PN Radiation and reaction in EFT - an EM example

Shahar Hadar

with Ofek Birnholtz and Barak Kol

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Outline

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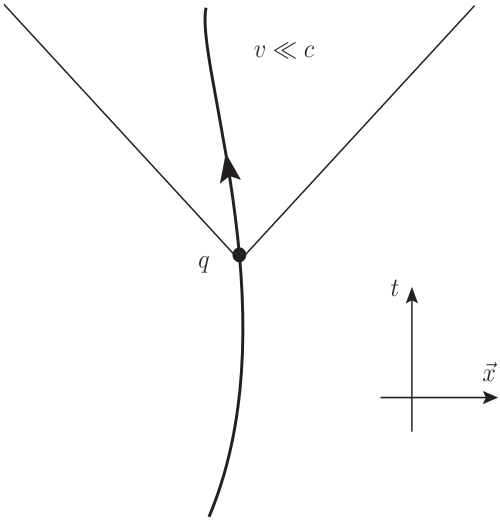
The problem



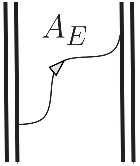
Motivation

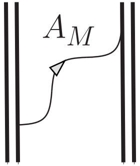
- 2-fold:
  - Observational: model PN GW sources.
  - Theoretical: analytic insight to GR 2-body problem.
- We have ALD:

$$\mathcal{F}^{\mu}_{ALD} = \frac{2}{3}\dddot{x}^{\mu}_{\perp}$$

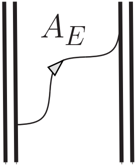
  Why look at EM in PN?
  - Composite systems of EM charges.
  - Lab for gravitational case: linear but many similarities.
- For gravity add nonlinearity (+ more gauge freedom).

Will simplify to:

$\hat{S}_{EM} =$ 

$+$ 

$\hat{S}_{EM} =$



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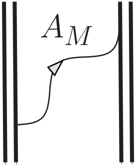
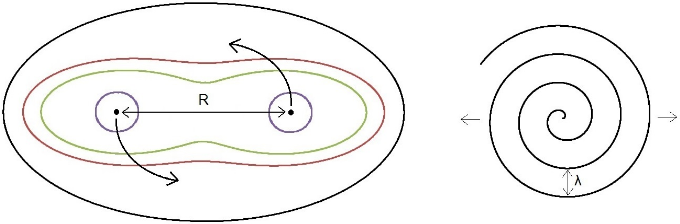


$$= \sum \left(Q \, G \, \hat{Q} \right)_E + \left(Q \, G \, \hat{Q} \right)_M$$

Zones & symmetries

In PN: $\lambda \propto \frac{R}{v} \gg R$.



- System zone: $\sim$ stationary.
- Radiation zone: $\sim$ spherical symmetry.

Odd propagation

- Dissipative RR force due to ϕ_{odd}.
- $\Box\phi_{odd} = 0 \Rightarrow \phi_{odd}$ determined by incoming waves.
- $\Rightarrow$ Matching regions automatically gives ϕ_{odd}.

Field doubling

- In order to treat classical dissipative effects need to double fields (Galley & Hu 2009, Galley 2012).
- In our version:

$$\hat{S} := \int dx \frac{\delta S[\phi]}{\delta \phi(x)} \hat{\phi}(x)$$

- Hatted in system zone $\Rightarrow$ linearization:

$$\hat{Q}[x, \hat{x}] = \int dt \, \frac{\delta Q[x]}{\delta x(t)} \, \hat{x}(t)$$

- EOM:

$$\frac{\delta \hat{S}}{\delta \hat{x}} = 0$$

The action

- Starting point: EM action

$$S = -\frac{1}{16\pi}\int d^4 x F^2 - \int d^4 x A_\mu J^\mu$$

- Reduce to 1D by decomposing

$$\begin{aligned} A_{t/r} &= \sum_{L\omega} A_{t/r}^{L\omega}\, x_L\, e^{-i\omega t} \\ A_\Omega &= \sum_{L\omega} \left(A_S^{L\omega}\, \partial_\Omega\, x_L + A_V^{L\omega}\, x_\Omega^L \right) e^{-i\omega t} \end{aligned}$$

Reduction to 1D

A few manipulations:

- eliminate ("integrate out") algebraic A^r
- $\partial_\mu J^\mu = 0$
- pack in E/M masterfunctions

$$S_{(E/M)} = \sum_{L\omega} \#(\ell) \int dr \left[A_L^* \hat{\mathcal{O}} A_L - \left(\rho_L^A A_L^* + c.c. \right) \right]$$

$$\hat{\mathcal{O}} := \frac{(2\ell+1)!!}{r^{2\ell+2}} \left[\omega^2 + \partial_r^2 + \frac{2(\ell+1)}{r} \partial_r \right]$$

Reading the far region Feynman rules

Propagator: solution of $\hat{\mathcal{O}} A_{L\omega} = \delta(r - r')$ with outgoing $\Leftrightarrow$ retarded BC.

$$G_{ret}(r', r) = \#(\ell)\, \omega^{2\ell+1}\, \tilde{j}_\ell(\omega r')\ \tilde{h}_\ell^{+}(\omega r)\ \delta_{LL'}$$

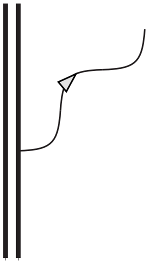
where:

$$\tilde{j}_l(x) := (2\ell + 1)!!\ \frac{j_\ell(x)}{x^\ell} \qquad\qquad \tilde{h}^{+/-} := \tilde{j} \pm i \tilde{h}$$

Reading the far region Feynman rules

- Work in far region $\Rightarrow$ eliminate near region.
- Vertices defined as

$$Q^{(E/M)}_{L\omega} \equiv$$



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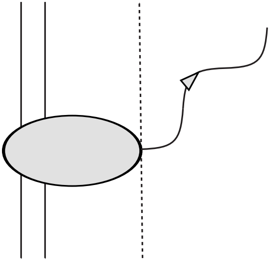


Reading the far region Feynman rules

- In far region source is at $r = 0$:

$$A^L_{EFT} = -Q^E_{L\omega}\#(\ell)\omega^{2\ell+1}\tilde{h}^+_\ell(\omega r)$$

- In the full theory the solution outside the sources is

$$A^L = -\left[\int dr' \tilde{j}_\ell(\omega r')\rho^A_{L\omega}(r')\right]\omega^{2\ell+1}\#(\ell)\tilde{h}^+_\ell(\omega r)$$

Reading the far region Feynman rules

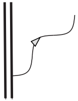
- Matching gives the E/M radiation source multipoles

$$Q_E^L = \int \frac{d^3x}{\ell+1}\left[\frac{1}{r^\ell}\left(r^{\ell+1}\tilde{j}_\ell(ir\partial_t)\right)'\rho(\vec{x}) - \tilde{j}_\ell(ir\partial_t)\,\partial_t\vec{J}(\vec{x}\,')\cdot\vec{x}\right] x_{TF}^L$$

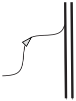
$$Q_M^L = \int d^3x\tilde{j}_\ell(ir\partial_t)\left[(\vec{r}\times\vec{J}(\vec{r}))^{k_\ell}x^{L-1}\right]^{STF}$$

Match to Thorne(1980), significantly economizing Ross(2012) in EFT.

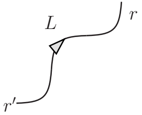
Feynman rules



$$= -Q_L$$



$$= -\hat{Q}_L^*$$

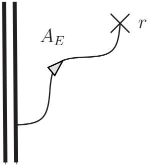


$$= G_{ret}(r', r) = \#(\ell)\omega^{2\ell+1}\, \tilde{j}_\ell(\omega r)\, \tilde{h}_\ell^+(\omega r')$$

- In GR - also interaction vertices...

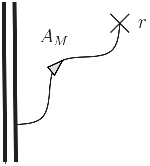
Radiation

$$A_E^L =$$



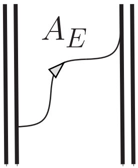
$$= -Q_E^{L'} G_{ret}(0,r) = \frac{\ell+1}{\ell}(-iw)^{\ell}\frac{Q_E^L}{r^{\ell}}\frac{e^{iwr}}{r}$$

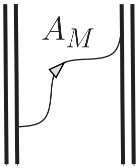
$$A_M^L =$$



$$= -Q_M^{L'} G_{ret}(0,r) = \frac{\ell}{\ell+1}(-iw)^{\ell}\frac{Q_M^L}{r^{\ell}}\frac{e^{iwr}}{r}$$

Reaction

$\hat{S}_{EM} =$ 

$+$ 

$$
= \frac{1}{2} \sum_{L, L', \omega} \left[Q^E_L \, G^{A_E}_{ret}(0,0) \, \hat{Q}^{E*}_{L'} + Q^M_L \, G^{A_V}_{ret}(0,0) \, \hat{Q}^{M*}_{L'} \right] + c.c.
$$

$$
= \int dt \sum_L \frac{(-)^{\ell+1}}{(2\ell+1)!!} \left[\frac{\ell+1}{\ell} \hat{Q}^E_L \partial_t^{2\ell+1} Q^L_E + \frac{\ell}{\ell+1} \hat{Q}^M_L \partial_t^{2\ell+1} Q^L_M \right]
$$

Now eliminated far region too!

Comparison to ALD

- Compare to ALD:

$$F^{\mu}_{ALD} \equiv \frac{dp^{\mu}}{d\tau} = \frac{2}{3}q^2 \left(\frac{d^3x^{\mu}}{d\tau^3} - \frac{d^3x^{\nu}}{d\tau^3}\frac{dx_{\nu}}{d\tau}\frac{dx^{\mu}}{d\tau} \right)$$

- Expand:

$$\vec{F}_{ALDLO} \quad = \quad \frac{2}{3}q^2\dot{\vec{a}}$$

- From $\hat{S}$:

$$\hat{S} = \frac{2}{3}\, q\, \hat{x}_i\, \partial_t^3\, q\, x^i \;\Rightarrow\; \vec{F} := \frac{\partial \hat{S}}{\partial \hat{x}} = \frac{2}{3}\, q^2\, \dddot{\vec{x}}$$

- Consistent with ALD.

Discussion

- Use of symmetry, matching & field doubling to obtain dissipative effective action.
- Simple, practical Feynman rules.
- 2 fields for 2 polarizations - optimal.

The road to gravity

- EM captures many aspects including (some) gauge issues, but:
- Main obstacle - nonlinearity. Possible interactions:
  - near-near: corrections to Q_L. Present at 3.5 - gravitating energy.
  - near-far: RW/Z type - replace Bessel with RW? $\geq 4PN$
  - far-far $\geq 5PN$
  - spin $\geq 4PN$ - replace Bessel with Teukolsky?
- $+1$ PN correction to all mass multipoles.
- Extra gauge issues.

Multi-index and spherical harmonics

$$\phi(r,t,\Omega) = \phi_L(r,t)\,x^L := \sum_{\ell=0}^{\infty} \frac{1}{\ell!} \sum_{i_1\ldots i_\ell=1}^{3} \phi_{i_1\ldots i_\ell}(r,t)\,x^{i_1}\ldots x^{i_\ell}$$

$$= \sum_m \phi_{\ell m}\, Y_{\ell m}(\Omega)$$

$$\int \quad x_{L_\ell} x^{L'_{\ell'}}\, d\Omega = \frac{4\pi r^{2\ell}}{(2\ell+1)!!}\delta_{\ell\ell'}\delta_{L_\ell L'_{\ell'}}\ ,$$

$$\int \quad g^{\Omega\Omega'}\partial_\Omega x_{L_\ell}\partial_{\Omega'} x^{L'_{\ell'}}\, d\Omega = \frac{4\pi\ell(\ell+1)r^{2\ell}}{(2\ell+1)!!}\delta_{\ell\ell'}\delta_{L_\ell L'_{\ell'}}\ ,$$

$$\int \quad x^{\Omega}_{L_\ell} x_{\Omega}^{L'_{\ell'}}\, d\Omega = \frac{4\pi\ell(\ell+1)r^{2\ell}}{(2\ell+1)!!}\delta_{\ell\ell'}\delta_{L_\ell L'_{\ell'}}\ ,$$

$$\int \quad g^{\Omega\Omega'} g^{PP'} D_\Omega x_P^{L_\ell} D_{\Omega'} x_{P'}^{L'_{\ell'}}\, d\Omega = \frac{8\pi\ell^2(\ell+1)^2 r^{2\ell}}{(2\ell+1)!!}\delta_{\ell\ell'}\delta_{L_\ell L'_{\ell'}}$$

Masterfunctions & sources

$$
\begin{aligned}
A_S &= \frac{(\ell+1) r^{1-\ell} (r^\ell \tilde{A}_S)'}{(\ell(\ell+1) - \omega^2 r^2)} \\
\rho_{L\omega}^{A_{(E/M)}} &= \frac{4\pi r^{\ell+1} (r^{\ell+2} \rho^S_{L\omega})'}{(\ell+1)(2\ell+1)!!} \\
&= \frac{1}{\ell+1} \int d\Omega \, r x_L \left[-r^2 \rho_\omega(\vec{r}) + \frac{i}{\omega} \left(r^2 \frac{\Lambda}{\Lambda - 1} \vec{J}_\omega(\vec{r}) \cdot \vec{n} \right)' \right]' \\
A_M &= \frac{\ell A_V}{r} \\
\rho_{L\omega}^{A_V} &= \frac{4\pi \ell r^{2\ell+3}}{(2\ell+1)!!} \rho^V_{L\omega}(r) = \ell r^2 \int \vec{J}_\omega(\vec{r}) \cdot \left[\vec{r} \times \vec{\nabla} x_L \right] d\Omega
\end{aligned}
$$