Dublin 2004 Workshop on

K-Theory, Algebraic Groups and Related Structures

University College Dublin

July 5 to 8, 2004

Information Leaflet

University College Dublin
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1. Welcome!

Welcome to the *Dublin 2004 Workshop on K-Theory, Algebraic Groups and Related Structures*!

This workshop is part of the activities of the European Research and Training Network "Algebraic K-theory, Linear Algebraic Groups and Related Structures" (HPRN-CT-2002-00287).

This Information Leaflet contains information on the following topics:

- How to get to UCD and to your accommodation from Dublin Airport;
- How to get to the Department of Mathematics;
- Computer account;
- Wine reception/Conference dinner;
- List of participants;
- Abstracts;
- Conference schedule.

We hope that you will enjoy the workshop and also your stay in Dublin.

If there are any problems, do not hesitate to contact one of us,

Kevin Hutchinson,
David Lewis,
Rachel Quinlan,
Thomas Unger.

Organizational support: Breda McMahon.

2. Registration

The Workshop Registration desk is located in front of the main entrance to the Mathematics Department on the second floor of the Science Lecture Block.

You are invited to collect a workshop pack on Monday morning before the first talk.

There are no fees for this workshop.

Participants who stay on campus and whose accommodation is not subsidized by us should **not** pay for their accommodation at the Merville Reception – we have already done that for you.

You are invited to settle your bill at the Workshop Registration desk on Monday morning. The price is €35.00 per night.

3. How to get to UCD and to your accommodation from Dublin Airport

The **Montrose Hotel** and UCD are opposite of each other.

In case you are staying on **campus**, the reception of **UCD Village** is located in the **Merville residence**: enter the UCD main entrance and turn left; continue until you reach the bus terminus; turn left – the location of the residence should be indicated on a sign post; continue and turn left again, you should see the Merville entrance now **11 on Map 1.**.

From the **airport** you can get to UCD and to your accommodation by one of the following methods:

Taxi: costs about €25.00, depending on duration of trip, number of people travelling and number of bags.

Dublin Bus: The 746 is an hourly service from the airport which stops at the Montrose hotel bus stop. If you are staying on campus, you should get off here as well and cross the fly-over to the UCD main entrance.

The fare is €1.75.

Note: Dublin Bus now requires exact fares on all routes. In other words the driver will not give you change!

Aircoach: The blue Aircoach busses depart from Dublin Airport (directly opposite the Arrivals Hall) every 20 minutes from 04:00 to midnight. (A special schedule is in operation from midnight to 04:00.)

The fare is €7.00 single/€12.00 return.

Aircoach operate two bus services. The Leopardstown/Sandyford/Stillorgan service passes UCD. (Confirm with the driver that you are on the right bus.) The bus stops at the Montrose hotel bus stop on the way from the airport and the UCD main entrance bus stop on the way to the airport.

The Aircoach busses are very comfortable and usually faster than the Dublin Bus 746.

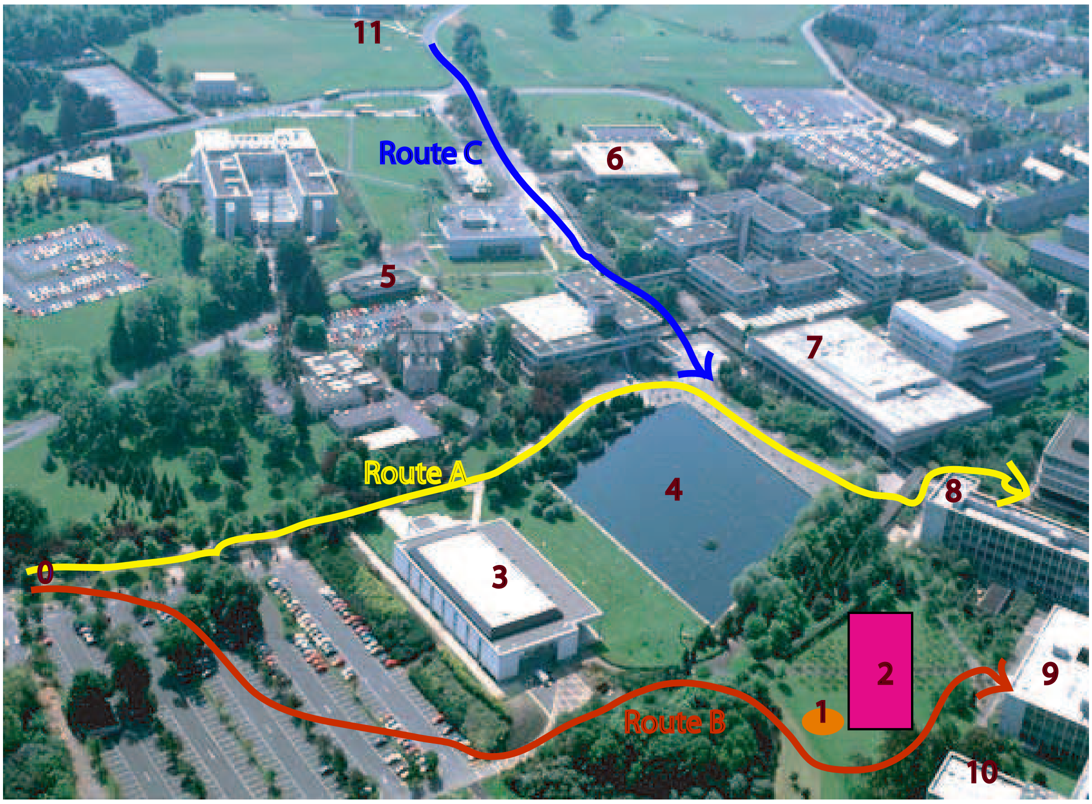
4. How to get to the department

The Department of Mathematics is located on the 1st and 2nd floor of the Science Lecture Building (**SLB**).

[Previous visitors note that the department relocated from the John Henry Newman Building to the SLB about a year ago.]

Below there is an aerial view of the UCD campus (slightly out of date) with the main routes to the Science Lecture Building $\boxed{9}$ indicated.

Because of **building works** the most direct route to the building may be blocked.



Map 1: routes to Science Lecture Building.

0	to Main Entrance	6	Main Restaurant
1	"Noah's Egg"	7	Main Library/Post Office/Shop
2	*Building Works*	8	Chemistry Building
3	O'Reilly Hall	9	Science Lecture Building
4	Lake	10	Physics Building
5	AIB Bank	11	Merville Residence

You can reach the SLB as follows:

From the Merville Residence: follow Route C in the direction of the Chemistry Building $\boxed{8}$ until it merges with Route A at the lake $\boxed{4}$. Then follow instructions for Route A from the lake onwards.

From the Montrose Hotel/Main Entrance of UCD: Follow Route A, or if you are adventurous, Route B.

Route A: When you arrive at the main entrance of UCD, do not enter, but turn right and walk towards the bus stop.

Behind the bus stop you will find two pedestrian crossings, one after the other, with a pavement in between. Cross both of them.

You are now at **0**. Keep on walking straight ahead. (The path goes up and has steps.) Walk beyond O'Reilly Hall **3** and turn right when you are beyond the lake **4**.

Walk straight ahead, keeping the lake to your right hand side.

Walk up the steps (or the bicycle slope). On your left you will find the Agriculture Building, on your right the Chemistry Building **8**.

Walk straight ahead until you see the entrance of the Chemistry Building.

Walk up the stairs and enter the Chemistry Building. You are now on the first floor of this building.

Inside, walk straight ahead and enter the Bridge connecting the Chemistry Building and the SLB.

Exit in the SLB. On your left is the SLB Information Desk and in front of you is a staircase (see **Map 3**).

Go up the stairs to the second floor: you have arrived. ■

Route B (only for the extremely adventurous): this is the most direct route to the SLB, but it may be blocked due to building works **2** (this varies from day to day, and seems to be getting worse).

When you arrive at the main entrance of UCD, do not enter, but turn right and walk towards the bus stop.

Behind the bus stop you will find two pedestrian crossings, one after the other, with a pavement in between. Cross both of them.

You are now at **0**. Turn right and cross the Veterinary Car Park diagonally until O'Reilly Hall **3** is on your left and the Veterinary Science Building (not indicated on this map) is on your right.

Follow the path until you reach *Noah's Egg* **1** and the building works **2**.

Carefully go beyond the works and turn left. The building in front of you is the Physics Building **10**. It is connected to the SLB **9** via a bridge.

Make your way to the entrance of the SLB and turn to **Map 2**.

You have reached the ground floor of the SLB.

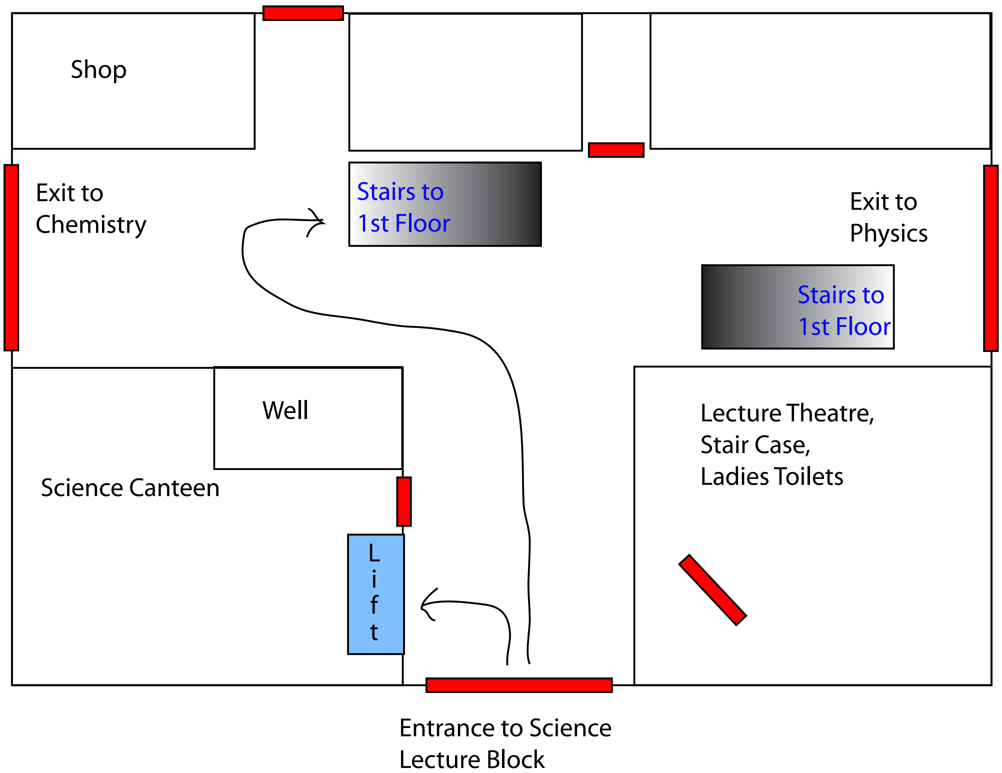
Upon entering the building, you will see a lift to the 1st and 2nd floor of the Mathematics Department.

You are welcome to try the lift and have a look around in the department. However, if this is your first visit, it will be easier if you walk straight ahead, follow the long arrow on Map 2, and take the stairs to the 1st and then the 2nd floor of the department: you have arrived. ■

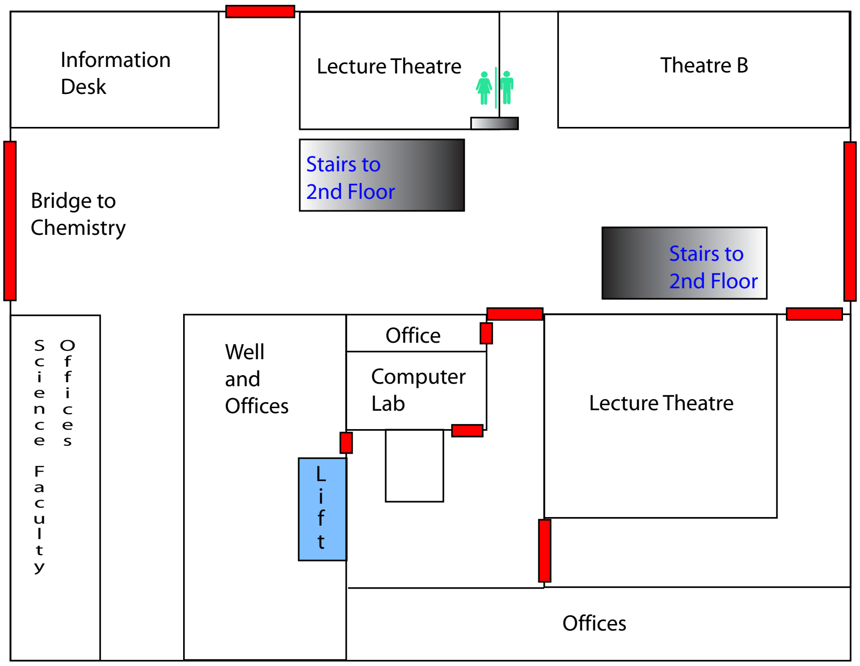
Below you will find plans of the Ground Floor, First Floor and Second Floor of the SLB.

Please pay attention to the following:

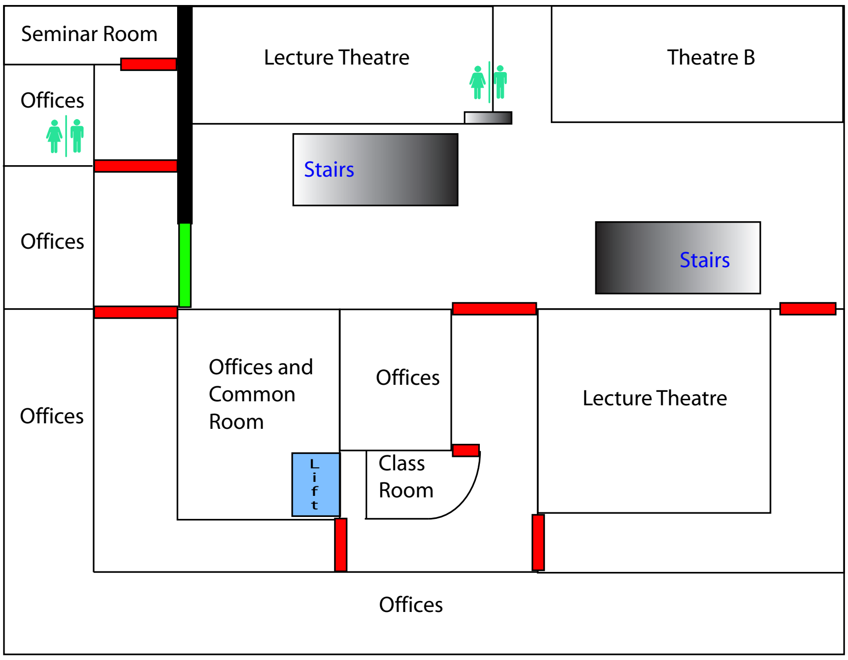
- the **registration desk** is on the 2nd floor;
- all lectures will take place in **Theatre B**, which is accessible from both the 1st and the 2nd floor;
- the **Computer Lab** (where you can check your email) is located on the 1st floor;
- there is a Chemistry conference going on at the same time;
- on Maps 2, 3 and 4 the most important doors are indicated by narrow red rectangles, the narrow green rectangle on Map 4 is the main entrance to the Mathematics Department.



Map 2: Ground Floor SLB.



Map 3: First Floor SLB.



Map 4: Second Floor SLB.

5. Busses to and from the City Centre

To City Centre: from bus stop at main entrance UCD: 10, 46A, 46B, 746, ... (if you are not certain, ask the driver). If you have never been to Dublin before, a convenient place to get off the bus is **St. Stephen's Green** (most people will get off here). The fare should be €1.45. **(You need exact change!)**

From City Centre: bus stops for the 10, 46A, 46B, 746 can be found in the area around **Trinity College**. 46A, 46B, 746 all stop at the Montrose bus stop opposite the UCD main entrance. The 10 stops on the fly-over and then continues on to the bus stop (terminus) inside UCD (this is convenient for people staying on campus). The fare should be €1.45. **(You need exact change!)**

6. Computer Account

You can check your email, etc. in the Computer Lab, which is located on the first floor of the SLB (see Map 3).

Other information: to be completed.

7. Workshop Photograph

We plan to take a workshop photograph, most likely on Monday afternoon. Details to be confirmed.

8. Wine Reception and Conference Dinner

A **wine reception** will take place after the last talk on Monday on the second floor of the SLB, in front of the main entrance to the Mathematics Department.

The **workshop dinner** will take place on Tuesday evening, starting at 8pm. The venue is *The Courtyard Cafe*, 1, Belmont Avenue, Donnybrook (behind Madigan's pub).

The (subsidized) price of the meal is €25.00 per person for the set menu (which has a choice of 4 starters and of 4 main courses, including meat, fish and vegetarian options). Drinks are not included in this price.

9. List of Participants

Name	Affiliation	Email address
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10. ABSTRACTS

Florence Soriano-Gafiuk

Application of logarithmic classes to K-Theory

We present the various groups of logarithmic classes in connexion with the wild kernel of the K-Theory for number fields. The main interest of this approach relies on the algorithmic aspects which we illustrate with a lot of examples we obtained by using the software MAGMA. ■

Karim Johannes Becher

Weakly isotropic forms

The notion of weak isotropy arises naturally in the context of local-global principles for quadratic forms over a real field F. It has been studied in this context in the 70ies by Prestel and Bröcker. However, little attention has been paid so far to the question what a weakly isotropic quadratic form actually may look like.

If a quadratic form φ over F contains a 2-dimensional torsion form, then it is certainly weakly isotropic. This sufficient condition for weak isotropy turns out to be necessary over certain (real) fields F, for example if I^3F (the third power of the fundamental ideal IF in the Witt ring WF) is torsion-free. A weakly isotropic form over F is said to be minimal, if none of its proper subforms is weakly isotropic. A construction due to Arason and Pfister shows that, if k is any real field such that I^2k is not torsion-free, then there exist minimal weakly isotropic forms of dimension 3 over $F = k(X, Y)$. The dimensions of minimal weakly isotropic forms over F are bounded (from above) by the Pythagoras number $p(F)$, further by 2^{n-2} if I^nF is torsion-free for some $n \geq 3$.

Considering the set of all anisotropic, weakly isotropic forms over F gives rise to a field invariant $\hat{u}(F)$, defined as the supremum of the dimensions of these forms. Comparison with the classical u-invariant $u(F)$ defined by Elman and Lam and with the Hasse number $\widetilde{u}(F)$ readily yields the inequalities $u(F) \leq \hat{u}(F) \leq \widetilde{u}(F)$. It is easy to see that, if $u(F) < 4$ then $u(F) = \hat{u}(F)$, and also that, if $\widetilde{u}(F) < \infty$ then $\hat{u}(F) = \widetilde{u}(F)$. Given any pair of integers (m, n) with $2 \leq m \leq n$, a field F can be constructed where $u(F) = 2m$, $\hat{u}(F) = 2n$, and $\widetilde{u}(F) = \infty$. ■

Janez Bernik

A generalization of Brauer's theorem on splitting fields to semigroups

Let A be a central simple algebra over a field F. Suppose A is spanned by a multiplicative semigroup S with the property that the minimal polynomial of every $s \in S$ splits over F. Does A represent the trivial class in the Brauer group of F? In general, this question is still open but the answer is known to be yes under some additional assumptions on S or F. For instance, if S is a finite group of invertible elements in A, then this is equivalent to the well-known Brauer's theorem on splitting fields.

I will report on another instance when the answer is known to be affirmative, namely if F is either a local or a global field of characteristic 0 (with no assumptions on S). The idea is, if F is local, to reduce the problem to the case when S is a compact group and then use the structural theory of compact subgroups of linear algebraic groups over local fields. ■

Raman Parimala

Rational points on homogeneous varieties

It is an open question whether a principal homogeneous space under a connected linear algebraic group which admits a rational point in coprime degree extensions has a rational point. A similar question for homogeneous varieties is formulated by Colliot-Thélène. We give examples of projective homogeneous varieties with a rational point under coprime degree extensions but have no rational point. ■

Jaka Cimprič

Higher product levels of skewfields

The level of a field is the length of the shortest representation of -1 as a sum of squares. Several generalizations of this notion appeared in the literature, in particular:

- around 1970 Joly introduced higher levels of a field,
- around 1980 Scharlau and Tschimmel introduced product levels of skewfields.

The aim of my talk is to survey my work on higher product levels of skewfields, a common generalization of levels in the sense of Joly and Scharlau-Tschimmel.

The following topics will be discussed:

- Artin-Schreier theory of higher level orderings on skewfields,
- inequalities between higher product levels of various degrees,
- that products of n-th powers and products of permuted n-th powers do not give the same notion of higher product levels. ■

Mohammad Mahmoudi

Hermitian forms and the u-invariant

The u-invariant is one of the most interesting invariants in the algebraic theory of quadratic forms. The purpose of this talk is to present some new results about the hermitian version of this invariant. In particular results about some bounds for this invariant and values taken by this invariant and some finiteness results are discussed. ■

Uzi Vishne

Representation theory and the spectrum of finite complexes

A finite graph is Ramanujan if the spectrum of the Laplacian acting on the graph is contained in the spectrum on its covering tree (plus trivial points). Such graphs were first constructed in 1986 by Lubotzky, Philips and Sarnak, and have many applications in finite mathematics. The notion of Ramanujan complexes is defined similarly using the colored Laplacians acting on the spherical Bruhat-Tits building assoicated to $\mathrm{PGL}(F)$. The recent proof of Ramanujan conjecture in positive characterisitic, by L. Lafforgue, enables us to construct finite Ramanujan quotients of the Bruhat-Tits building, using the algebraic groups defined by division algebras.

Joint work with Alex Lubotzky and Beth Samuels. ■

Susanne Pumplün

U-invariants of forms of higher degree

A form of degree d over a base field k is a homogeneous polynomial of degree d over k. Contrary to the situation of quadratic forms ($d = 2$), forms of degree higher than 2 are not all isomorphic to a "diagonal" form. Hence we introduce both a diagonal u-invariant as well as a general u-invariant for forms of higher degree. Artin's conjecture that the u-invariant for forms of degree d over a p-adic field should be less than or equal to d^2 (which turned out to be wrong) triggered lots of research about the u-invariants of forms over such fields. Apart from the results obtained here, the well- known results of Chevalley on the u-invariant of a finite field, and the results obtained using Tsen-Lang theory, not much is known about these invariants.

We present some general results, some of them classical ones in the $d = 2$ case, for u-invariants of forms of higher degree. ■

Dejan Velušček

Higher product Pythagoras numbers of skewfields

In a commutative field F the n-th Pythagoras number $p_n(F)$ is the smallest integer k such that each sum of n-th powers in F is already a sum of k n-th powers. By analogy we extend this notion to skewfields: the n-th product Pythagoras number $p_n(D)$ of a skewfield D is the smallest integer k, such that every sum of permuted n-th powers is already a sum of k permuted n-th powers in D.

In the talk we will show that the connection between product Pythagoras number of the generalised Laurent series skewfield $K((G))$, where K is a skewfield and G a totally ordered group, is the same as in commutative case, namely $p_n(K((G))) = p_n(K)$. This has a consequence, that for every $n \in \mathbb{N}$ there exists a formally real noncommutative skewfield D with $p_2(D) = n$. ■

Nikita Semenov

$(2, 3)$ ***and Hurwitz generation of Chevalley groups***

It is intended to give a brief survey of the problem of $(2, 3)$ and Hurwitz generation of Chevalley groups. In particular, the recent developments, concerning explicit generators of exceptional groups in their minimal representations, are of our main interest. ■

Mikaël Lescop

On the 2-rank of the wild kernel

We aim at explaining the calculation of the 2-rank of the wild kernel of number fields, using the genus formula for the 2-primary part of the wild kernels proved by M. Kolster and A. Movahhedi. More precisely, E/F being a relative quadratic extension of number fields, we show how to compute the 2-rank of the wild kernel of E provided that the 2-primary part of the wild kernel of F is trivial. ■

Ricardo Baeza

***Behaviour of quadratic forms under function fields of Pfister quadrics in characteristic* 2**

Let $W_q(F)$ be the $W(F)$-module of quadratic forms over a field F of characteristic two and IF the maximal ideal of $W(F)$ of even dimensional bilinear forms. If q is an anisotropic n-fold quadratic Pfister form and $F(q)$ its function field, we compute the kernel of $I^m W_q(F) \longrightarrow I^m W_q(F(q))$ for any m. ■

	Monday 5 July	Tuesday 6 July	Wednesday 7 July	Thursday 8 July
09:20 – 09:30	**Daily Announcements**			
09:30 – 10:20	J.-P. Tignol, *Algebras with Involution* (Short Course)			
10:20 – 11:00	**Coffee Break**			
11:00 – 11:50	M. Kolster, *K-Theory and Arithmetic* (Short Course)			
12:00 – 12:50	U. Rehmann, *Linear Algebraic Groups* (Short Course)			
12:50 – 14:30	**Lunch Break**			
14:30 – 14:55	F. Soriano-Gafiuk *Application of logarithmic classes to K-theory*	J. Cimprič *Higher product levels of skewfields*	**Free Afternoon**	D. Veluščček *Higher product Pythagoras numbers of skewfields*
15:00 – 15:25	K.J. Becher *Weakly isotropic forms*	M. Mahmoudi *Hermitian forms and the u-invariant*		N. Semenov $(2,3)$ *and Hurwitz generation of Chevalley groups*
15:30 – 15:55	J. Bernik *A generalization of Brauer's theorem on splitting fields to semigroups*	U. Vishne *Representation theory and the spectrum of finite complexes*		M. Lescop *On the 2-rank of the wild kernel*
16:00 – 16:30	**Coffee Break**			**Coffee Break**
16:30 – 17:20	R. Parimala *Rational points on homogeneous varieties*	S. Pumplün *U-invariants for forms of higher degree*		R. Baeza *Behaviour of quadratic forms under function fields of Pfister quadrics in characteristic* 2
17:30 – ☺	**Wine Reception**			
20:00 – ☺		**Workshop Dinner**		