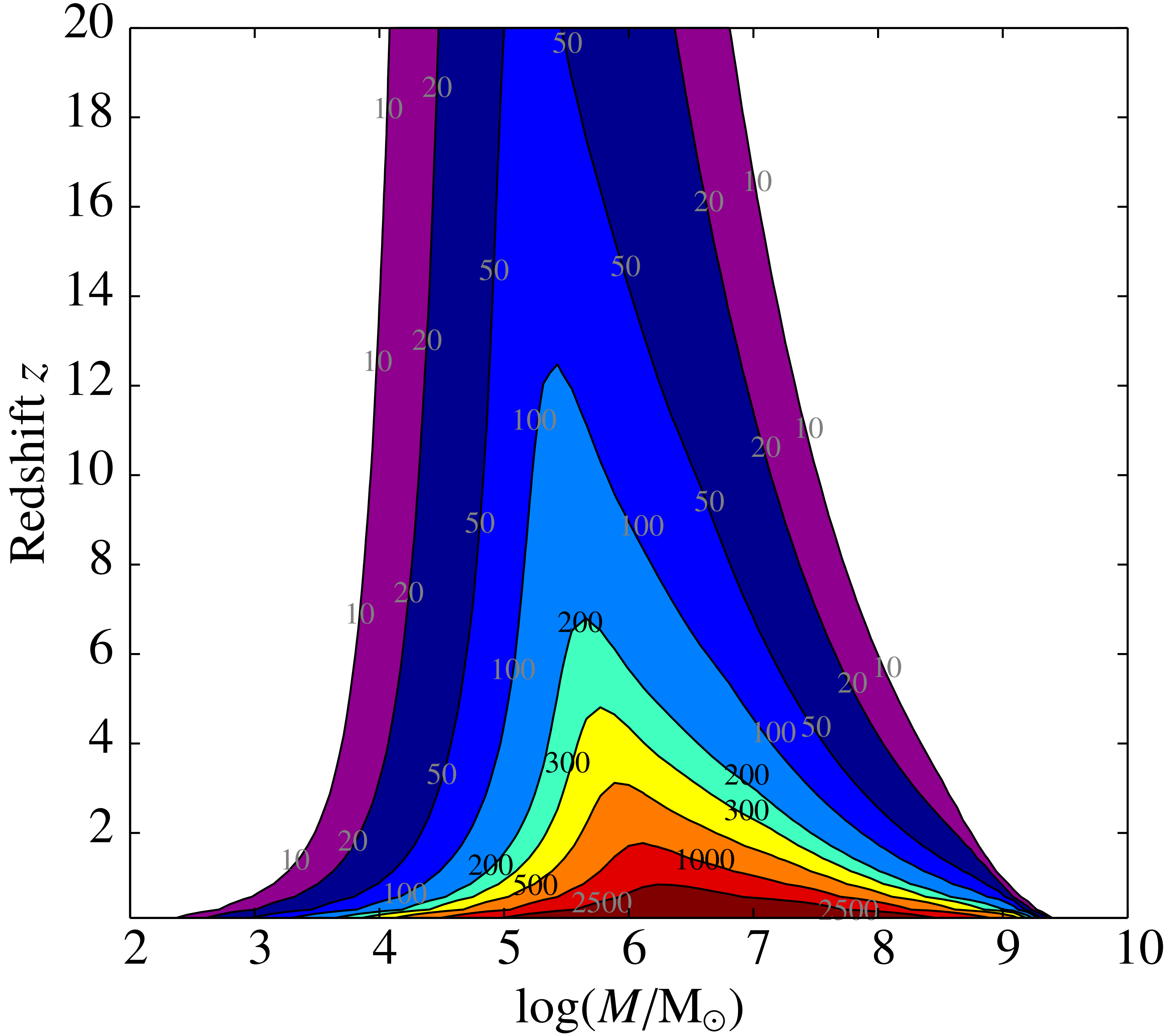
Redshift distribution of rate for LISA CMCB inspiral events (Sesana et al. 2005)

LISA Event Rate



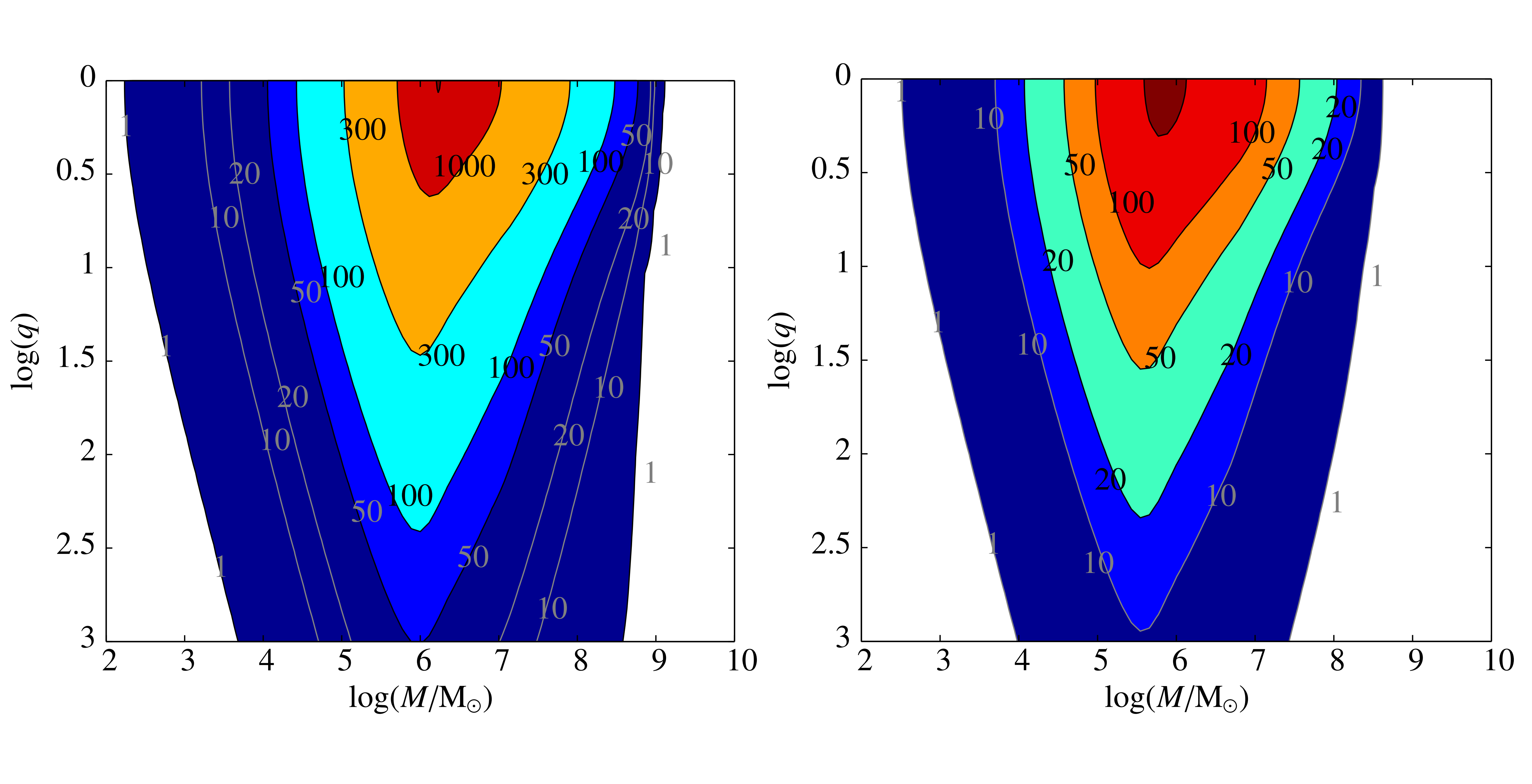
Distribution of primary mass and mass ratio for LISA CMCB inspiral events (Sesana et al. 2005)

LISA SNRs



Redshift z
$\log(M/M_\odot)$
20
18
16
14
12
10
8
6
4
2
2
3
4
5
6
7
8
9
10
10
20
50
100
200
300
500
1000
2500

LISA SNRs

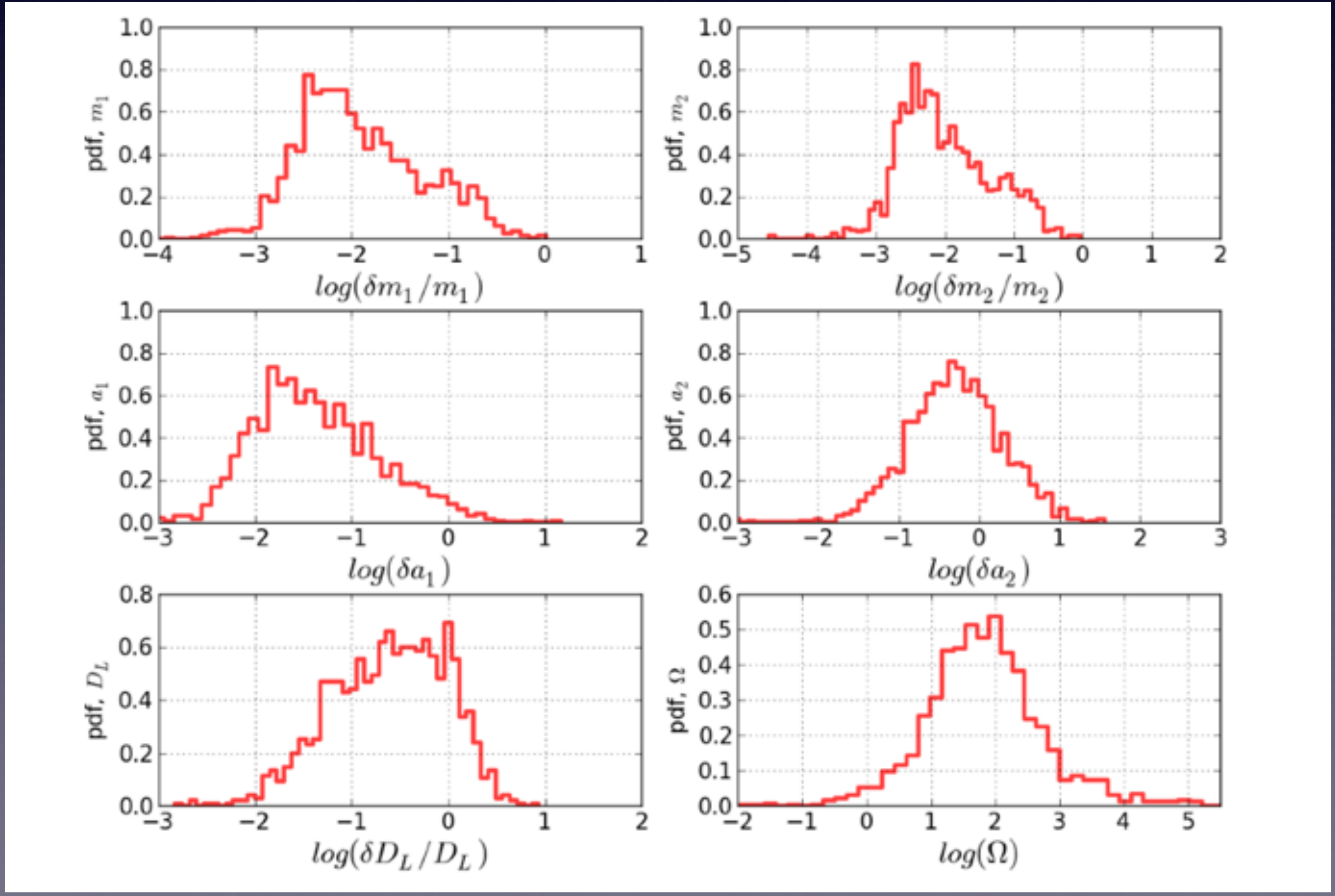


LISA SNRs

- Expected event rate depends on assumptions about black hole population (Klein+, 2016)
  - Light pop-III seed model: 2Gm/6-link configuration expected to see ~350 events.
  - Heavy seed model, no delay in binary formation: ~550 events.
  - Heavy seed model, with delays: ~50 events.
- Current baseline 2.5Gm/6-link configuration would see 150/300/4 events at $z > 7$ under the different models.
- 1 Gm/4-link detector would see ~15/185/3 events.
- 5 Gm/6-link detector would see ~400/350/4 events.

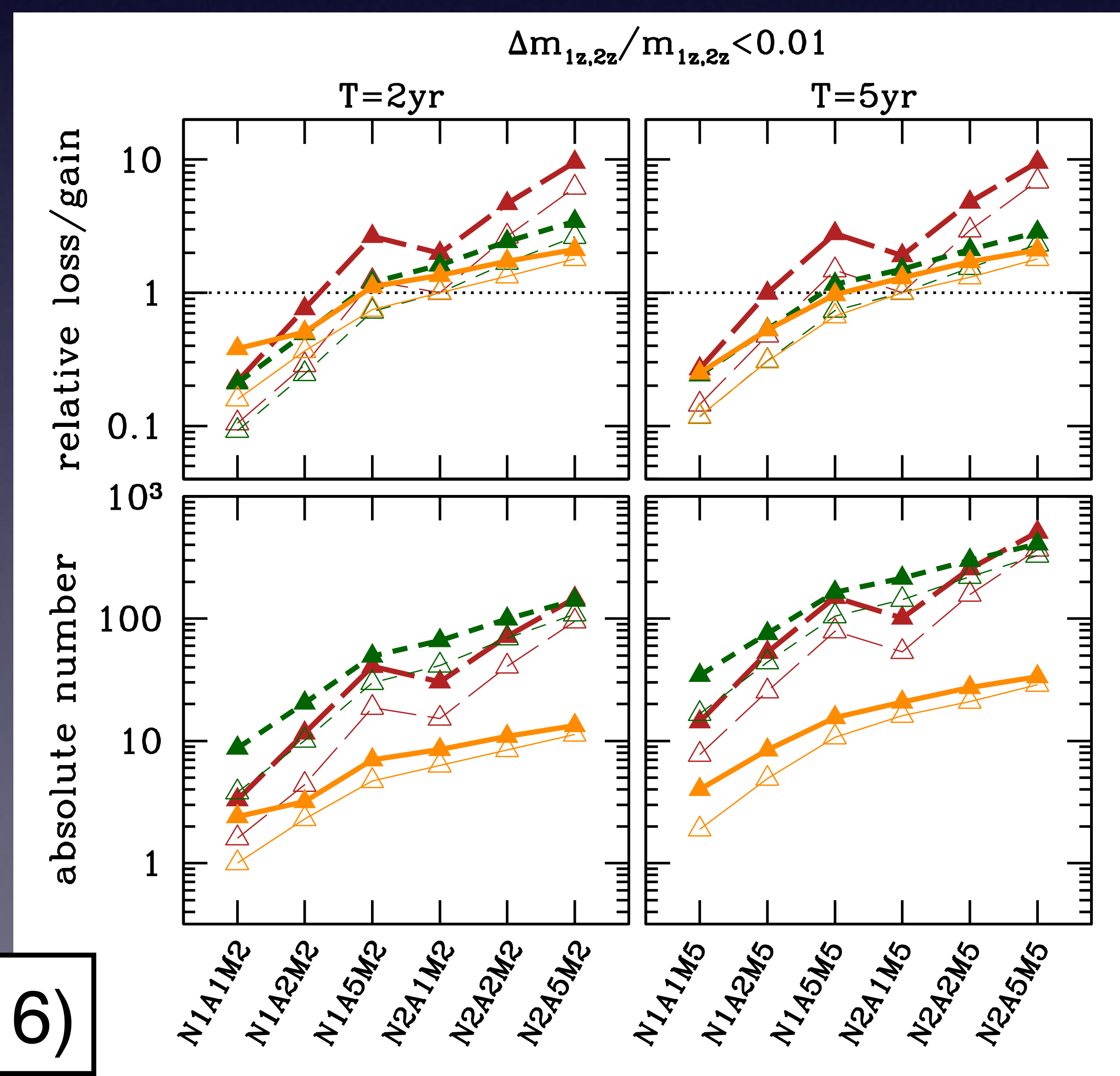
LISA Event Rate

- LISA will measure the parameters of black hole mergers to high precision. Typical errors for 1Gm/6-link configuration are
  $$\Delta m_1/m_1, \Delta m_2/m_2 \sim 10^{-3} - 10^{-2}, \Delta a_1 \sim 10^{-2}$$
  $$\Delta a_2 \sim 10^{-1}, \Delta\Omega \sim 100\ \mathrm{deg}^2, \Delta D_L/D_L \sim \mathrm{few} \times 10^{-1}$$



LISA Event Rate

- In two years, 2Gm/6-link configuration could determine
  - *both* redshifted masses to 1% for ~70/100/10 systems.
  - the spin of the primary to 1% for ~30/50/2 systems.
  - sky location to 10 deg^2 *and* distance to 10% for ~7/23/4 systems.



Klein et al. (2016)

Detection

- These are sources for space-based interferometers.
- Detection methods include *matched filtering* and *stochastic (MCMC) searches.*
- Waveform modelling as for LIGO binaries
  - Post-Newtonian expansion for inspiral.
  - Numerical relativity for merger/ringdown.
  - Combined in IMR Phenom and EOB models.
- *But* higher signal-to-noise of LISA events places more stringent requirements on the phase accuracy of templates - open challenge.

Science Applications of LISA binaries

- Applications of LISA observations of SMBH mergers are similar to those of LIGO observations of stellar mass mergers.
  - ➡ *Astrophysics*. Number of mergers and redshift distribution tell us about galaxy evolution and occupation fractions. Masses and spins probe black hole growth mechanisms.
  - ➡ *Strong field test of general relativity*. Even more powerful test than LIGO since we will detect the signals without needing any templates. High SNR allows very accurate comparisons to numerical relativity.
  - ➡ *Cosmology*. Single observation of an SMBH merger with a counterpart will determine luminosity distance to ~1%, which would determine cosmological parameters better than supernovae observations. Weak lensing limit is a concern.
  - ➡ *Kicks*. Will observe/measure black hole kicks to constrain models.

LISA Science - Astrophysics

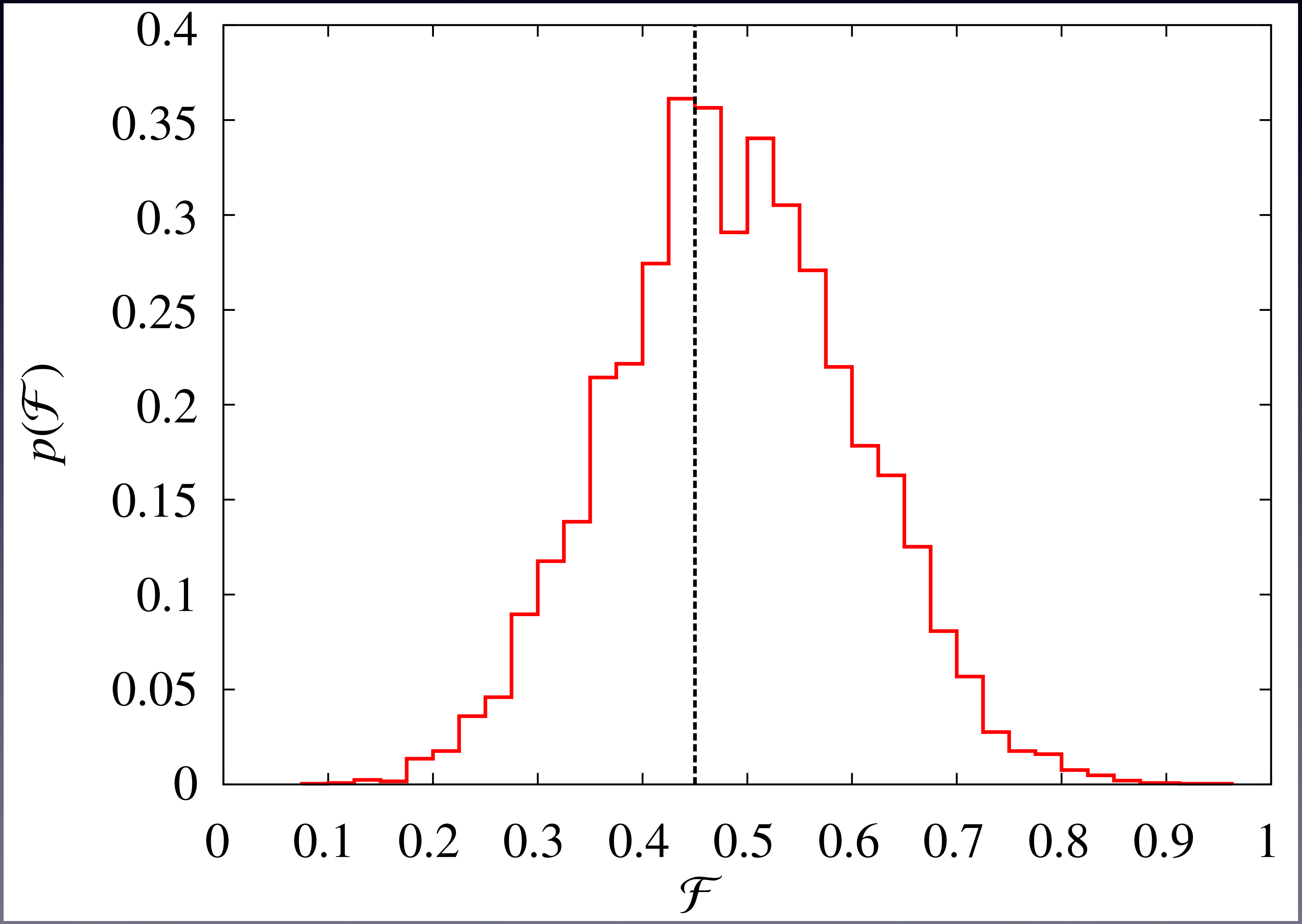
- Use LISA observations of SMBH mergers to constrain models for hierarchical growth of structure.
- Compare four different models
  - SE: VHM (Volonteri, Haardt & Madau 2003: light seed model) plus coherent accretion.
  - SC: VHM plus chaotic accretion.
  - LE: BVR (Begelman, Volonteri & Rees 2006: massive seed model) plus coherent accretion.
  - 4LC BVR plus chaotic accretion.

	Without spins			
	SE	SC	LE	LC
SE	×	0.48	0.99	0.99
SC	0.53	×	1.00	1.00
LE	0.01	0.01	×	0.79
LC	0.02	0.02	0.22	×

	With spins			
	SE	SC	LE	LC
SE	×	0.96	0.99	0.99
SC	0.13	×	1.00	1.00
LE	0.01	0.01	×	0.97
LC	0.02	0.02	0.06	×

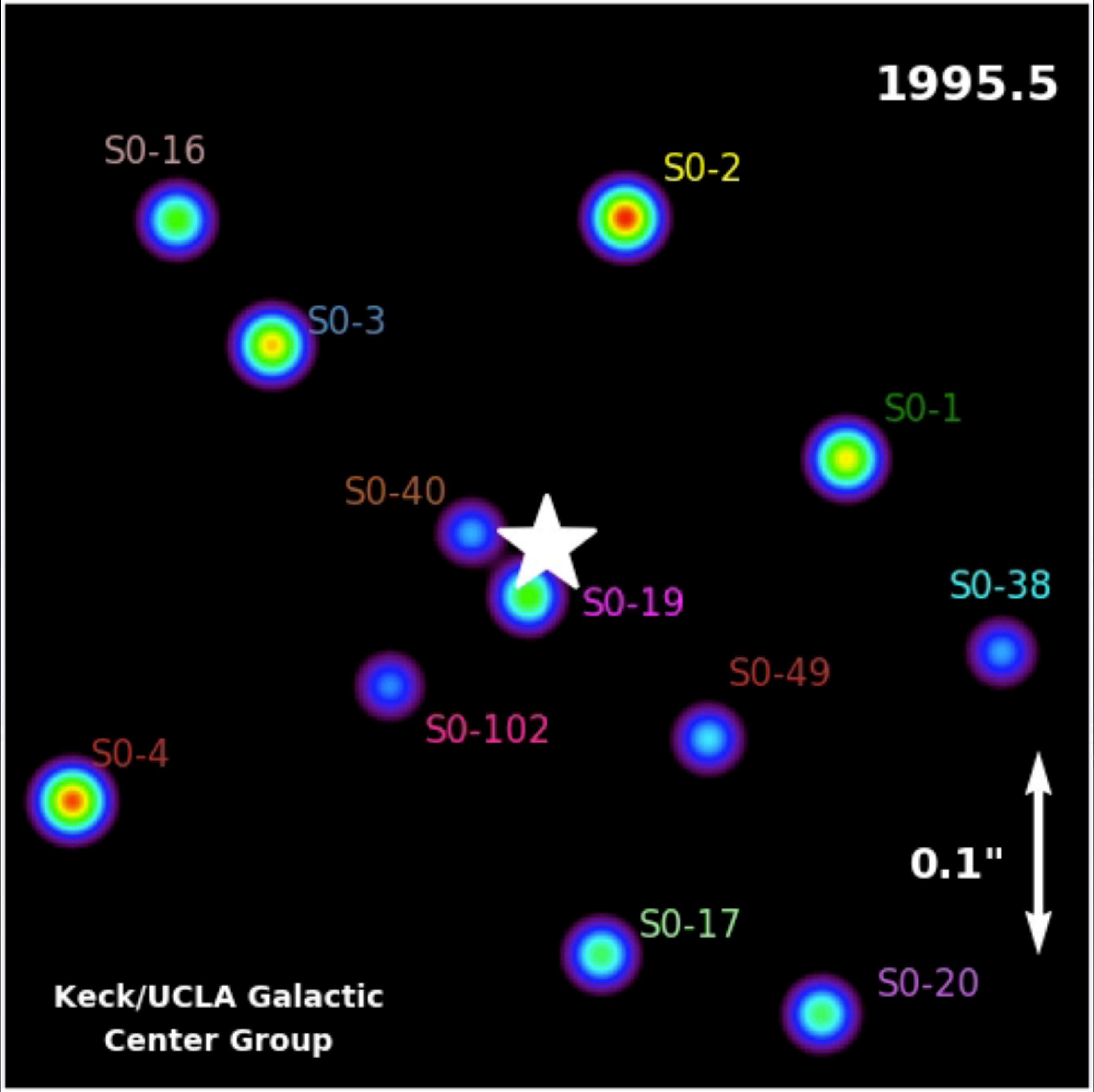
LISA Science - Astrophysics

- Can also constrain mixed models - take a certain fraction, F, of SE and the remaining fraction, $1-F$, of LE.



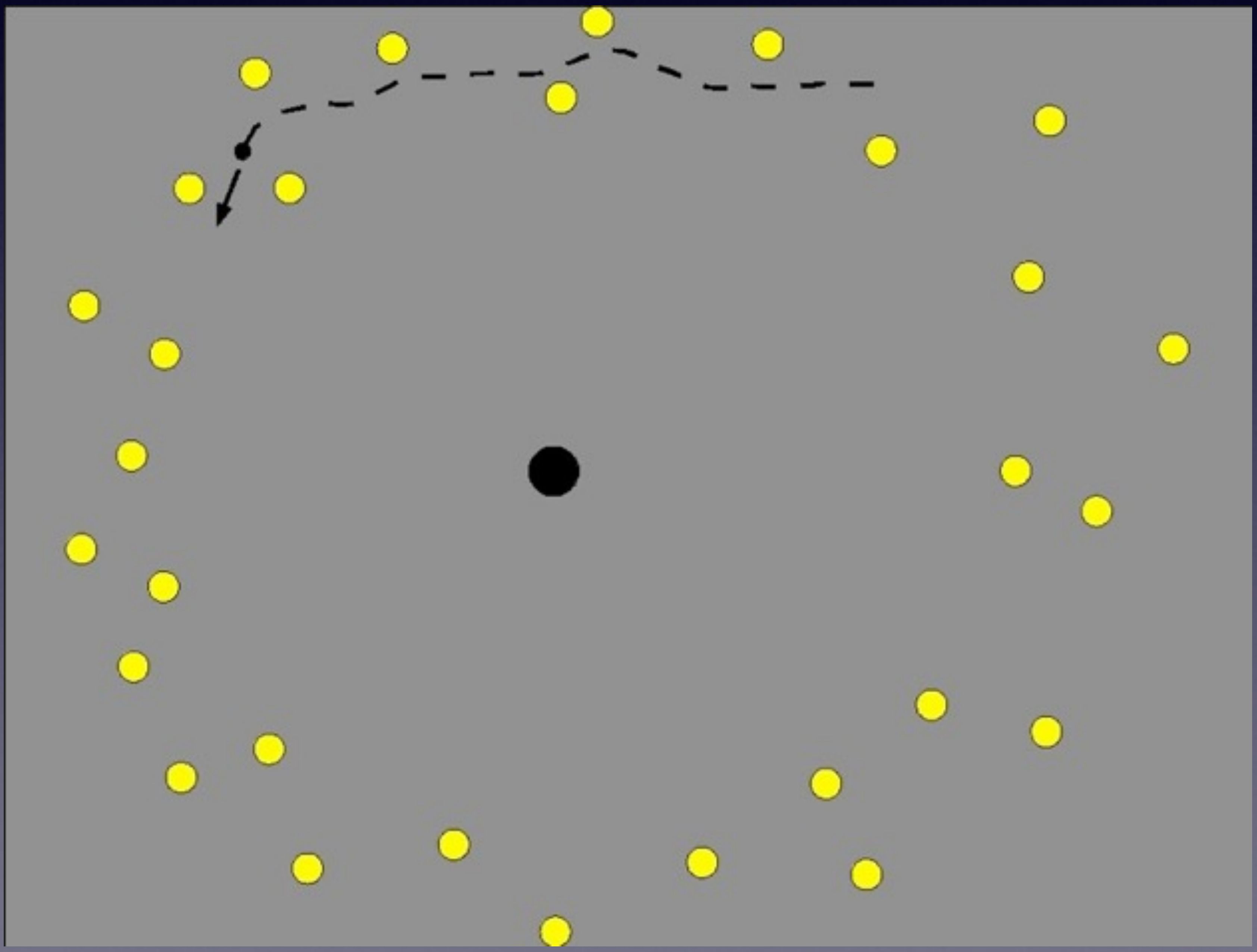
Extreme Mass Ratio Inspirals

- Supermassive black holes in the centres of most galaxies are typically surrounded by clusters of stars.
- The centre of our own galaxy is a good example. Orbits of "S-stars" indicate presence of black hole coincident with radio source Sgr A*.



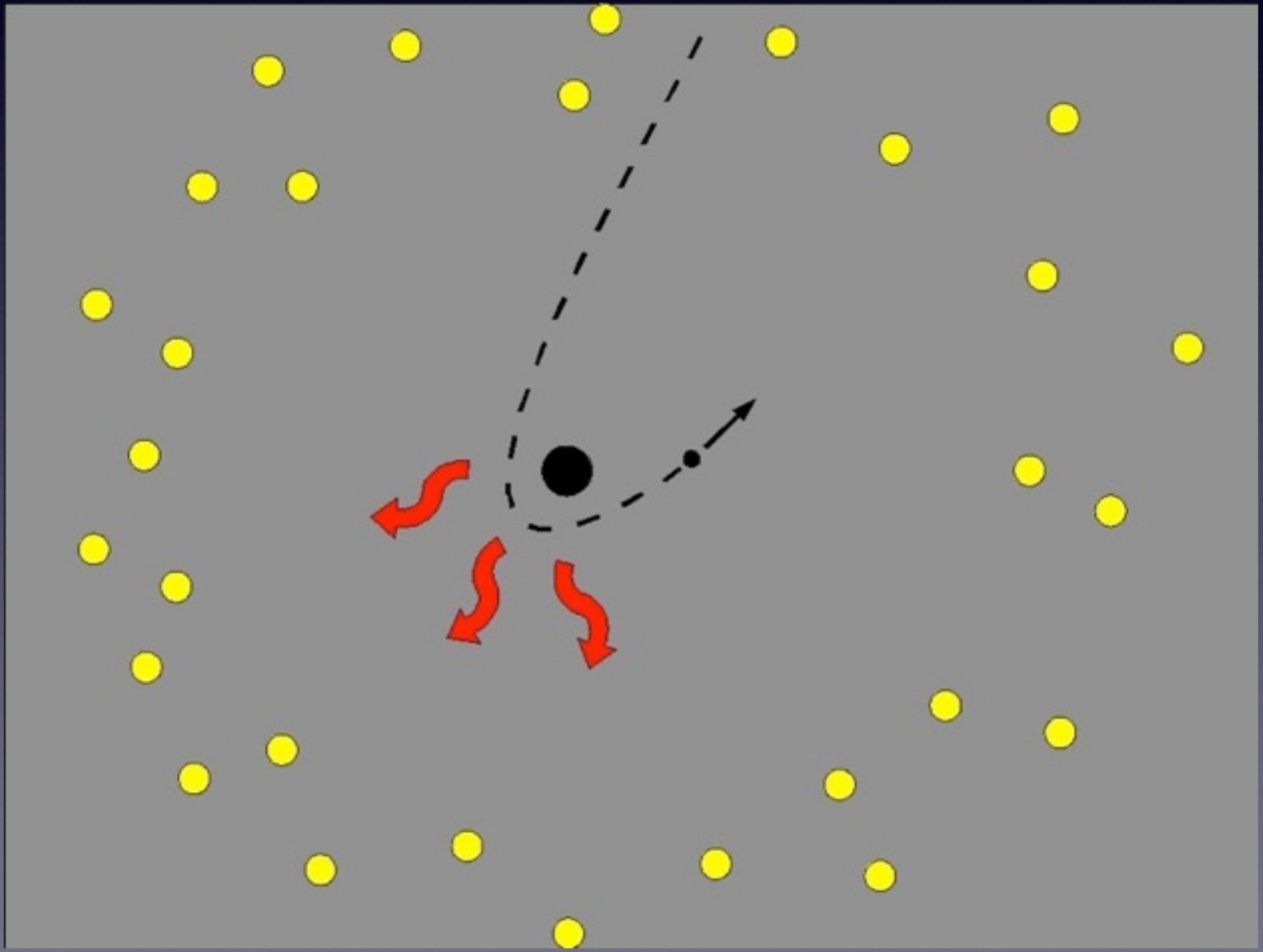
Extreme Mass Ratio Inspirals

- Stars in the cluster interact gravitationally. Orbits evolve over time, diffusing in energy and angular momentum.



Extreme Mass Ratio Inspirals

- Stars in the cluster interact gravitationally. Orbits evolve over time, diffusing in energy and angular momentum.
- This can put a star on an orbit that passes very close to the SMBH.



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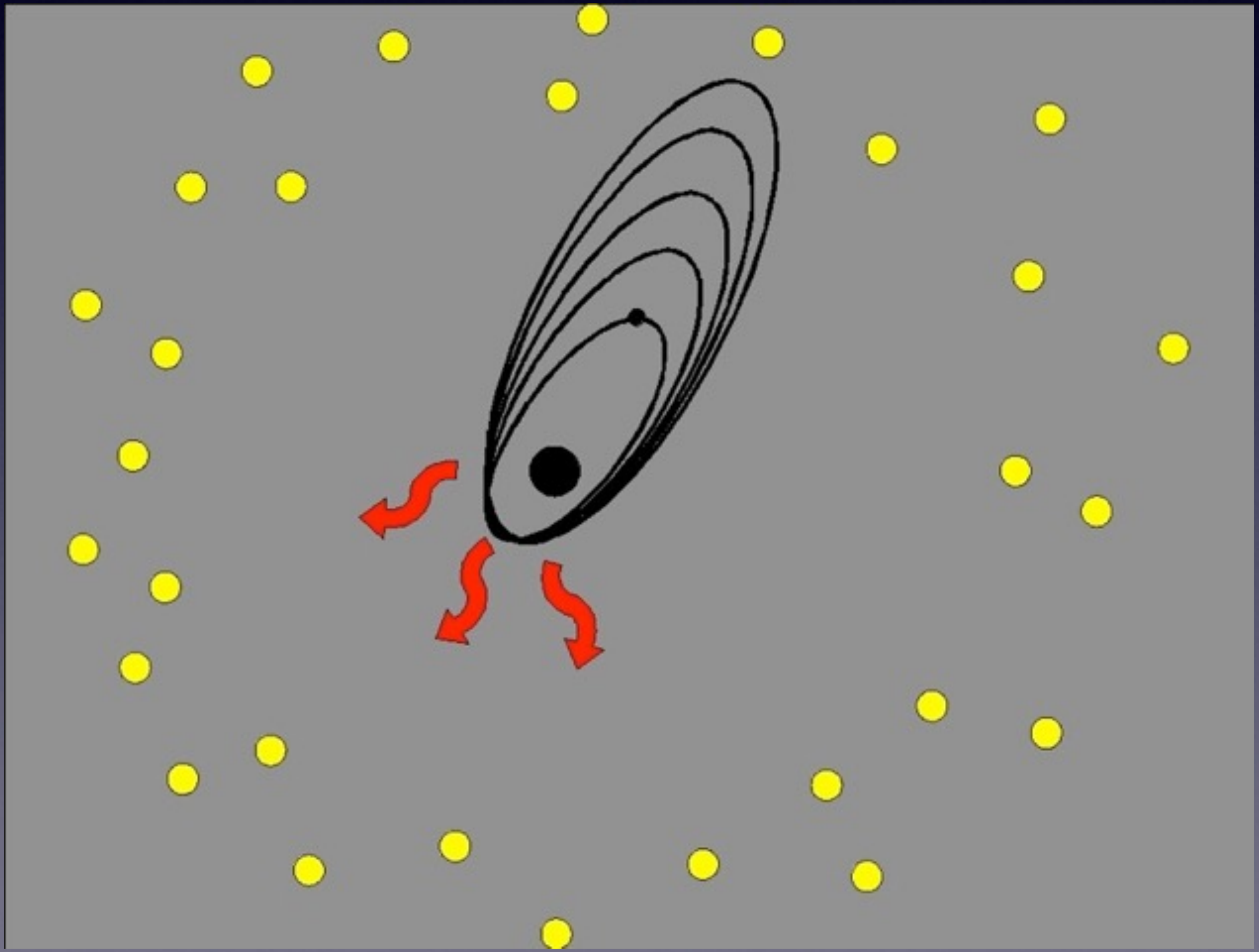


Extreme Mass Ratio Inspirals

- Stars in the cluster interact gravitationally. Orbits evolve over time, diffusing in energy and angular momentum.
- This can put a star on an orbit that passes very close to the SMBH.
- An extreme mass ratio inspiral (EMRI) is the inspiral of a compact stellar remnant (a white dwarf, neutron star or black hole) into a SMBH in the centre of a galaxy.
- For black holes of mass $M \sim 10^5 M_\odot - 10^7 M_\odot$, the gravitational waves emitted during the last few years of inspiral will be detected by LISA.
- Inspiral is slow - the compact object will complete up to 100,000 orbits during the observation. Systems are ‘clean’ - the small object acts like a test particle in the background spacetime.

EMRIs - Event Rates

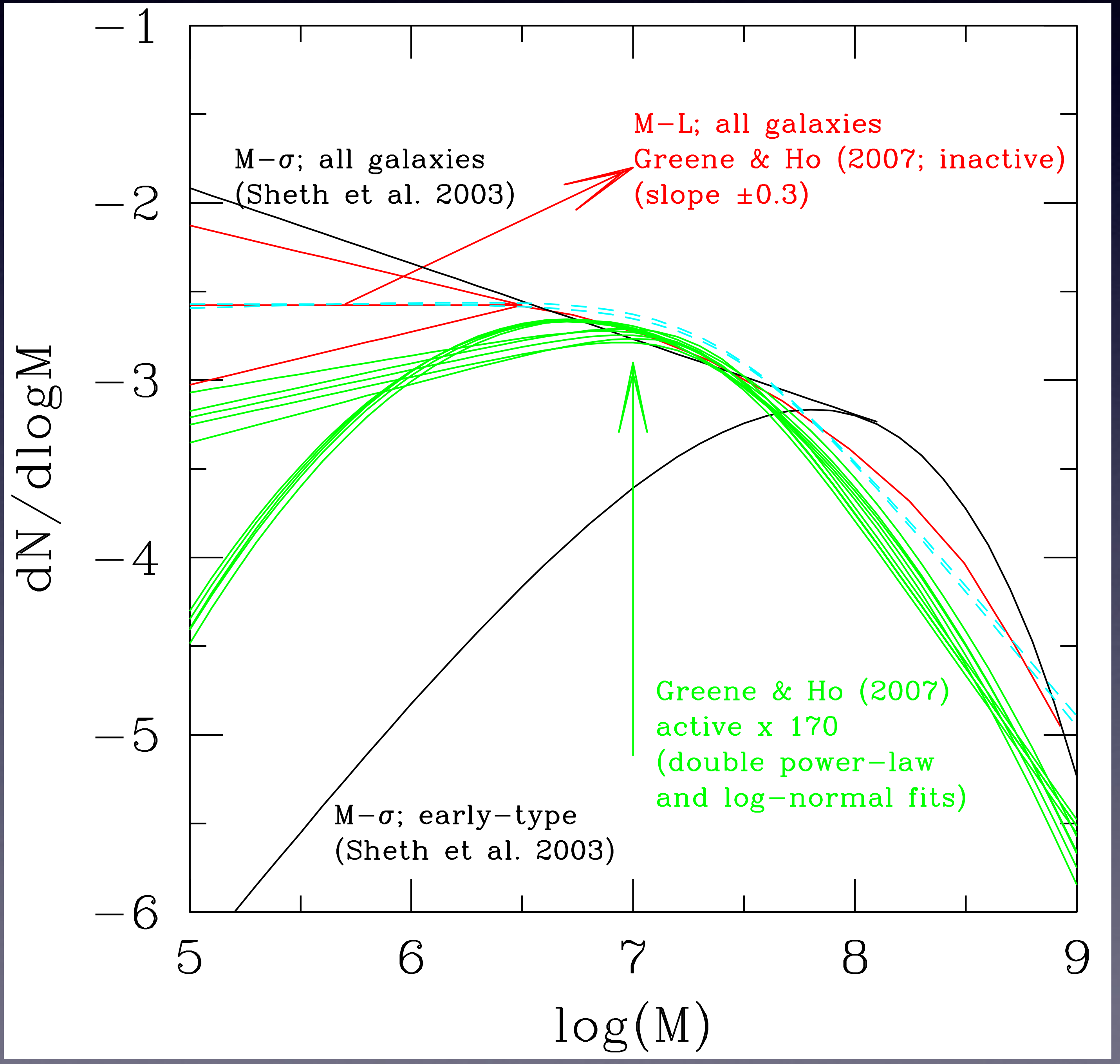
- To estimate EMRI event rates need several ingredients

- Mass function of black holes: for $10^4 M_\odot \lesssim M \lesssim 10^7 M_\odot$ the BH mass function is not well constrained observationally.

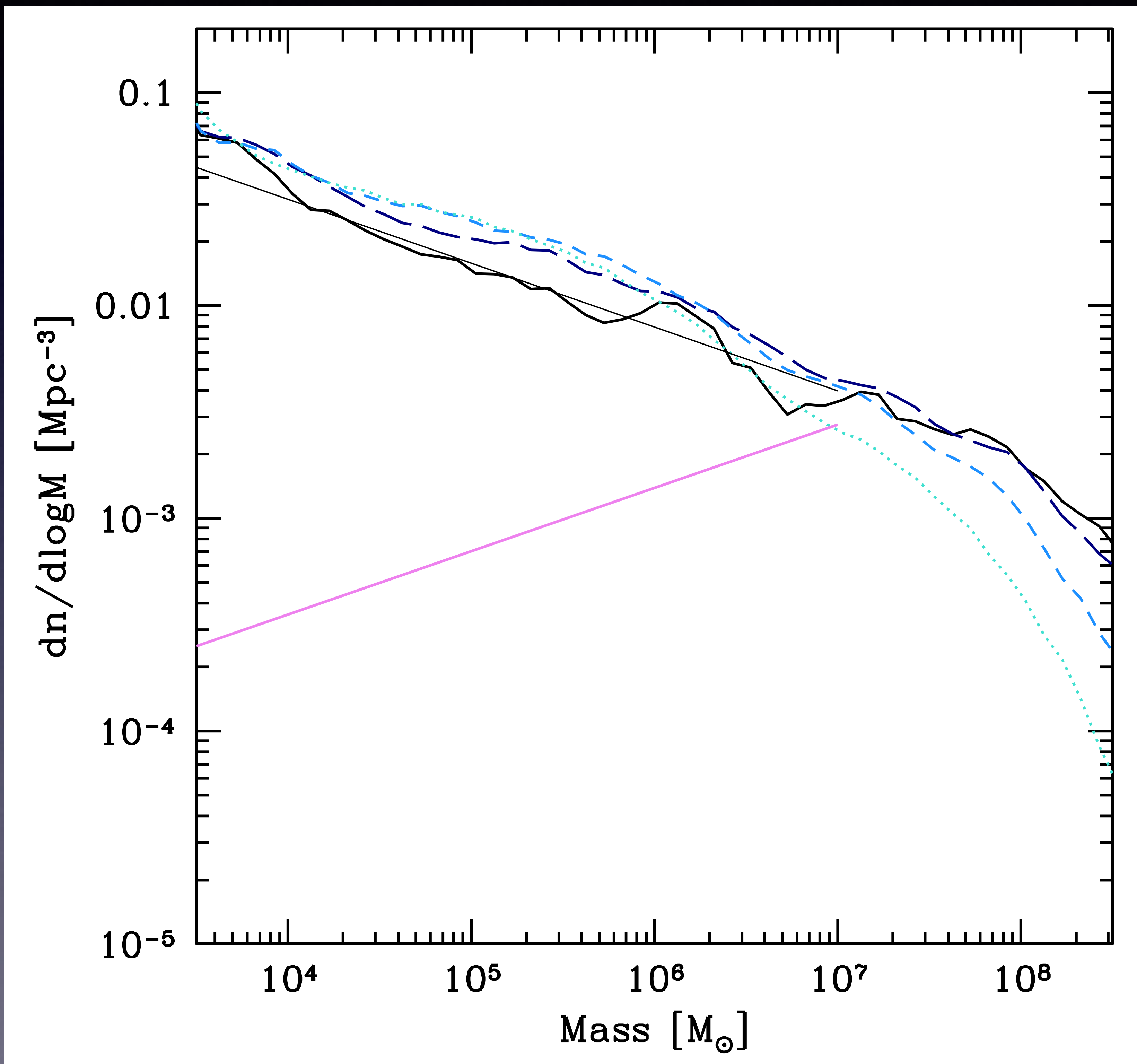
- Traditionally have assumed a flat distribution

$$\frac{\mathrm{d}N}{\mathrm{d}\ln M} = 0.002\mathrm{Mpc}^{-3}$$

- Uncertainty in slope +/-0.3. Models for MBH mergers favour slopes close to -0.3.



EMRIs - Event Rates



- Consider two cases
  - a numerically simulated population, evolved consistently from pop III seeds: slope ~ -0.3
  - a pessimistic analytic model: slope = 0.3.

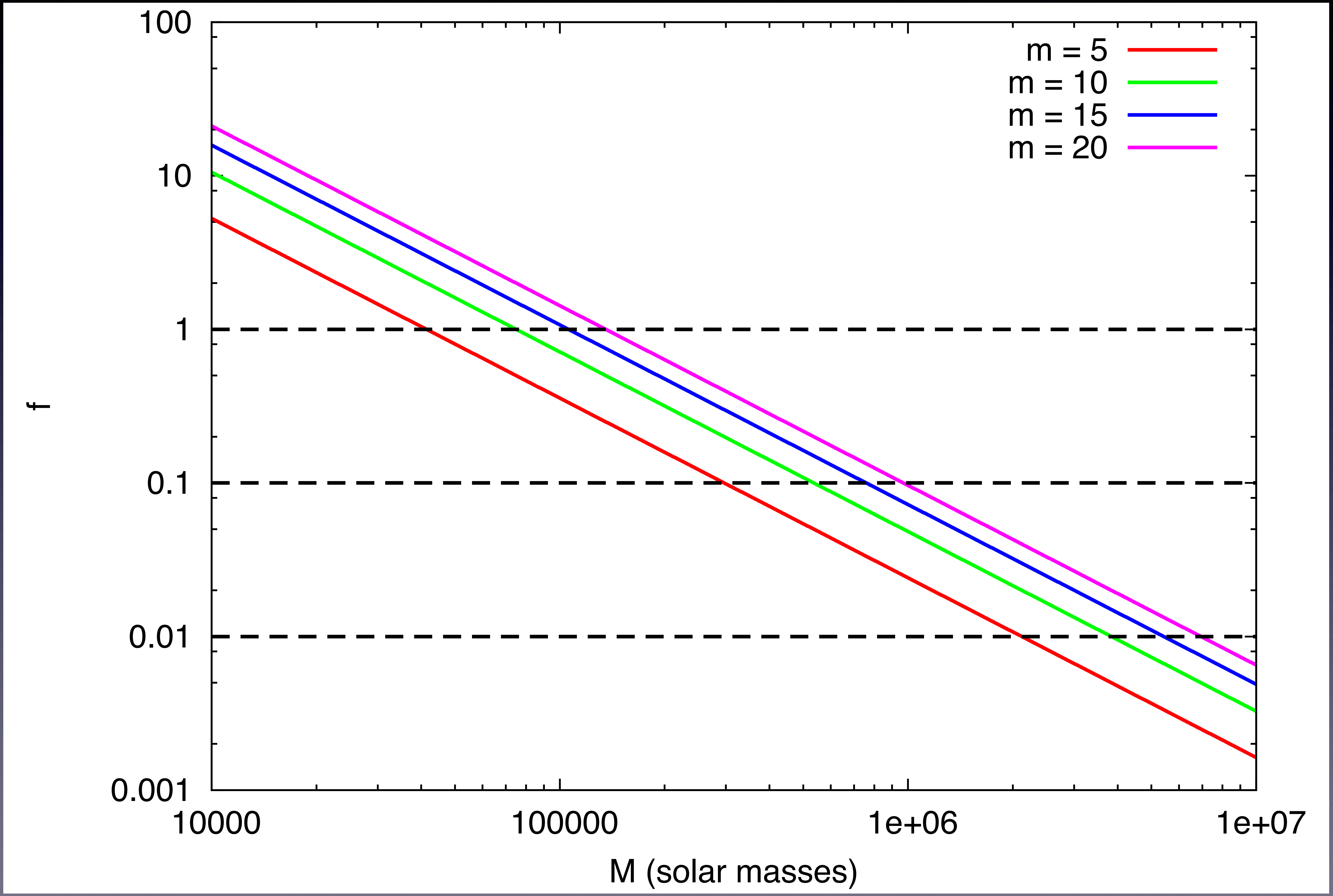
EMRIs - Event Rates

- To estimate EMRI event rates need several ingredients
  - EMRI rate per galaxy numerical simulations suggest rate of black hole mergers (Hopman 2009, Amaro-Seoane & Preto 2011)

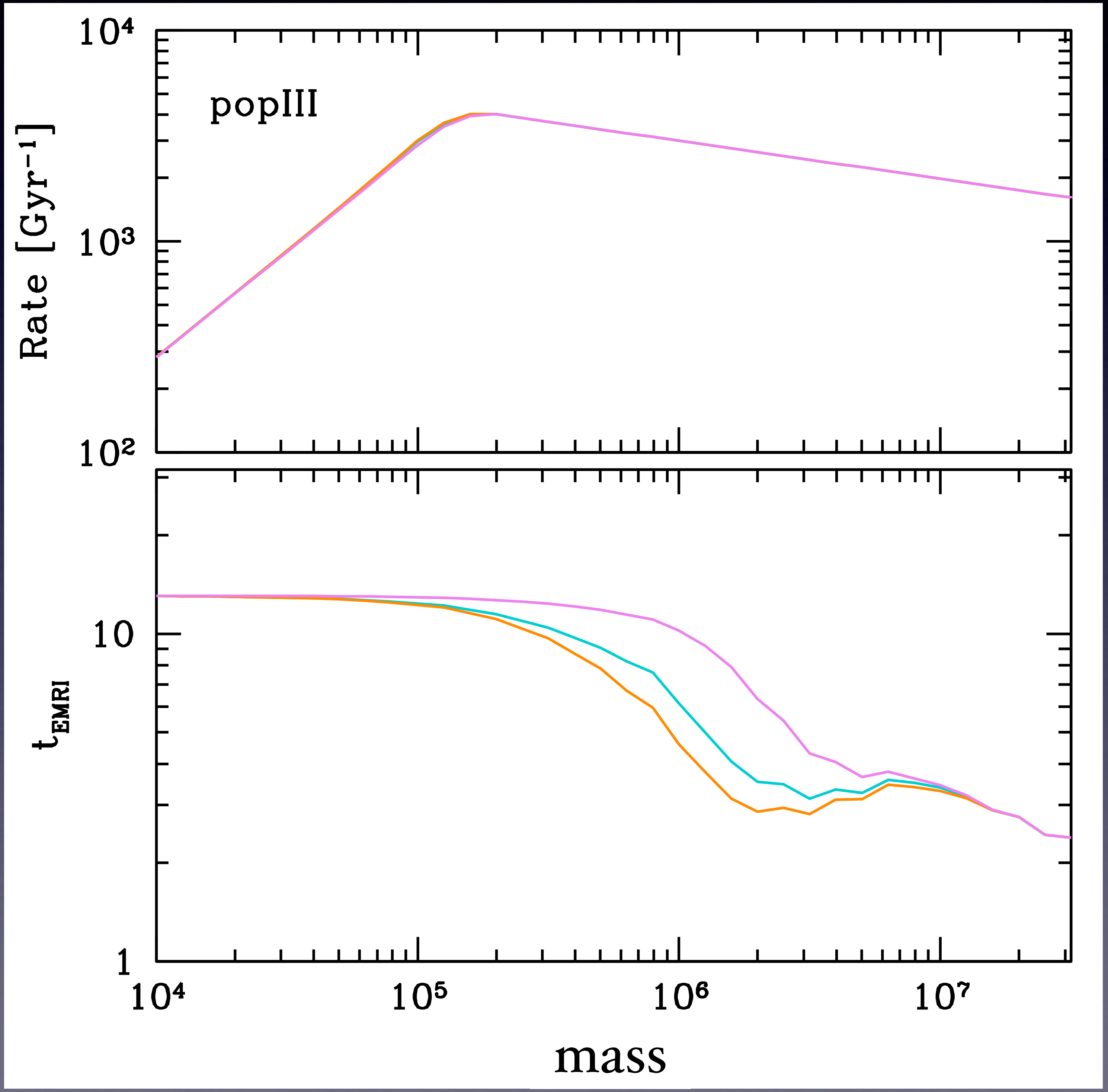
$$R = 400\mathrm{Gyr}^{-1}\left(\frac{M}{3\times 10^{6} M_{\odot}}\right)^{-0.19}$$

  - **But** cannot have such a high rate over whole cosmic history for light massive black holes to avoid overgrowth. Assume maximum of one e-fold of mass from EMRI accretion.
  - Host galaxy mergers also disrupt stellar cusps - massive black hole is not available as EMRI host until cusp has regrown.
  - Black hole spin/inclination influence capture cross-section - enhanced rate for spinning black holes and prograde EMRIs (Amaro-Seoane et al. 2013).

EMRIs - Event Rates



EMRIs - Event Rates

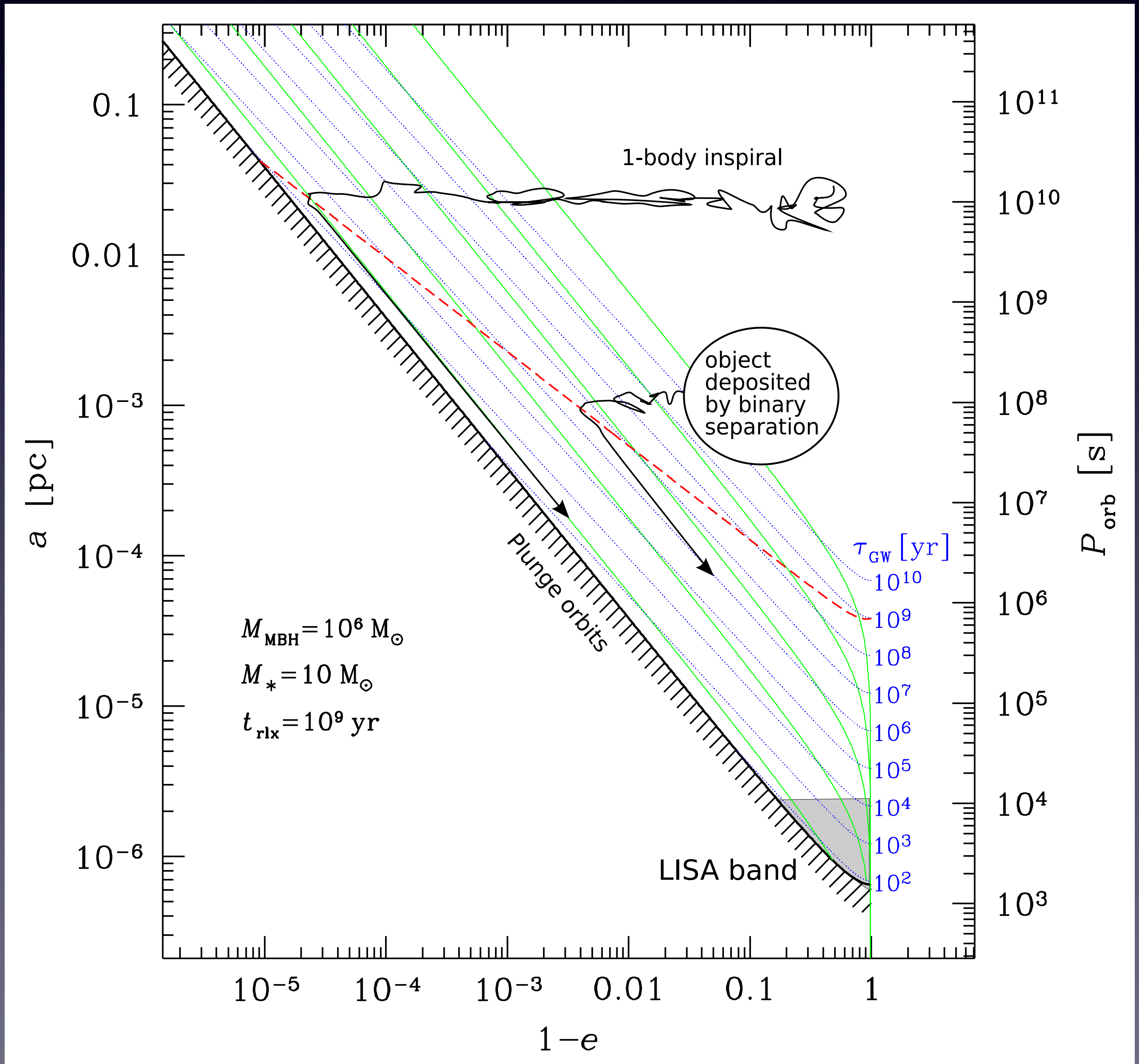


- Consider three scenarios for cusp regrowth
  - fiducial, t ~ 6 Gyr
  - optimistic, t ~ 2 Gyr
  - pessimistic, t ~ 10 Gyr
  - Here t is the cusp regrowth time for a $10^6 M_\odot$ black hole following an equal-mass merger.

$$t_{\rm cusp} \approx 6 M_6^{1.19} q^{0.35} {\rm Gyr}$$

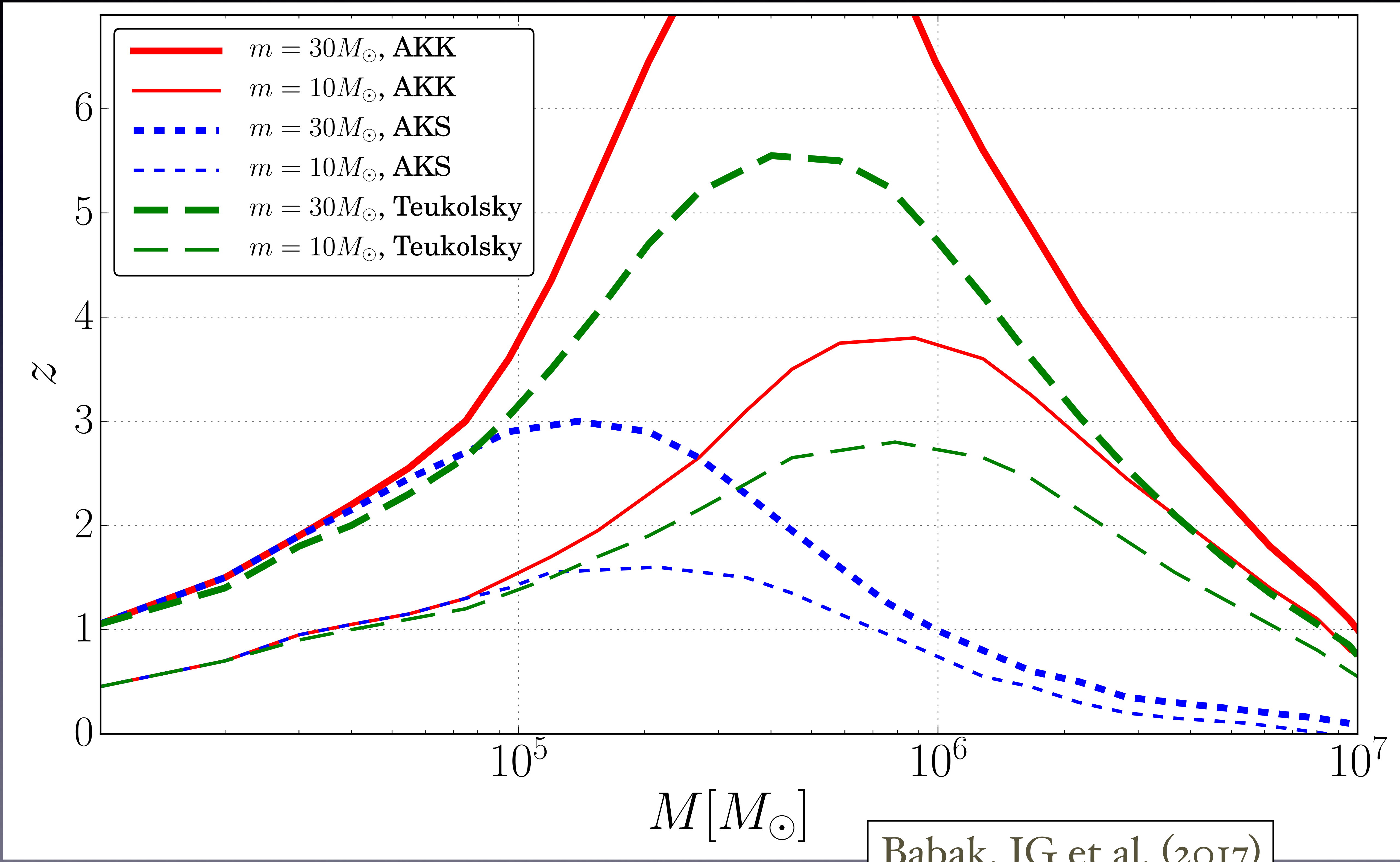
EMRIs - Event Rates

- To estimate EMRI event rates need several ingredients
  - Compact object properties
  - Mass: consider only black holes. Assume $m = 10M_\odot$ (usual assumption) or, given GW150914, $m = 30M_\odot$.
  - Eccentricity distribution: assume capture through diffusion. Eccentricities mostly moderate at plunge.
  - Inclination distribution: random at capture, but prograde EMRIs preferentially inspiral.



Amaro-Seoane et al. (2007)

EMRIs - Event Rates



Babak, JG et al. (2017)

EMRIs - Event Rates

- Reference model and baseline configuration predicts ~15 to ~3000 events.

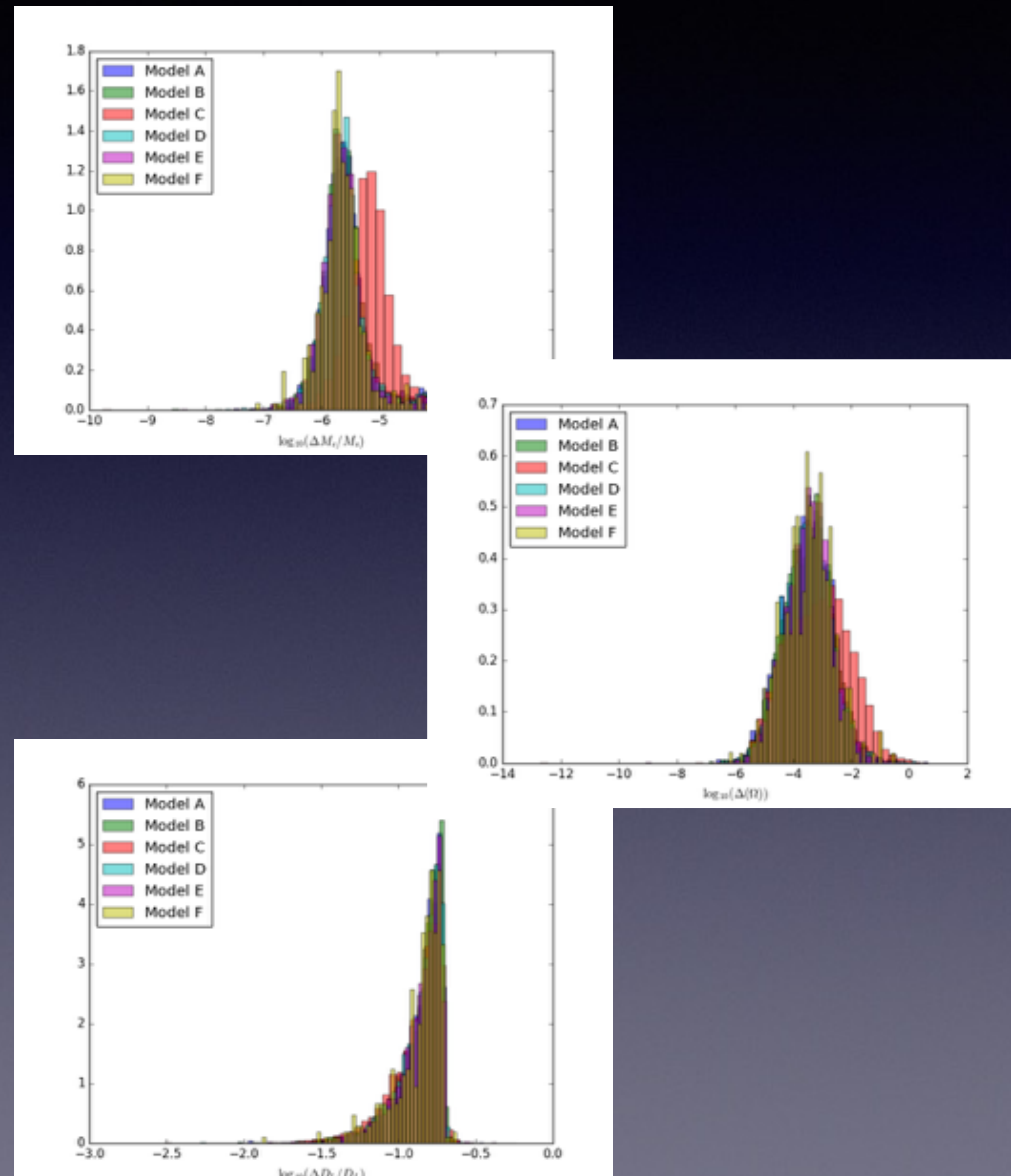
	1 Gm		2 Gm		5 Gm	
	4-link	6-link	4-link	6-link	4-link	6-link
m=10, T=6Gyr	0.1	0.2	0.4	1	1.9	2.9
m=10, T=10Gyr	0.1	0.2	0.4	0.9	1.7	2.7
m=30, T=6Gyr	0.3	0.7	1.1	1.6	2.2	2.6
m=10, T=2Gyr	0.1	0.3	0.6	1.2	2.3	3.4
m=10, T=0Gyr	0.1	0.4	0.7	1.5	2.6	3.9
m=10, T=0, power law mf	0.01	0.04	0.08	0.16	0.3	0.4

EMRIs - Event Rates

- LISA expected to observe a few to a few hundred EMRIs. Each observation will yield very precise parameter estimates

$$\frac{\Delta M_z}{M_z},\ \frac{\Delta \mu_z}{\mu_z}, \Delta\chi,\ \Delta e_{\rm pl}$$
$$\sim 10^{-6} - 10^{-4}$$
$$\Delta\Omega \sim 10^{-5} - 10^{-3}$$
$$\frac{\Delta D_L}{D_L} \sim 0.05 - 0.2$$

- Precision arises from tracking GW phase over many cycles. Not strongly dependent on detector, at fixed signal-to-noise.



Alternative Channels

- *Binary splitting* can also create EMRIs. A close encounter between a binary and a SMBH can split the binary. This leaves one component tightly bound to the SMBH (an EMRI) and the other is ejected as a *hypervelocity star*. Rate could be comparable to standard scenario.
- *Star formation in an accretion disc* and *tidal stripping of massive stars* can also produce EMRIs.

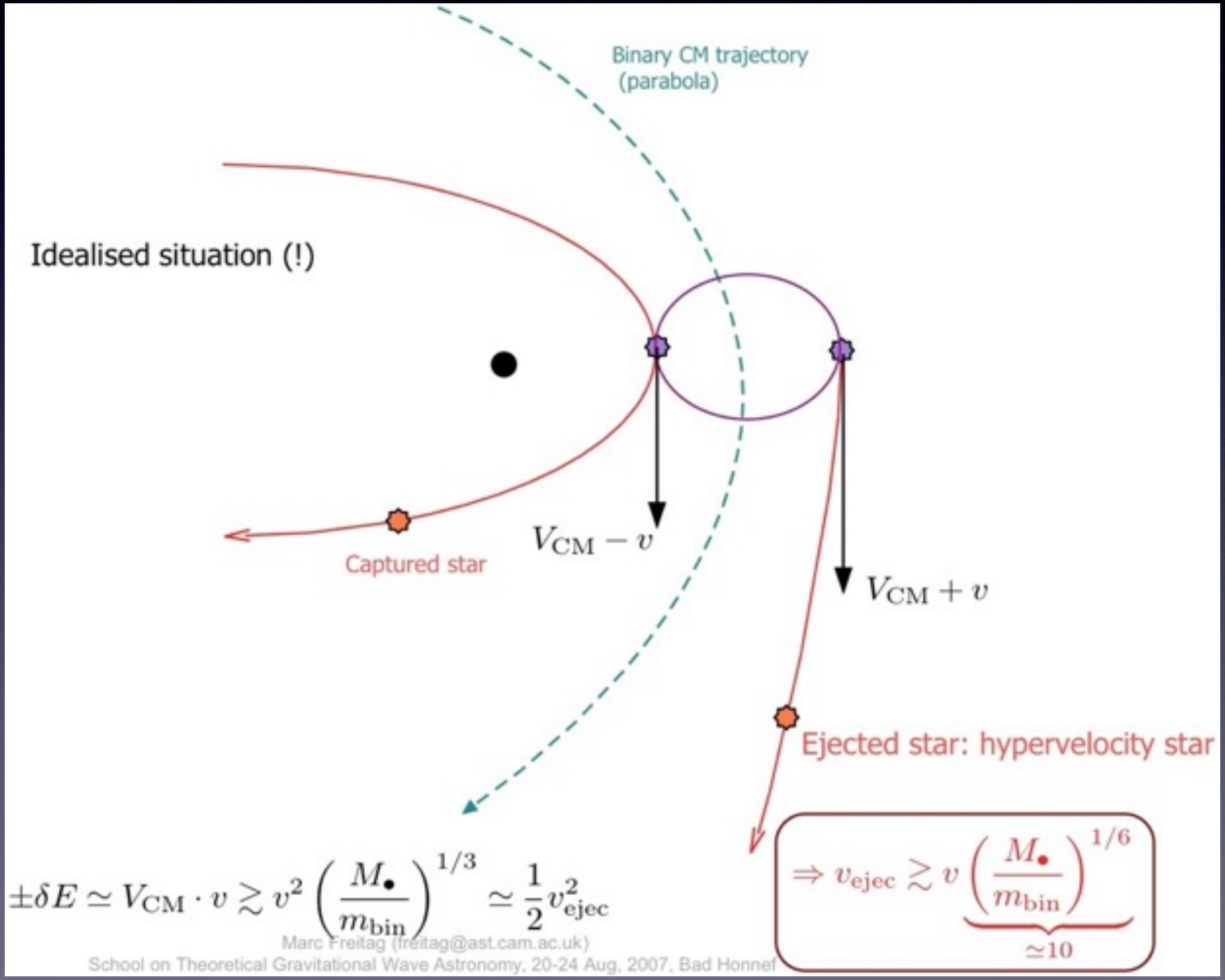


Diagram from Freitag (2007)

Related Source Types

- *Extreme Mass Ratio Bursts*
  - Compact objects passing close to supermassive black holes emit bursts of gravitational radiations. Both EMRIs and failed EMRIs.
  - Burst sources for LISA, but only for events in the Galactic Centre.
- *Intermediate Mass Ratio Inspirals.* Two different categories
  - *LIGO IMRIs.* Inspiral of ~1.4 solar mass neutron star into a ~100 solar mass intermediate mass black hole in a globular cluster.
  - *LISA IMRIs.* Inspiral of 100-1000 solar mass intermediate mass black holes into supermassive black holes.
  - Rates highly uncertain, as existence of intermediate mass ratio black holes has not been firmly established.
- Science applications of IMRIs are similar to EMRIs. Source modelling still unknown.

Detection

- These are sources for space-based interferometers.
- Detection methods include *matched filtering, stochastic (MCMC) searches, time-frequency analysis* and *semi-coherent searches.*
- Waveform modelling
  - Use black-hole perturbation theory with mass ratio as small expansion parameter, i.e., self-force.
  - Require results to second-order in mass ratio to track phase to required 1 cycle in 10^5.
  - Data analysis will use phenomenological models, hermes "kludges". Various flavours - *analytic kludge, numerical kludge, augmented analytic kludge, osculating elements* etc.

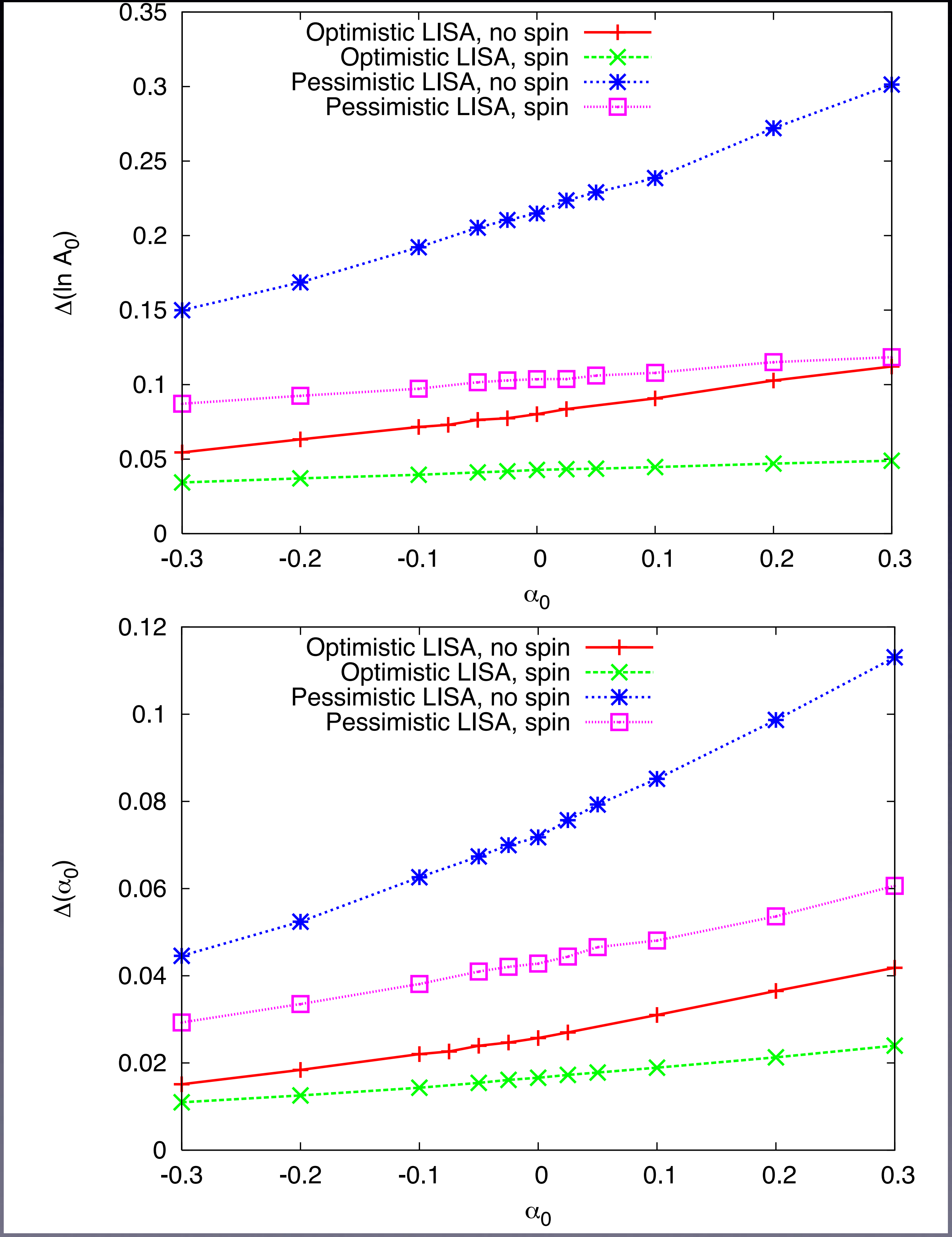
EMRI Science - Probe of Black Holes

- Can use set of observed EMRI events to probe the properties of black holes in the LISA range.
- Model BH mass function as a power law

$$\frac{\mathrm{d}N}{\mathrm{d}\ln M} = AM^{\alpha}$$

- We estimate that EMRIs will provide estimates with precision

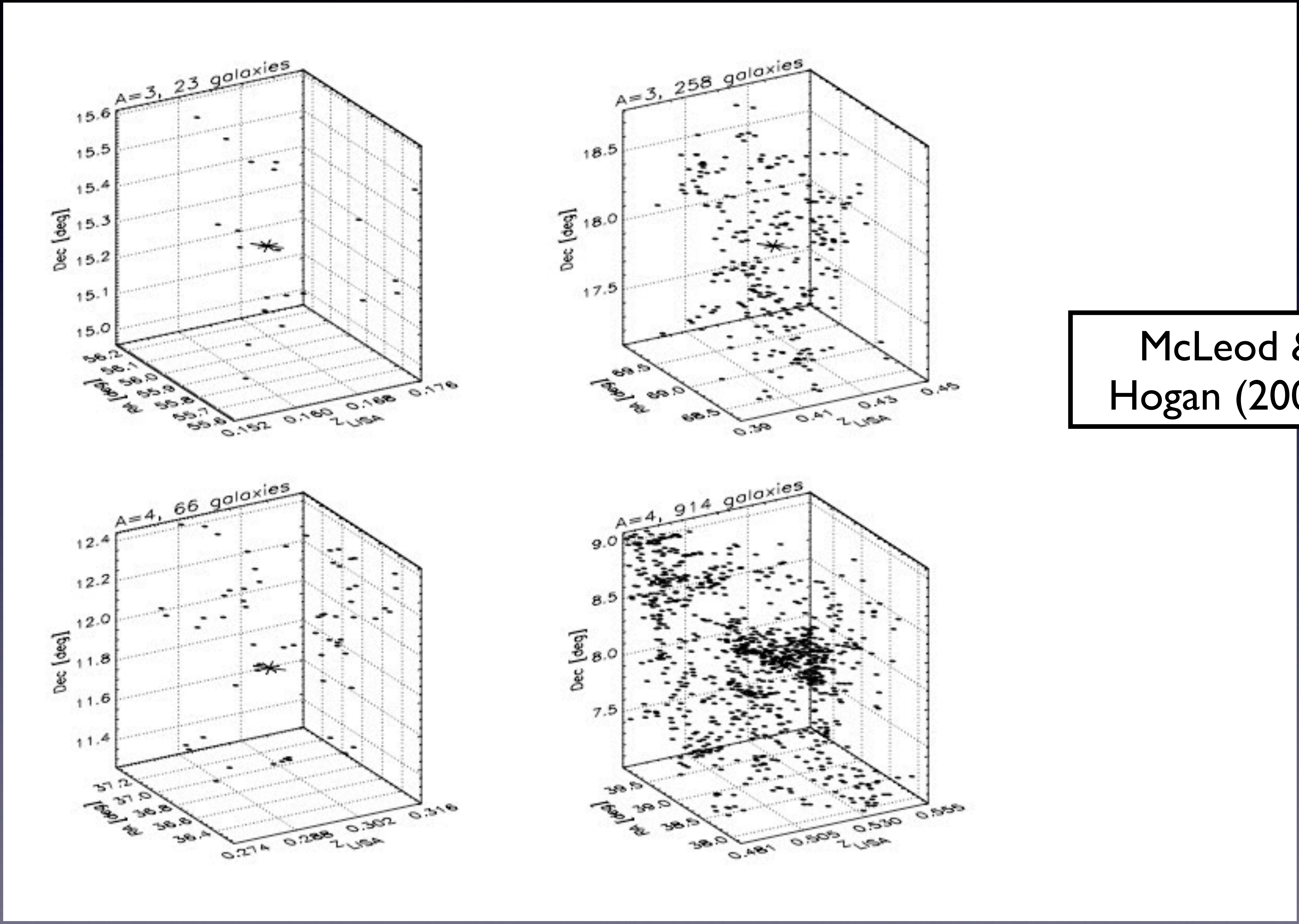
$$\Delta(\ln A) \approx 1.1\sqrt{10/N_{\mathrm{obs}}}$$
$$\Delta(\alpha) \approx 0.35\sqrt{10/N_{\mathrm{obs}}}$$



EMRI Science - Cosmology

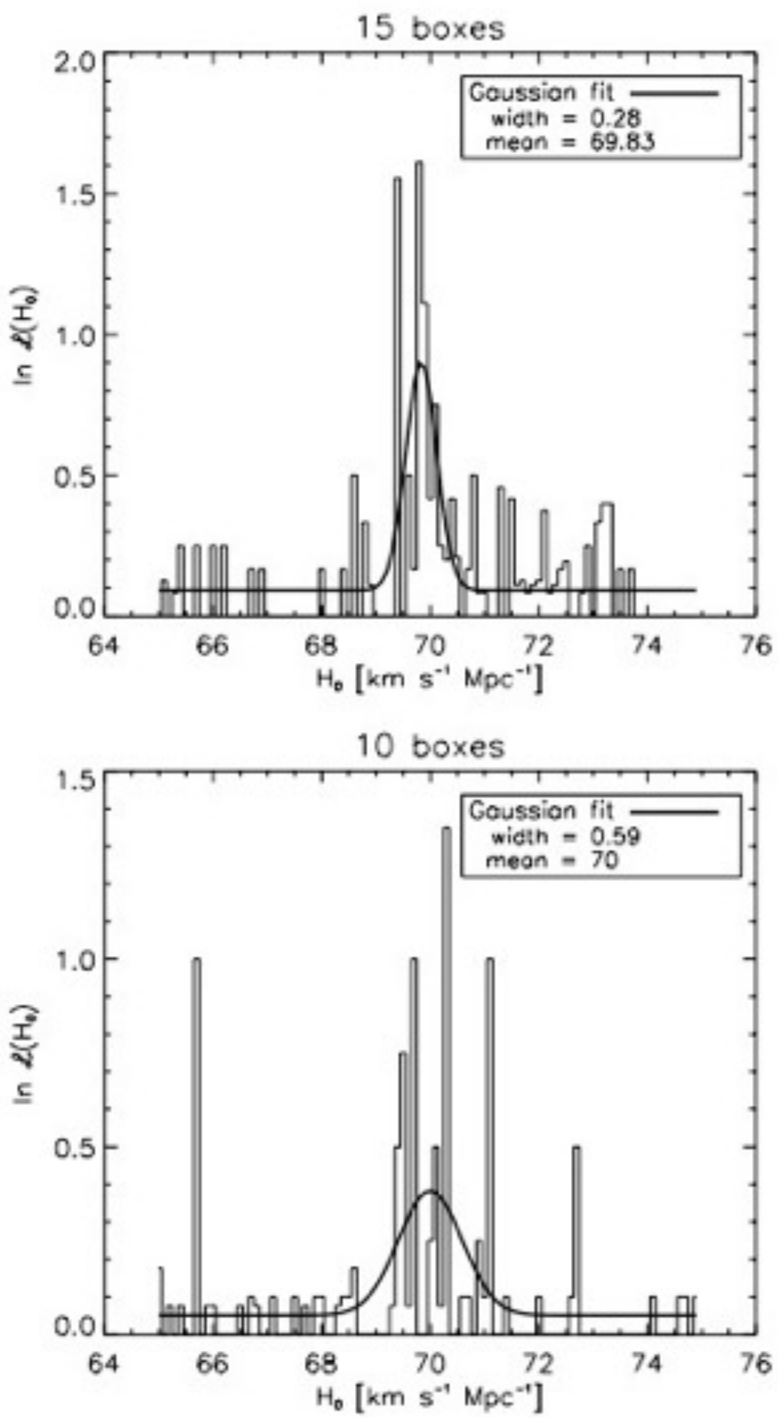
- A single EMRI event with an electromagnetic counterpart (and hence a redshift measurement) will give the Hubble constant to an accuracy of ~3%. N events give an accuracy of ~ $3/\sqrt{N}$ %.
  - Even without a counterpart, can estimate Hubble constant statistically (McLeod & Hogan 08)
  - Let every galaxy in the LISA error box "vote" on the Hubble constant.

EMRI Science - Cosmology



McLeod & Hogan (2008)

EMRI Science - Cosmology



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  - Even without a counterpart, can estimate Hubble constant statistically (McLeod & Hogan 08)
  - Let every galaxy in the LISA error box "vote" on the Hubble constant.
- If ~20 EMRI events are detected at $z < 0.5$, will determine the Hubble constant to ~1%.
- Analysis assumed typical distance uncertainties for Classic LISA. Pessimistically, LISA could have a factor 2 larger distance error; ~20 events at $z < 0.5$ would provide ~2% Hubble measurement, ~80 events would provide 1% precision

Detection

- A single EMRI event with an electromagnetic counterpart (and hence a redshift measurement) will give the Hubble constant to an accuracy of ~3%. N events give an accuracy of ~ %.
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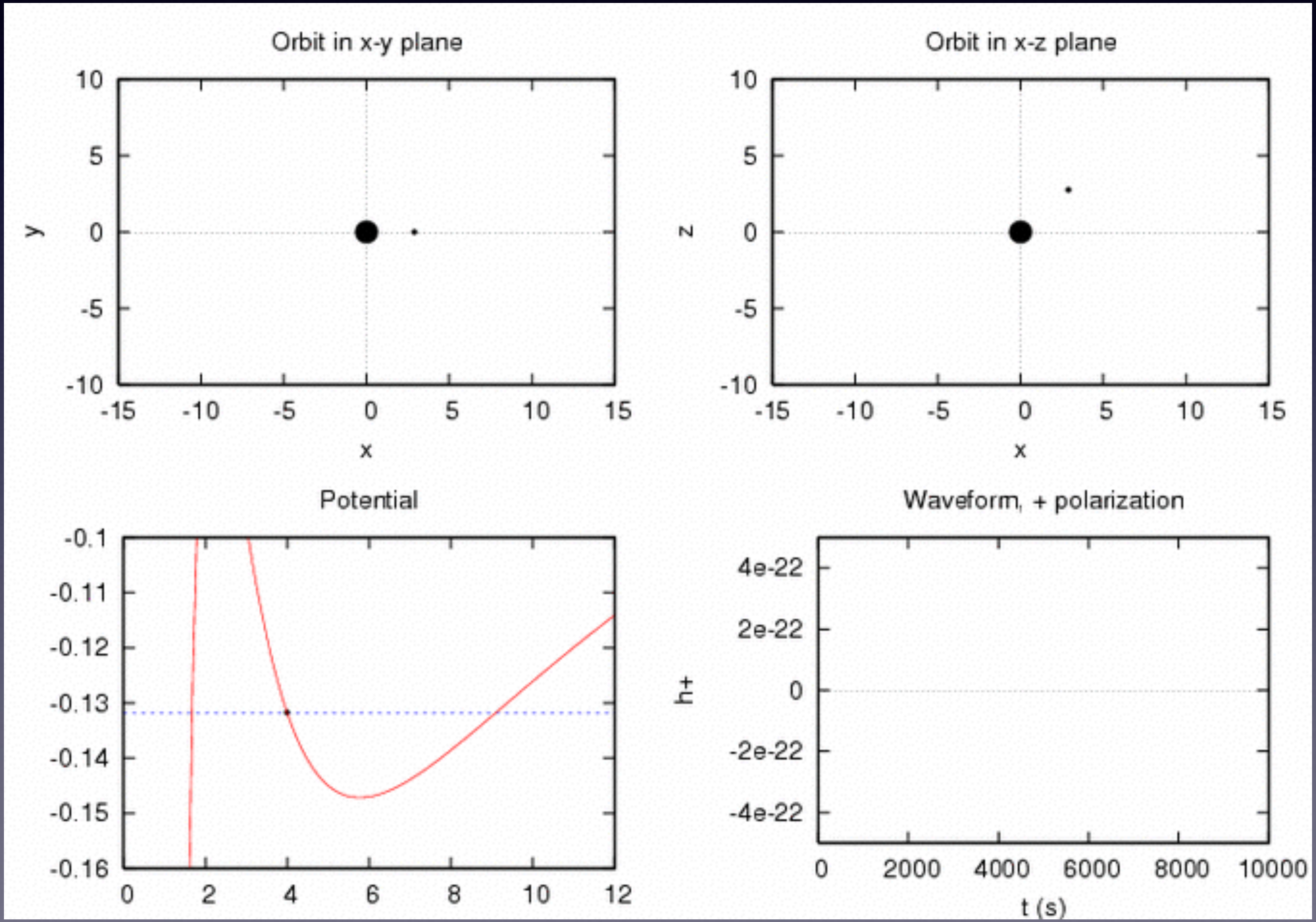
Probing the nature and structure of BHs

- GW emission from EMRIs encodes a map of the space-time structure outside the central massive black hole.

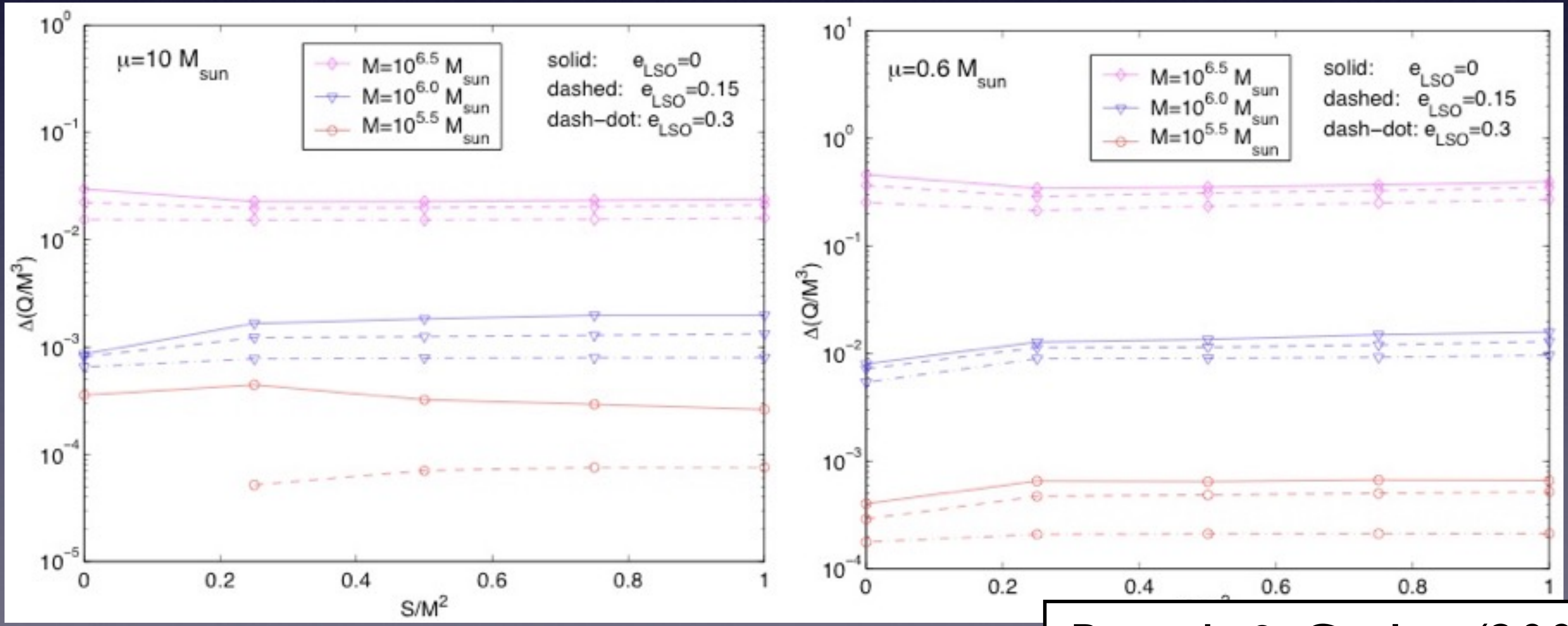


Probing the nature and structure of BHs

- GW emission from EMRIs encodes a map of the space-time structure outside the central massive black hole.
- Can characterize a vacuum, axisymmetric spacetime in GR by its multipole moments. For a Kerr black hole, these satisfy the 'no-hair' theorem:
$$M_l + \mathrm{i}S_l = M(\mathrm{i}a)^l$$
- Multipole moments are encoded in gravitational wave observables - precession frequencies & number of cycles spent near a given frequency (Ryan 95).
$$\Delta\mathcal{N}(f) = \frac{f^2}{\mathrm{d}f/\mathrm{d}t} = f^2\frac{\mathrm{d}E/\mathrm{d}f}{\mathrm{d}E/\mathrm{d}t}$$
- Multipole moments enter at different orders in $M\Omega$
$$\frac{\Omega_p}{\Omega} = 3(M\Omega)^{\frac{2}{3}} - 4\frac{S_1}{M^2}(M\Omega) + \left(\frac{9}{2} - \frac{3}{2}\frac{M_2}{M^3}\right)(M\Omega)^{\frac{4}{3}} + \cdots$$
- Also encoded in frequency and damping time of quasi-normal modes.

Probing the nature and structure of BHs

- Need infinite number of multipoles to describe Kerr. Instead, consider "bumpy" black holes with small departures from Kerr.
  - Many studies, e.g., Collins & Hughes (2004), Glampedakis & Babak (2005), Barack & Cutler (2007), JG, Li & Mandel (2008), Sopuerta & Yunes (2009), Canizares, JG & Sopuerta (2012).



Barack & Cutler (2007)

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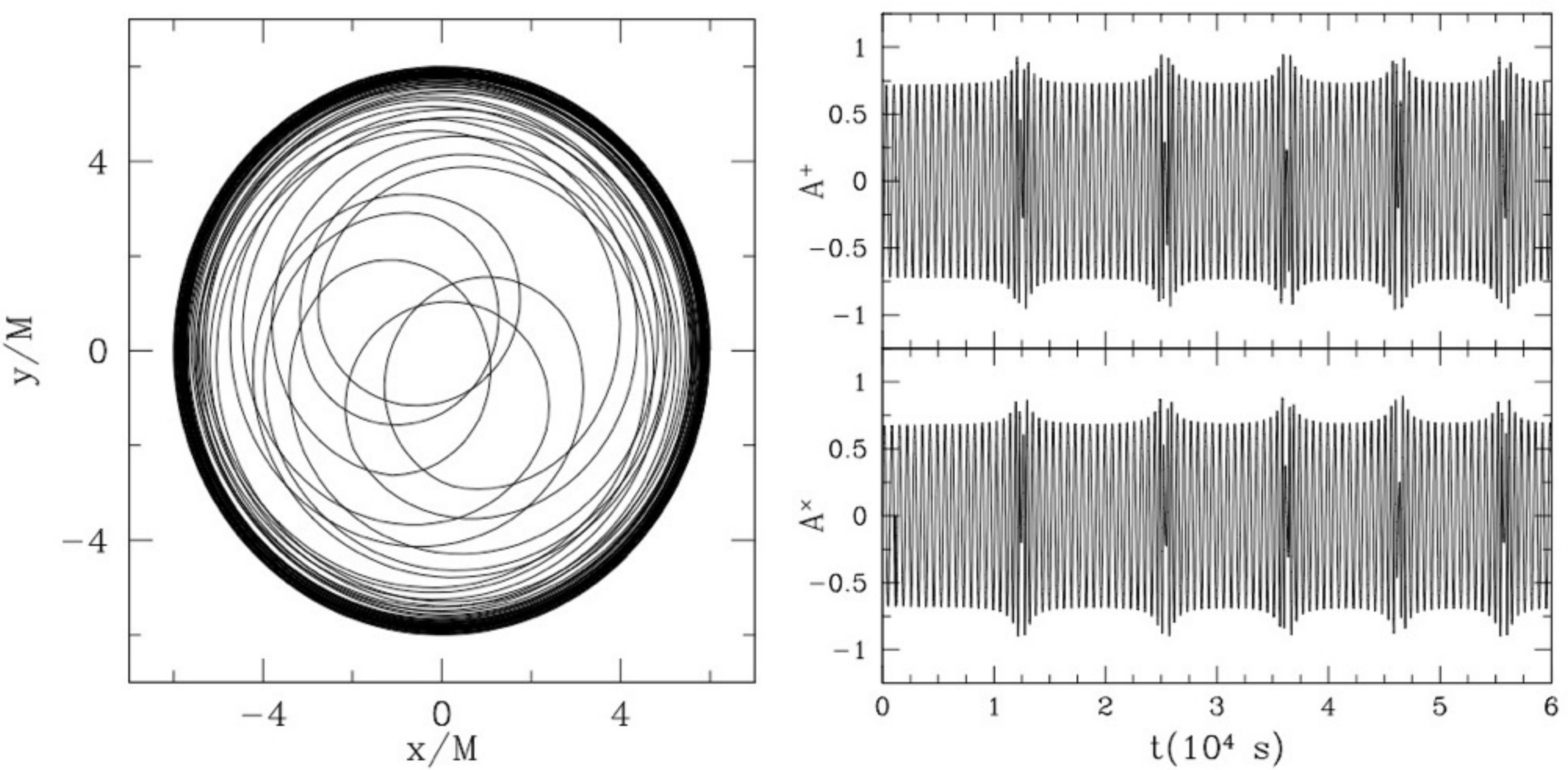
  - Can simultaneously measure M, a to ~0.01% and excess quadrupole to ~0.1%.
- Other information is also encoded in emitted GWs
  - Horizon: presence/absence of a horizon indicated by cut-off/continuation of emission at plunge, e.g., persistent emission for an inspiral into a Boson-Star.

Probing the nature and structure of BHs



Kesden, Gair & Kamionkowski (2004)

Probing the nature and structure of BHs

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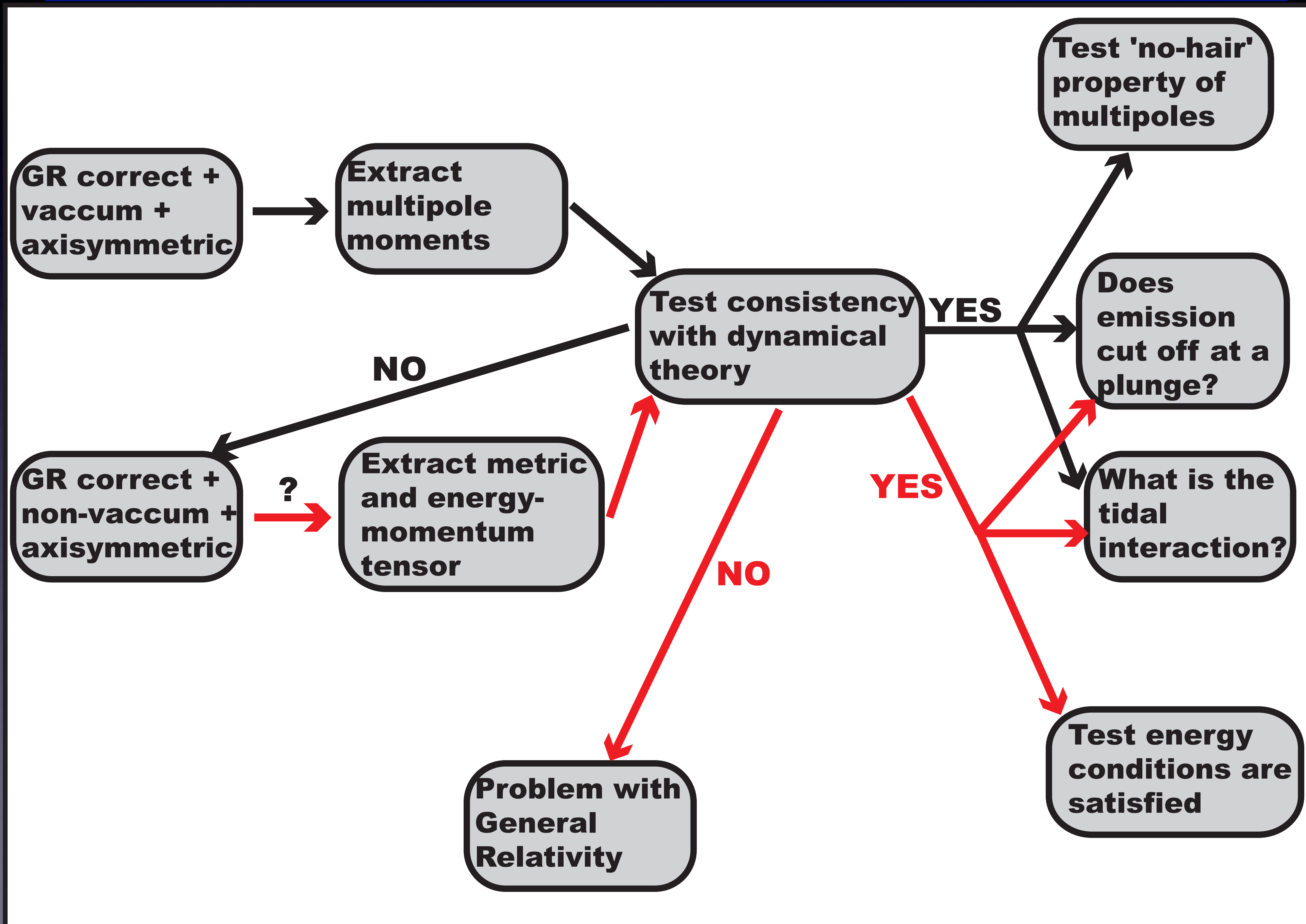
- Other information is also encoded in emitted GWs
  - Horizon: presence/absence of a horizon indicated by cut-off/continuation of emission at plunge, e.g., persistent emission for an inspiral into a Boson-Star.
  - Tidal coupling: Energy is lost 'into the horizon' through tidal heating. Infer strength of tidal interaction (Li & Lovelace 07).
  - Presence of matter: gas, accretion disc, second SMBH or exotic matter can leave measurable imprint on signal. Can't be confused with no-hair violation.

Spacetime Mapping



Spacetime Mapping

	No-Hair	Plunge? Tidal int.	Conclusion
Vacuum	Y	Y	Kerr Black Hole
	Y	N	Exotic object, Kerr exterior
	N	Y	“Bumpy” Black Hole
	N	N	Exotic object or naked singularity, non-Kerr exterior

	Energy conditions	Plunge? Tidal int.	Conclusion
Non Vacuum	Y	Y	Kerr Black Hole + accretion disc?
	Y	N	Exotic object, e.g., Boson star?
	N	Y	Exotic matter with horizon (?)
	N	N	Extended distribution of exotic matter

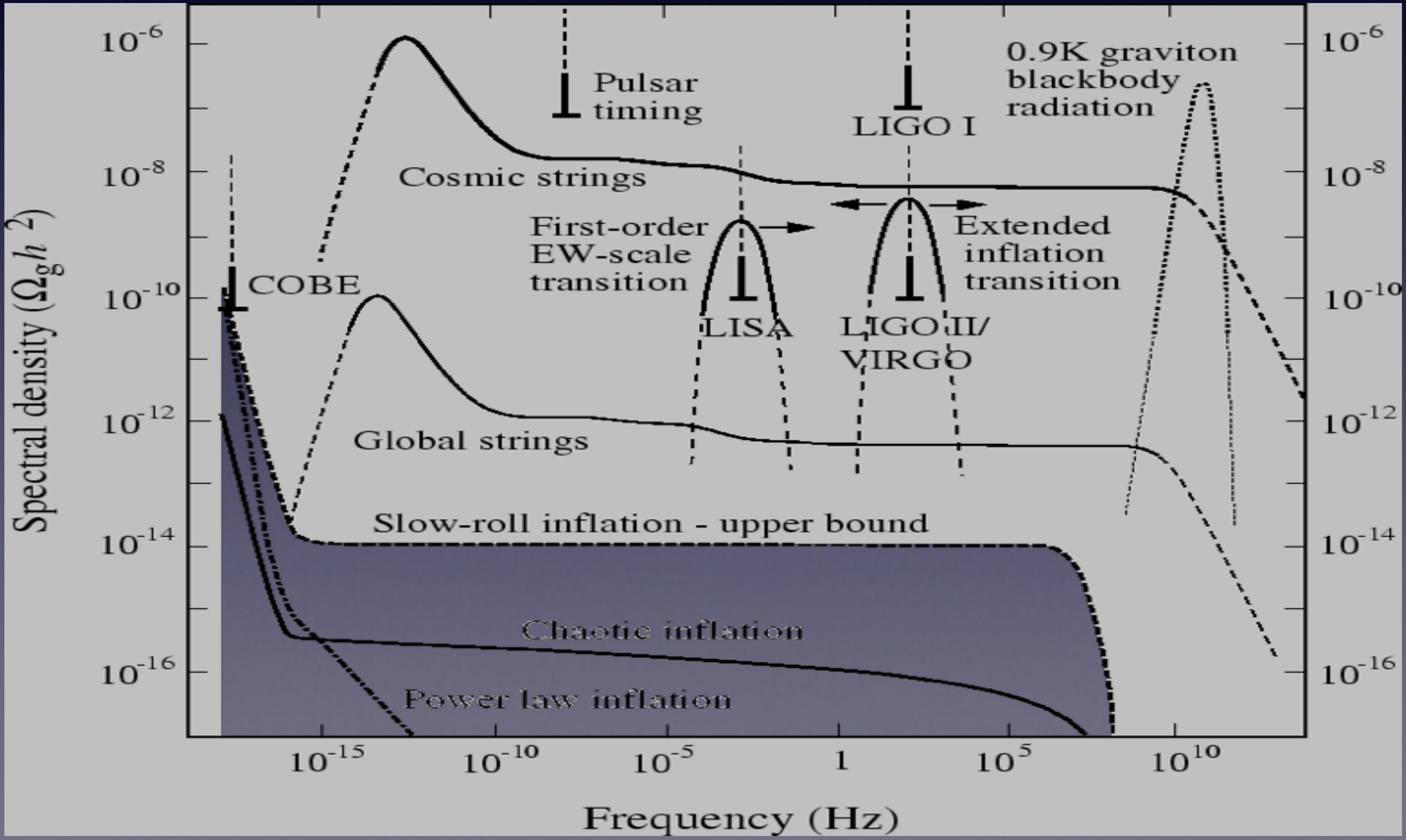
Gravitational Wave Sources: Backgrounds, Bursts, Continuous Waves

Stochastic Gravitational Wave Fore/Backgrounds

- As in electromagnetic wavebands, we expect to detect diffuse gravitational wave emission that is roughly homogeneous on the sky.
- This background will be *stochastic*, i.e., random, but with a characteristic spectrum in frequency.
- There are two distinct types of stochastic gravitational wave background
  - *Cosmological Gravitational Wave Background.* Relic gravitational waves, generated in the first second of the lifetime of the Universe.
  - *Astrophysical Foregrounds.* A population of gravitational wave sources that are numerous, but sufficiently weak to be unresolvable individually, will also create a diffuse foreground that appears stochastic.

Cosmological Gravitational Wave Background

- Quantum fluctuations in the early Universe are stretched during inflation and appear as a GW background today. Background is broadband, but the exact amplitude and spectral shape depends on the details of inflation.

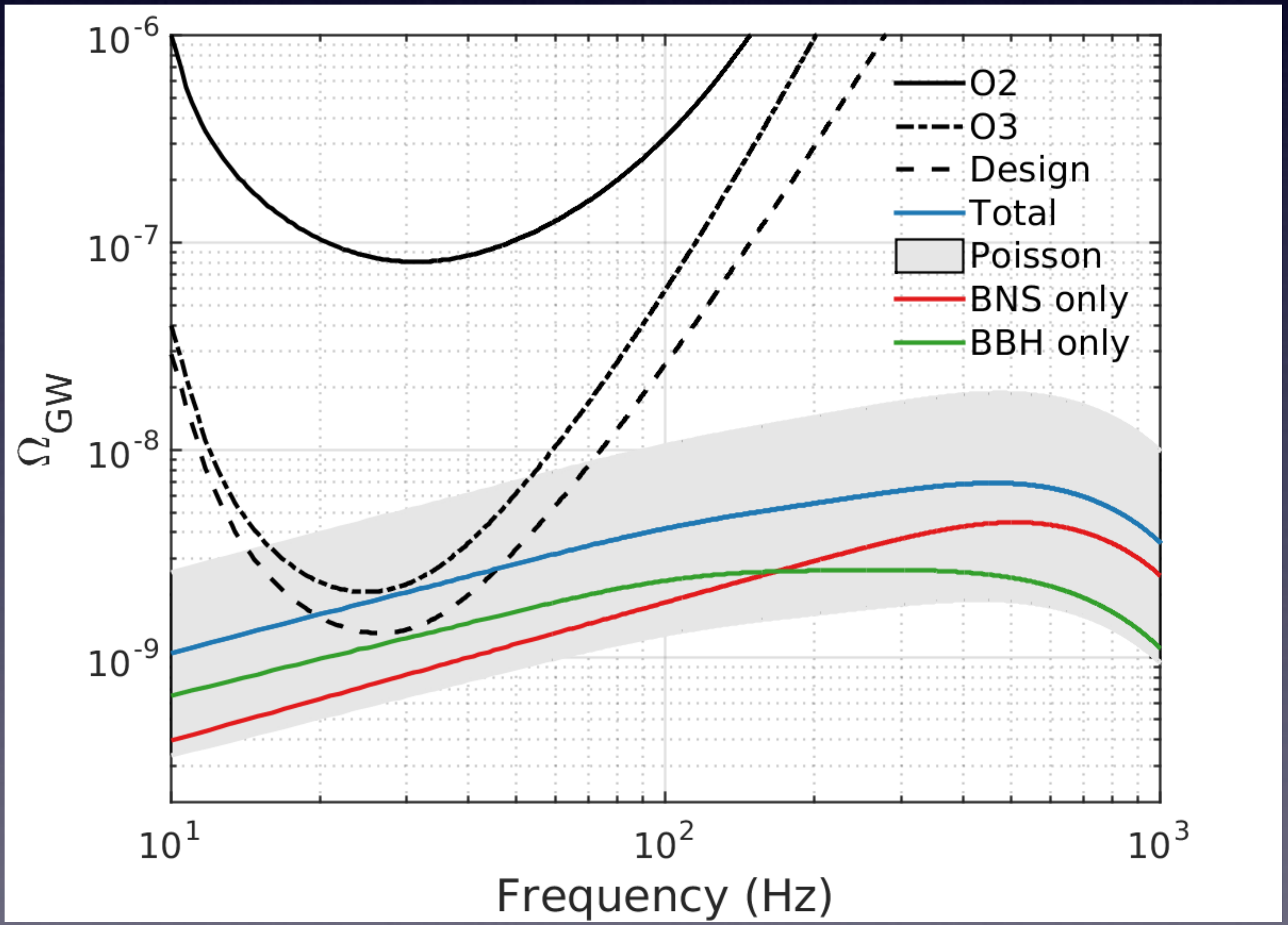


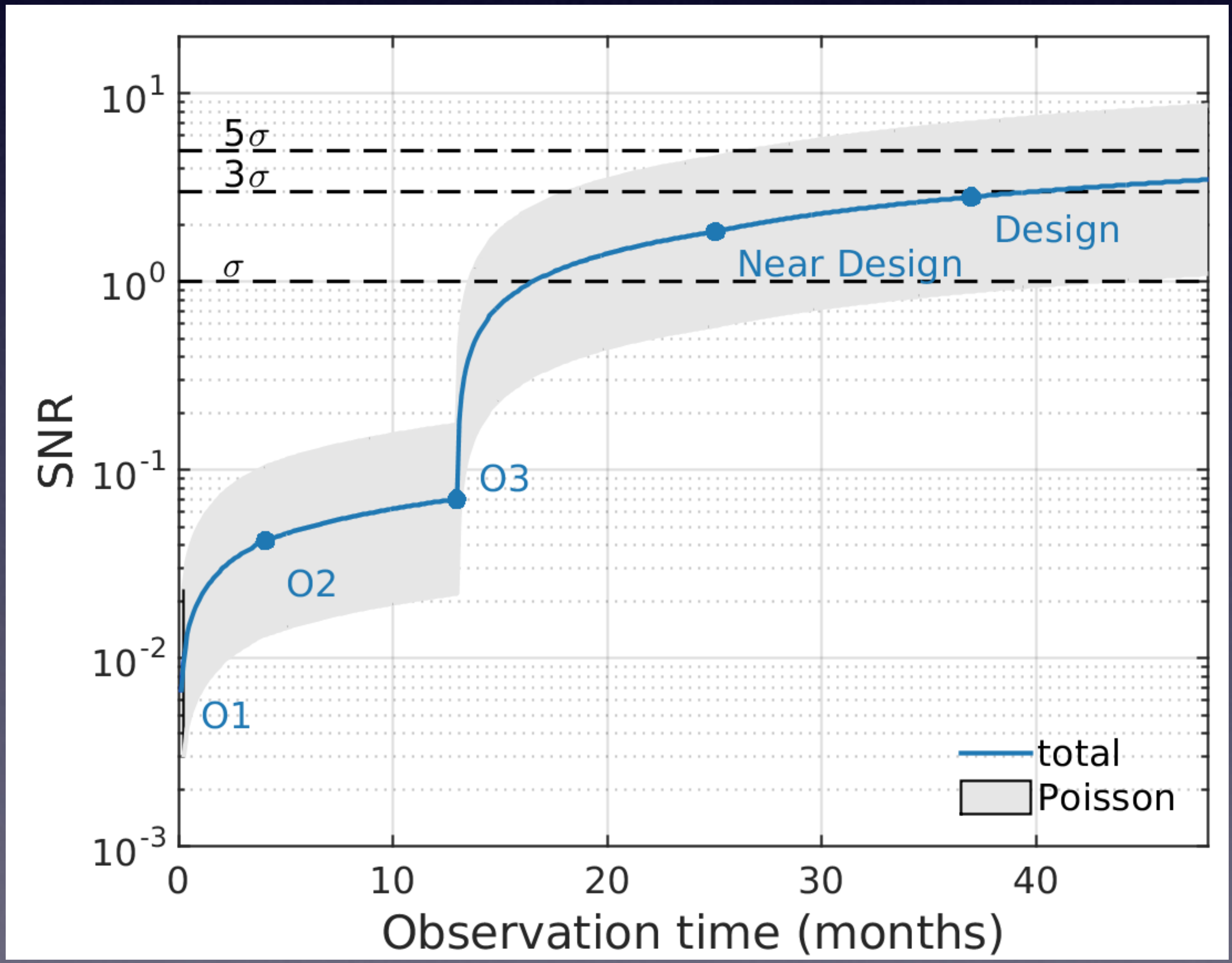
Astrophysical GW Foregrounds

- Any cosmological population of sources will produce a background.
- Examples include
    - *Stellar mass binary mergers*. LIGO sources at higher redshift are not individually resolvable and form a background.
    - *Supermassive black hole mergers*. Inspirals of very massive black hole binaries are the dominant sources in the pulsar timing band. Expect to observe as a background.
    - *Galactic compact binaries*. Approximately 10 million compact binaries (WD-WD, NS-WD, NS-NS) emit in the LISA band.
    - *Extreme mass ratio inspirals*. Gravitational waves from white dwarfs inspiralling into supermassive black holes can only be resolved in the nearby Universe, but are very numerous. May form a background for LISA.

Astrophysical GW Foregrounds

- Stellar binary stochastic background expected to be detected by LIGO in a few years.





LVC, *Phys. Rev. Lett.* 120 091101 (2017)

Astrophysical GW Foregrounds

- The *White Dwarf Background* for LISA has spectral density

$$S_h^{\rm gal}(f) = 2.1 \times 10^{-45} \left(\frac{f}{1{\rm Hz}}\right)^{-\frac{7}{3}} {\rm Hz}^{-1}$$

$$S_h^{\rm ex.gal}(f) = 4.2 \times 10^{-47} \left(\frac{f}{1{\rm Hz}}\right)^{-\frac{7}{3}} {\rm Hz}^{-1}$$

- Here "gal" and "ex. gal" denote contributions from galactic and extra-galactic binaries respectively.

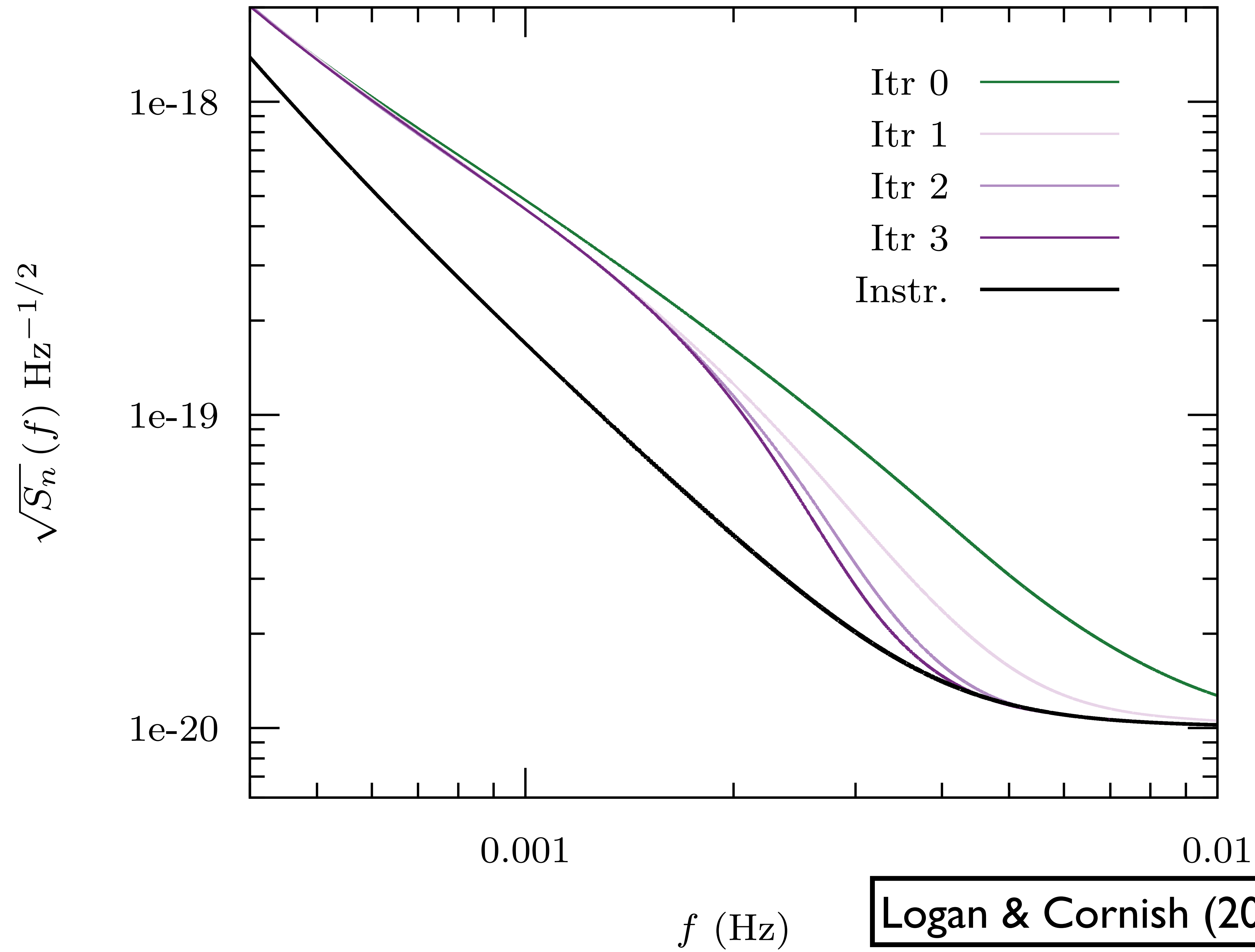
- We will be able to resolve binaries that are well separated in frequency. Quantify this using κ, the number of bins that are "lost" fitting out one source.

$$S_h^{\rm inst+gal} = \min\left\{ S_h^{\rm inst}(f)/\exp\left(-\kappa T_{\rm mission}^{-1} {\rm d}N/{\rm d}f\right), S_h^{\rm inst}(f) + S_h^{\rm gal}(f)\right\}$$

$${\rm d}N/{\rm d}f = 2 \times 10^{-3} {\rm Hz}^{-1} \left(1{\rm Hz}/f\right)^{11/3}$$

- Confusion dominates the noise between 0.1mHz and 3mHz.

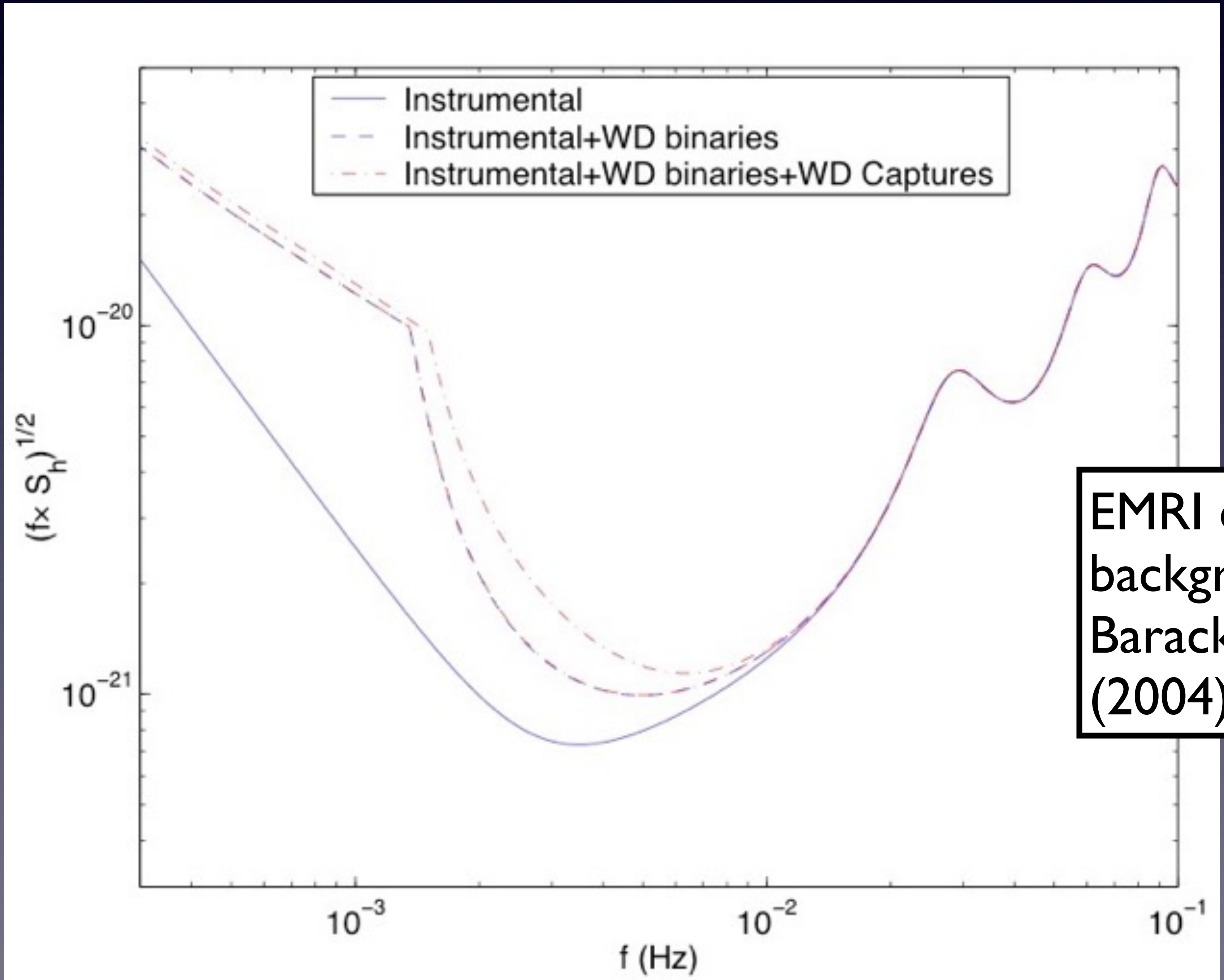
Astrophysical GW Foregrounds



Logan & Cornish (2017)

Astrophysical GW Foregrounds

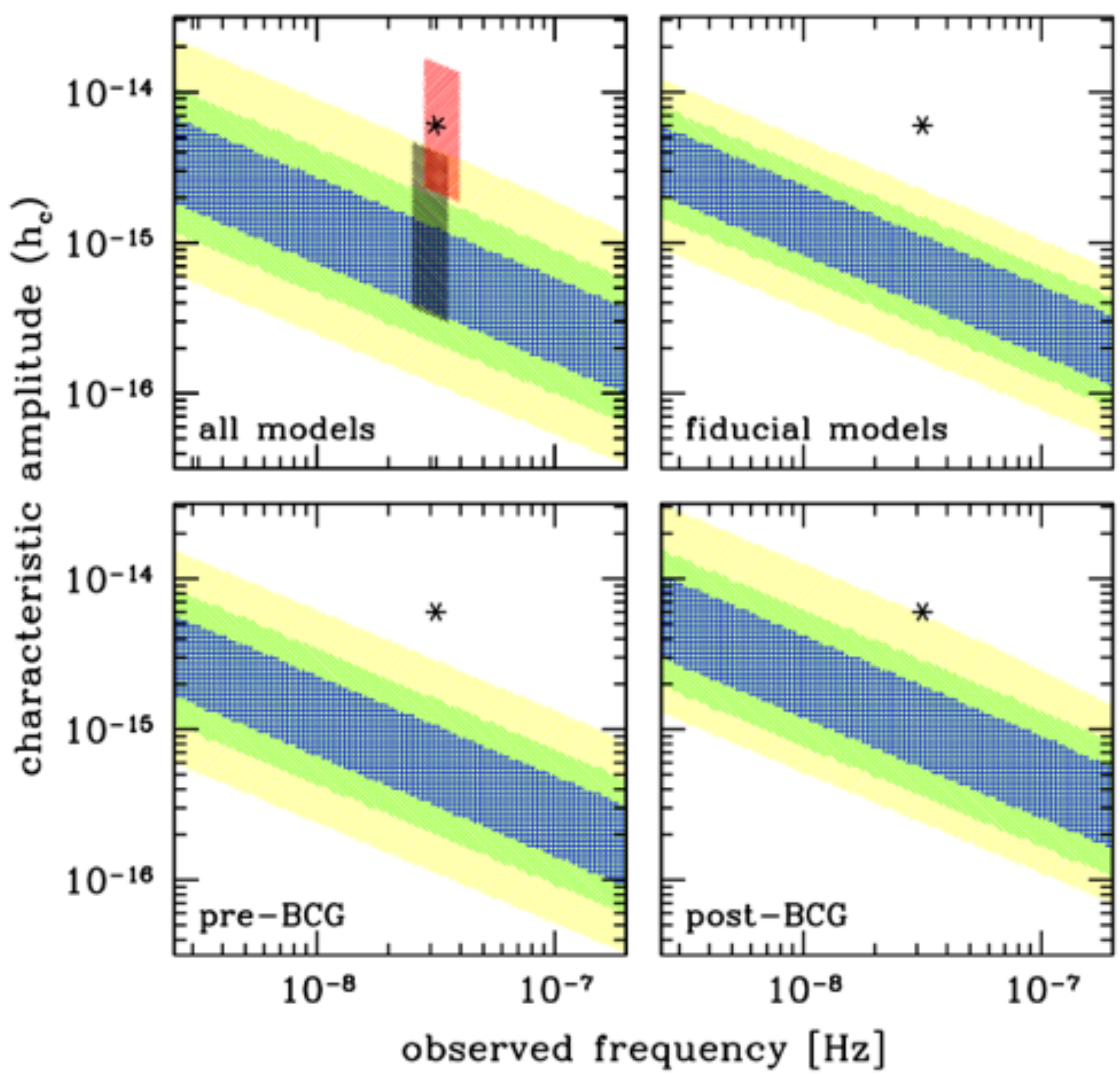
- The importance of the *Extreme Mass Ratio Inspiral Background* depends on the (very uncertain) astrophysical rates of EMRI events. Most likely not observable.



EMRI confusion background from Barack and Cutler (2004).

Astrophysical GW Foregrounds

- For pulsar timing arrays, there will be a stochastic foreground from supermassive black hole binaries at low redshift (z<1). This is the primary PTA source.



Sesana (2012)

Stochastic Gravitational Wave Fore/Backgrounds

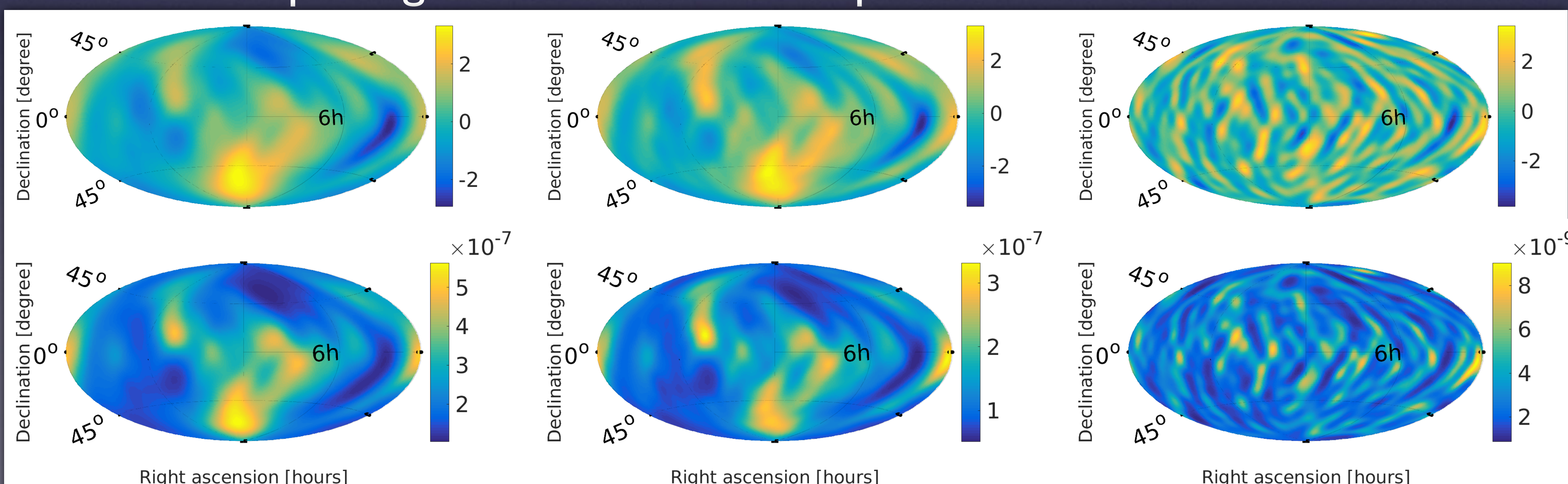
- Using LIGO only, have set limit on energy density in an isotropic stochastic gravitational wave background (LVC, *Phys. Rev. Lett.* 118 121101, 2017) of

$$\Omega_{\rm GW}(f) < 1.7 \times 10^{-7}$$

- This beats the Big Bang Nucleosynthesis upper limit

$$\Omega_{\rm GW} < 1.1 \times 10^{-5}$$

- Also placing directional limits on point-like emitters



Stochastic Gravitational Wave Fore/Backgrounds

- There are several science applications of detection of a stochastic GW background.

  - ➡ *Inflation Probe*. Most cosmological models predict weak GW backgrounds. If one is detected this will be a significant constraint on inflation models.

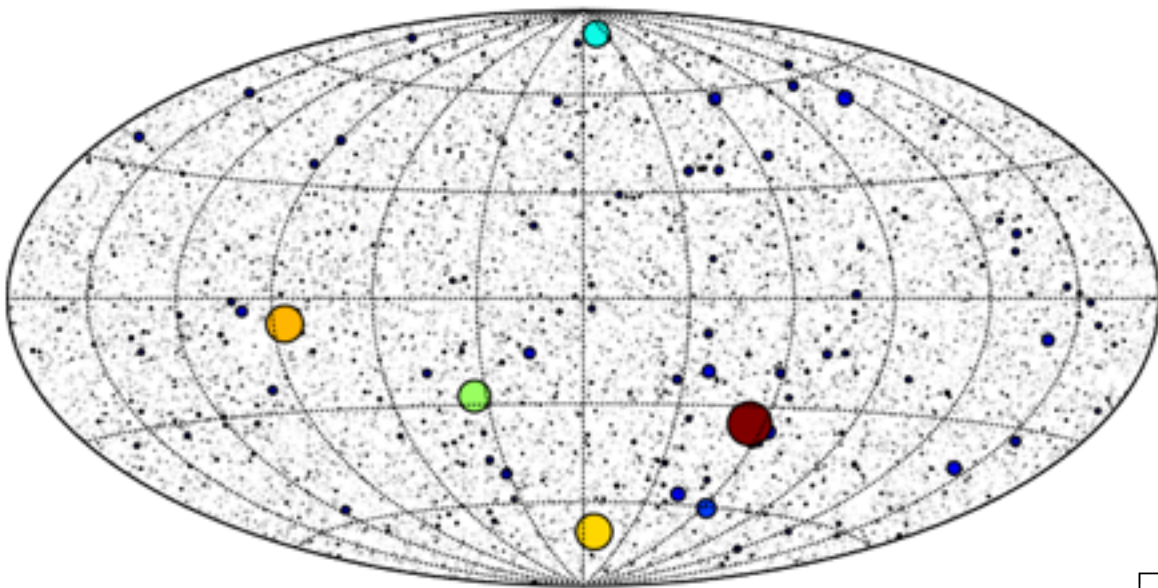
  - ➡ *Stellar populations*. Star formation models predict varying numbers of binaries, and compact objects due to differences in initial mass function, stellar evolution etc. The shape of the galactic binary background will rule out some of these models.

  - ➡ *Cluster properties*. Absolute and relative numbers of EMRIs containing WDs, NSs or BHs depend on cluster properties, e.g., stellar population and efficiency of mass segregation.

  - ➡ *Formation channels*. Three formation mechanisms for EMRIs have been proposed - *capture via relaxation, tidal splitting of binaries, star formation in a disk*. The shape of the EMRI background will constrain the relative importance of these.

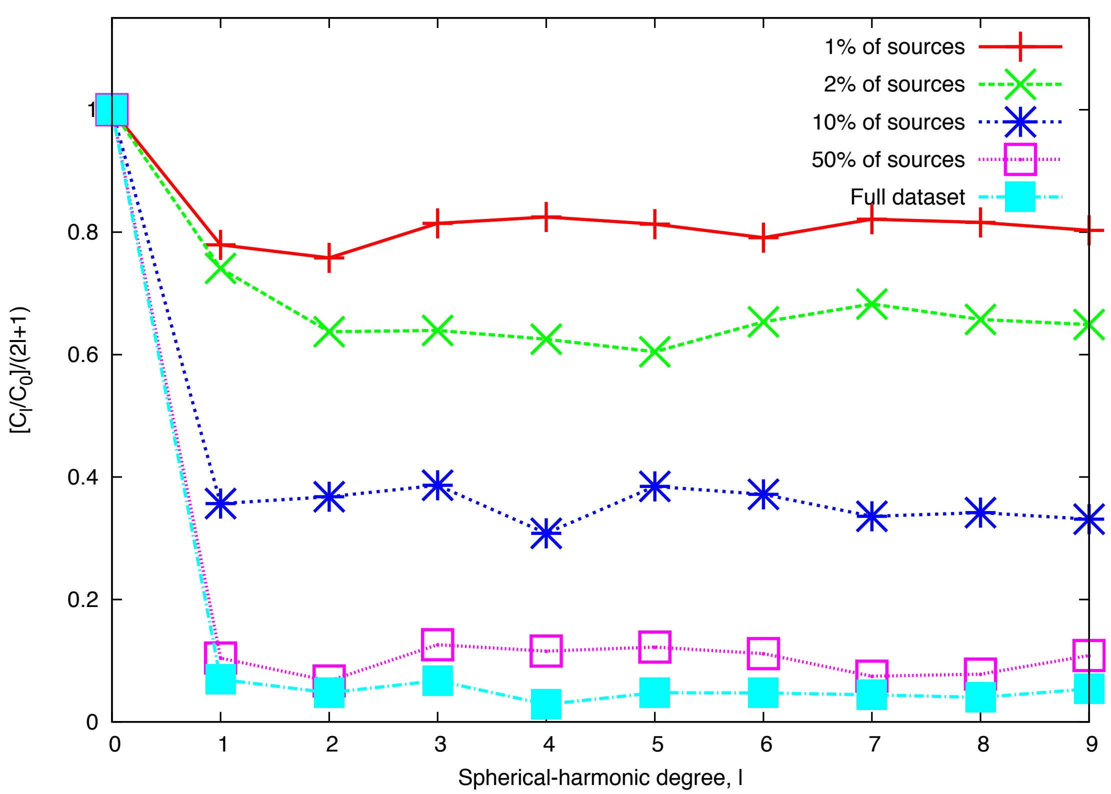
Stochastic Gravitational Wave Fore/Backgrounds

- Anisotropy in PTA foreground encodes information about SMBH binary distribution.



Sources comprising background

Power in different multipoles

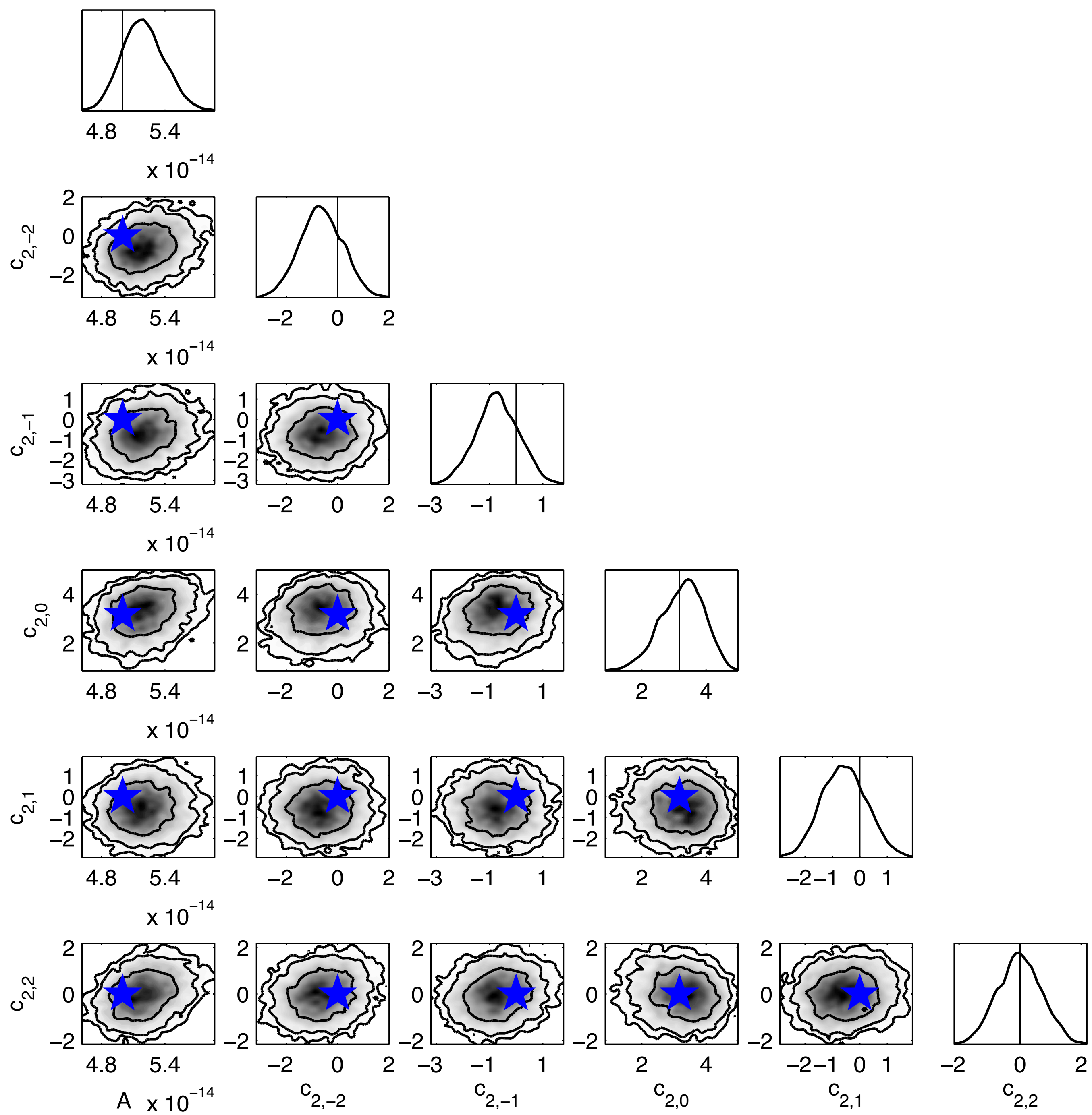


Stochastic Gravitational Wave Fore/Backgrounds

- Anisotropy in PTA foreground encodes information about SMBH binary distribution.

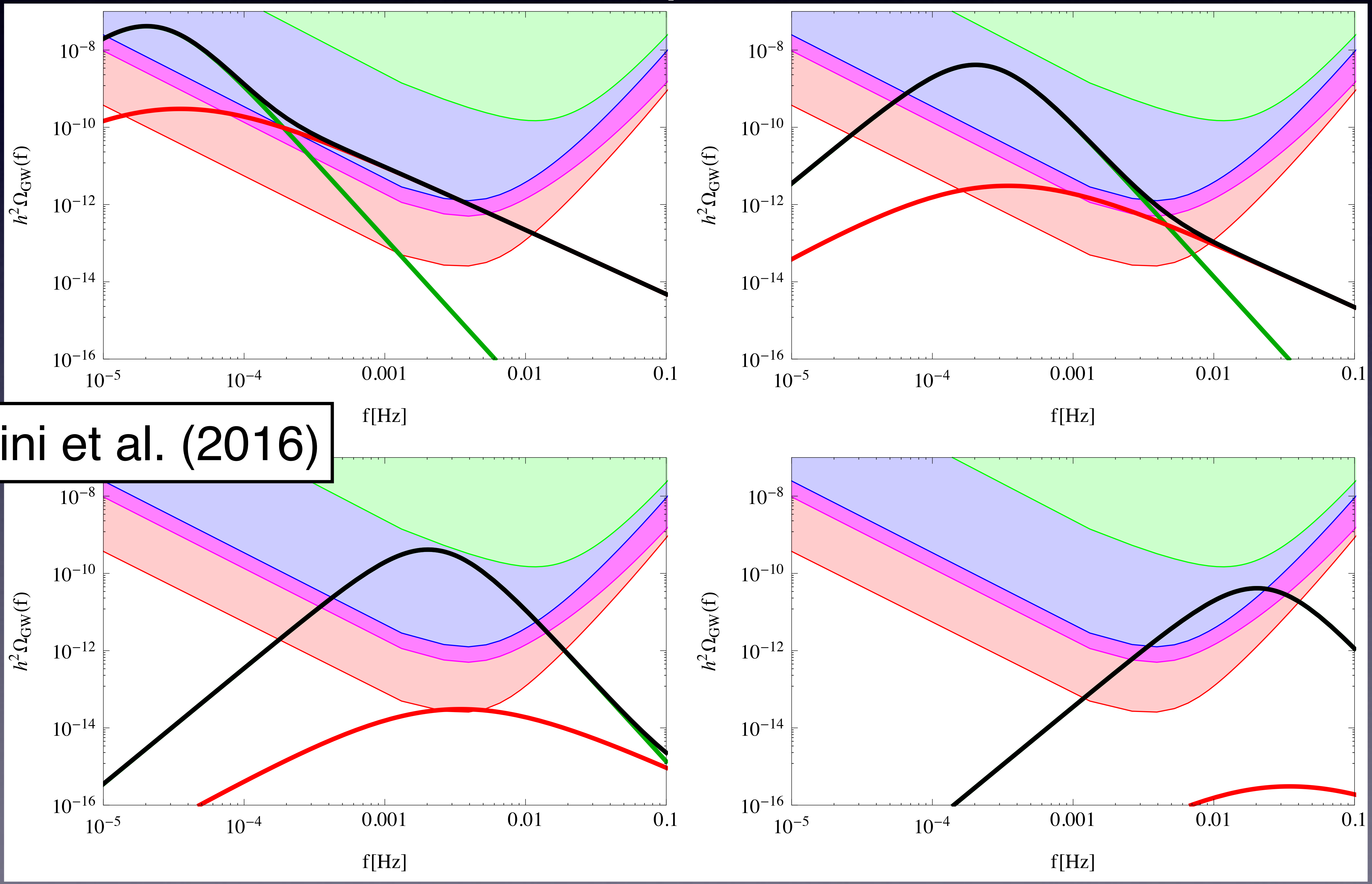
Anisotropy in the background can be measured.

This figure illustrates the successful recovery of the components of a pure quadrupole anisotropy.



Stochastic Gravitational Wave Fore/Backgrounds

- A mHz background detected by LISA would probe physics at the TeV scale in the early Universe.



Caprini et al. (2016)

Gravitational Wave Bursts

- Both LIGO and LISA will look for transient "burst" events, that are short in duration and generally broadband in frequency.

- Possible burst sources include

  - ➡ *Core collapse supernovae*. Core collapse involves large amounts of mass being accelerated. If collapse is asymmetric, would expect gravitational waves to be produced.

  - ➡ *Cosmic strings*. Topological defects created in the early Universe radiate GW bursts from *kinks* and cusps.

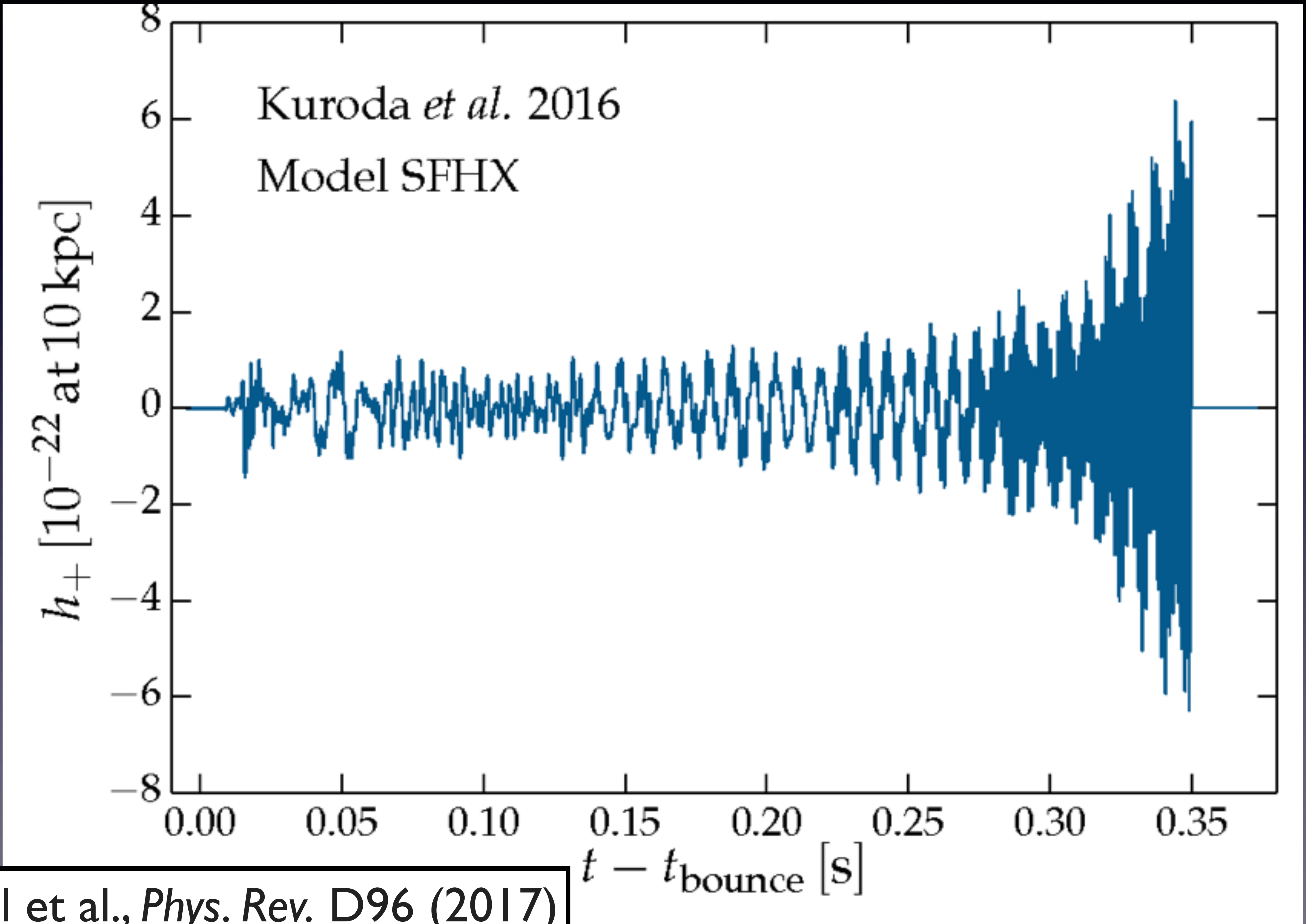
  - ➡ *Neutron star quakes*. Neutron stars have crusts, which can crack and reform in quakes. Observed in e.m. as *glitches*.

  - ➡ *Black hole mergers*. The merger phase is the most energetic of a BH-BH inspiral and the least well modelled. Might see mergers of sources not detected during the inspiral phase.

Supernovae

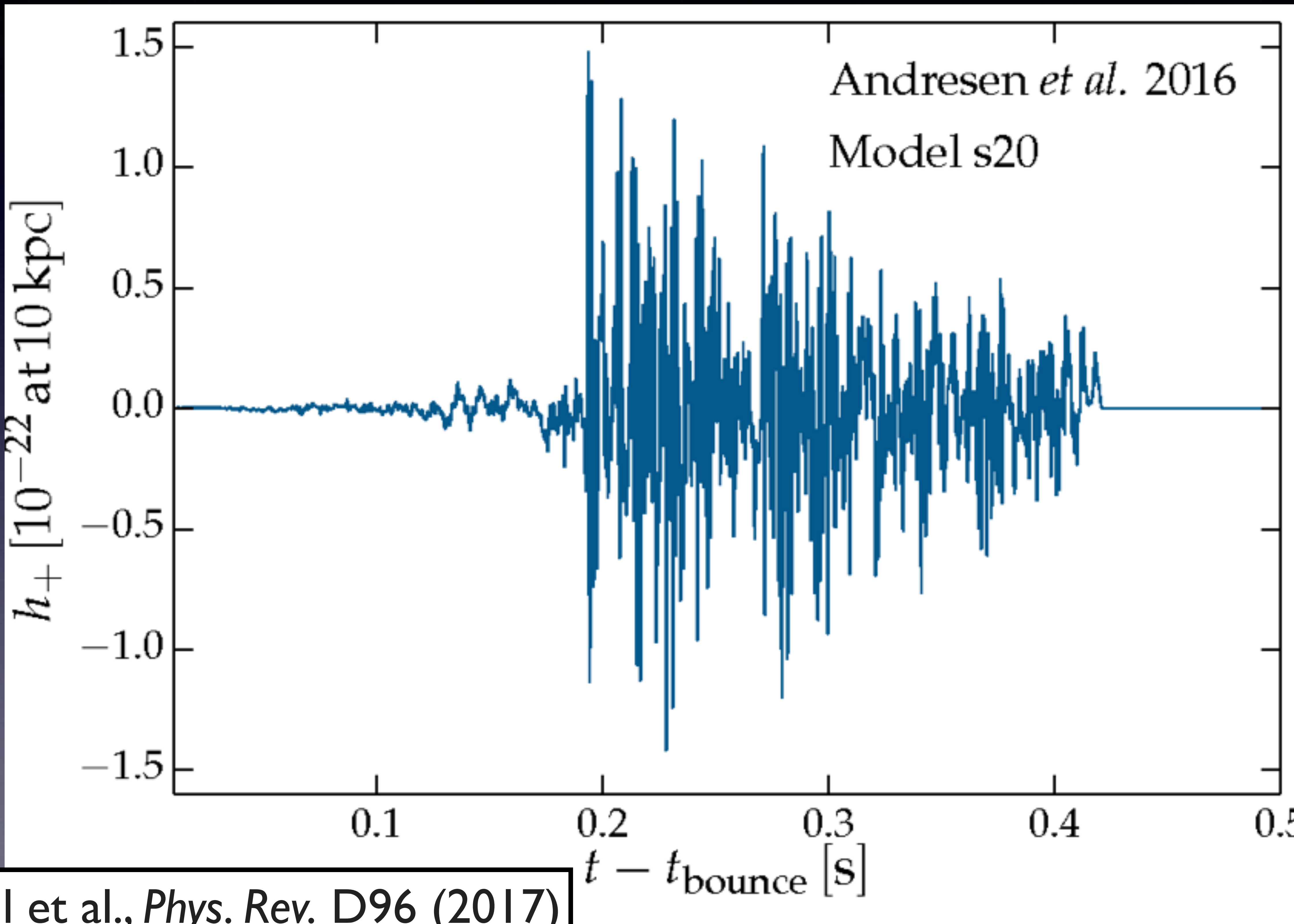
- Various processes have been proposed that might lead to generation of gravitational radiation in a core-collapse
    - Axisymmetric rotating core collapse, core bounce, postbounce convection and anisotropic neutrino emission.
    - Oscillations of the proto-neutron star core.
- Supernovae simulations have been used to predict the waveforms and energy density of gravitational waves produced during supernovae events.
- Power is spread over a wide range of frequencies, but emission lasts only a short time (~ 1s).

Supernovae



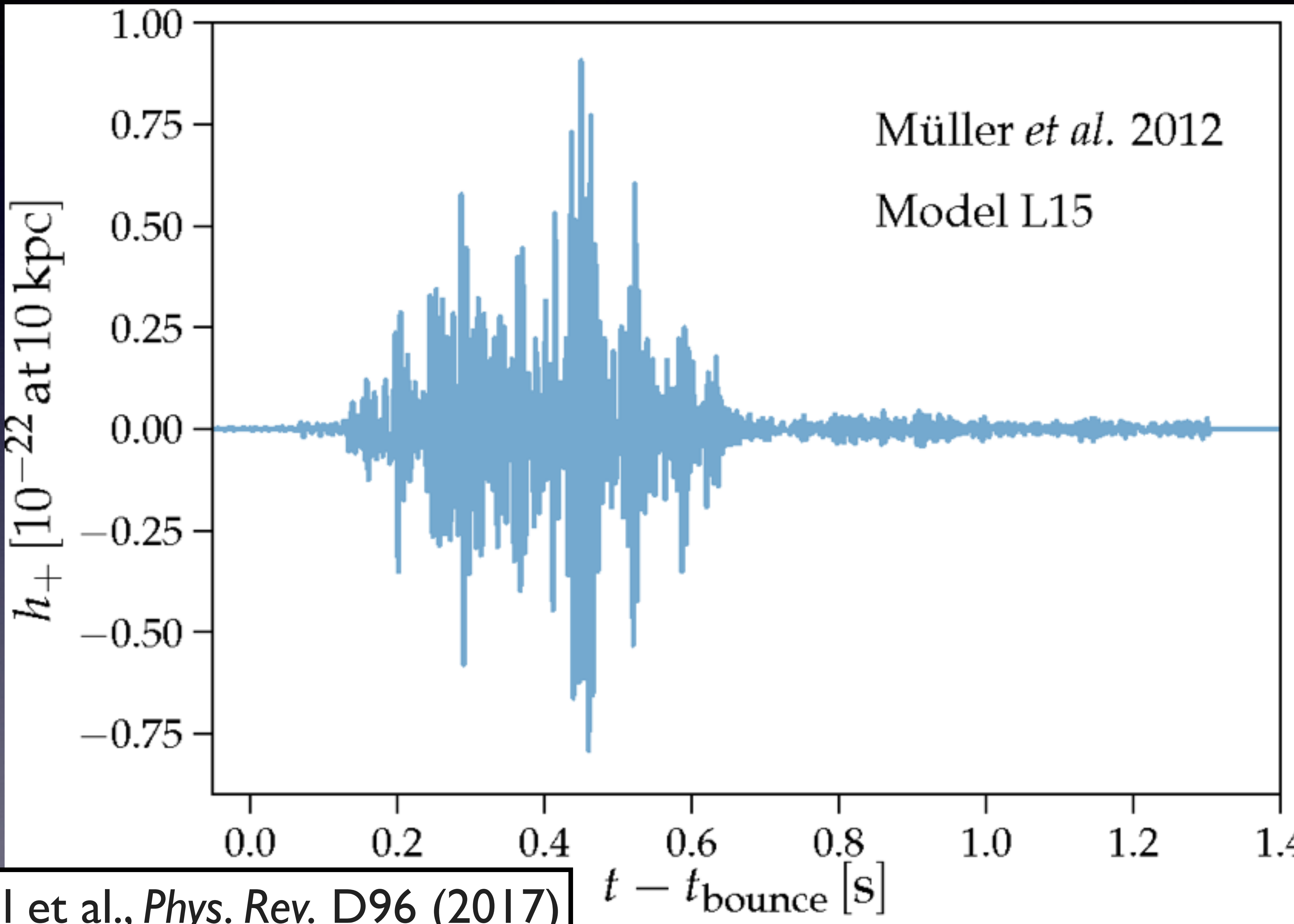
Powell et al., *Phys. Rev.* D96 (2017)

Supernovae



Powell et al., *Phys. Rev.* D96 (2017)

Supernovae



Powell et al., *Phys. Rev.* D96 (2017)

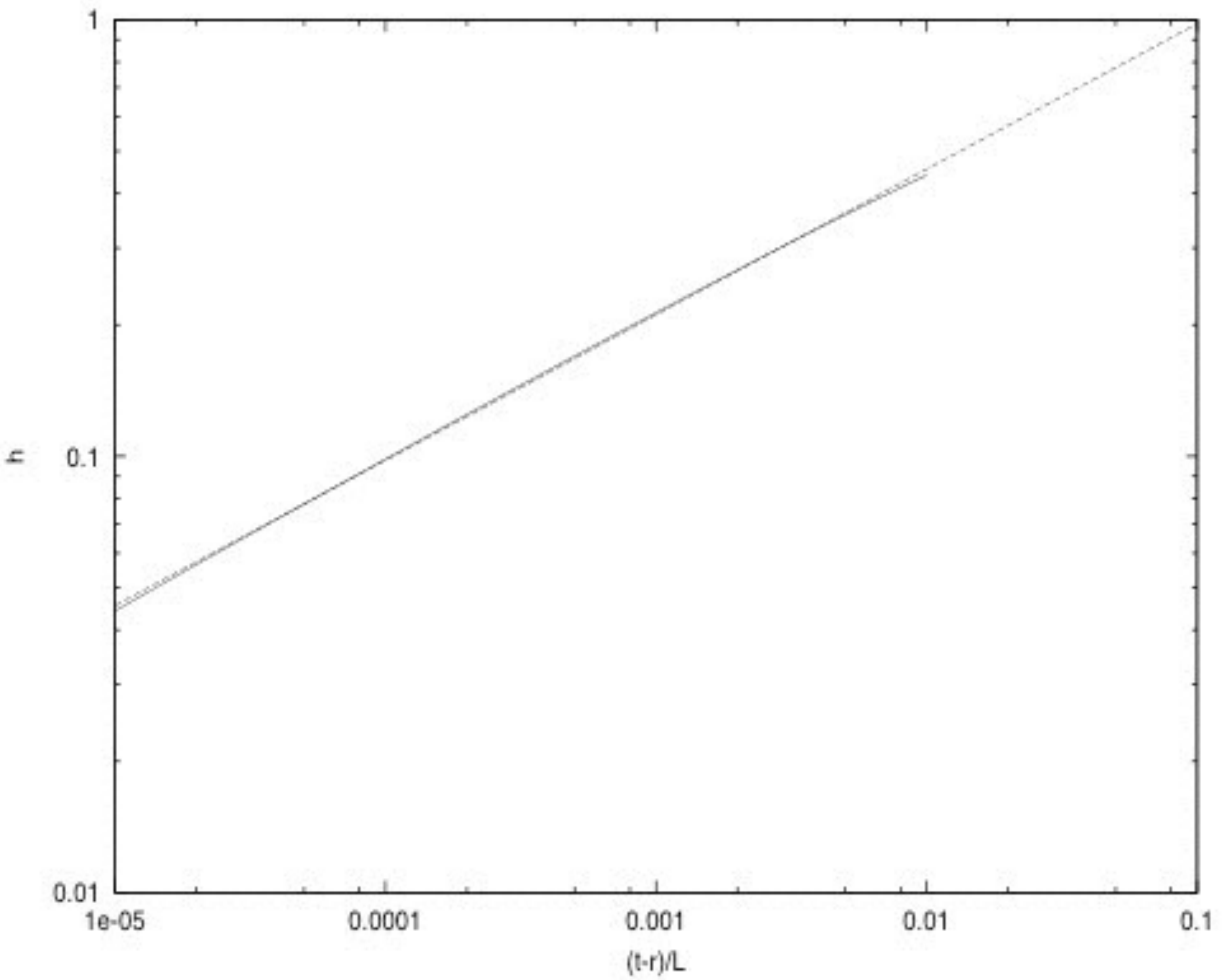
Cosmic String Cusps

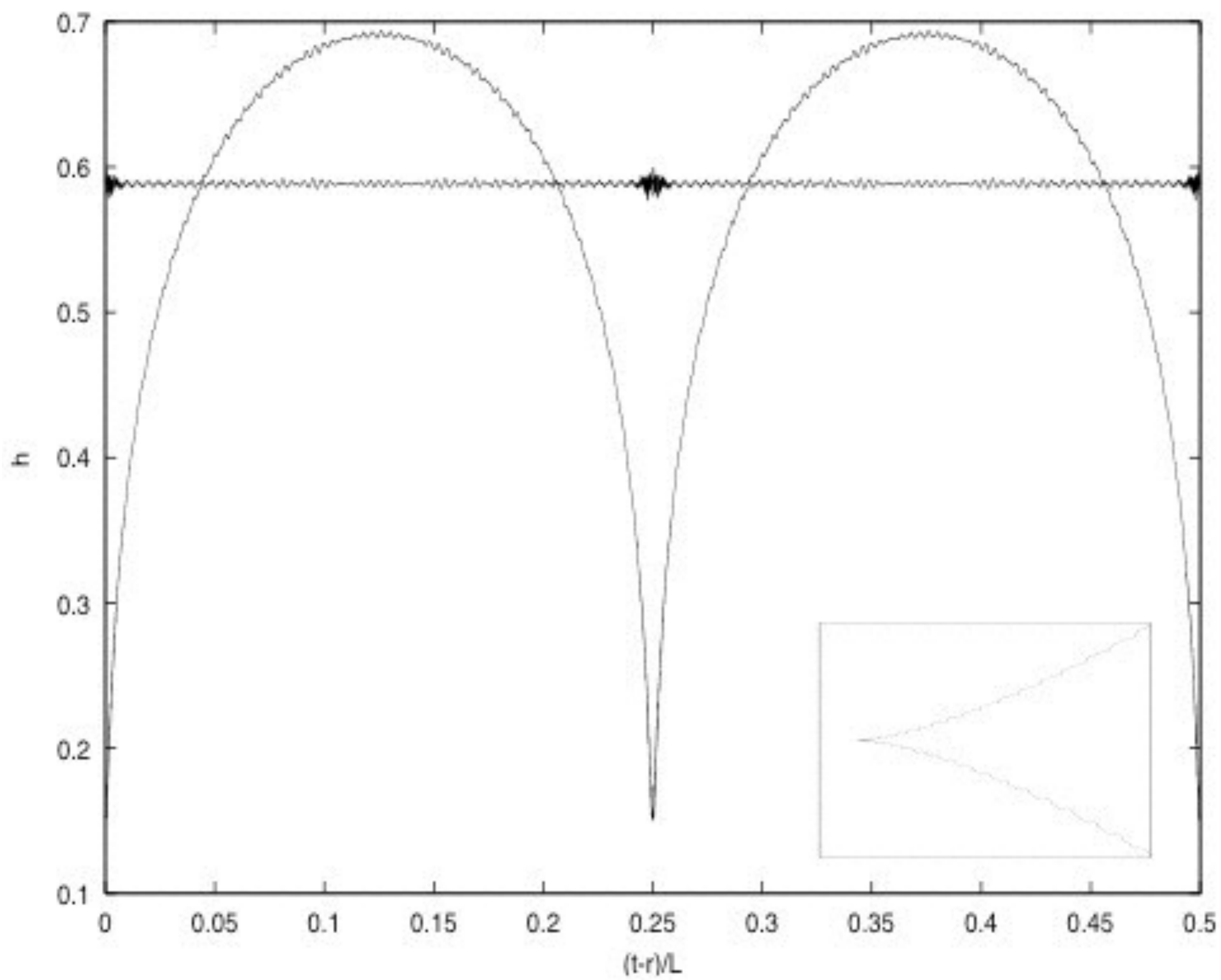
- Cosmic strings oscillate and lose energy to gravitational waves. This generates a stochastic background. In addition, cosmic strings generate infrequent, highly beamed radiation from *cusps*.
- A string in general has both *left* and *right* moving modes. When these meet, cusps form which can be moving close to the speed of light. These cusps generate bursts of gravitational radiation.
- The strength of gravitational wave emission depends on the *string length*, l , the *string tension,* μ, (mass per unit length of the string) and the redshift

$$h(f) \sim \frac{G\mu l}{((1+z)fl)^{1/3}} \frac{1+z}{z} \qquad\qquad h(t) \propto |t-r|^{\frac{1}{3}}$$

Cosmic String Cusps

Gravitational waveforms from cosmic string cusps





Near cusp waveform

Waveform from periodic source

Cosmic String Cusps

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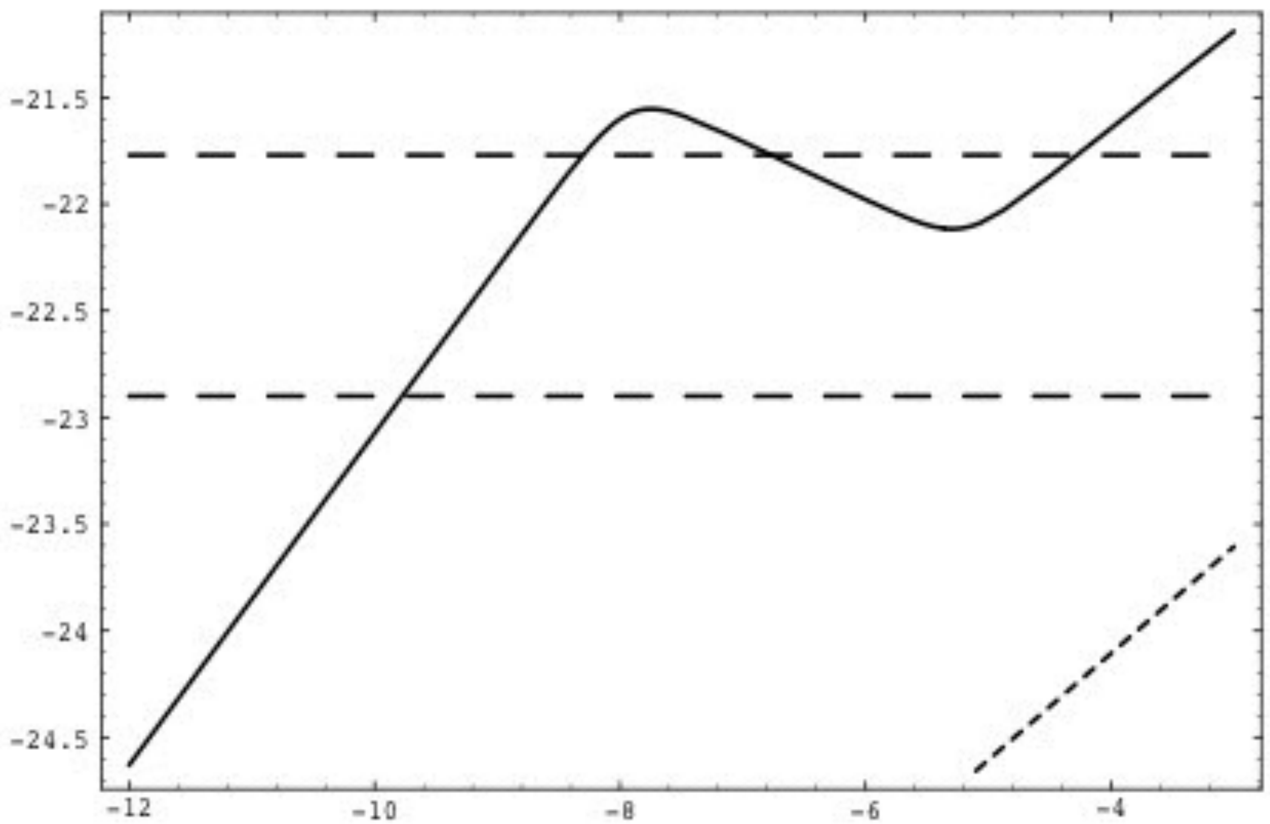
$$h(f) \sim \frac{G\mu l}{((1+z)fl)^{1/3}} \frac{1+z}{z} \qquad h(t) \propto |t-r|^{\frac{1}{3}}$$

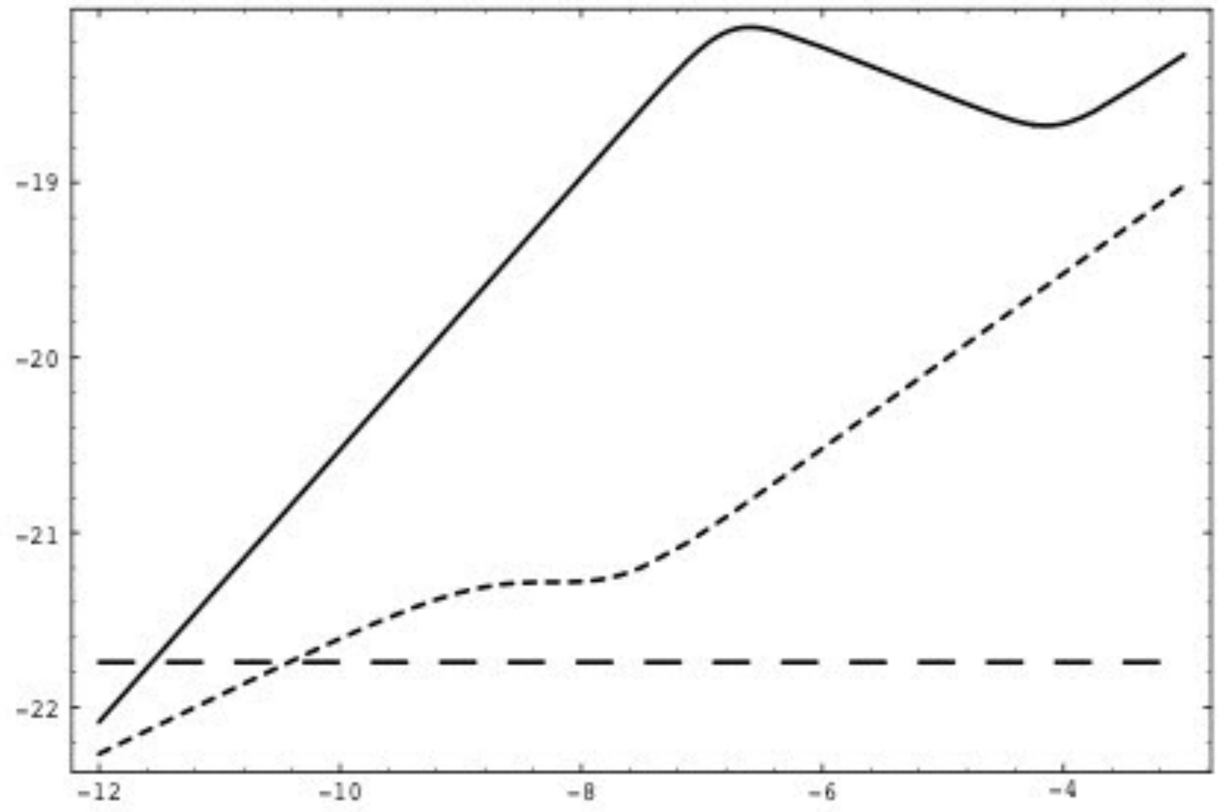
- Radiation from cosmic string cusps could be seen by both LIGO and LISA.

Cosmic String Cusps

Cosmic string burst amplitudes



LIGO band

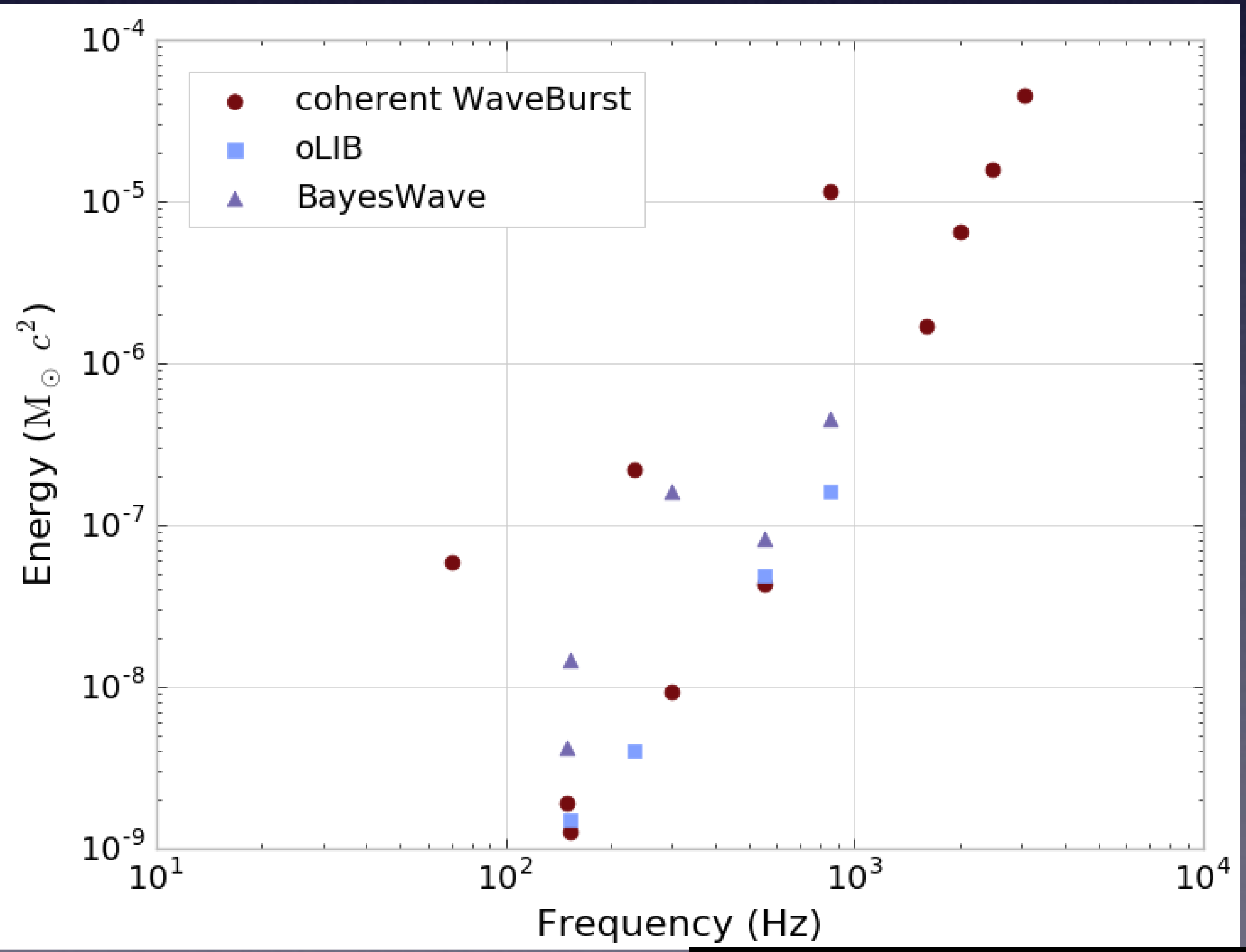


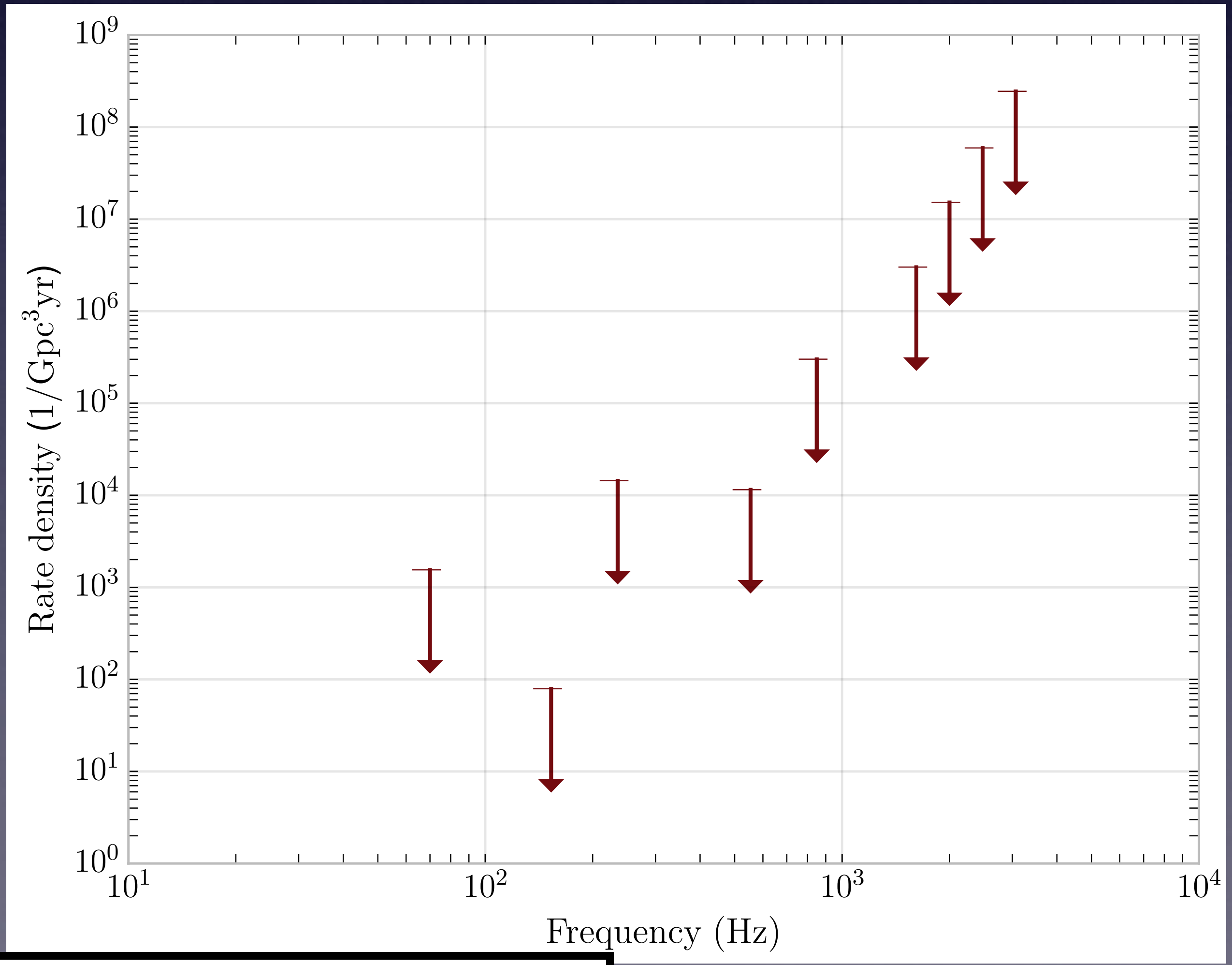
LISA band

Lines denote burst amplitudes (solid), background amplitude (short dash) and detector sensitivity (long dash)

Gravitational Wave Bursts - LIGO results

- No bursts detected as yet, but have upper limits on short bursts.
- In O1, LIGO had 50% detection efficiencies for sine-Gaussian bursts with $h \sim 1.7 \times 10^{-22}$
- Constrain rates at level of 10^{-7} Mpc^{-3} yr^{-1} (frequency dependent)

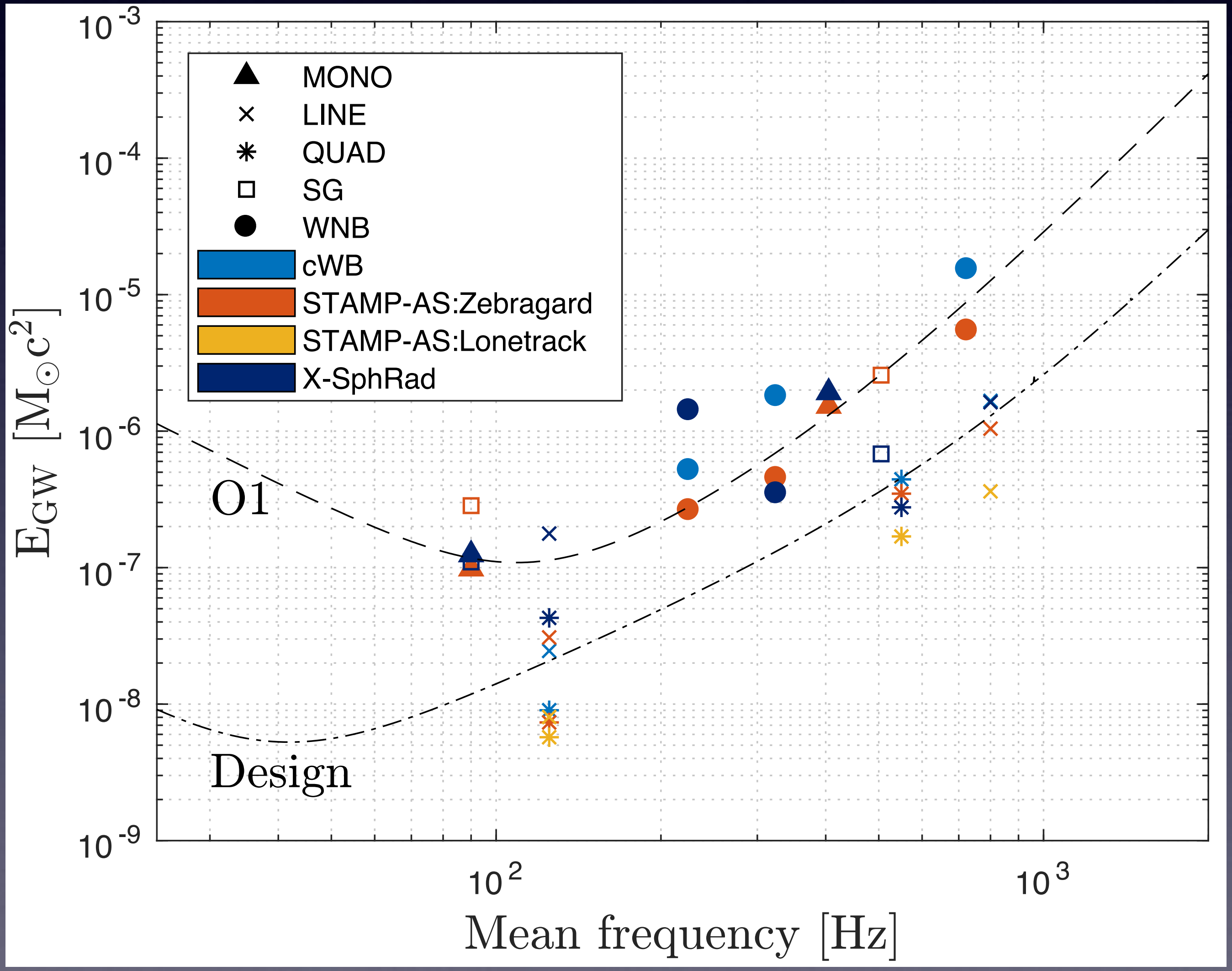


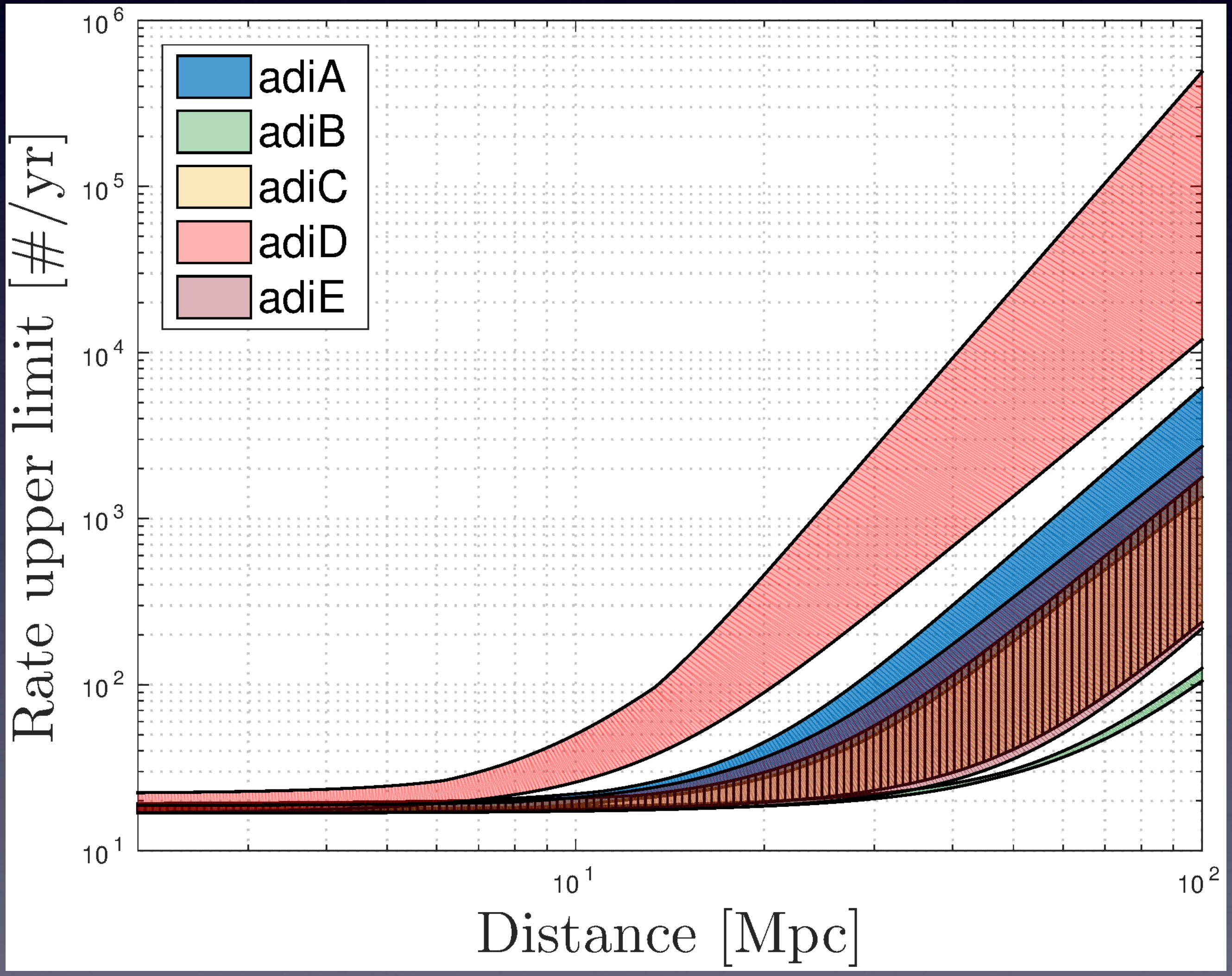


LVC, *Phys. Rev.* D **95** 042003 (2017)

Gravitational Wave Bursts - LIGO results

- Also have upper limits on long bursts.

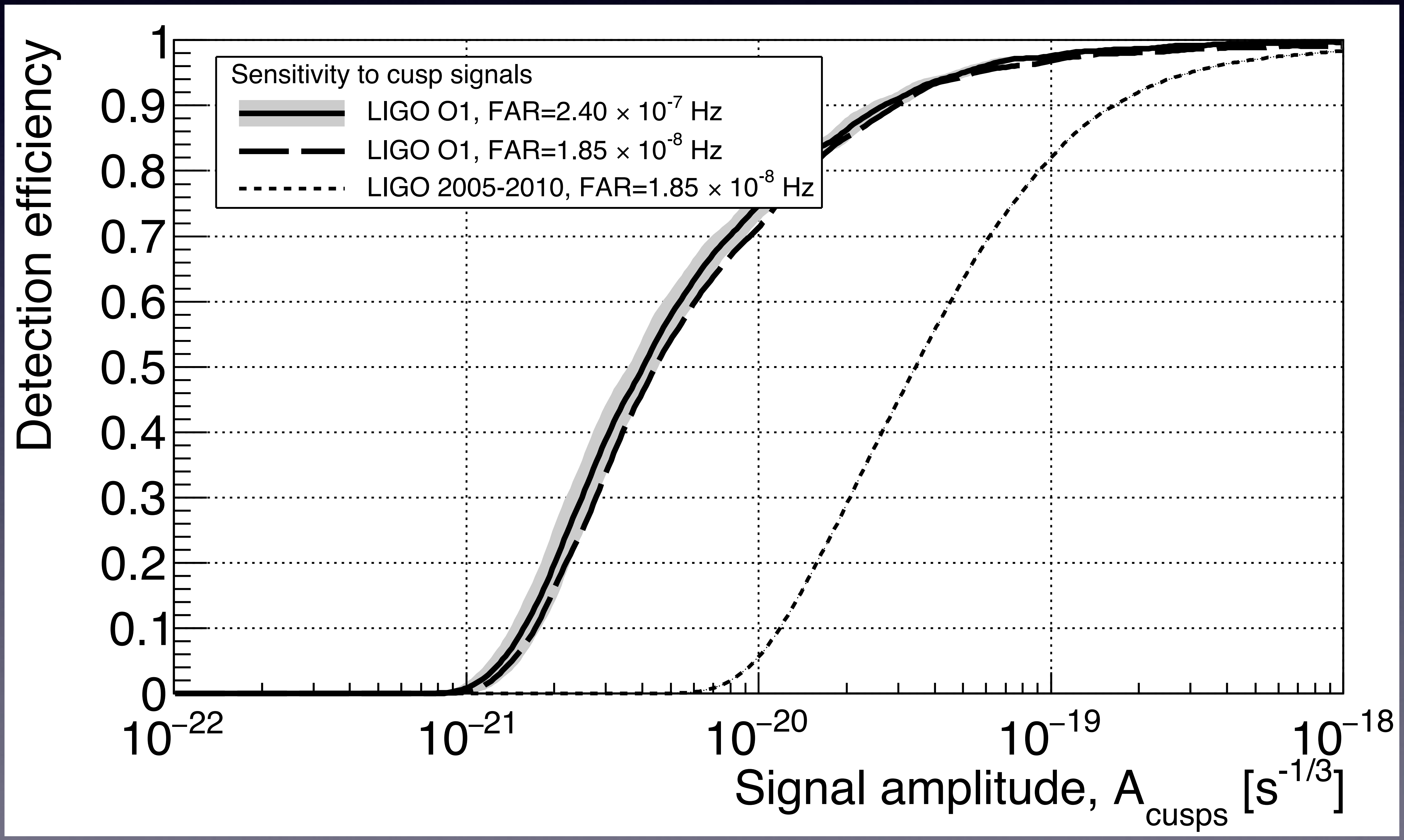




LVC, *Class. Quantum Grav.* 35 065009 (2018)

Gravitational Wave Bursts - LIGO results

- S6 limits on cosmic string cusps are starting to constrain the cosmic string parameter space.



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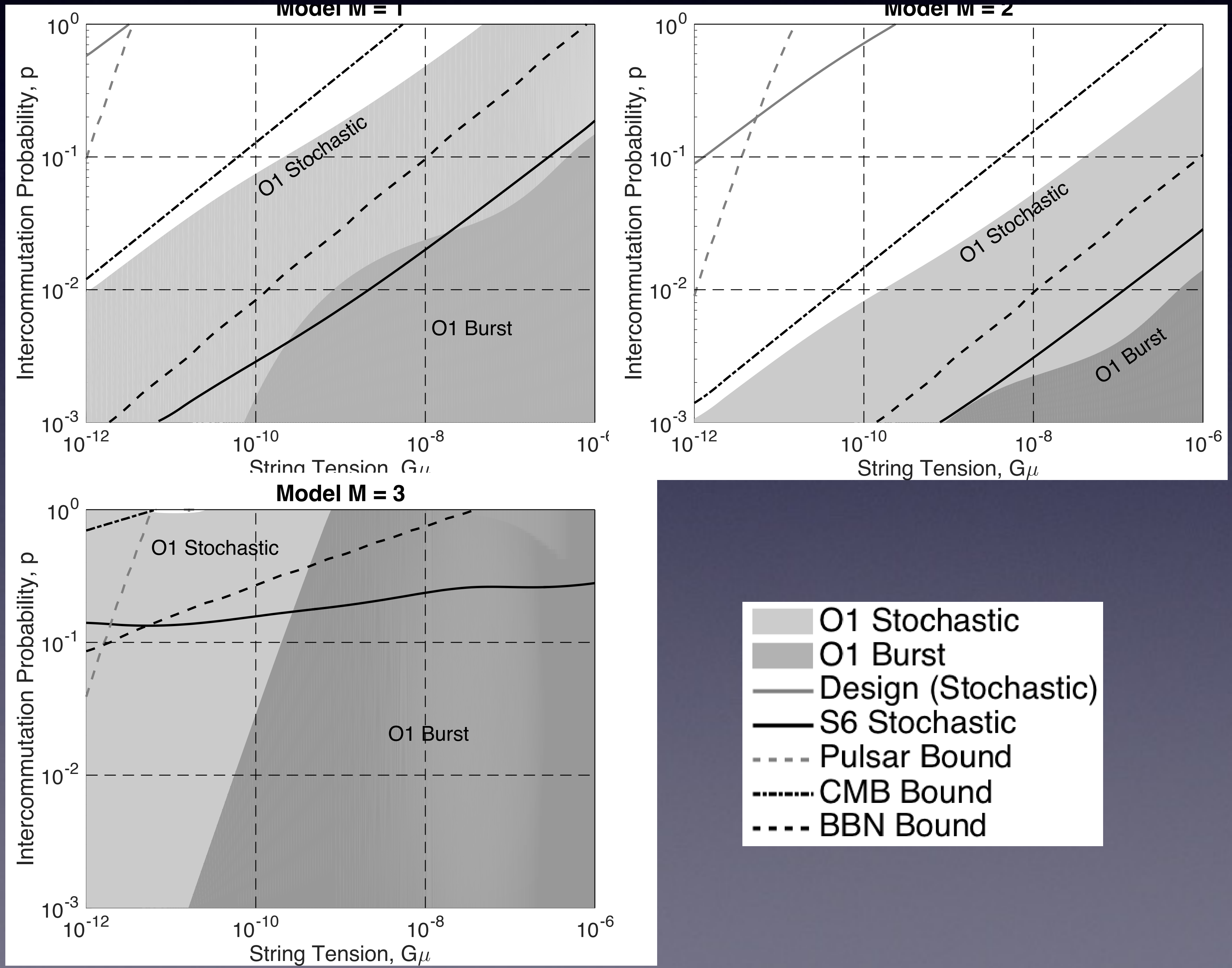


Gravitational Wave Bursts

- Science applications of burst events include
  - *Supernovae models*. There is great uncertainty in current models of stellar collapse. Detection and measurement of the gravitational waves from an event will help constrain these.
  - *Absolute distance scale*. Distances to local supernovae are tied to the Cepheid distance scale. If GWs are detected from an event this will provide an independent distance measure which will improve statistics on other supernovae distances. Difficulty is in computing intrinsic GW amplitude.
  - *Cosmic string detection*. The detection of a burst from a cosmic string will be remarkable, providing information on the number density and tension of these defects.
  - *Unknown sources*. Looking for bursts is one of the few ways we might be able to see an unexpected type of GW source. Many advances have come from serendipitous observations.

Continuous Wave Sources

- Rotating Neutron Stars that have deformities may have an associated changing quadrupole moment and will generate continuous, periodic gravitational wave emission.
- There are three main mechanisms that could lead to quadrupole deformations
  - *Non-axisymmetric crustal deformities.* Asymmetries in the crust, i.e., "mountains" on the surface.
  - *Unstable fluid modes in the core of the star.* Unstable fluid oscillations in the fluid core of the star, particularly r-modes.
  - *Free precession of the whole star.* If the rotation axis and symmetry axis of the neutron star do not coincide, the neutron star will "wobble" and generate gravitational waves.

Crustal Deformities

- Characterize Neutron Star deformation in terms of an *ellipticity*

$$\epsilon = \frac{I_{xx} - I_{yy}}{I_{zz}}$$

- in which the I_{ii}'s are the principle moments of inertia of the star.

- The shear modulus of a Neutron Star crust is small relative to the pressure, so the maximum ellipticity that can be supported by a neutron star crust is only

$$\epsilon_{\mathrm{max}} \approx 5 \times 10^{-7} \left(\frac{\sigma}{10^{-2}} \right)$$

- The breaking strain, $\sigma \approx 10^{-2}$ for the best terrestrial alloys. It could be a factor of ten higher for a perfect crystal, or several orders of magnitude lower for an amorphous solid.

Crustal Deformities

- More exotic Neutron Star models can support larger ellipticities, e.g., a solid strange-quark star

$$\epsilon_{\rm max} \approx 4 \times 10^{-4} \left(\frac{\sigma}{10^{-2}} \right)$$

- or a hybrid star with a solid quark-baryon core and a normal Neutron Star exterior

$$\epsilon_{\rm max} \approx 9 \times 10^{-6} \left(\frac{\sigma}{10^{-2}} \right)$$

- Regardless of the maximum ellipticity, the mechanism by which deformities form is somewhat uncertain.

- For Neutron Stars in binaries, accretion creates hot spots, guided by the magnetic fields of the star. *Electron capture* can form mountains at the hot spots with net ellipticities as high as 10^{-5}. Ellipticities of 10^{-6} can be created by internal toroidal magnetic fields in newborn NSs.

Fluid Instabilities - r-Modes

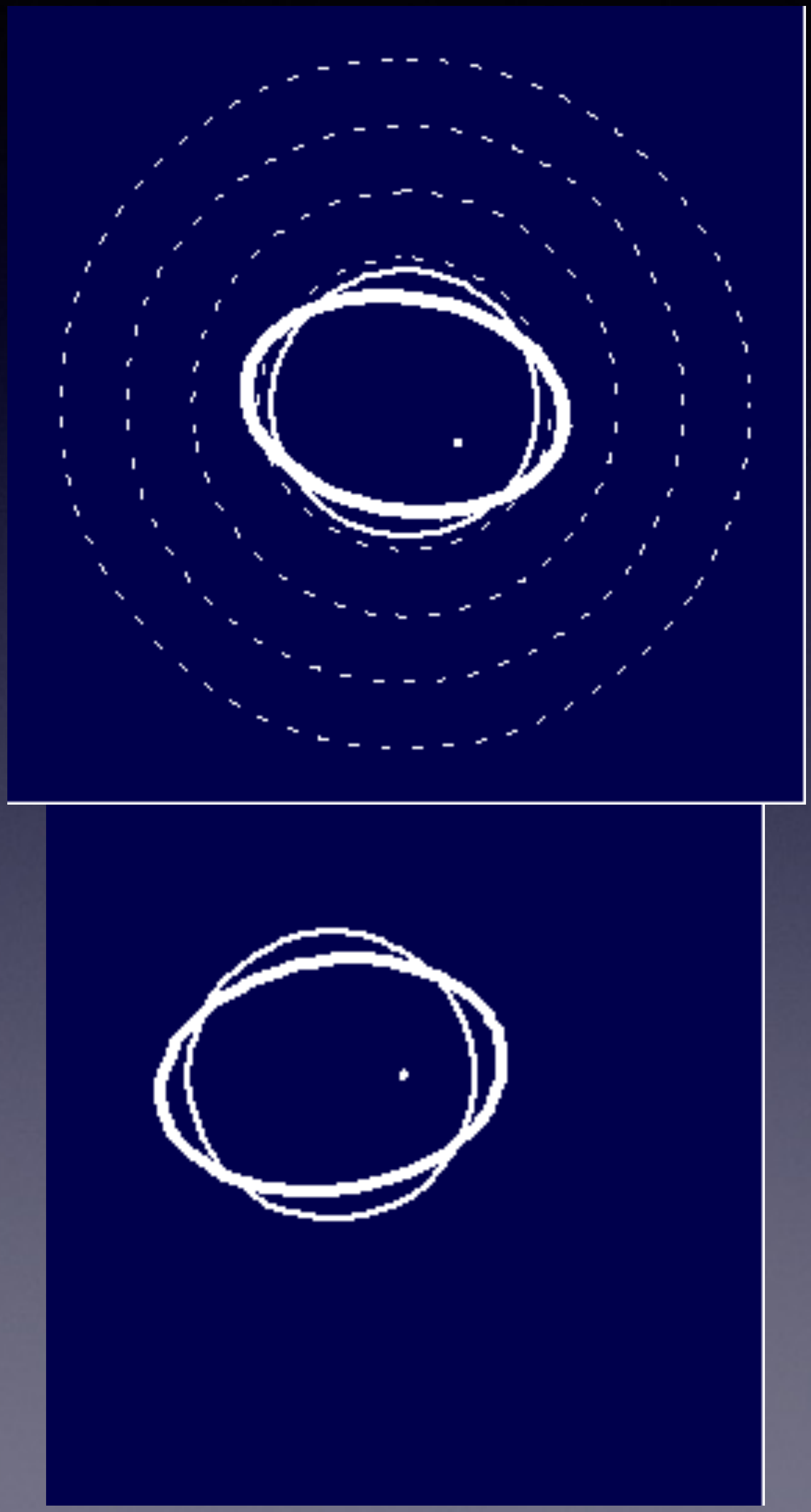
- Bulk motions of the fluid that makes up the Neutron Star can also create quadrupole deformities that lead to gravitational wave emission.

- The favoured mechanism is the *r-mode instability*. r-modes are rotation modes, for which the restoring force is the Coriolis force.

- These are *prograde* in an *inertial frame* but *retrograde* in a corotating frame and suffer the CFS instability.

$$\omega_i = -m\Omega\left(1 - \frac{2}{l(l+1)}\right)$$

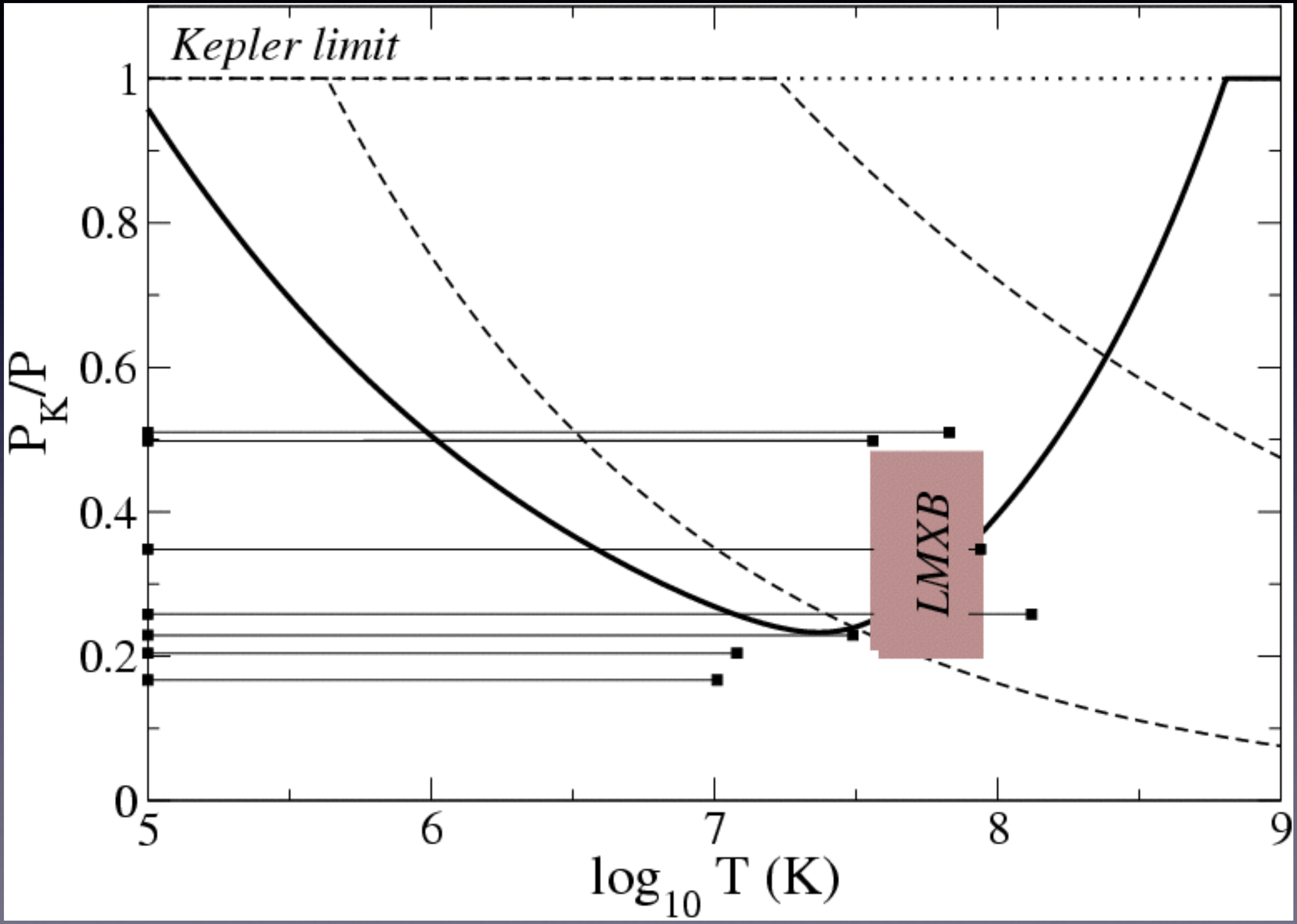
$$\omega_r = \omega_i + m\Omega$$



Fluid Instabilities - r-Modes

- The *Chadrasekhar-Friedman-Schutz instability* arises because in the inertial frame, the mode is prograde and therefore carries away positive angular momentum.
- In the corotating frame, the mode is retrograde. As positive angular momentum is radiated, the angular momentum becomes more negative and the mode amplitude increases.
- The mode amplitude grows and the star temperature increases until the waves break and shocks form.
- The growth of the r-mode was suggested as a mechanism for the observed maximum rotation rate of millisecond pulsars.

Fluid Instabilities - r-Modes



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- The mode amplitude grows and the star temperature increases until the waves break and shocks form.
- The growth of the r-mode was suggested as a mechanism for the observed maximum rotation rate of millisecond pulsars.
- *Magnetic fields, hyperon bulk viscosity* and *nonlinear saturation* might limit the maximum r-mode amplitude.

Free Precession

- If the symmetry axis of a Neutron Star is not aligned with its rotation axis, the rotation will create a wobble.
- The expected strain from a freely-precessing Neutron Star is

$$h_0 \sim 10^{-27} \left(\frac{\theta_w}{0.1}\right) \left(\frac{1\mathrm{kpc}}{d}\right) \left(\frac{\nu}{500\mathrm{Hz}}\right)^2$$

- where θ_w is the wobble amplitude in radians, d is the distance to the source and ν is the star rotation frequency.
- These wobbles may not be very long-lived. Even if they are, the amplitude is so low that they will not be detected by LIGO, although perhaps by Advanced LIGO.

Detection

- These are sources for ground-based detectors.
- Waveform modelling is simple

$$h(t) = \frac{1}{2}F_{+}(t;\psi)h_0(1+\cos^2\iota)\cos 2\phi(t) + F_{\times}(t;\psi)h_0\cos\iota\sin 2\phi(t)$$

- where F_{+}, $F_{\times}$ are the detector beam pattern functions, ι is the inclination of the source to the line of sight and the waveform amplitude, h_0, and phase, $\phi(t)$, are given by

$$h_0 = 16\pi^2\frac{\epsilon I_{zz}\nu^2}{d}$$

$$\phi(T) = \phi_0 + 2\pi\left\{\nu_0(T-t_0) + \frac{1}{2}\dot{\nu}_0(T-t_0)^2 + \frac{1}{6}\ddot{\nu}_0(T-t_0)^3 + \cdots\right\}$$

- Must also relate time at the detector to time at the barycentre, including Einstein delay and Shapiro delay

$$t_{\mathrm{b}} = t + \mathbf{r}\cdot\tilde{\mathbf{n}} + \Delta_{E_\odot} + \Delta_{S_\odot}$$

Detection of Continuous Wave Sources

- LIGO is searching for gravitational waves from pulsars. These searches fall into two categories
  - ➡ *Known pulsars*. Targeted searches for pulsars which have accurate radio timing data.
  - ➡ *Unknown pulsars*. Blind search across parameter space for pulsars that have not been observed previously.
- Known pulsar search uses radio timing data to determine the phase evolution and then heterodynes the gravitational wave data with this phase function. A posterior of the unknown parameters is then constructed for each pulsar in the set.
- S3/S4 data estimated upper limits for 76 known pulsars. Limit for Crab pulsar was a factor of 2.2 above spindown limit.

Detection of Continuous Wave Sources

- LIGO searches use *matched filtering* (known pulsars) and *semi-coherent* methods (unknown pulsars).
- Have obtained limits on emission from known pulsars.

LVC, *Phys. Rev.* D **96** 122006 (2017)

Detection of Continuous Wave Sources

- For nearby pulsars, have now beaten spin down limit

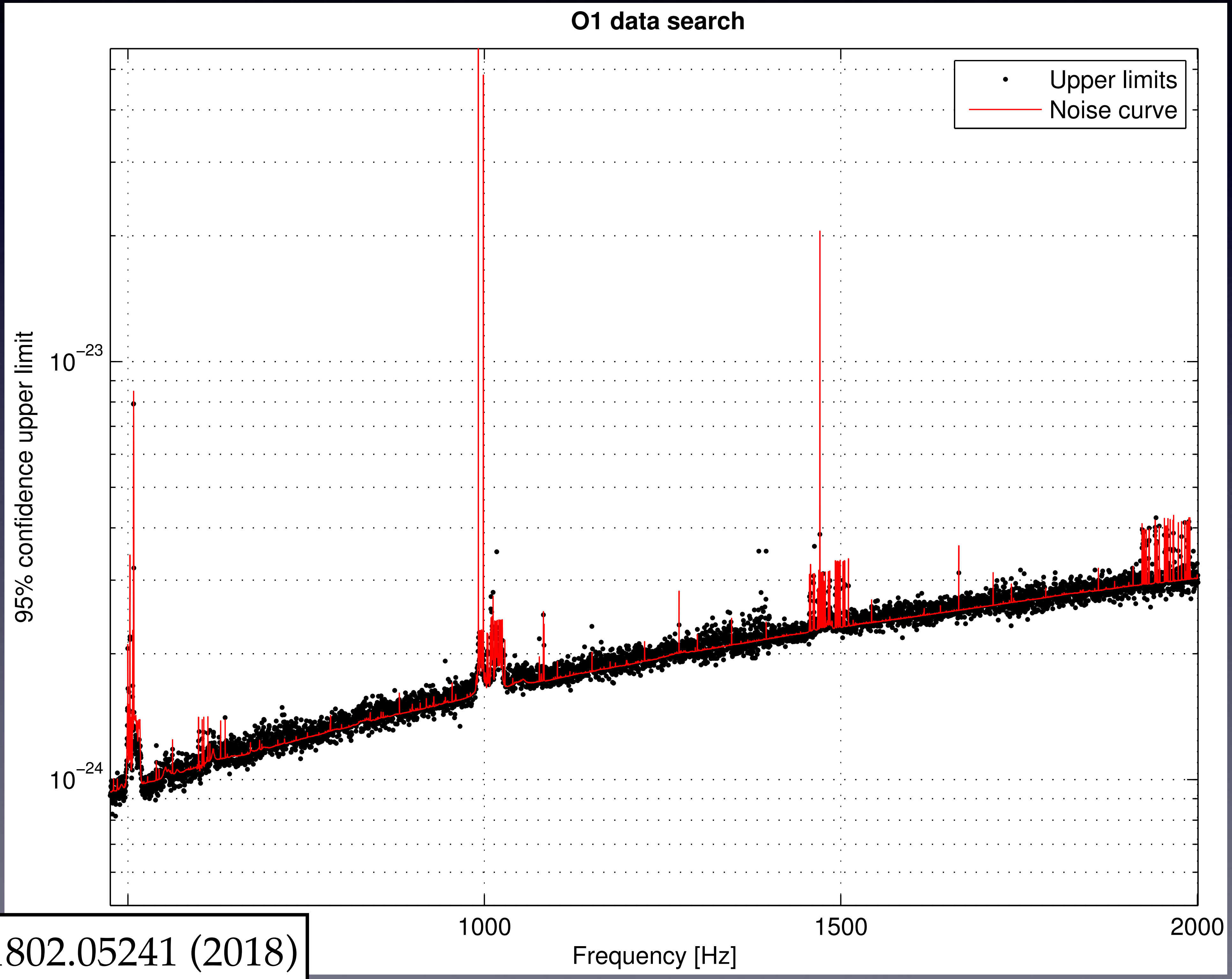
$$h_{\rm sd} = \left(\frac{5}{2} \frac{G I_{zz} |\dot{\nu}|}{c^3 r^2 \nu} \right)^{\frac{1}{2}}$$

Name	$h_{\rm ul}$ $\cdot 10^{-25}$	$\epsilon_{\rm ul}$ $\cdot 10^{-4}$	$h_{\rm ul}/h_{\rm sd}$	$\dot{E}_{\rm rot}/\dot{E}_{\rm GW}$
J0205+6449	3.76	7.7	0.54 ± 0.09	0.29
J0534+2200 (Crab)	1.08	0.58	0.07 ± 0.02	0.005
J0835-4510 (Vela)	9.28	5.3	0.27 ± 0.02	0.07
J1400-6326	1.17	2.7	1.3 ± 0.4	-
J1813-1246	1.80	2.5	> 1.0	-
J1813-1749	1.9	4.8	0.64 ± 0.04	0.41
J1833-1034	3.08	13	0.99 ± 0.09	-
J1952+3252	1.31	1.4	1.31 ± 0.22	-
J2022+3842	1.90	11	1.77 ± 0.35	-
J2043+2740	14.4	47	2.07 ± 0.83	-
J2229+6114	1.78	3.4	0.54 ± 0.35	0.30

LVC, *Phys. Rev.* D **96** 122006 (2017)

Detection of Continuous Wave Sources

- Also have limits on emission from unknown pulsars.



LVC, arXiv:1802.05241 (2018)

Continuous Wave Sources

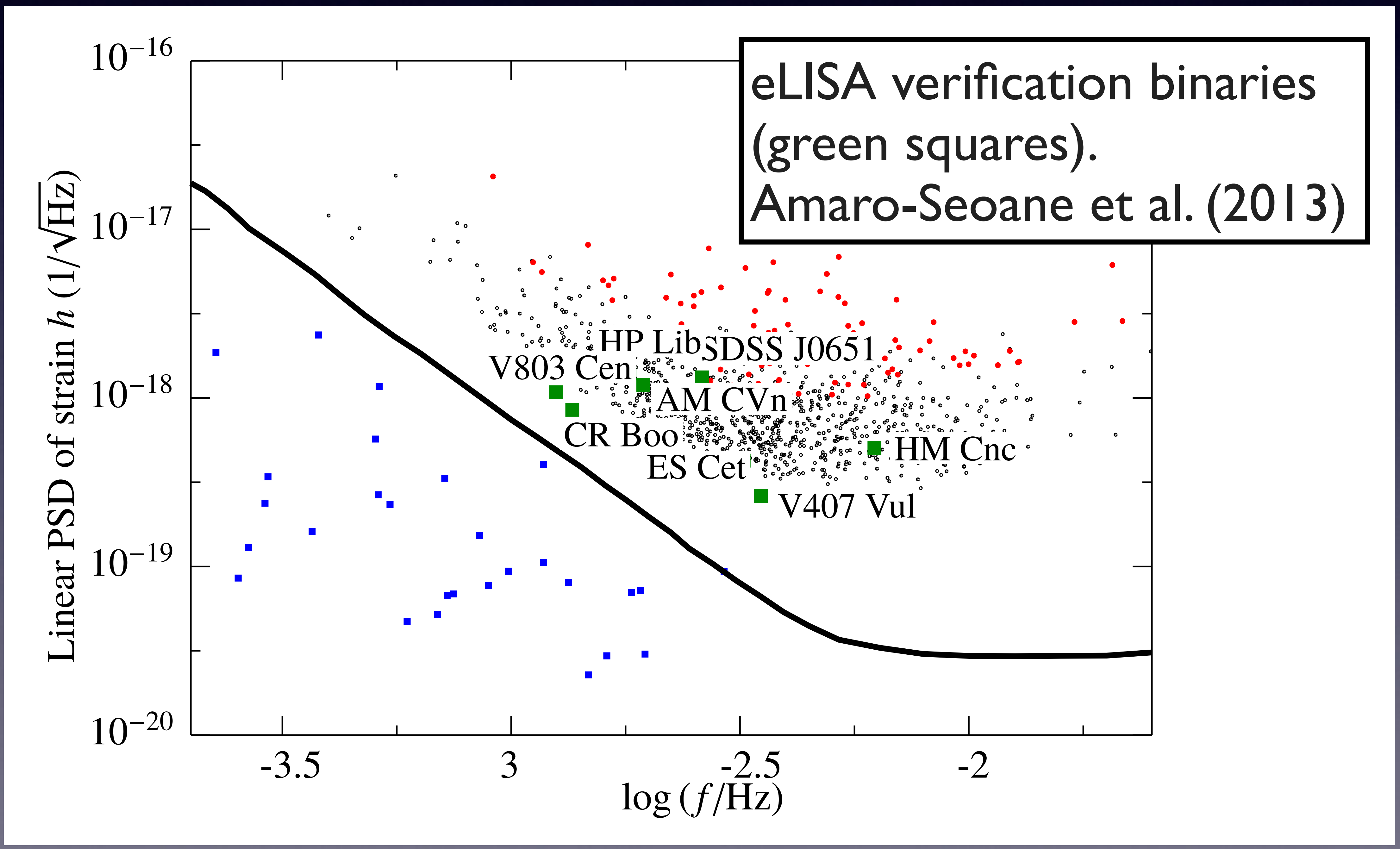
- Detections of continuous wave sources will tell us many things about Neutron Stars
  - *Ellipticities*. If these are high, it will suggest that strange matter may play an important role in Neutron Star structure.
  - *Neutron Star survey*. With a significant number of observations, we will learn about the distribution of Neutron Star spins, and locations.
  - *Spin down mechanisms*. Comparison of observed gravitational wave amplitude to spin down rates of known pulsars will constrain other physical processes in the systems.
  - *Neutron Star structure*. Amplitudes, rotation rates and glitch rates are all probes of Neutron Star structure. Current models are very uncertain, and this will help.

Galactic Binaries

- Binary systems in the Milky Way and nearby galaxies composed of two compact stars (WD, NS or BH) are periodic, continuous wave sources for eLISA and will be seen as inspiral and merger sources by BBO and LIGO (NS-NS, NS-BH, BH-BH only).
- The majority of the sources are not individually resolvable and combine to form the diffuse stochastic foreground discussed earlier.
- Sources that are close and therefore louder, or at high frequency, where there are less systems, will be individually resolved.
- Several binary systems have been observed electromagnetically that must be radiating in the eLISA band - these are *verification binaries*.

Verification Binaries

- Several WD-WD binaries are known which have frequencies and distances such that eLISA should be able to resolve them.



Verification Binaries

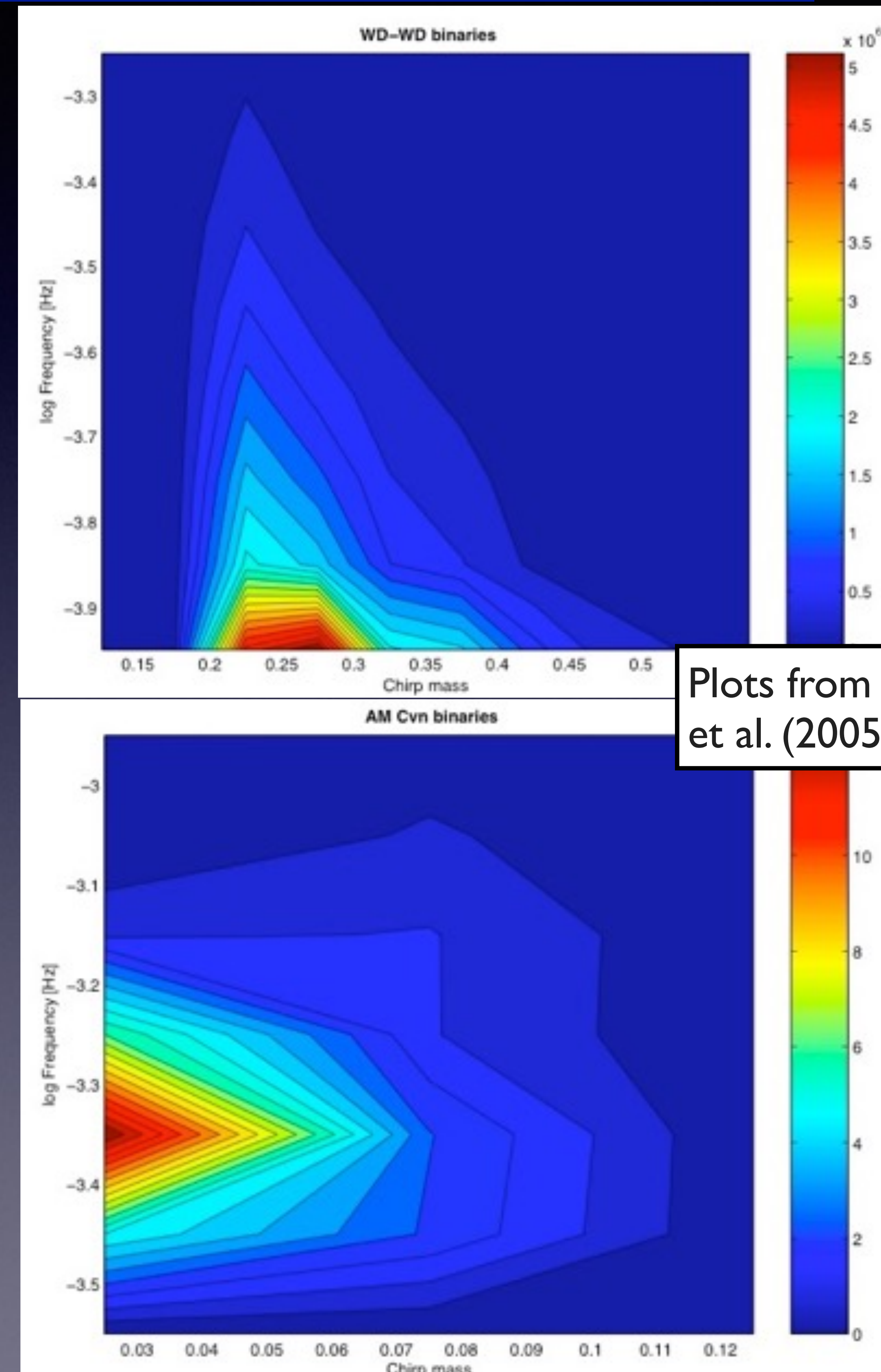
- Several WD-WD binaries are known which have frequencies and distances such that eLISA should be able to resolve them.
- The other system parameters are somewhat uncertain, so only four sources are absolutely guaranteed to be detected in one year of observation - HM Cnc (RXJ0806.3+1527), V407 Vul, ES Cet and SDSS J0651+28.
- HM Cnc should be seen within a couple of weeks - verification of instrument performance.
- Several other systems are likely to be detectable and surveys such as Pan-Starrs, EGAPS and (in the future) LSST are expected to find more.

Other Binaries

- Population models predict distribution of binary parameters as endstate of stellar evolution. Divide into *detached* and *AM CVn* (mass transfer) systems.
- In these plots *chirp mass*

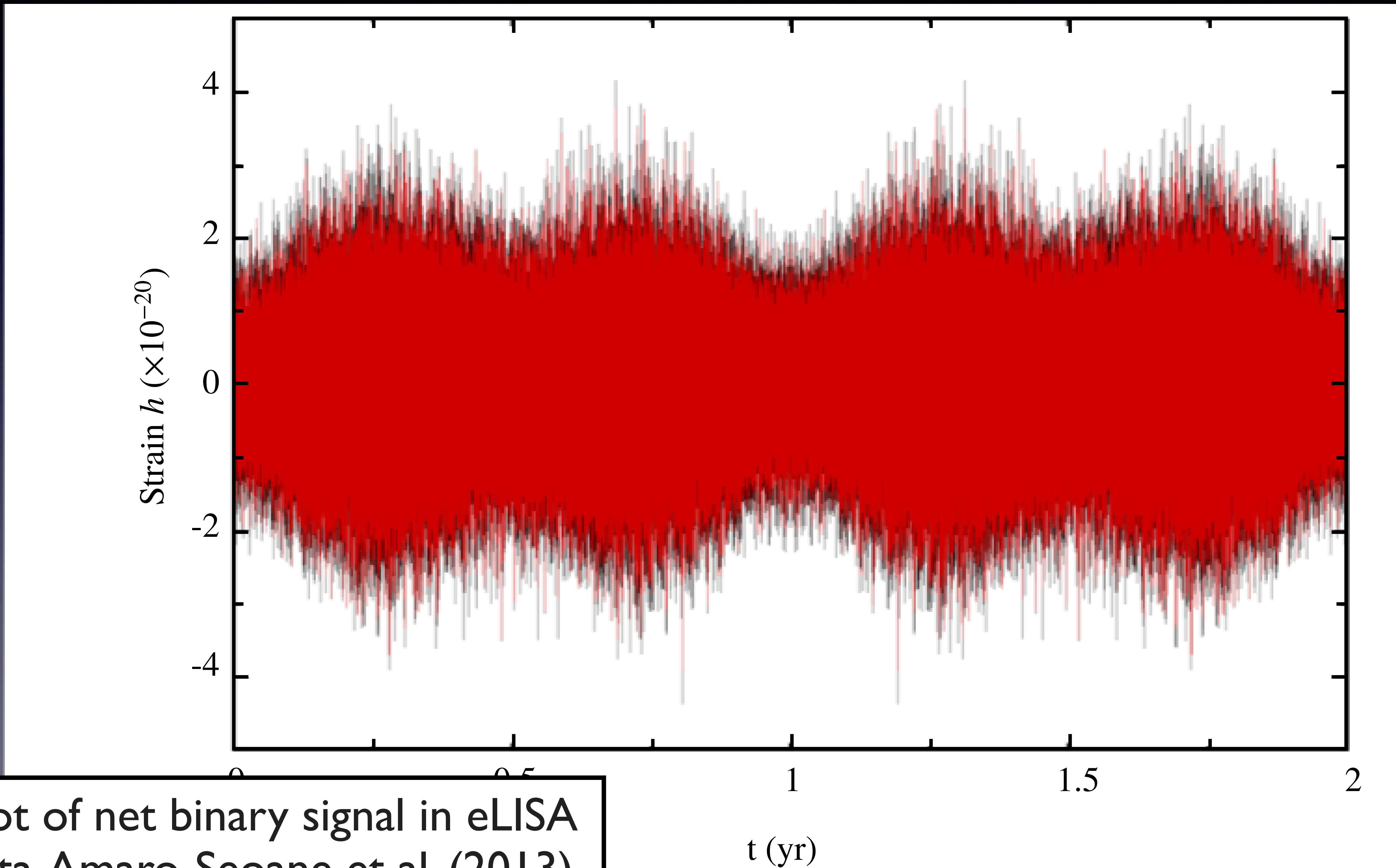
$$\mathcal{M}_c = \frac{(m_1 + m_2)^{2/5}}{(m_1 m_2)^{3/5}}$$

- The superposition of all the sources produces a loud signal in eLISA.



Plots from Edlund et al. (2005)

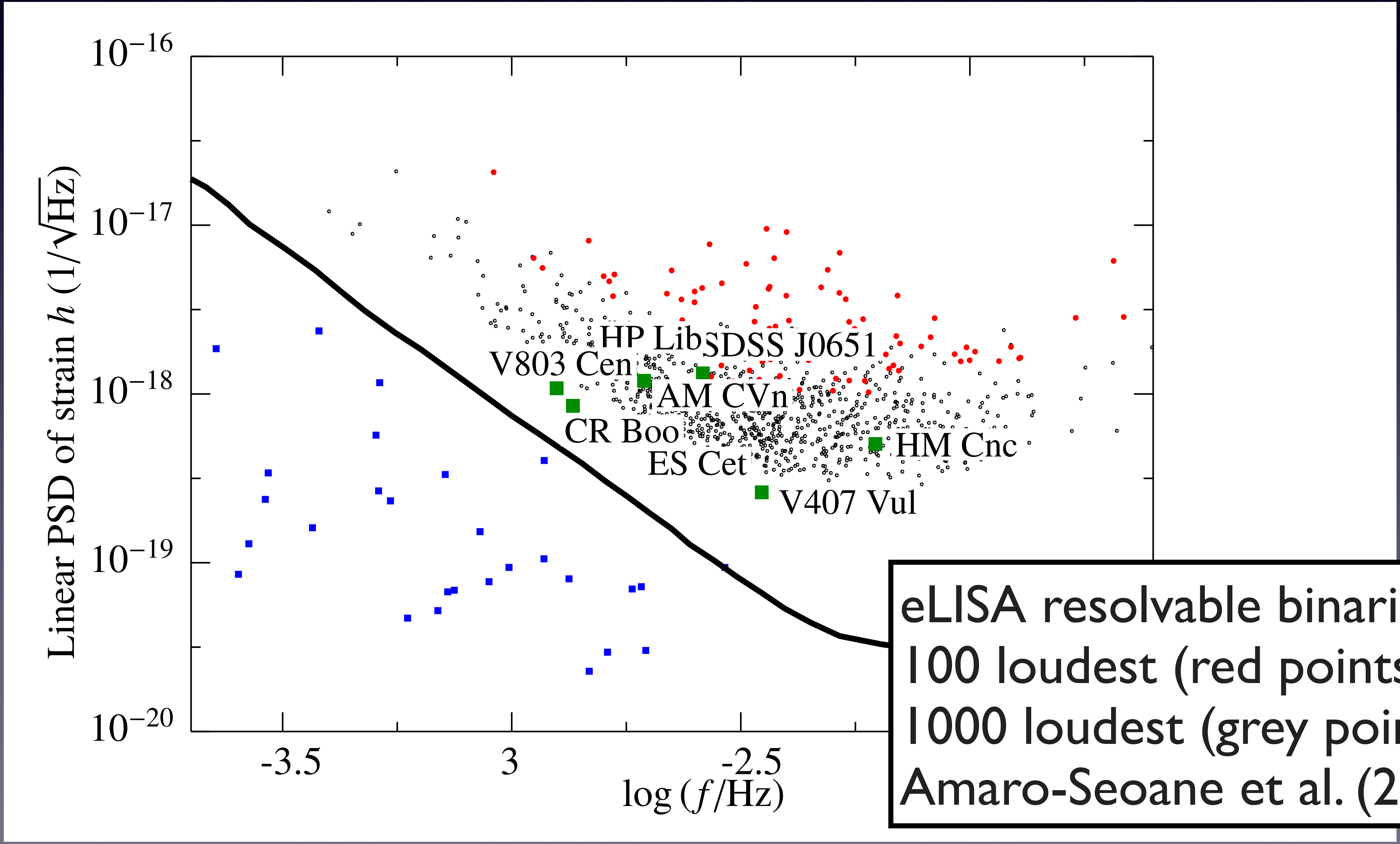
Galactic Binaries



Plot of net binary signal in eLISA data. Amaro-Seoane et al. (2013)

Galactic Binaries

- Expect to see a few thousand individually resolved binaries with eLISA, of which ~ a thousand will have measurable frequency derivatives.



eLISA resolvable binaries:
100 loudest (red points)
1000 loudest (grey points)
Amaro-Seoane et al. (2013)

Galactic Binaries - Detection

- LISA Source. Data analysis will primarily use *matched filtering*.

- Gravitational wave signal is intrinsically very simple

$$h_+(t) = \mathcal{A}(1+\cos^2\iota)\cos(2\pi f t + \phi_0) \qquad \cos\iota = \hat{\mathbf{L}}\cdot\hat{\mathbf{n}}$$

$$h_\times(t) = -2\mathcal{A}(\cos\iota)\sin(2\pi f t + \phi_0) \qquad \mathcal{A} = 2\frac{\mathcal{M}_c^{5/3}}{\mathcal{D}}(\pi M f)^{2/3}$$

- Expect frequency to be constant for the majority of sources. May see some sources *chirping* due to gravitational wave induced inspiral or *anti-chirping* due to mass transfer.

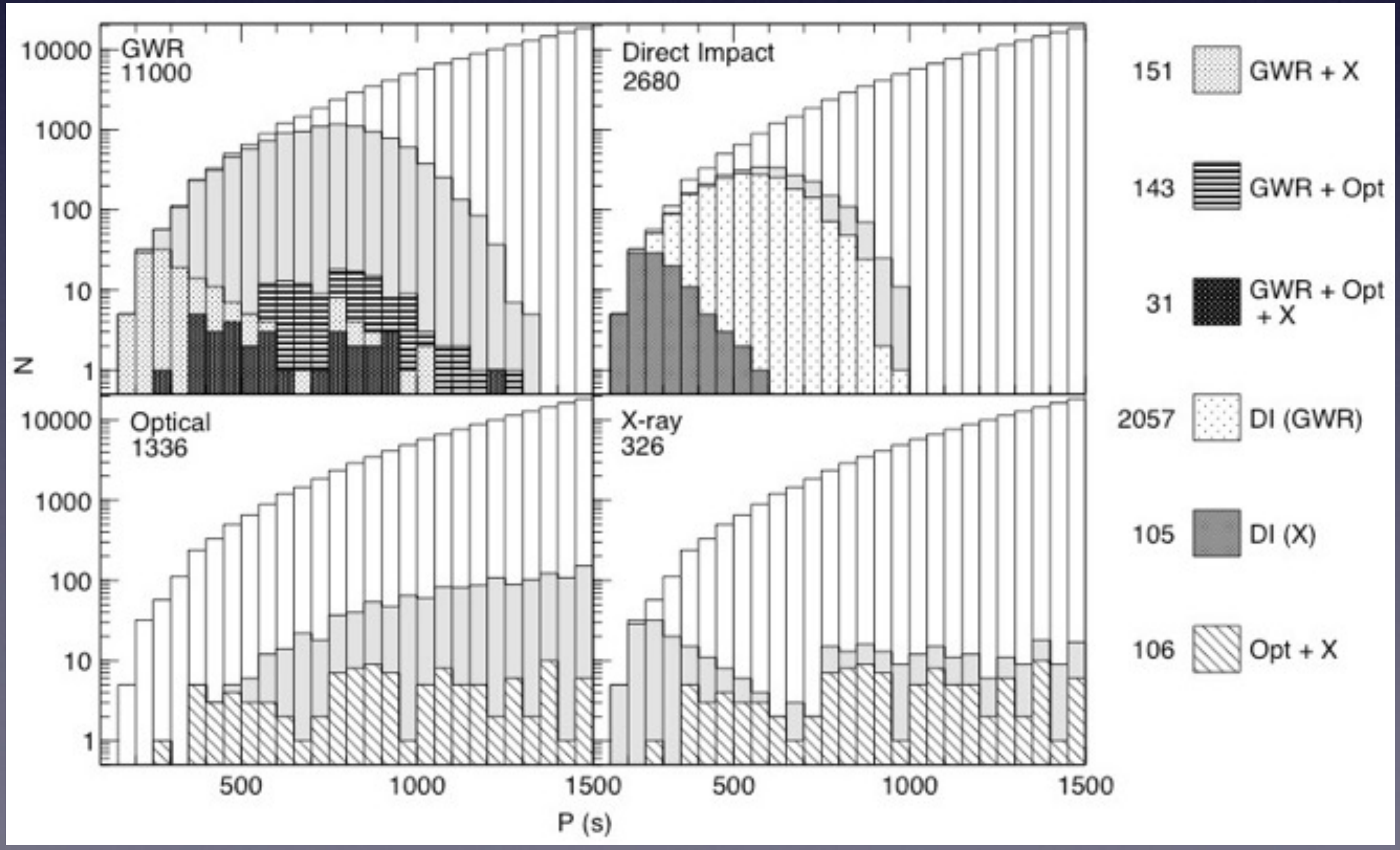
- Detector modulations impose a periodic signal on the response. Single source typically contributes to ~8 frequency bins over a one year observation.

Galactic Binaries

- Galactic binary observations will constrain stellar population models, will probe the initial mass function of stars and help us identify globular clusters.
- Measurements of frequency derivatives probe tidal interactions, which are thought to be important in detached compact binary systems.
- Observations of mass-transfer systems will probe the physics of mass-transfer stability.
- eLISA will not observe WD-WD mergers, but the observed short period systems can be used to infer the merger rate.
- eLISA will also detect binaries containing NSs and BHs. There is the possibility of simultaneous eLISA and LIGO observations of ultra-compact X-ray binaries.

Galactic Binaries

- Combined GW and EM observations could be used to calibrate the local distance scale, and AM CVn models.
- The plot below is for Classic LISA. Current configuration will see only ~1/2 of these events, but fractions observed electromagnetically will be comparable.



Plot of expected number of observed WD-WD binaries, in each waveband. Nelemans (2004).

Ringdown Radiation

- After a binary merger, the newly formed black hole takes time to settle down after merger. It settles into a Kerr black hole state, radiating higher multipole perturbations at the *quasi-normal frequencies* of the black hole.

- The ringdown waveform takes a simple form

$$h(t-t_0) = Ae^{-\frac{\pi f_0}{Q}t}\cos(2\pi f_0 t + \phi_0)$$

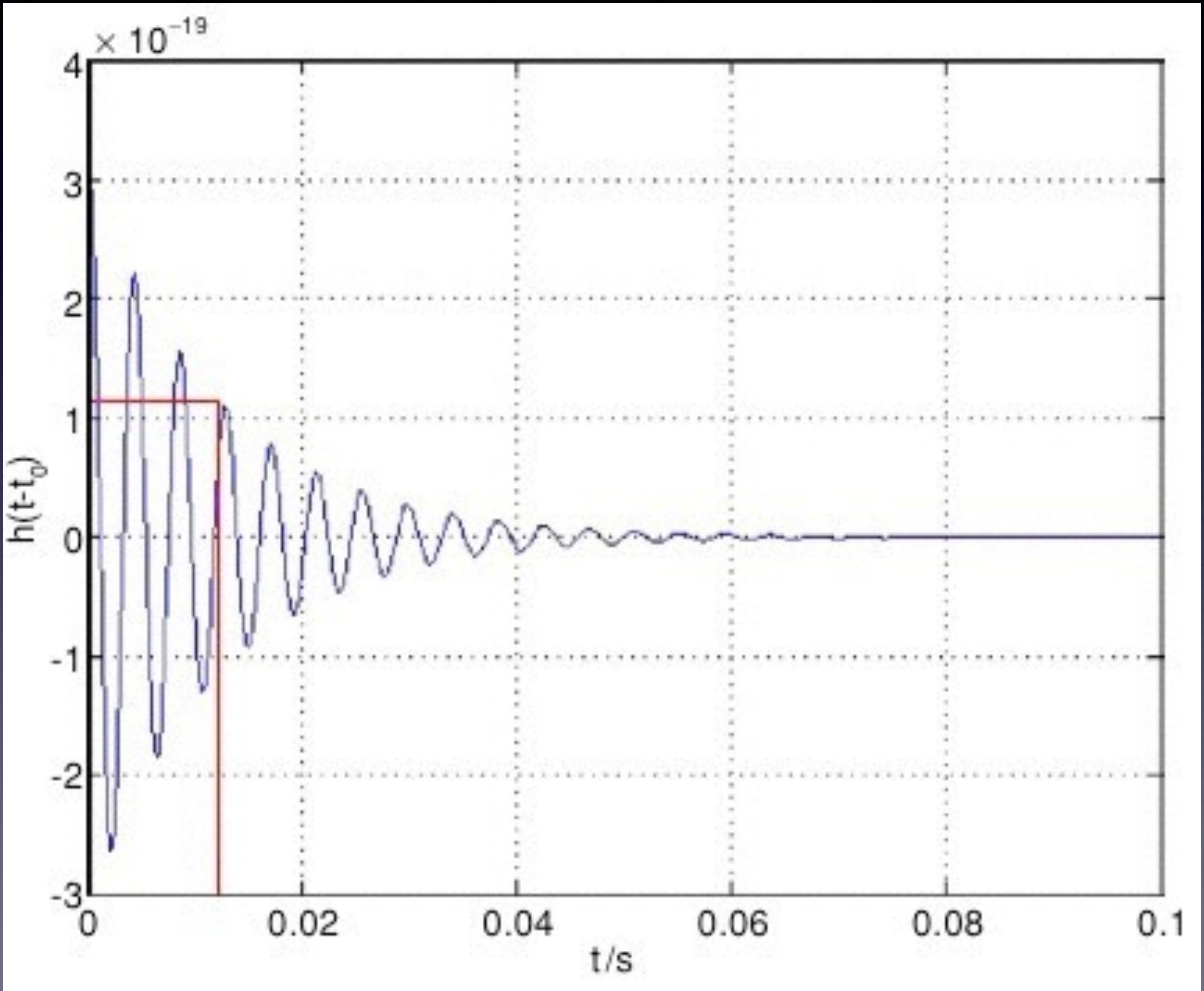
- and the ringdown frequency and *quality factor* can be approximated by

$$f_0 \approx \frac{1}{2\pi GM}\left[1 - 0.63(1-\chi)^{0.3}\right] \qquad Q \approx 2(1-\chi)^{-0.45}$$

- where M and χ are the mass and spin of the merged remnant.

- Search for ringdowns separately since they are at higher frequency - detect sources that merged out of band.

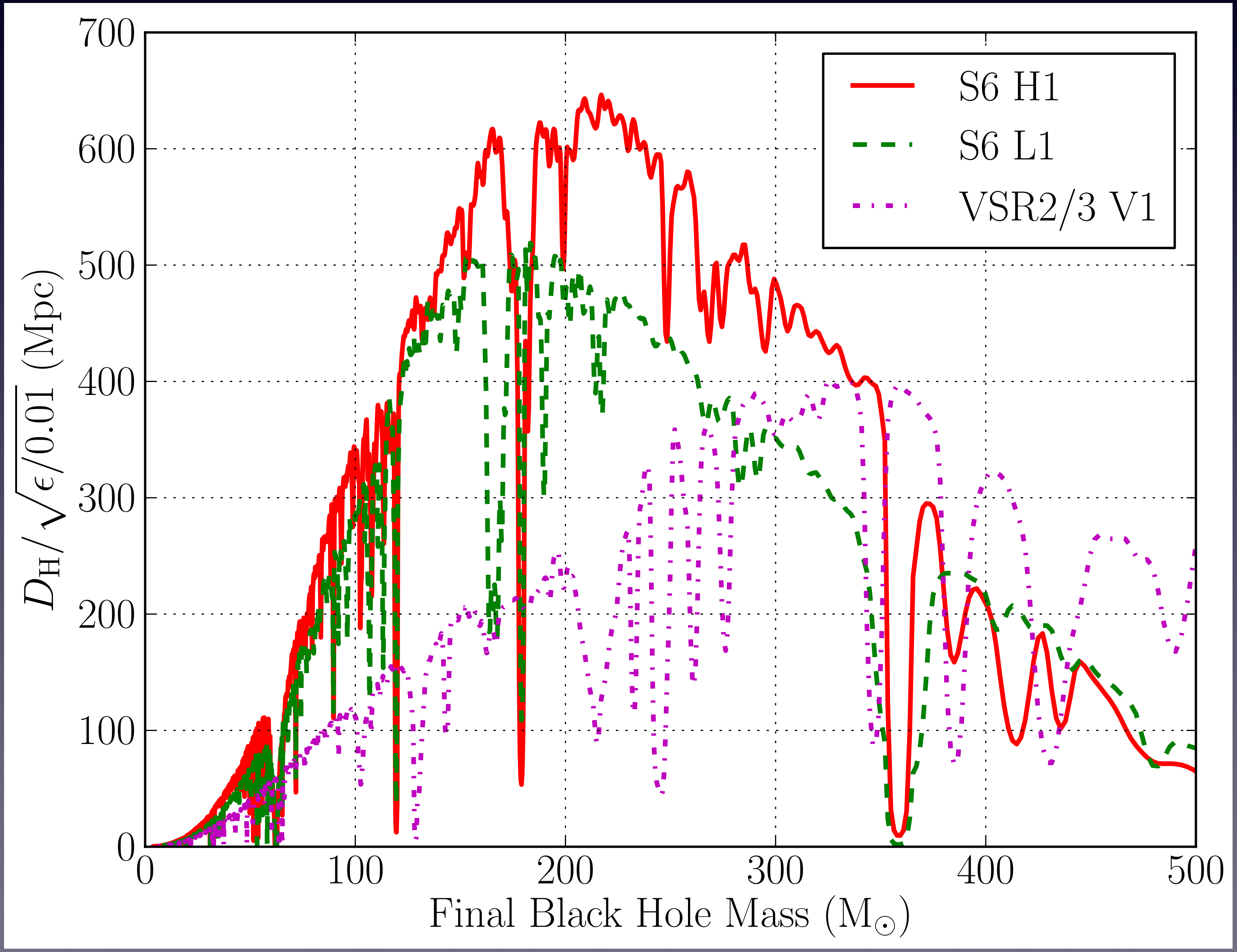
Ringdown Radiation



Ringdown waveform from Goggin et al. (2006)

Ringdown Searches

- LIGO bringdown only searches were carried out in S6 These again used matched filtering. Template placement is analytical in this case since waveform is simple.



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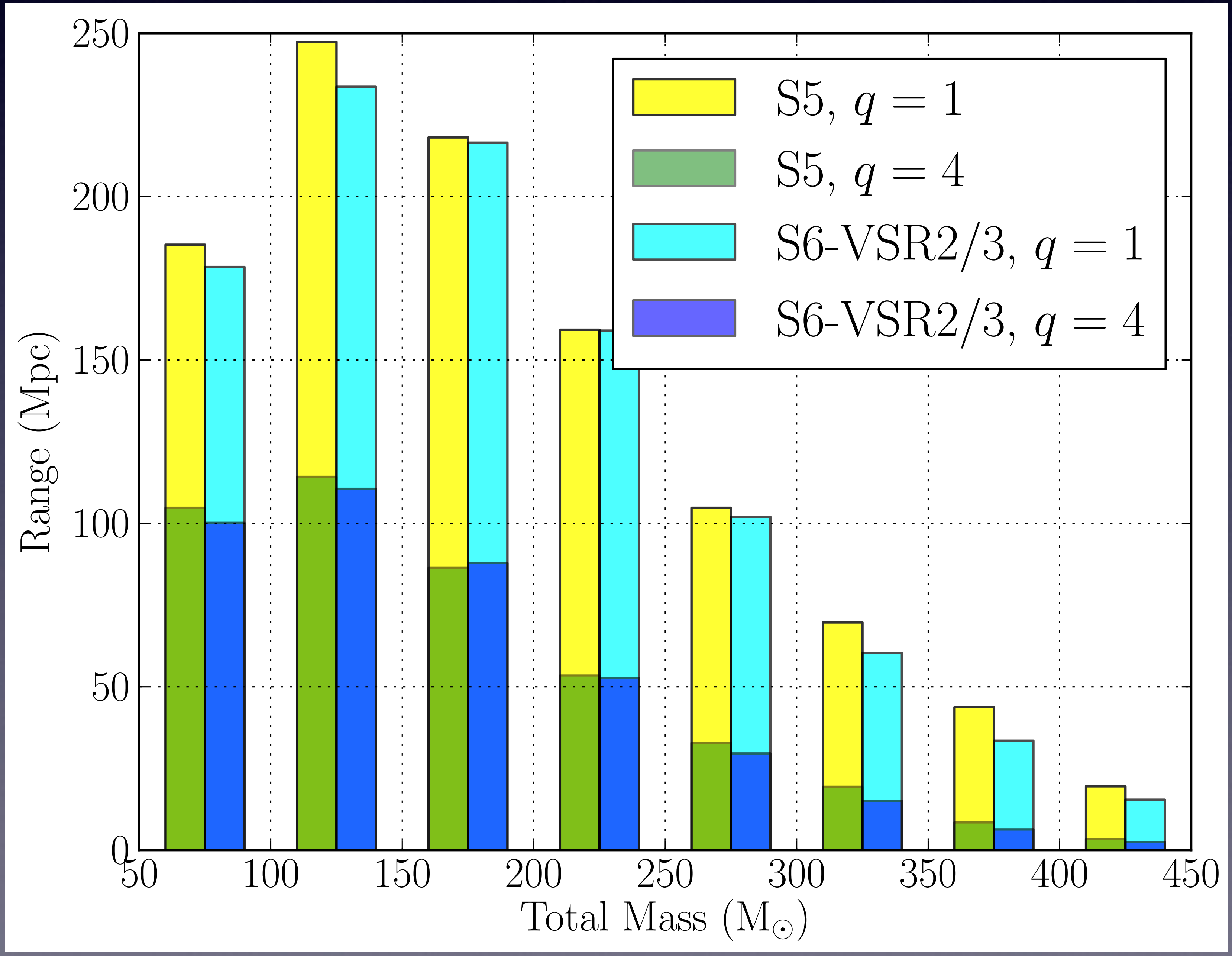
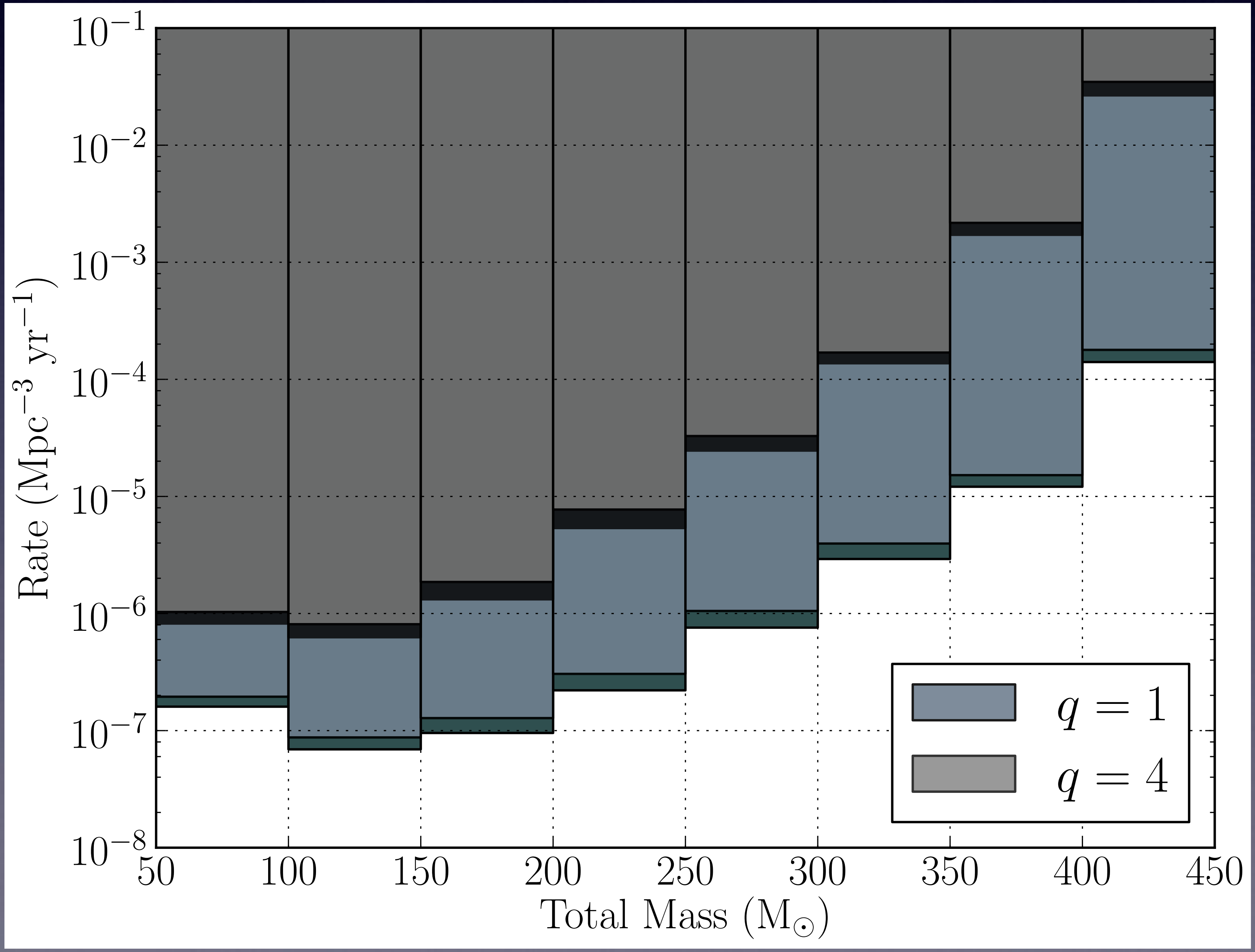


Ringdown Searches

- LIGO bringdown only searches were carried out in S6 These again used matched filtering. Template placement is analytical in this case since waveform is simple.



LVC, *Phys. Rev.* D 89 122003 (2014)