ORBIT EVOLUTION WITH THE SELF-FORCE

PROGRESS AND CHALLENGES

Niels Warburton
University College Dublin

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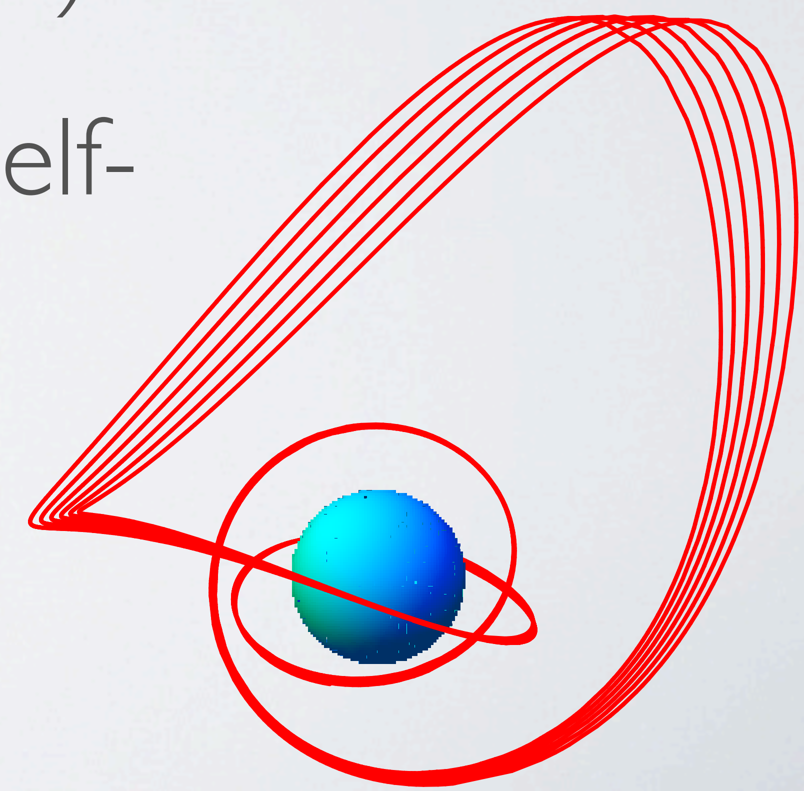
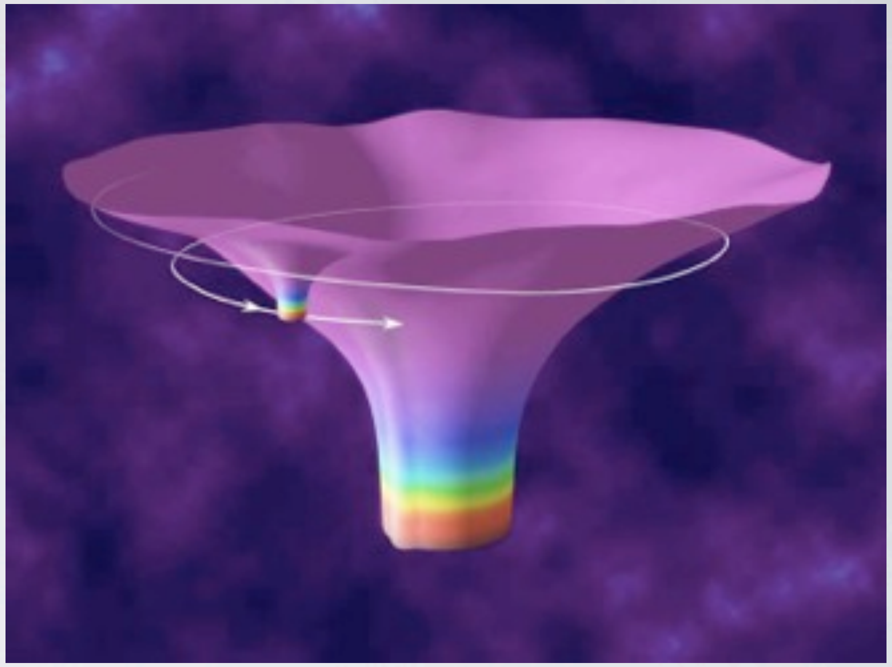
Overview

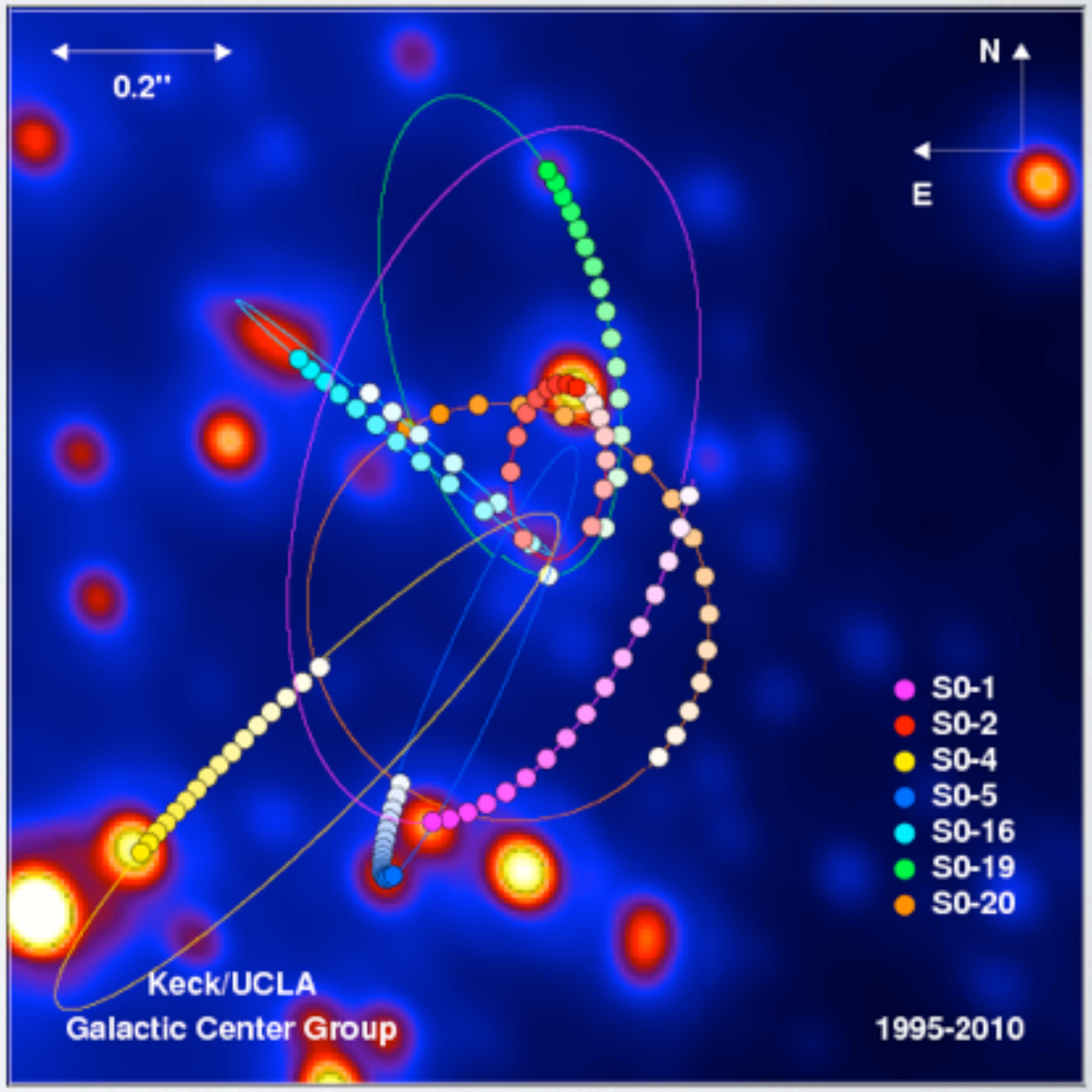
- Motivation and progress
- Phase evolution
- Orbit evolution with osculating orbits and geodesic gravitational self-force (GSF)
- Inspiral comparisons for the scalar self-force (SSF)
- Future directions



Motivation



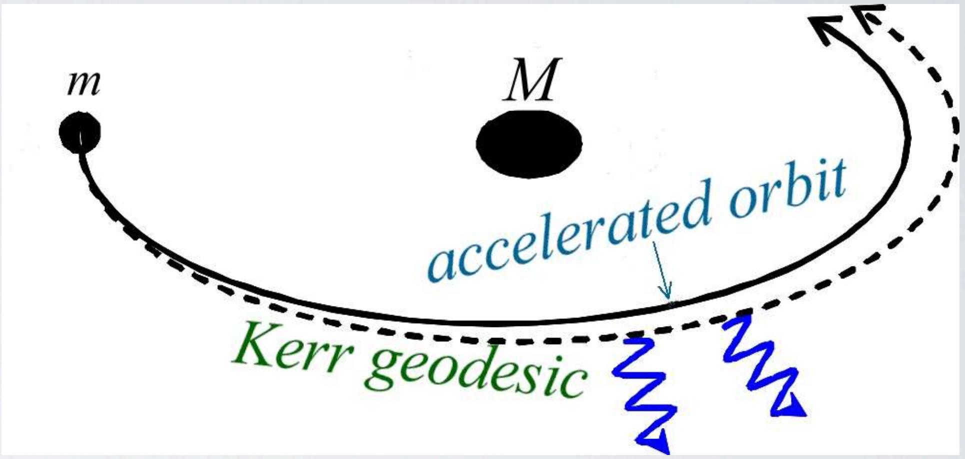
- Model gravitational waves from Extreme/Intermediate-mass-ratio inspirals
- Allows the spacetime around the central black hole to be mapped out. Resolve the Kerr hypothesis.
- Compare perturbative results with other approaches to the GR two-body problem



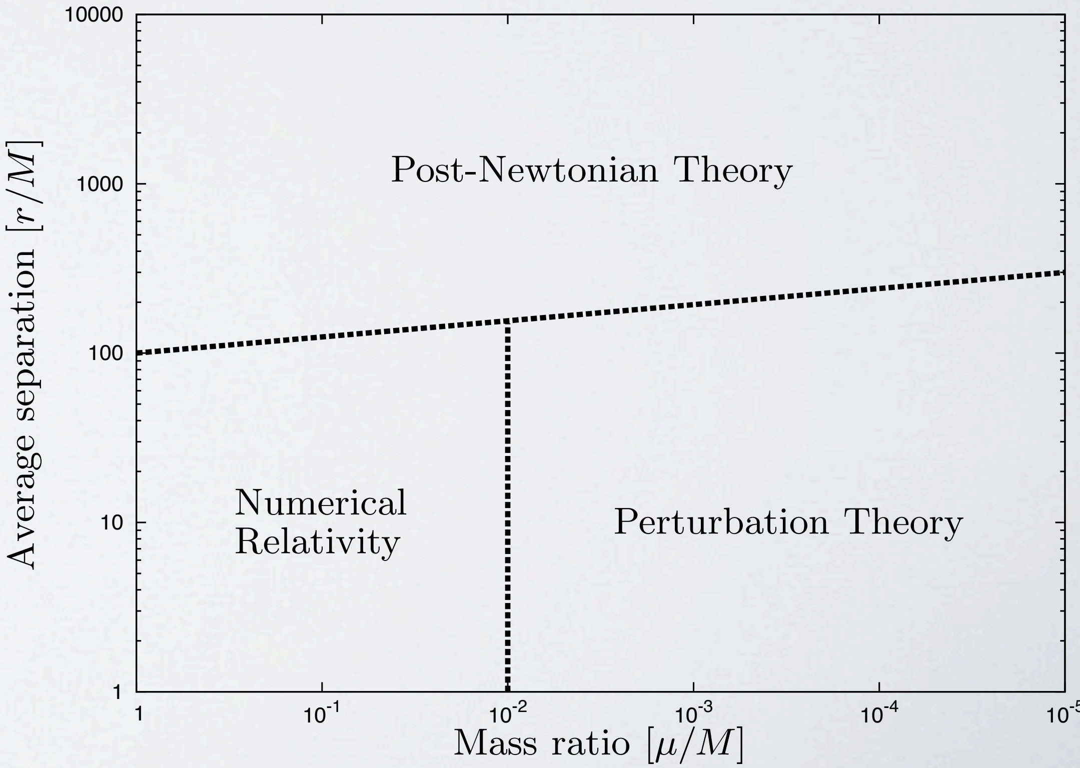
See Gair's talk for eLISA motivation

Self-force: overview

$$F_\alpha(\tau) = q^2 \int_{-\infty}^{\tau^-} \nabla_\alpha G_{\rm ret}(z(\tau), z(\tau'))\, d\tau'$$

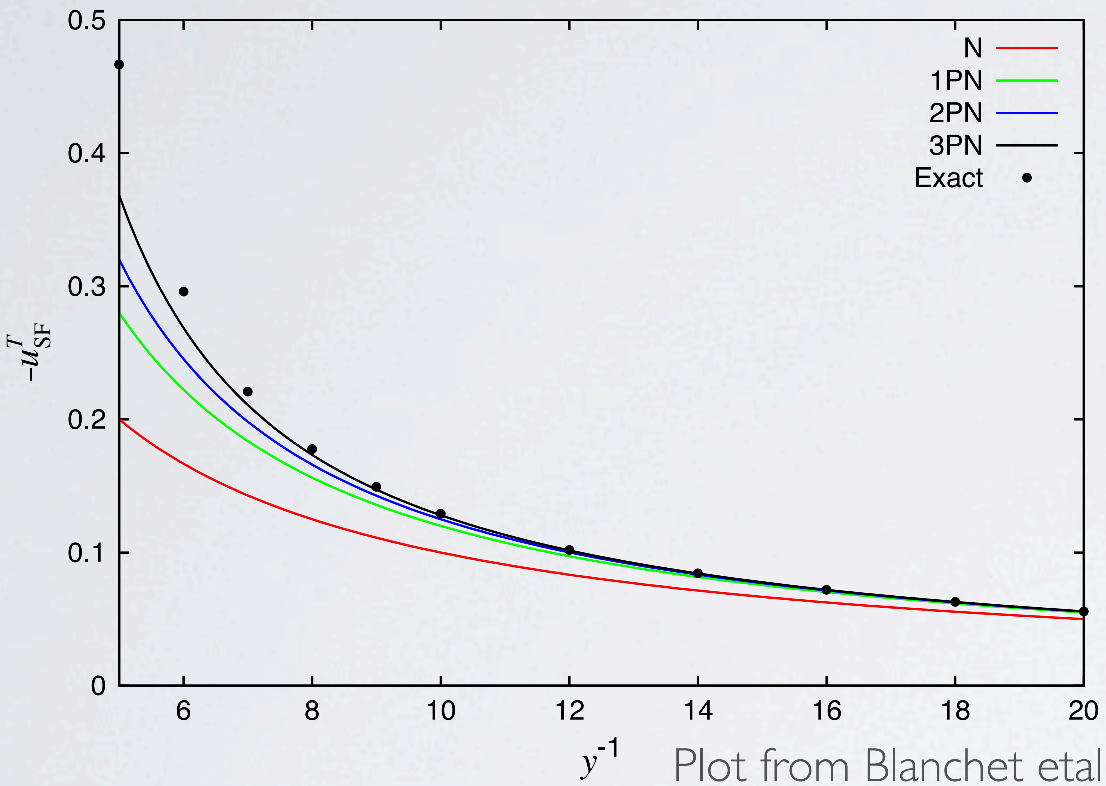


- Gravitational self-force is known for generic bound geodesic orbits in Schw. spacetime
- Progress in Kerr spacetime (see Dolan's talk)
- Progress in non-Lorenz gauges (see Merlin's talk)
- Calculation of gauge invariant quantities and comparisons with other approaches to the two body problem



Self-force: comparisons

Comparison with post-Newtonian

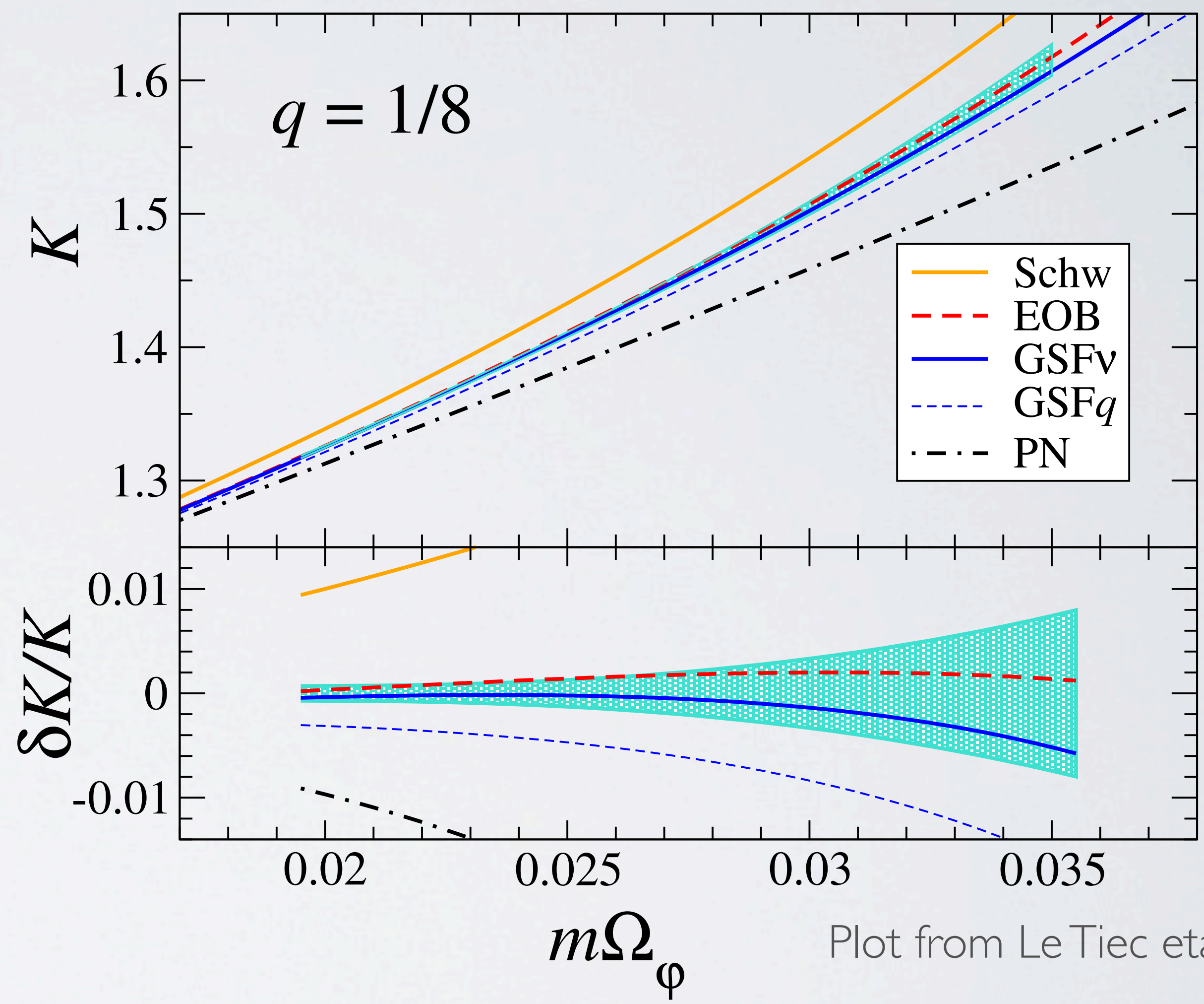


Plot from Blanchet etal.

Assess PN in the strong-field and extract higher-order PN parameters

See Shah's talk

Comparison with numerical relativity



Plot from Le Tiec etal.

Self-force: comparisons

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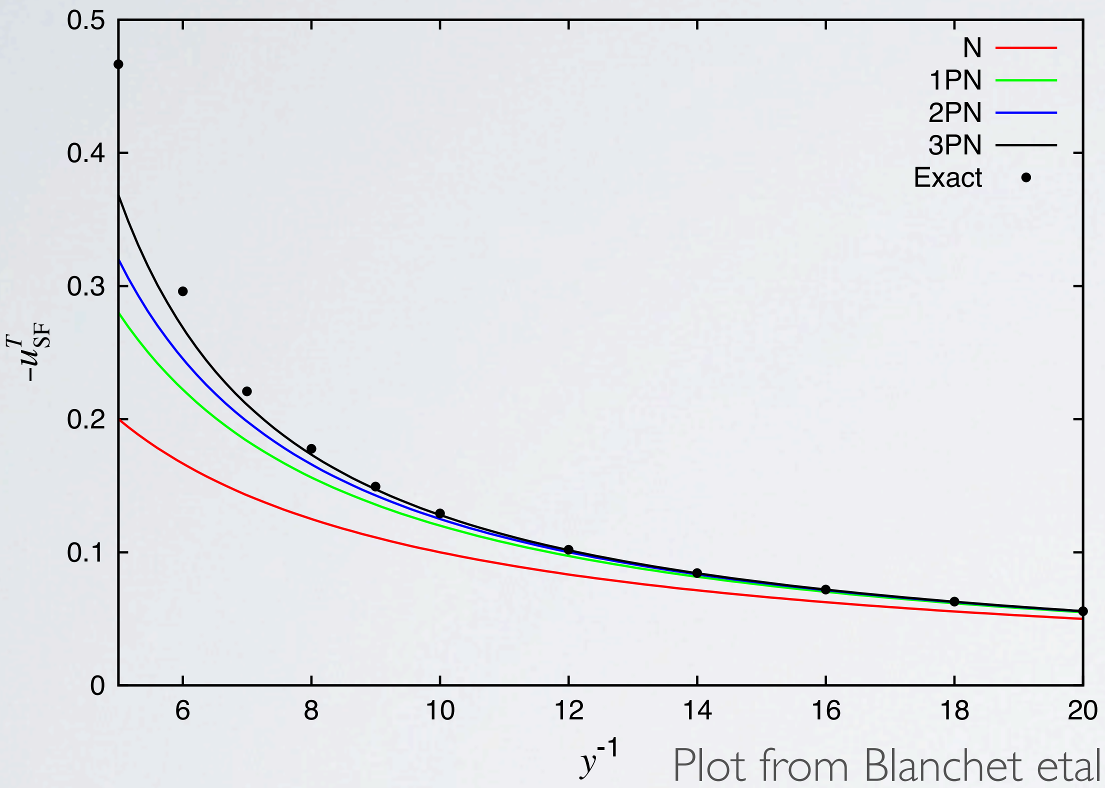

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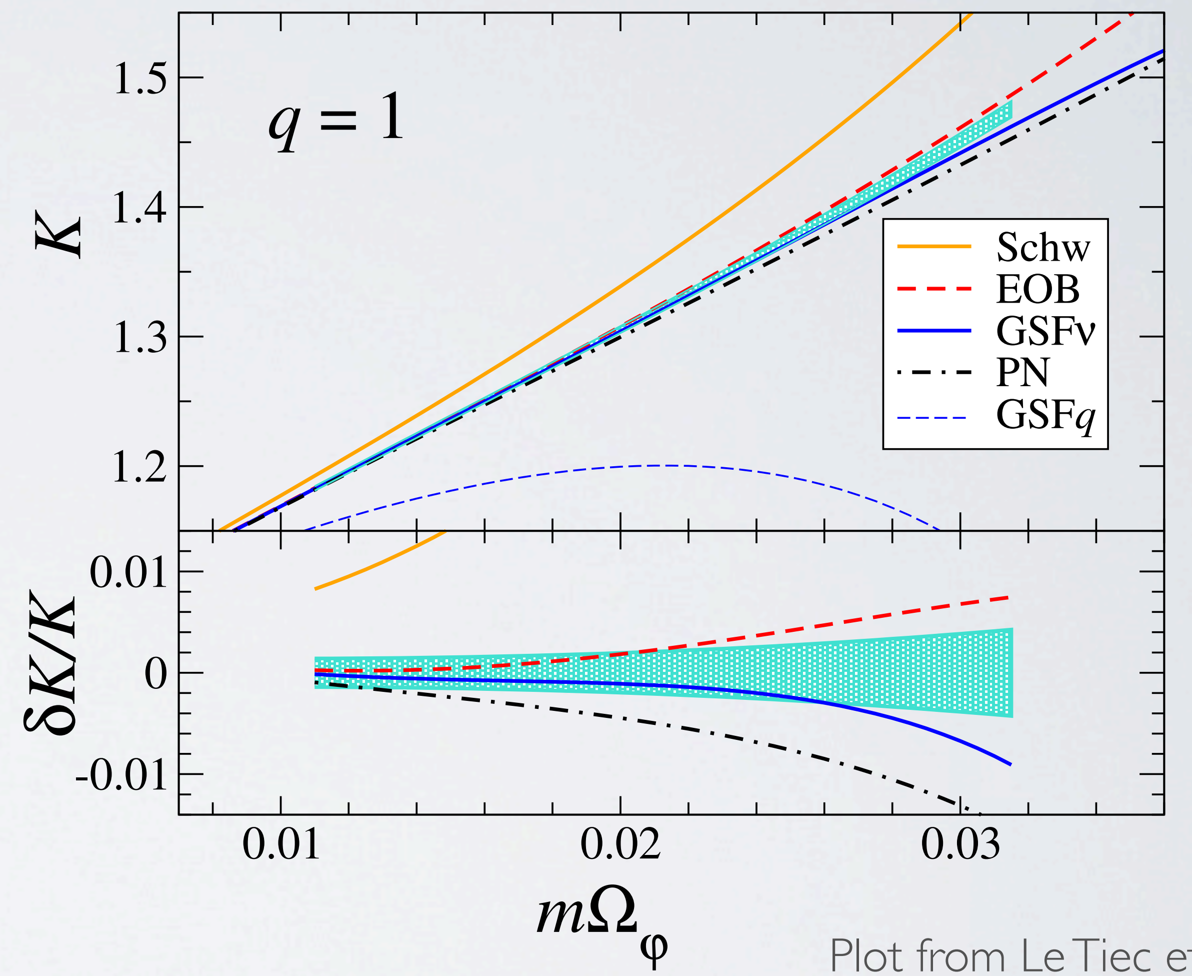


Plot from Le Tiec etal.

Domain of validity of perturbation theory may be much greater than first thought

Phase evolution

With $\epsilon = \mu/M$ Hinderer and Flanagan showed that the phase evolution of a generic inspiral about a Kerr black hole scales as:

$\mathcal{O}(\epsilon^{-1})$: Orbit-averaged dissipative component of the SF

$\mathcal{O}(\epsilon^{-1/2})$: Resonances, oscillating SF no longer averages out

$\mathcal{O}(\epsilon^{0})$: $\begin{cases} \text{Oscillatory component of the dissipative SF} \\ \text{Conservative component of the SF} \\ \text{Orbit-averaged dissipative piece of the second-order SF} \end{cases}$

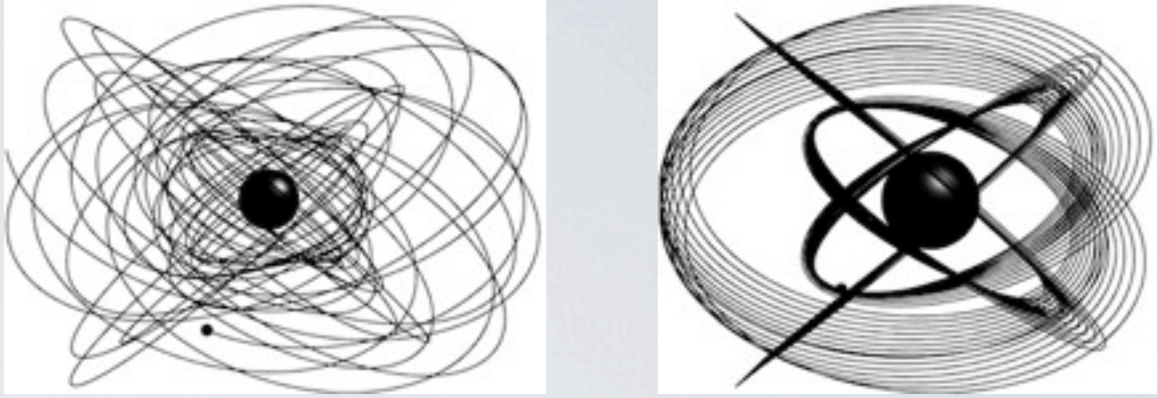
(See Pound's talk)

Leading order inspiral

Movie from S. Drasco

Change in $\mathcal{E}$ and $\mathcal{L}$ from flux balance. Adiabatic change in $\mathcal{Q}$ building on Mino's work

Resonances



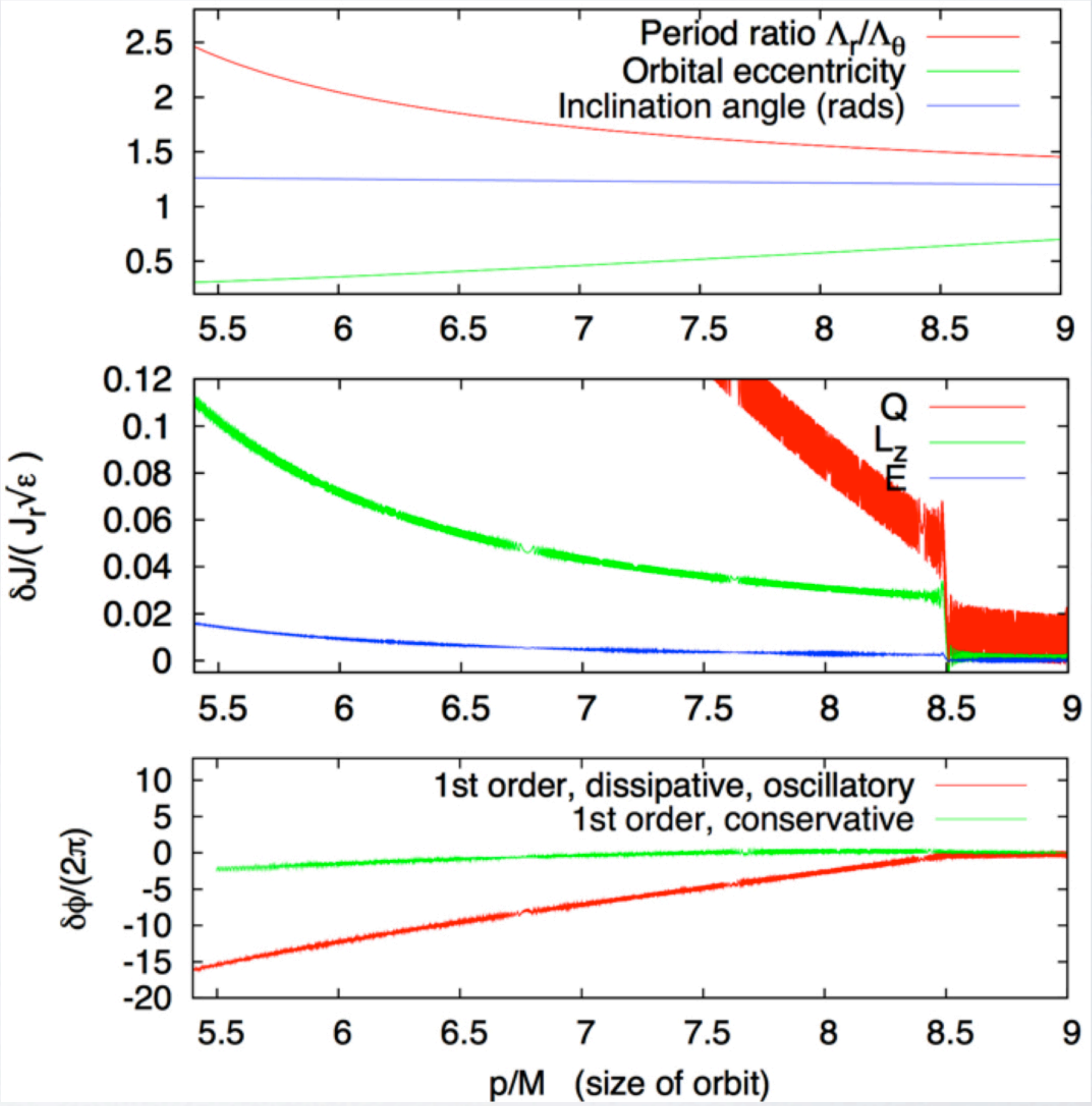
Images from S. Drasco

- Only occurs for generic orbits about Kerr black holes
- Occurs when:

$$\frac{\Omega_r}{\Omega_\theta} = \frac{n_1}{n_2}$$

where n_1 and n_2 are coprime with small ratio

- Causes a large change in the orbital parameters, but only when the oscillatory parts of the self-force are included
- Location of resonance will be a good test of the Kerr metric



Plot from Hinderer and Flanagan

See van de Meent, Cañizares, Tanaka and Isoyama's talks

Inspirals in Schwarzschild spacetime using geodesic self-force data

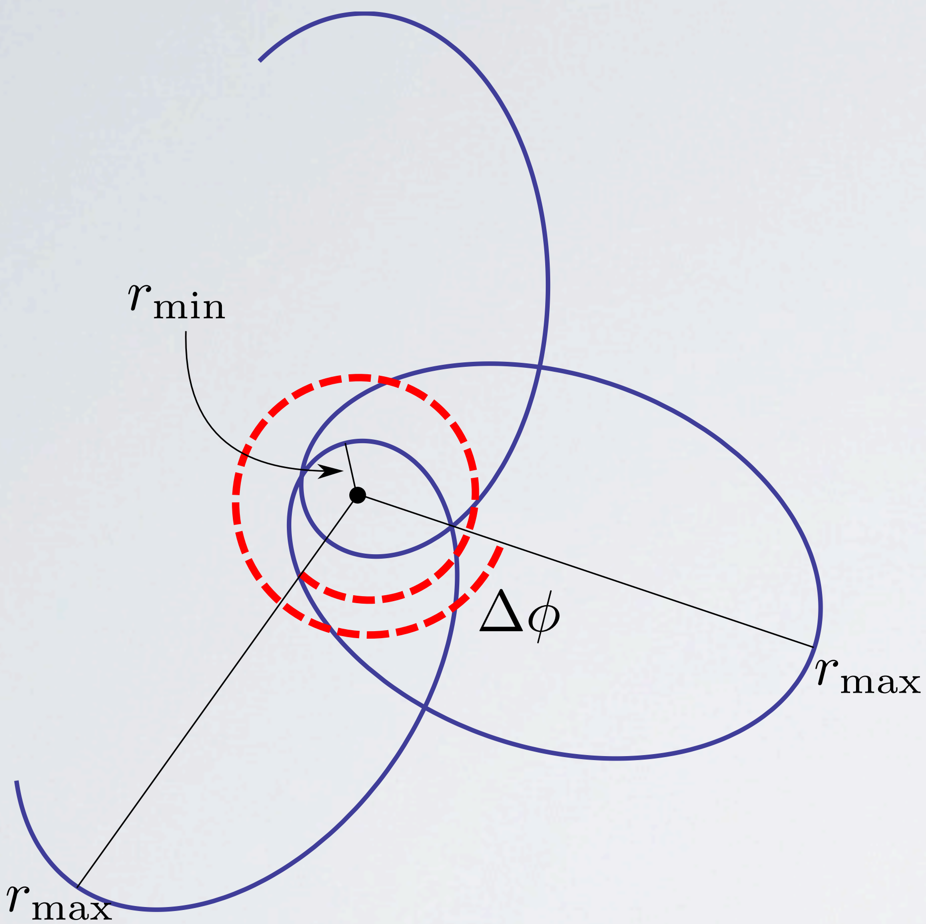
NW, S. Akcay, L. Barack, J. Gair and N. Sago
Phys. Rev. D 85.061501(r)

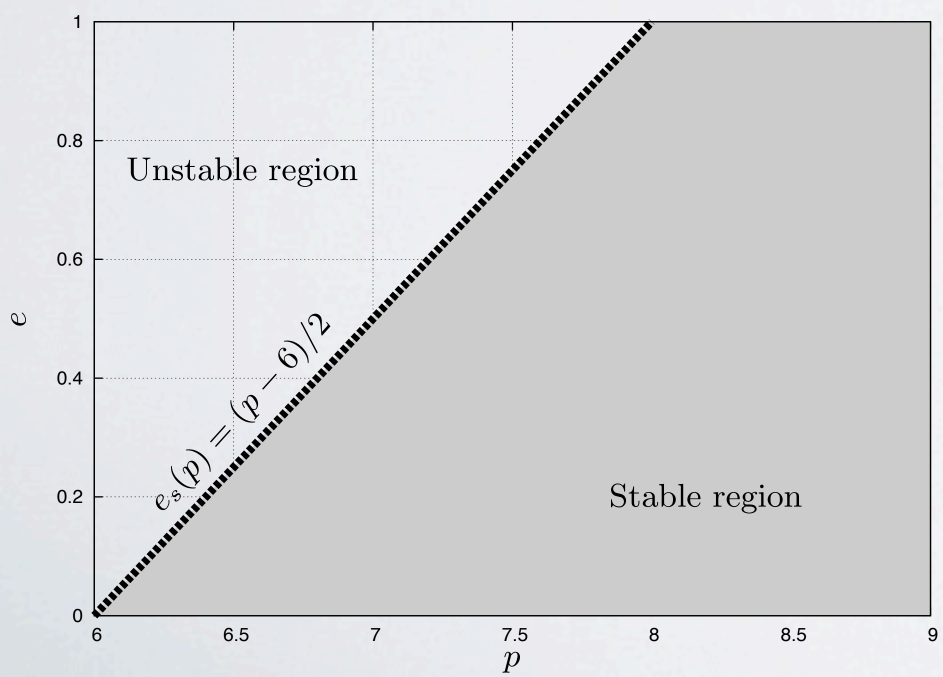
$\mathcal{O}(\epsilon^{-1})$: Orbit-averaged dissipative component of the SF

$\mathcal{O}(\epsilon^{-1/2})$: ~~Resonances, oscillating SF no longer averages out~~

$\mathcal{O}(\epsilon^{0})$:
- Oscillatory component of the dissipative SF
- Conservative component of the SF
- ~~Orbit-averaged dissipative piece of the second-order SF~~

Orbital parametrization





Up to orientation, bound geodesic orbits in Schw. spacetime are uniquely specified by $\mathcal{E}$ and $\mathcal{L}$.

We use an alternative parametrization: the semi-latus rectum, p, and the orbital eccentricity, e.

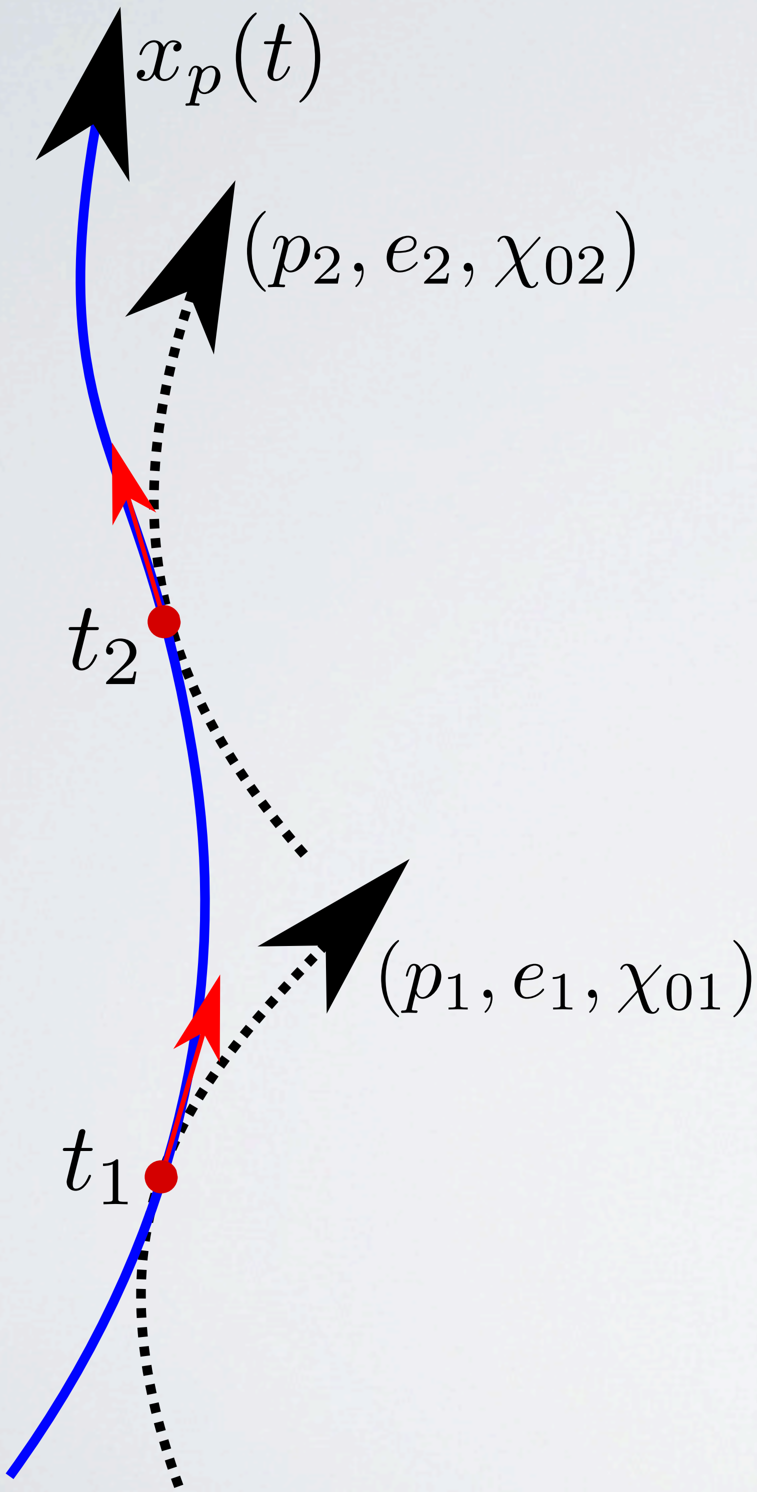
$$p \equiv \frac{2 r_{\max} r_{\min}}{M(r_{\max} + r_{\min})} \qquad e \equiv \frac{r_{\max} - r_{\min}}{r_{\max} + r_{\min}}$$

Also introduce a relativistic anomaly parameter, χ , such that

$$r(t) = \frac{pM}{1 + e\cos[\chi(t) - \chi_0]}$$

where χ_0 is the periastron phase

Orbit evolution: osculating orbits



At t_1 the position and velocity of the inspiralling particle corresponds to that of an osculating ('kissing') geodesic. In general, at later times $x_p(t)$ and the osculating geodesic will diverge. If instead $\{p, e, \chi_0\} \to \{p(t), e(t), \chi_0(t)\}$ the trajectory can be described by a sequence of osculating geodesics.

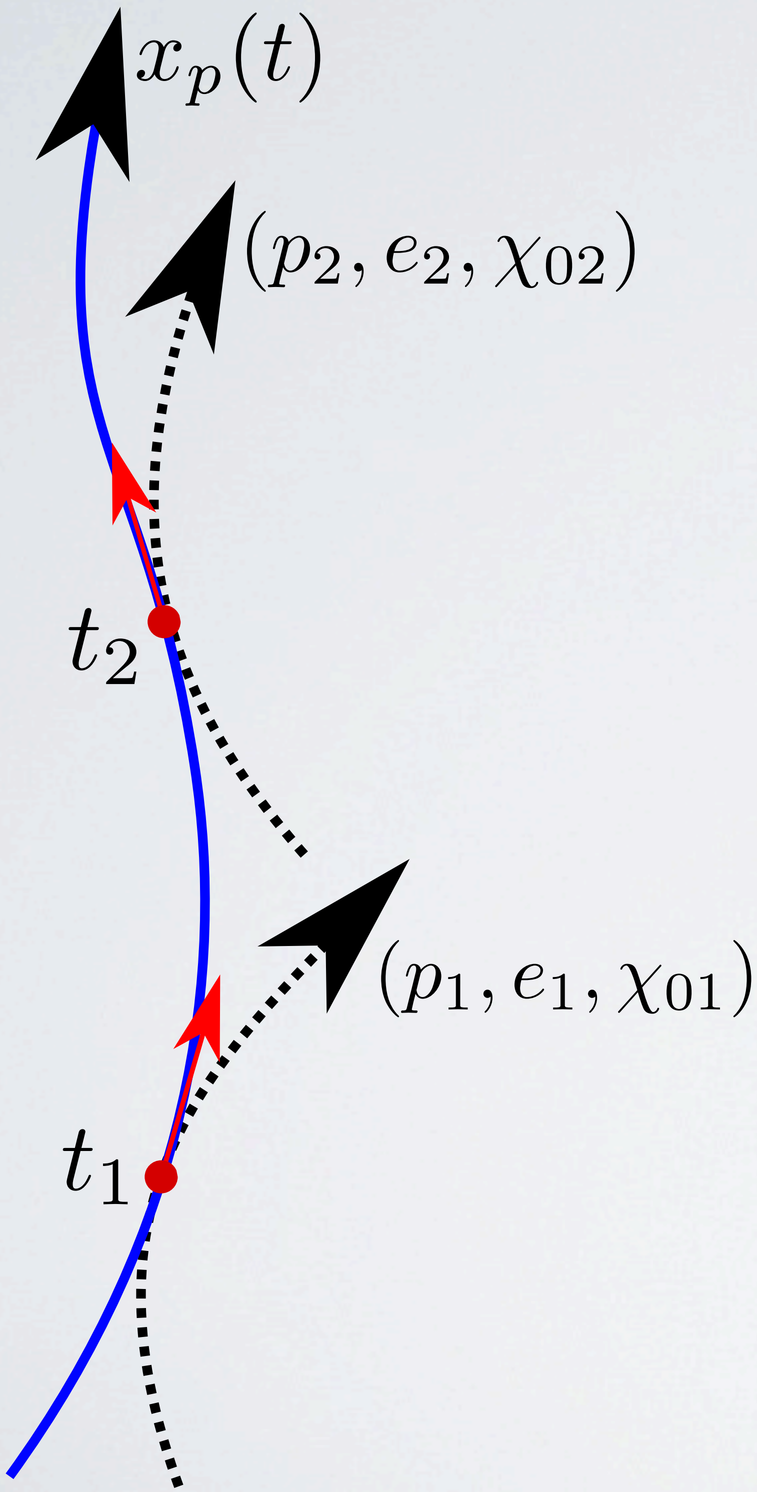
There is no requirement in this formulation that the force be small

Pound and Poisson derived the evolution equations in Schw. spacetime

$$\dot{p} = \mathcal{F}_p[p, e, \chi - \chi_0, F_{\rm self}^{\rm diss}(t)]$$
$$\dot{e} = \mathcal{F}_e[\cdots]$$
$$\dot{\chi_0} = \mathcal{F}_{\chi_0}[p, e, \chi - \chi_0, F_{\rm self}^{\rm cons}(t)]$$

Gair etal. gave the extension to forced motion in Kerr spacetime

Orbit evolution: key assumption



The true self-force is given by an integral over the entire past history of the particle

$$F_\alpha(\tau) = q^2 \int_{-\infty}^{\tau^-} \nabla_\alpha G_{\rm ret}(z(\tau), z(\tau'))\, d\tau'$$

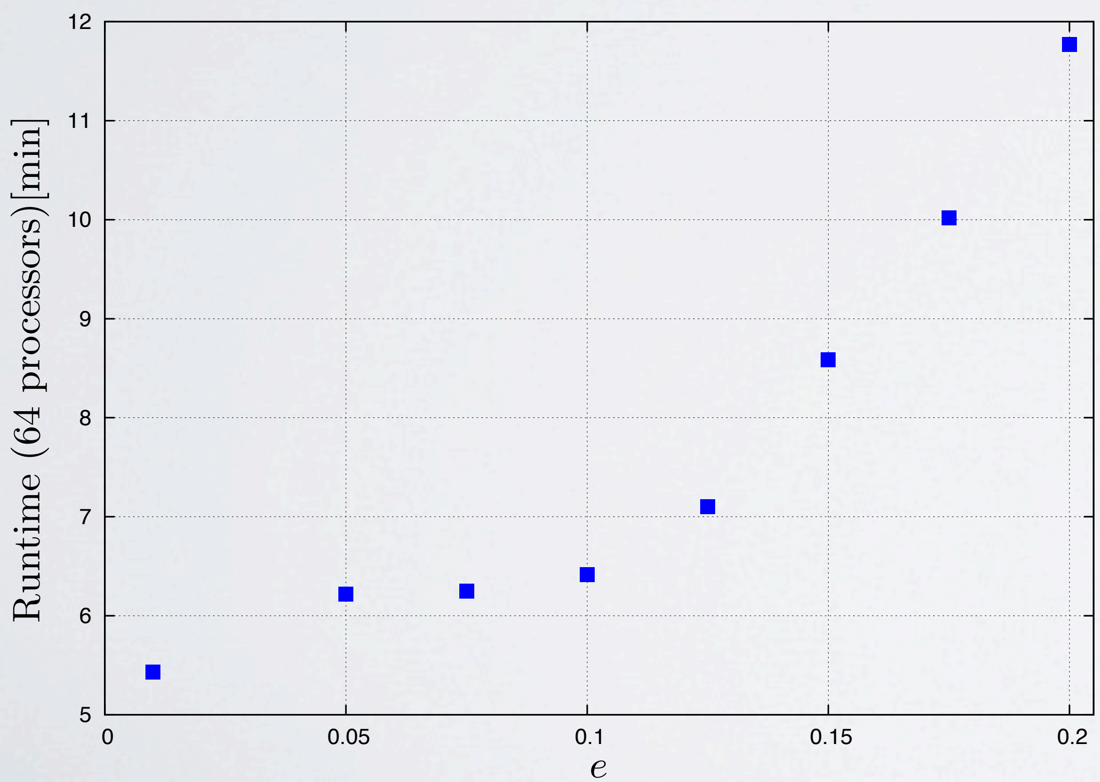
In this work we approximate the self-force at a give instance by that of the self-force of a particle that has spent its entire history on the corresponding osculating geodesic

$$\begin{aligned} \dot{p} &= \mathcal{F}_p[p, e, \chi - \chi_0, F^{\rm diss}_{\rm self}(p, e, \chi - \chi_0)] \\ \dot{e} &= \mathcal{F}_e[\cdots] \\ \dot{\chi_0} &= \mathcal{F}_{\chi_0}[p, e, \chi - \chi_0, F^{\rm cons}_{\rm self}(p, e, \chi - \chi_0)] \end{aligned}$$

It is not yet known how the phase error from the approximation scales with ϵ . See later on.

Self-force interpolation model

- Though we now have fast FD GSF codes it is still impractical to calculate F^r and F^φ for given $(p, e, v \equiv \chi - \chi_0)$ at each time step of the orbital evolution
- Instead we fit as much data as we can produce (see next slide) to a model



Efficiency of S. Akcay, NW and L. Barack code.
See also talks by Hopper, Forest and Osburn

We use a Fourier representation and fit the four GSF $\{F^r_{\rm cons}, F^\varphi_{\rm diss}, F^r_{\rm diss}, F^\varphi_{\rm cons}\}$ components as

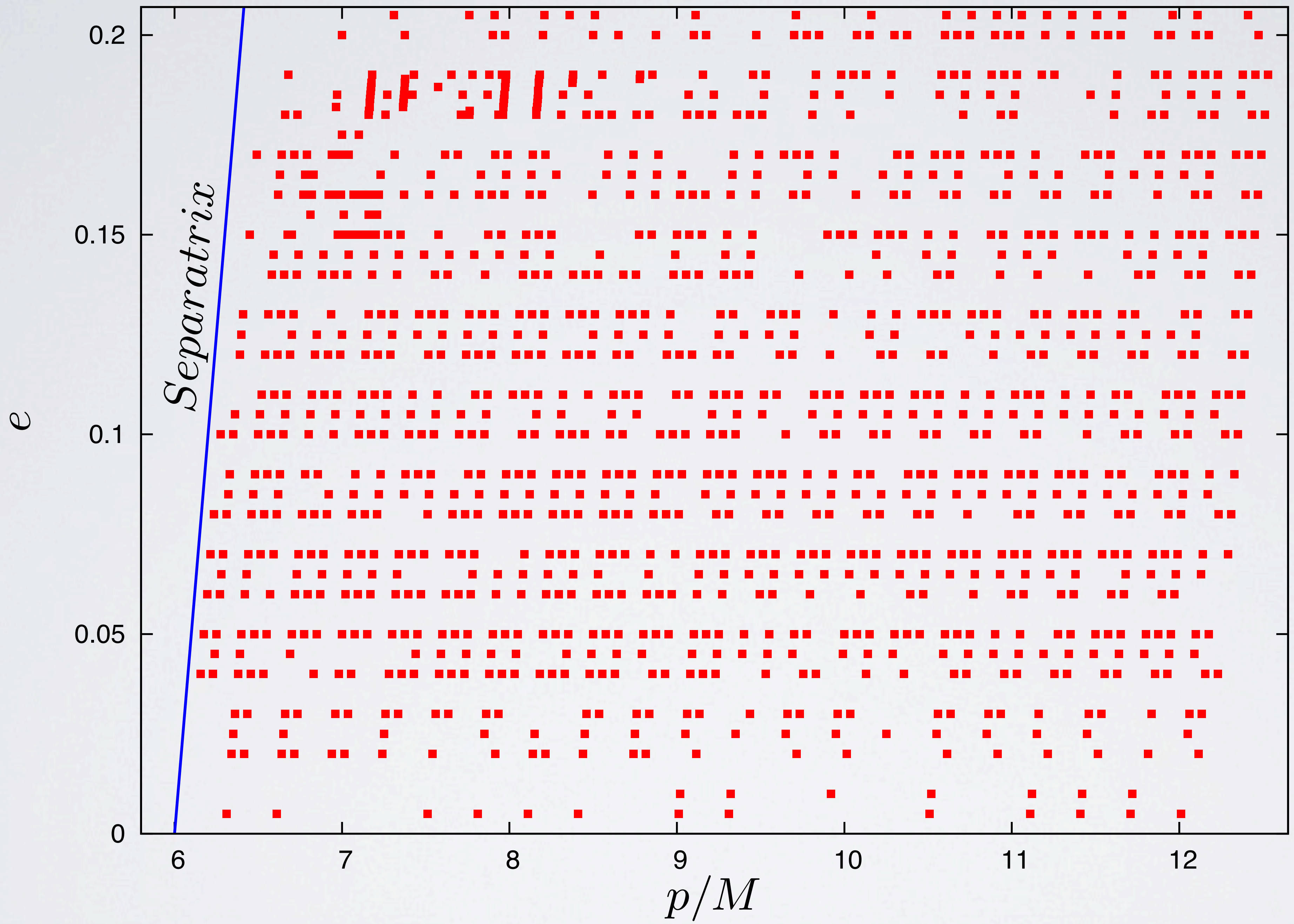
$$F^r_{\rm cons} = (\mu/M)^2 \sum_{n=0}^{\bar{n}} A_n(p,e)\cos(nv)$$

$$A_n(p,e) = p^{-2} \sum_{j=n}^{\bar{j}_a} \sum_{k=0}^{\bar{k}_a} a^n_{jk} e^j p^{-k}$$

Fit the a^n_{jk} using standard χ^2 minimization and seek a global accuracy of

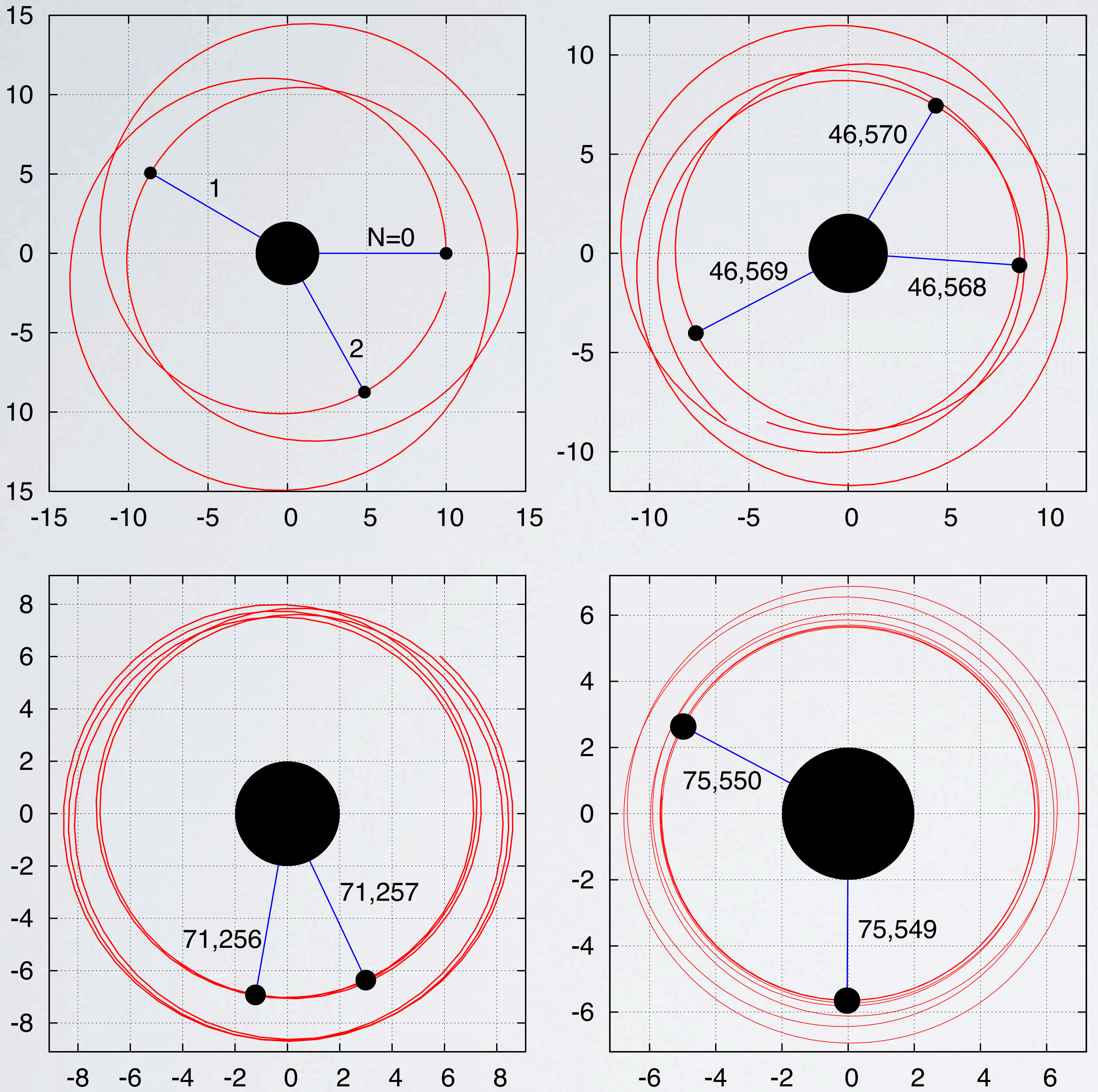
$$\delta F \equiv \frac{F(\text{model}) - F(\text{data})}{F(\text{data})} < 10^{-3}$$

Self-force interpolation model



Fit the model with data from over 1000 geodesics. Verified the fit by checking the results of the model against Barack and Sago's TD code.

Sample Inspiral: snapshots



One hour snapshots of orbital motion at (reading from top left) 1443 days, 500 days, 75 days and 1 hour to plunge

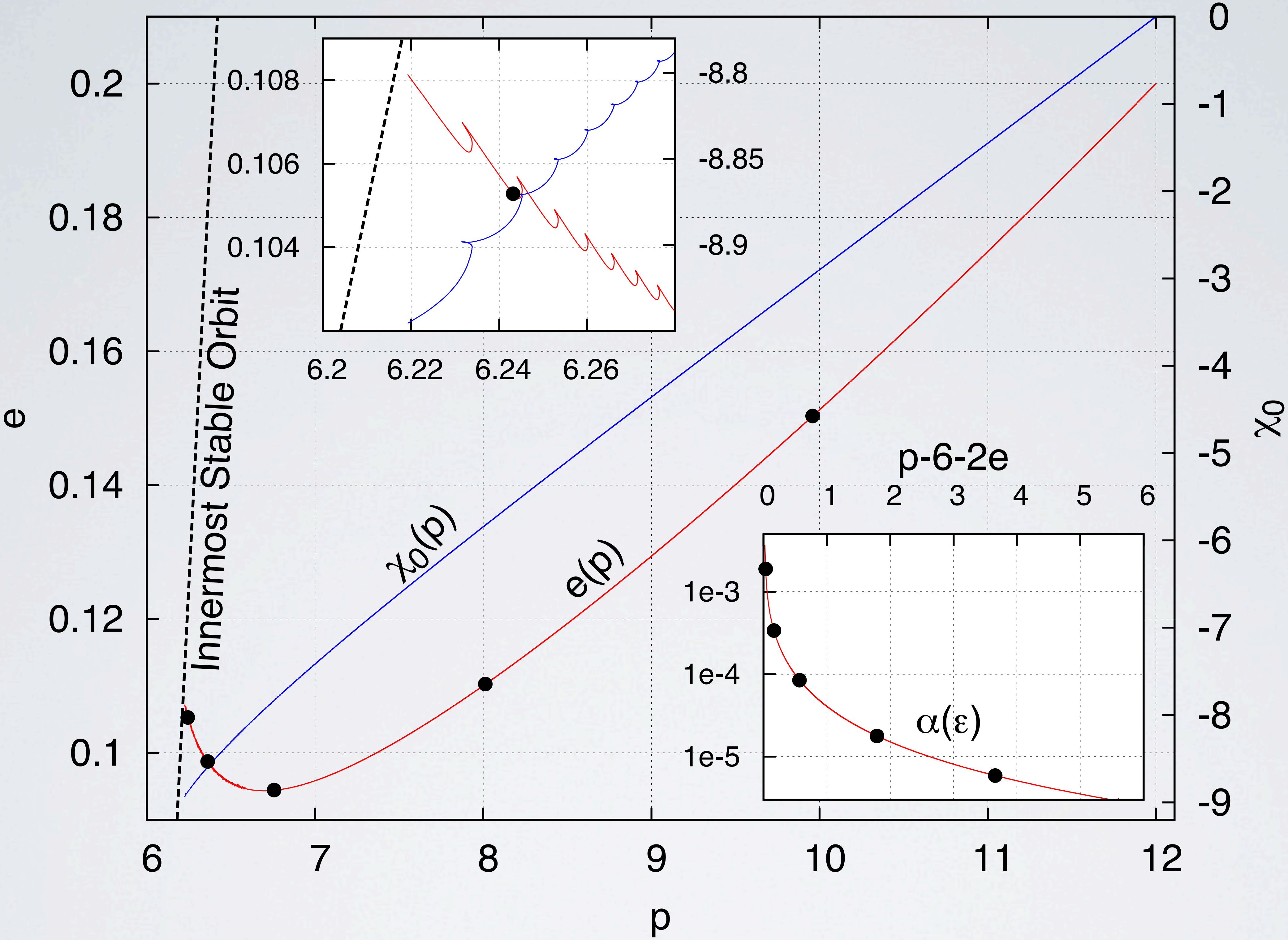
Masses:

$$\mu = 10 M_\odot, M = 10^6 M_\odot$$

Initial conditions:

$$(p_0, e_0, \chi_{00}) = (12, 0.2, 0)$$

Inspiral completes 75,550 periastron passages

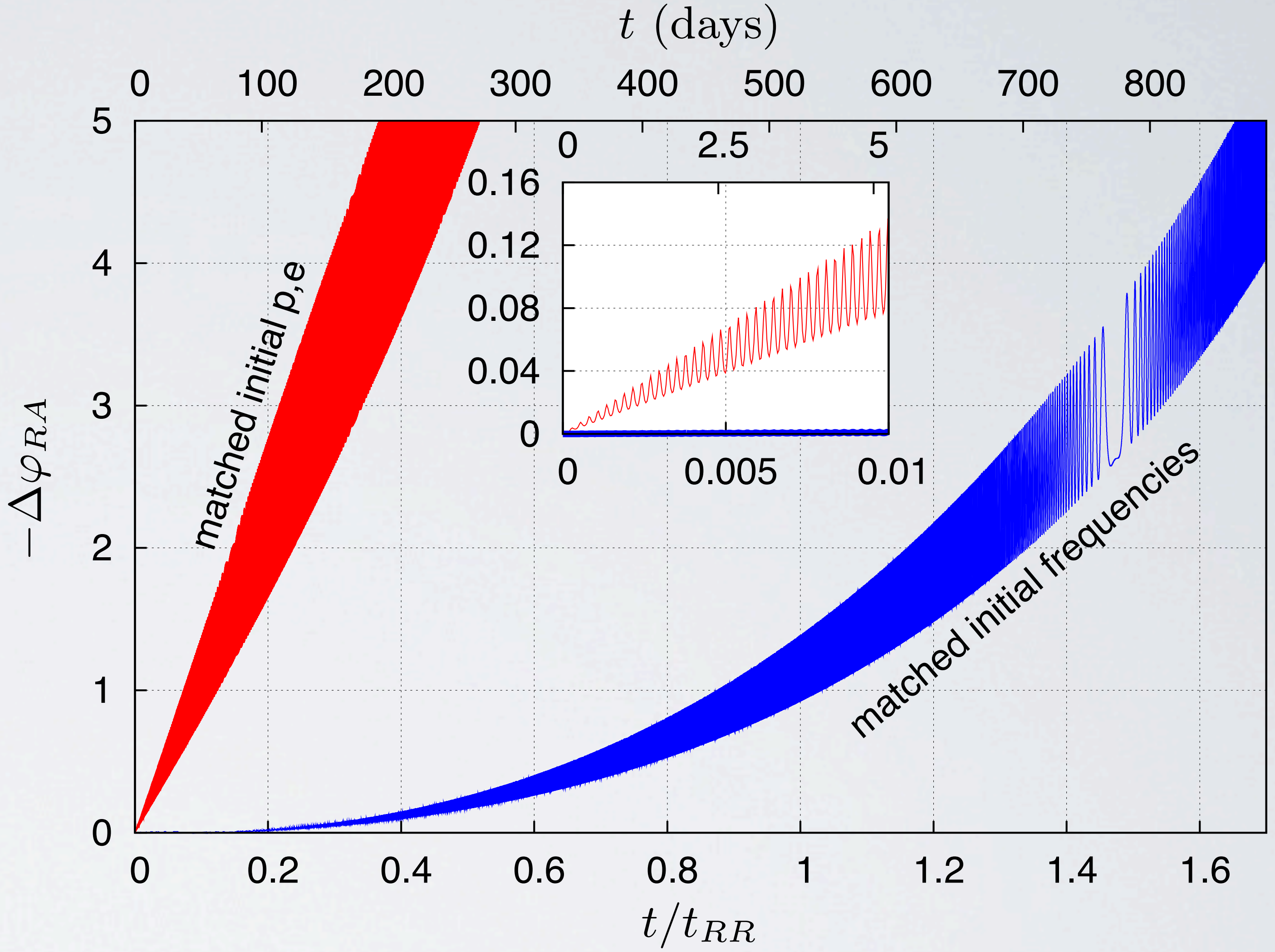


Black dots mark (from right) 500 days, 100 days, 10 days and 1 hour to plunge (note the left and right hand axes)

Construct a radiative approximation:

$$\dot{p} = \langle \mathcal{F}_p[p, e, v, F_{\rm self}^{\rm diss}] \rangle$$
$$\dot{e} = \langle \mathcal{F}_e[\cdots] \rangle$$
$$\dot{\chi_0} = \langle \mathcal{F}_{\chi_0}[p, e, v, F_{\rm self}^{\rm cons}] \rangle = 0$$

where $\langle \cdot \rangle$ is a t-average over the instantaneous osculating geodesic



Then consider difference in accumulated phase:

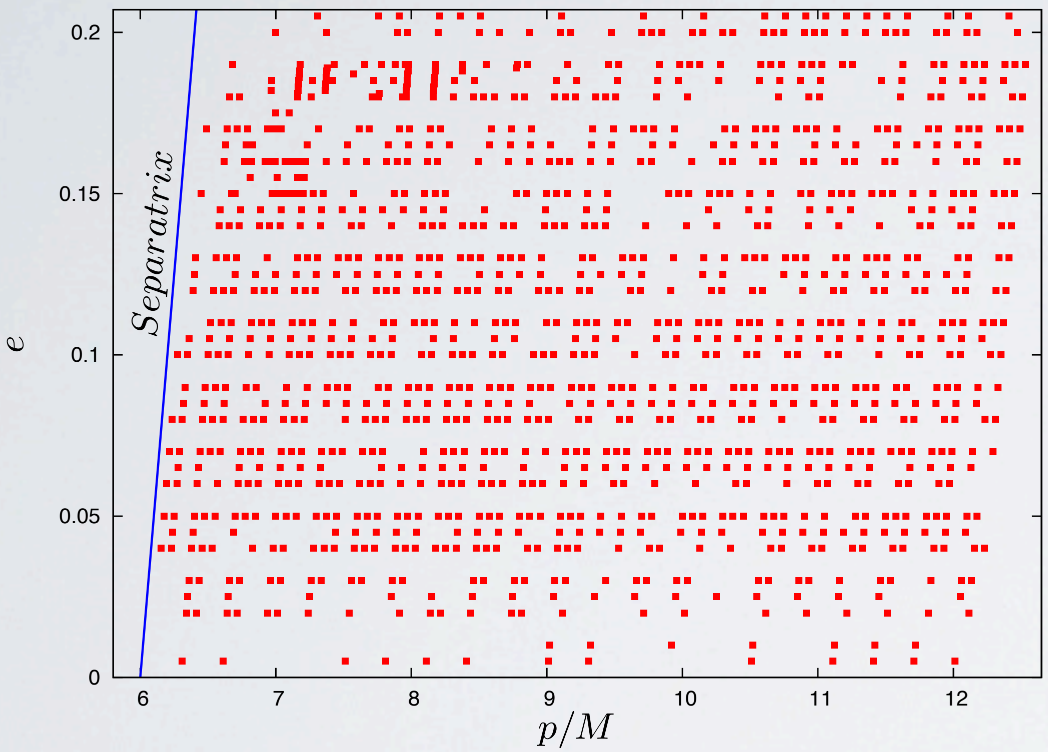
$$t_{RR} = (M/\mu)T_c, \qquad T_c = 2\pi/6^{3/2}M$$

$$\Delta\varphi_{\rm RA} \equiv \varphi^{\rm full} - \varphi^{\rm RA}$$

'Beating' waveforms

Inspiral comparison: scalar case

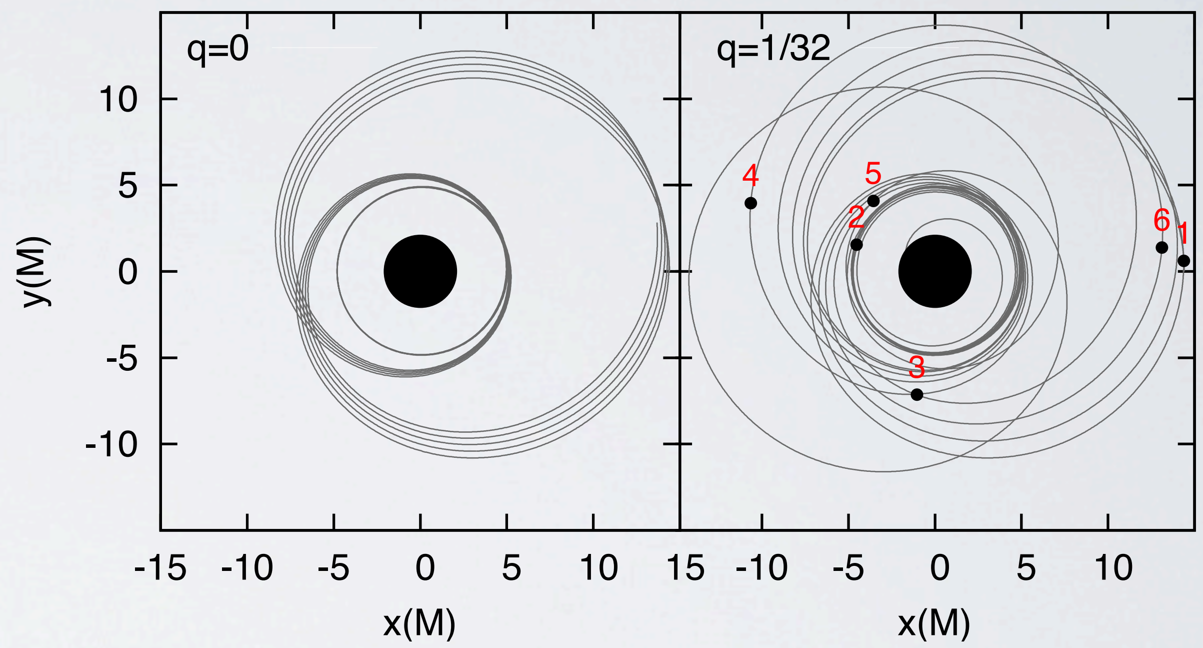
Osculating orbits with geodesic SSF



Fit model with 1000 geodesics worth of SSF data computed using FD code using mode-sum regularization [see Heffernan's talk]

NW

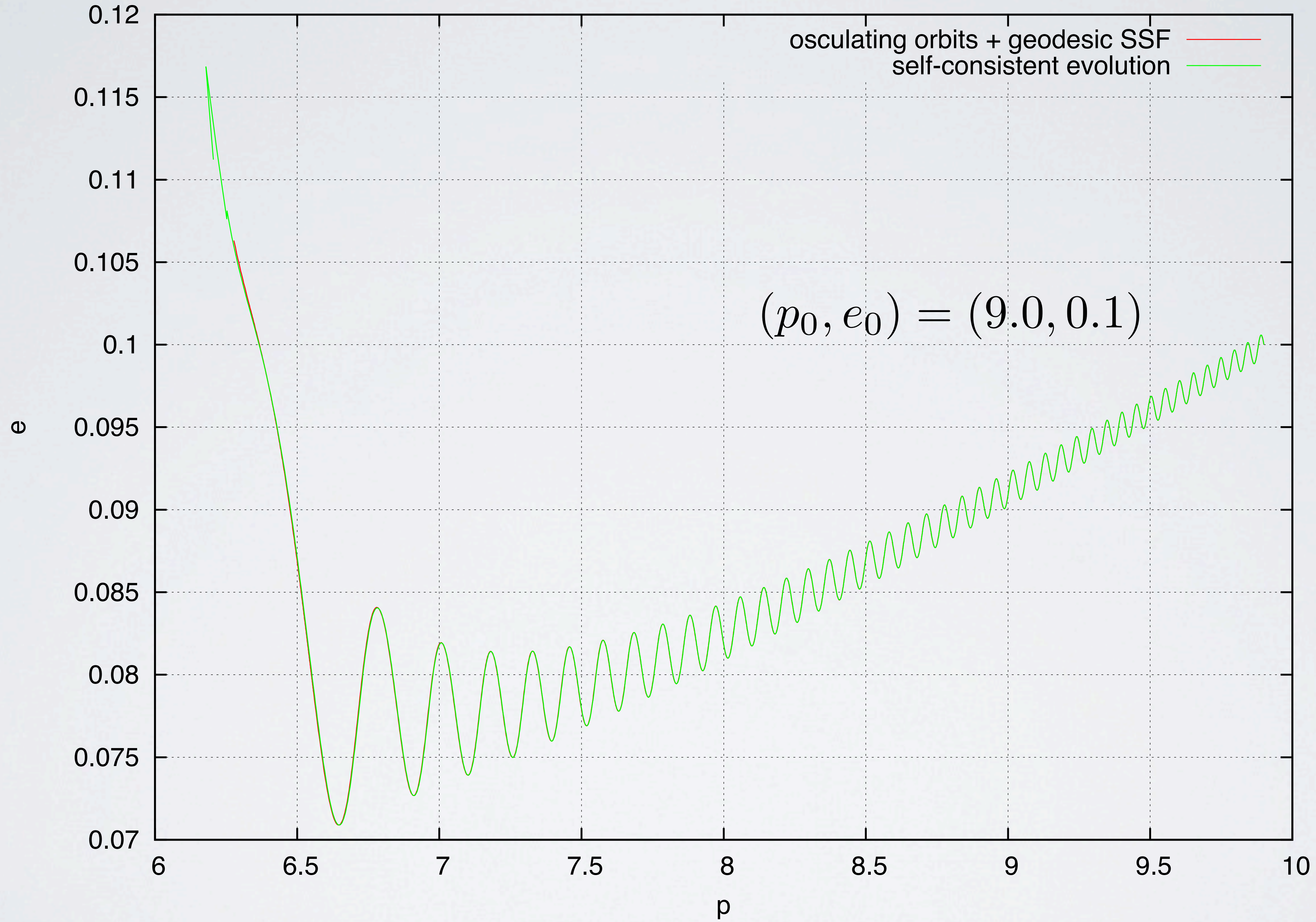
Self-consistent evolution



3+1 time domain code using the effective source approach [see Diener's talk]

Deiner, Vega and Wardell

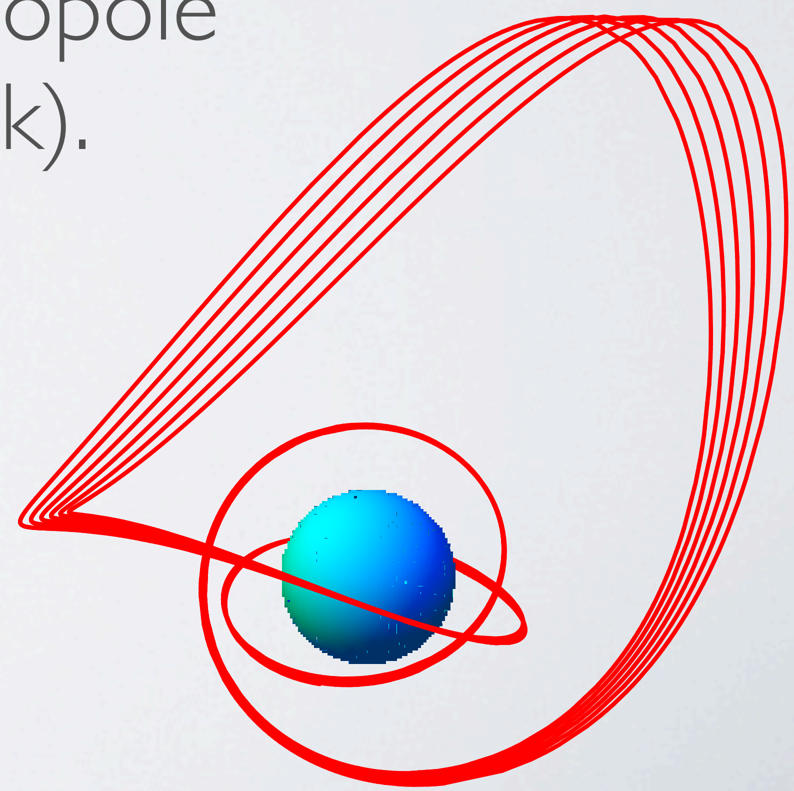
Inspiral comparison: preliminary results



Evolutions are indistinguishable to within the numerical error from the 3+1 code

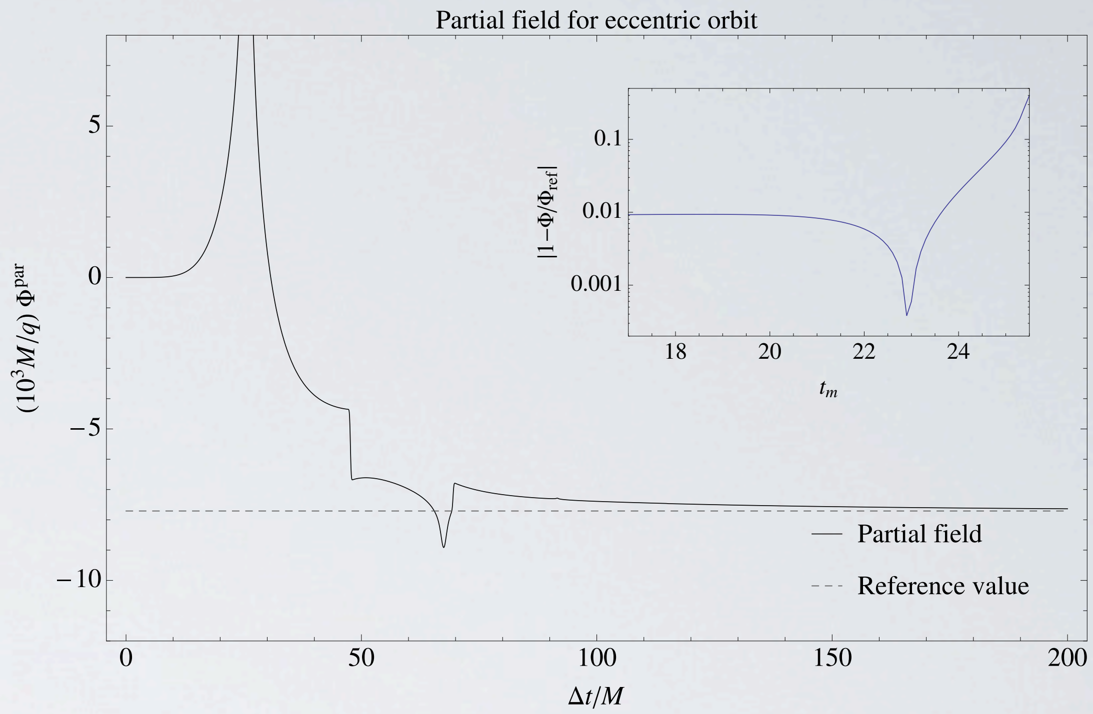
Future directions

- Increase accuracy of self-consistent evolution. Progress on a 1+1D effective-source code (See Wardell's talk)
- Ascertain the scaling with ϵ of the error from using osculating orbits. Can we come up with an analytical argument for the scaling?
- Extension to gravity. Stability issues of the monopole and dipole in the time-domain (see Dolan's talk).
- Extension to Kerr. Is osculating orbits with geodesic SF practical here?



Future directions

Recent work by Casals etal. allows for the calculation of the retarded Greens function

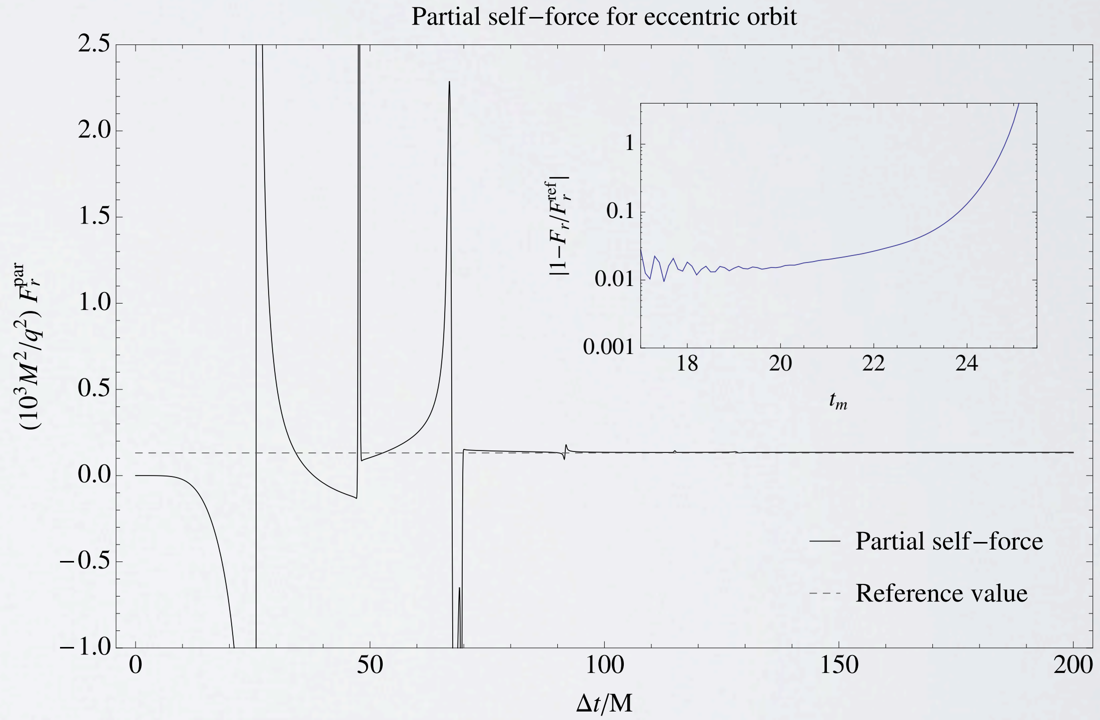


Self-force along a tangent geodesic

$$F^g_\alpha(\tau) = q^2 \int_{-\infty}^{\tau^-} \nabla_\alpha G_{\rm ret}(z(\tau), z^g(\tau'))\, d\tau'$$

Self-force along the inspiral

$$F^i_\alpha(\tau) = q^2 \int_{-\infty}^{\tau^-} \nabla_\alpha G_{\rm ret}(z(\tau), z^i(\tau'))\, d\tau'$$



Plots courtesy of Casals, Dolan, Ottewill and Wardell
orbit parameters (p,e) = (7.2, 0.5)

How do the Greens functions differ?

For details see Casals' talk