Scalar self-force for highly eccentric orbits in Kerr spacetime

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in collaboration with

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Outline

Goals, overall plan of the computation

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Brief review of effective-source (puncture-function) regularization

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- *m*-mode decomposition
- separate 2+1D time-domain evolution for each mode
- worldtube scheme
  - finite differencing across the worldtube boundary
  - moving the worldtube
- computing the effective source and puncture function
- finite differencing near the particle (where fields are only C^2)
- mesh refinement

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Sample results

Conclusions, Plans, Lessons Learned

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Work in progress: some goals accomplished, some not yet!

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- worldtube moves in (r, θ) to follow the particle around the orbit
- Cauchy evolution
- fixed mesh refinement; some (finer) grids follow the worldtube/particle
- (almost) causally-disconnected spatial boundaries
  (with mesh refinement this isn't very expensive)

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Even though $\varphi_p \neq \varphi_{\text{singular}}$, then self-force ie exact to $\mathcal{O}(\mu)$

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- define "numerical field" $\varphi_{\text{num}} = \begin{cases} \varphi_r & \text{inside the worldtube} \\ \varphi & \text{outside the worldtube} \end{cases}$

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- finite differencing must locally "adjust" (a copy of) φ_{num} by $\mp\varphi_p$ across the worldtube bndry to undo the jump discontinuity in φ_{num}

m-mode decomposition

Instead of numerically solving $\Box\varphi_{\text{num}} = \begin{cases} S_{\text{effective}} & \text{inside the worldtube} \\ 0 & \text{outside the worldtube} \end{cases}$

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- in practice, solve numerically for $0 \le m \le m_{\text{max}} \sim 20$;
  fit large-m asymptotic series to estimate "tail sum" $\sum_{m=m_{\text{max}}+1}^{\infty}$

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- with mesh refinement, having very large domain is not expensive

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- recall that our numerically-evolved field is
$$\varphi_{\text{num}} := \begin{cases} \varphi - \varphi_p & \text{inside the worldtube} \\ \varphi & \text{outside the worldtube} \end{cases}$$
this means then if we move the worldtube,
  - a given (r, θ) may change from being inside the worldtube to being outside $\Rightarrow$ must add φ_p
  - a given (r, θ) may change from being outside the worldtube to being inside $\Rightarrow$ must subtract φ_p
- I was worried that this would be a source of numerical noise
$\Rightarrow$ not a problem in practice (modulo bugs!)

Current Status

Equatorial eccentric orbits:

- elliptic-integral puncture fn & effective src
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- fixed mesh refinement with "hollow grids";
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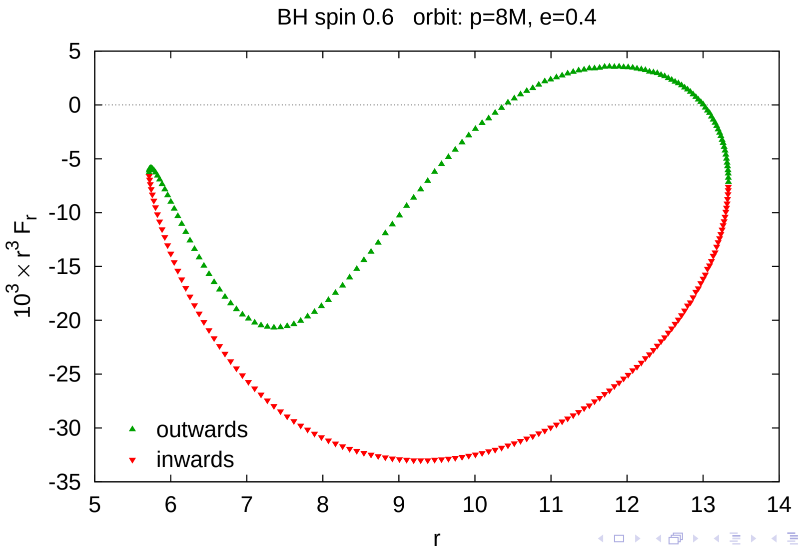
Generic (inclined eccentric) orbits:

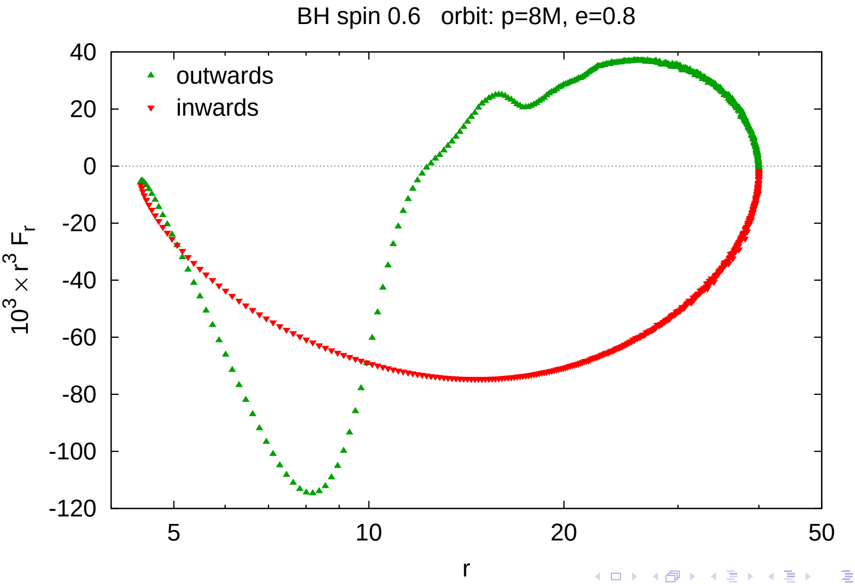
- our first attempt at an effective source had $\sim$ 20 million terms
  $\Rightarrow$ impractical to compile machine-generated C code
- we are starting to explore various ideas to reduce the complexity,
  and are optimistic we can solve this

Self-force for $e = 0.4$ orbit

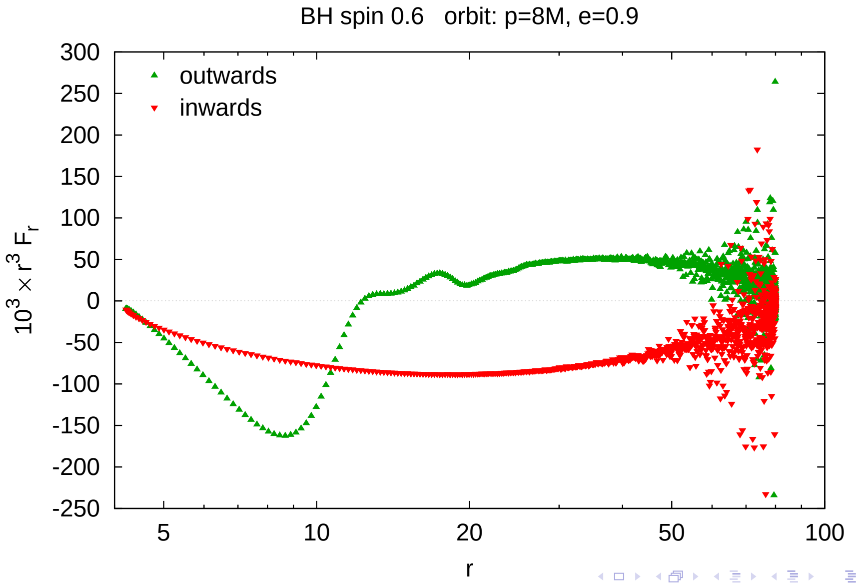
[[tail fit]]



Self-force for $e = 0.8$ orbit (preliminary)



Self-force for $e = 0.9$ orbit (**very** preliminary)



Conclusions

Things that work well:

- puncture-function regularization
- worldtube
- m-mode decomposition and 2+1D evolution
  - gives moderate parallelism "for free"
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Things that don't yet work well (a.k.a. directions for further research)

- evolved fields only $C^2 \Rightarrow$ hard to get higher-order finite-diff convergence
- inclined eccentric orbits $\Rightarrow$ effective src is too complicated to be usable