Gravitational self-force in the ultra-relativistic regime

Chad Galley, California Institute of Technology

with Rafael Porto (IAS)

arXiv: 1302.4486

v2 soon! (with details)

Capra16; Dublin, Ireland; July 15, 2013

Ultra-relativistic regime & limit

Ultra-relativistic speeds parametrized by *boost factor*

$$\gamma = \frac{1}{\sqrt{-g_{\alpha\beta} v^\alpha v^\beta}} \gg 1 \qquad\qquad v^\mu = \frac{dz^\mu}{dt} = \left(1, \vec{v}\right) \sim 1$$

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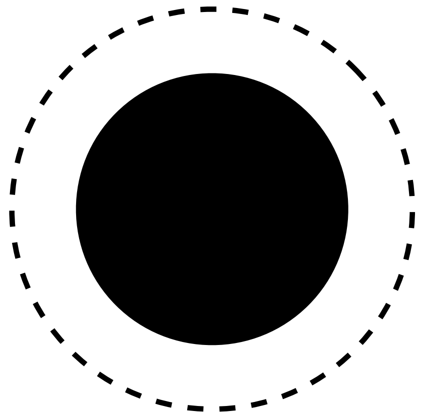
Contexts:

- Circular orbit near Schwarzschild light ring



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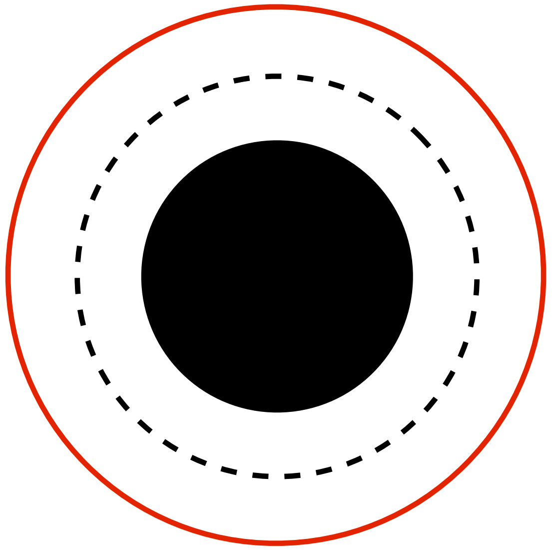

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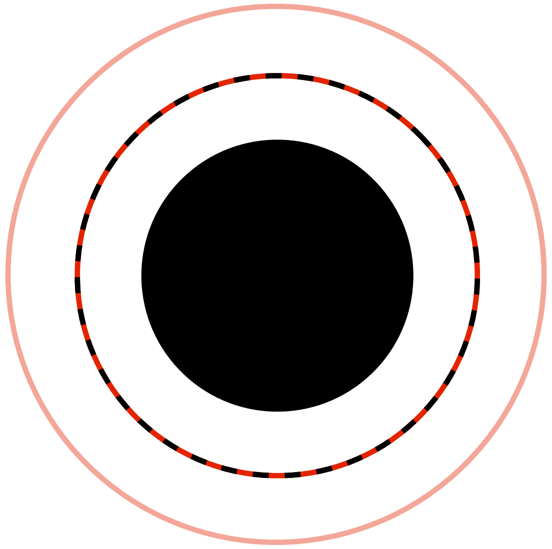

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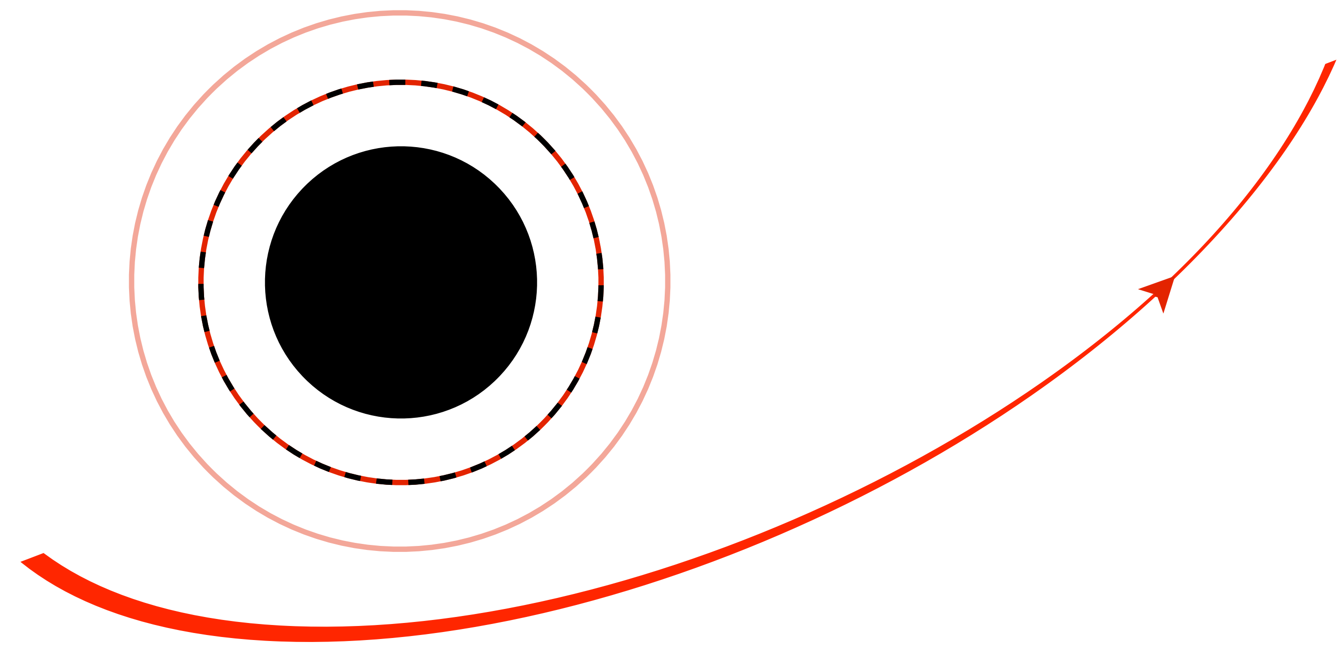
Contexts:

- Circular orbit near Schwarzschild light ring
- Fast "fly-by" motions
- Fast motion in any background



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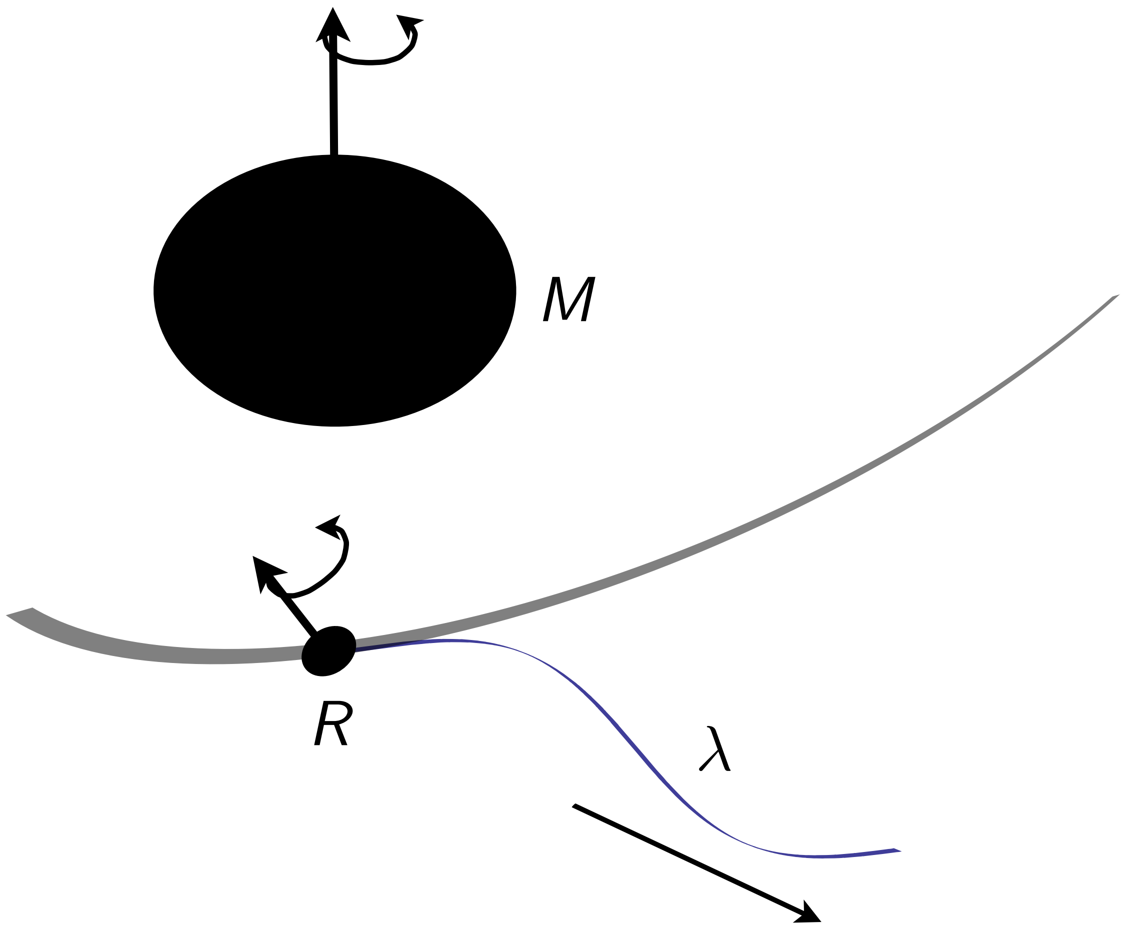
Help calibrate semi-analytical merger models *Akcay + (2012)*

Effective field theory & perturbation theory

CRG & Hu (2009)



Black hole

Neutron star

White dwarf

Star,...

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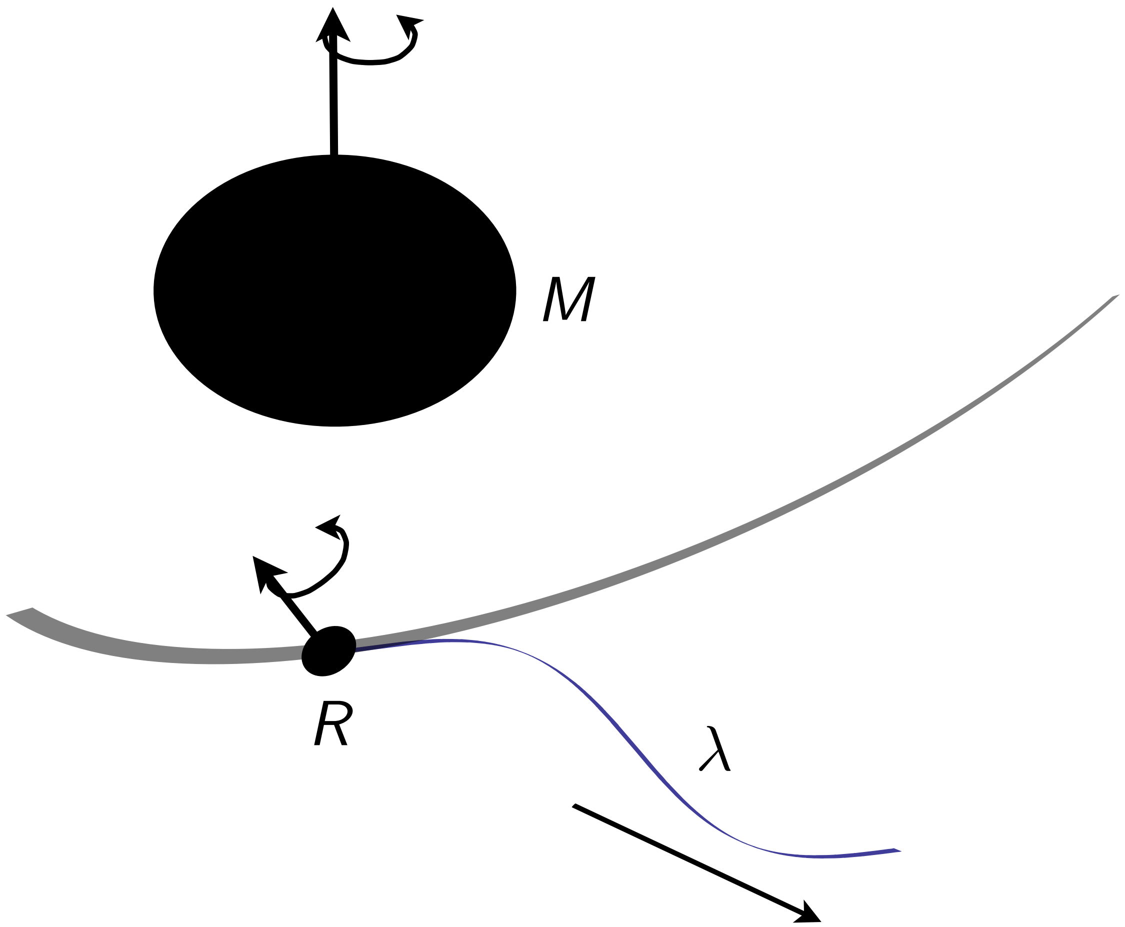

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Expansion parameter:

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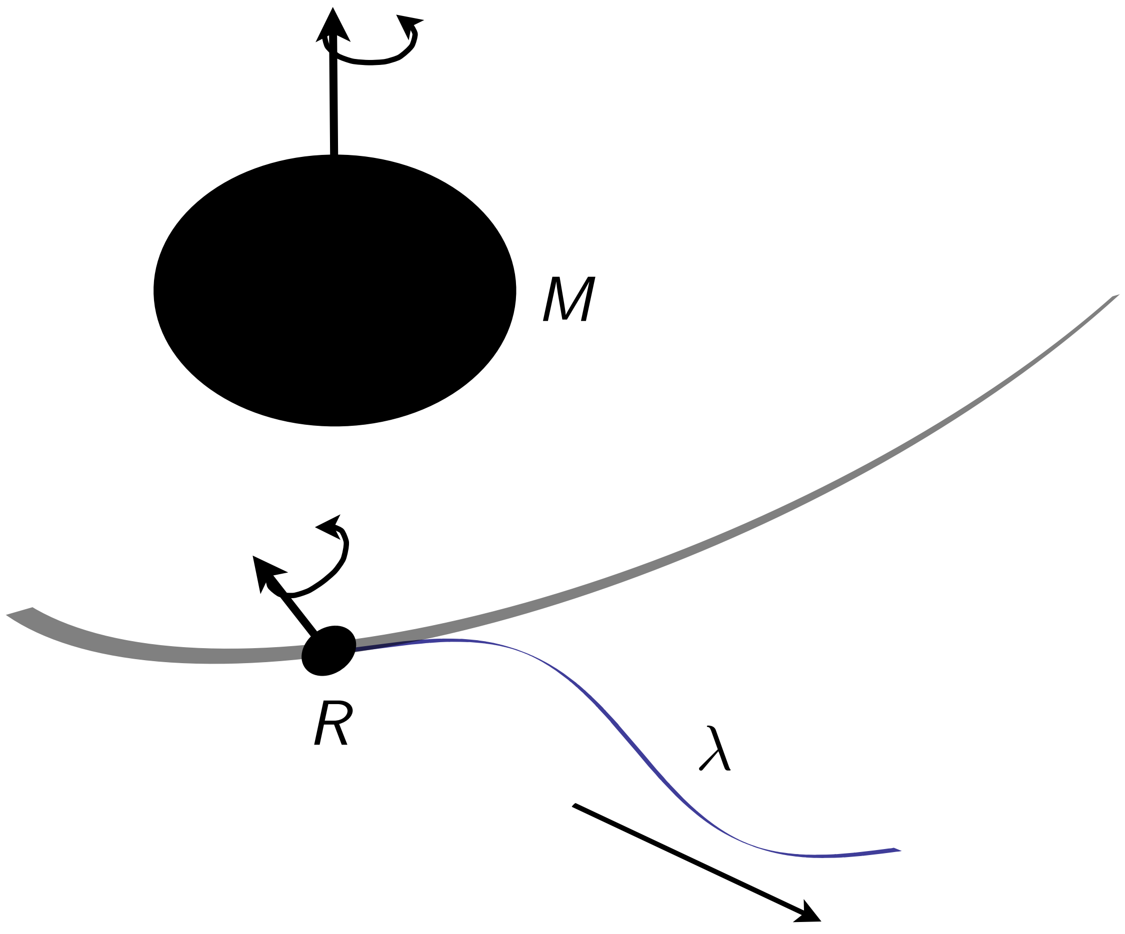

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Relevant degrees of freedom:

$$z^{\mu}(\lambda)\ ,\quad h_{\alpha\beta}(x^{\mu})$$

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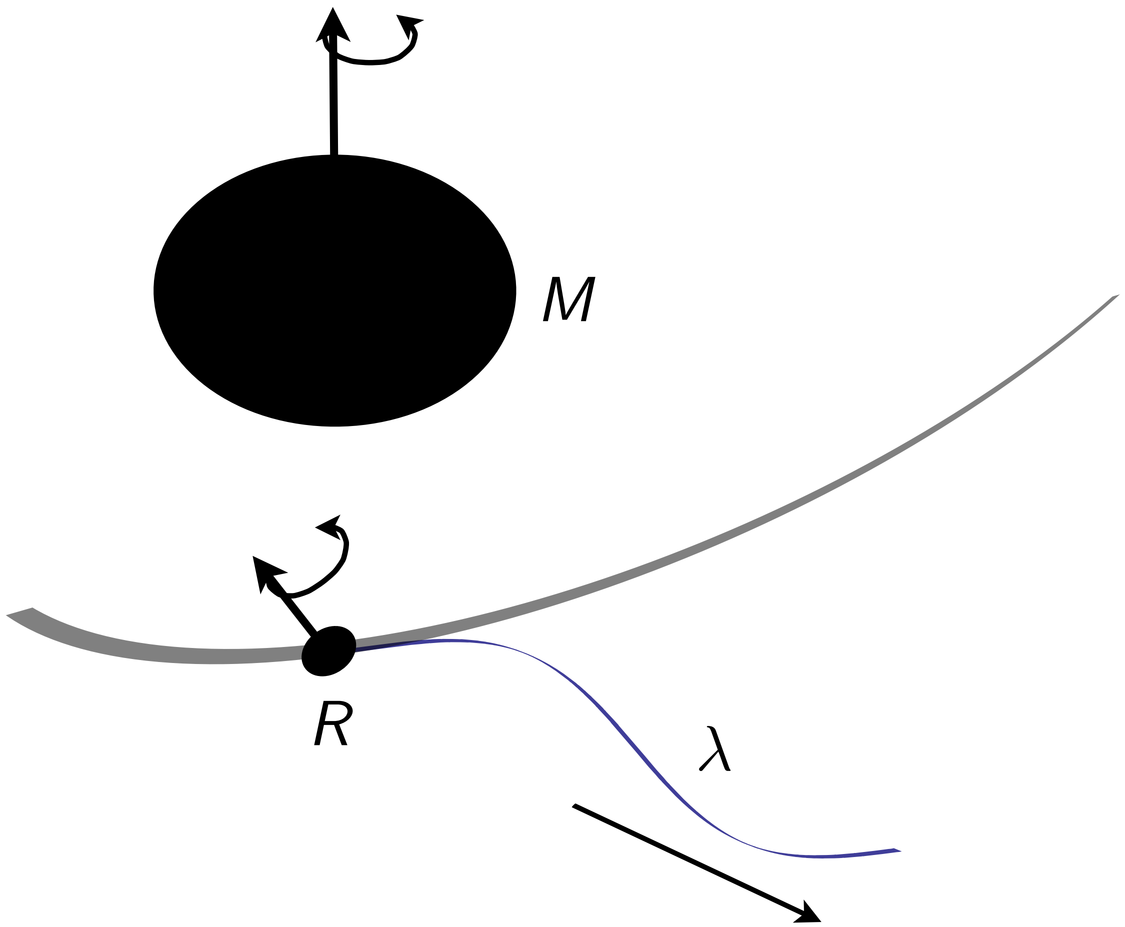

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Finite size effects at $(m/M)^4$

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Gravity & effective particle action:

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$$\boxed{\begin{array}{l}\text{Ultra-relativistic limit:}\\ \gamma \to \infty,\, m \to 0 \qquad \lambda = \text{constant}\end{array}}$$

Which terms contribute to effective action in ultra-relativistic regime?

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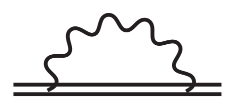
$\sim \dfrac{L}{N}$ $+$



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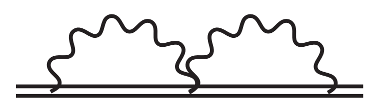

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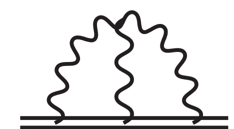

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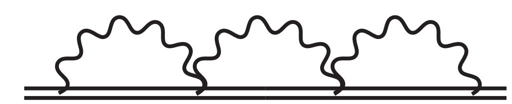

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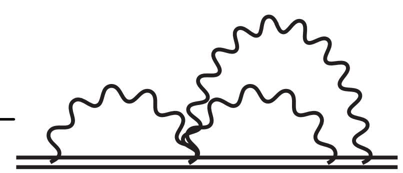

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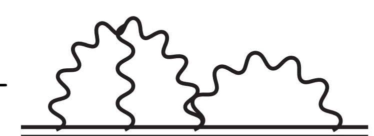

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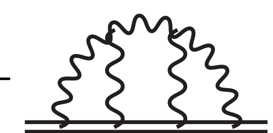

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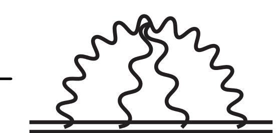

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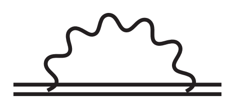
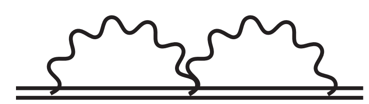
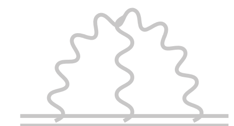
$S_{\text{eff}}[z^\mu] =$  $+$  $+$  $+$  $+$ 

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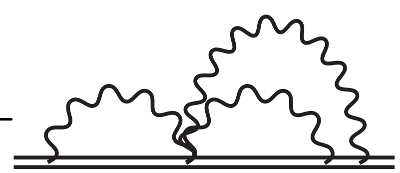
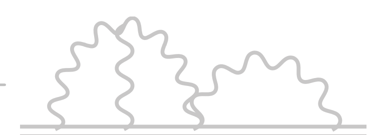
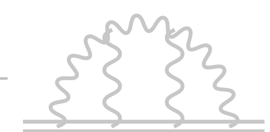
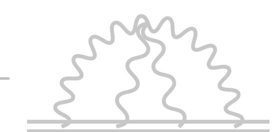
$+$  $+$  $+$  $+$  $+\cdots$

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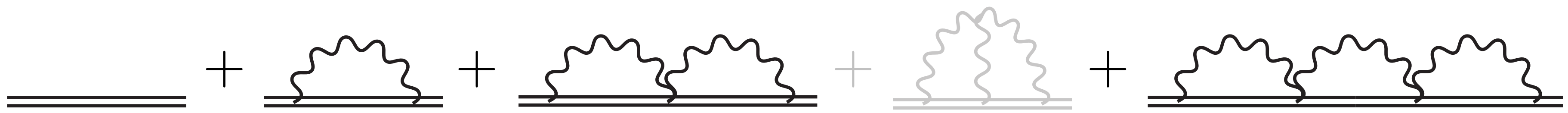

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Only diagrams without interactions in bulk contribute to ultra-relativistic limit!

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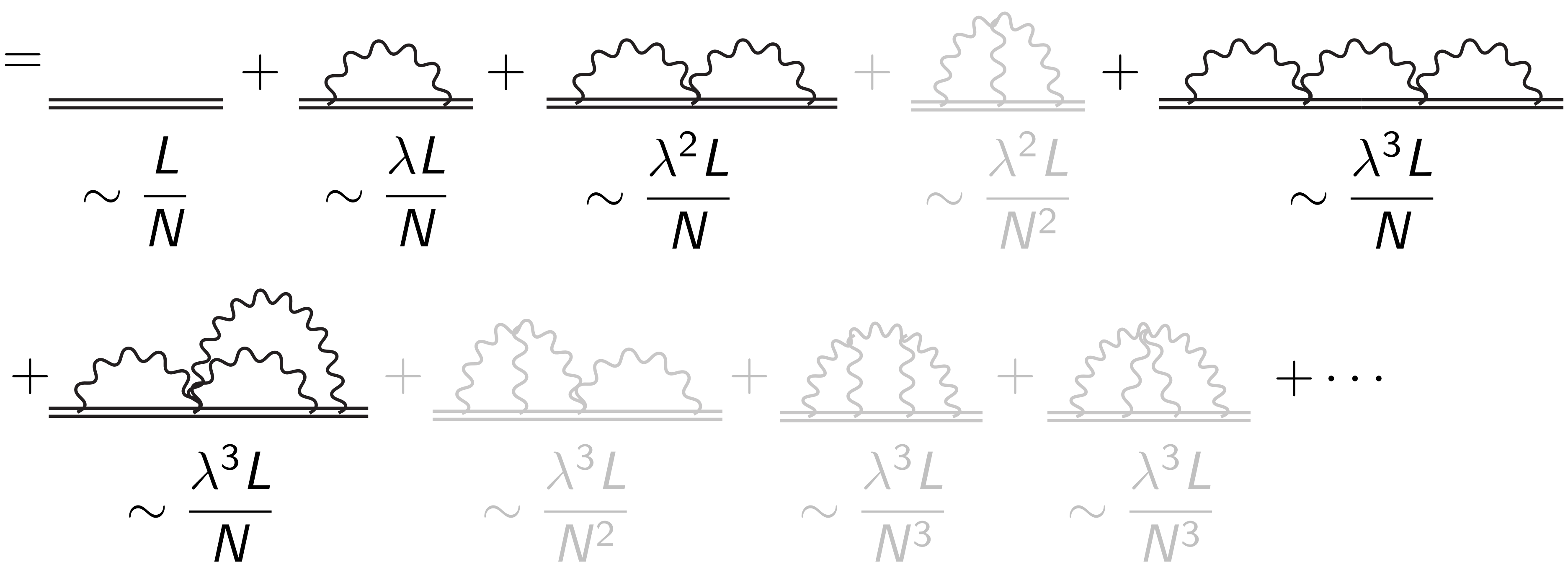

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Resembles large-N expansions in QFT

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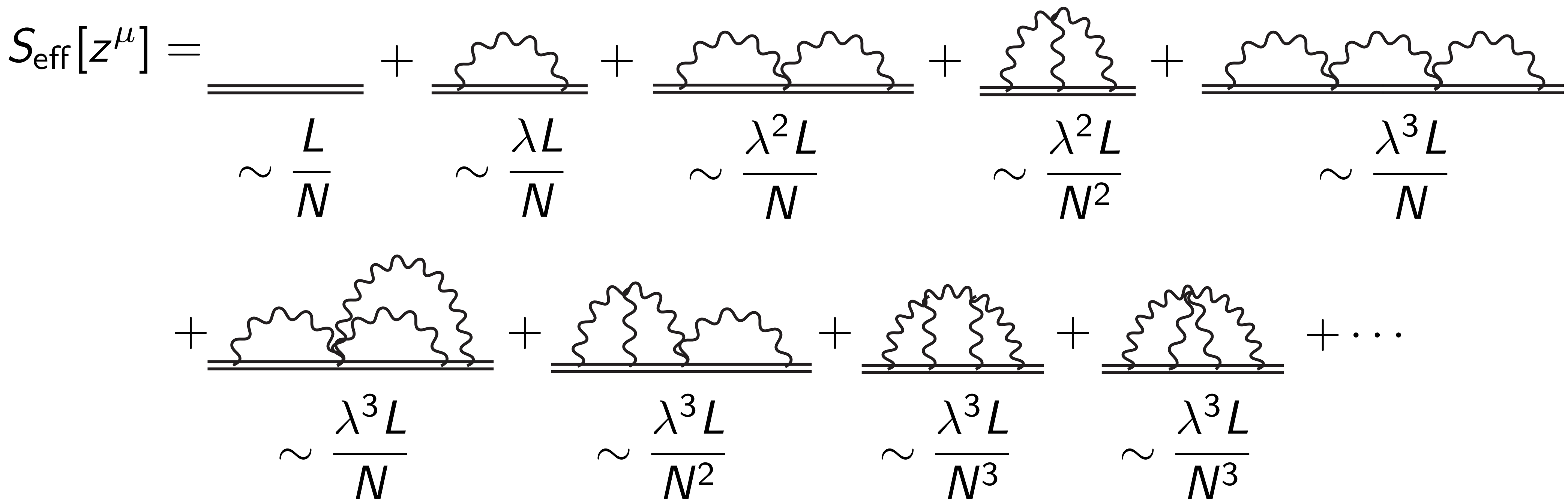
Domain of validity of ultra-relativistic regime



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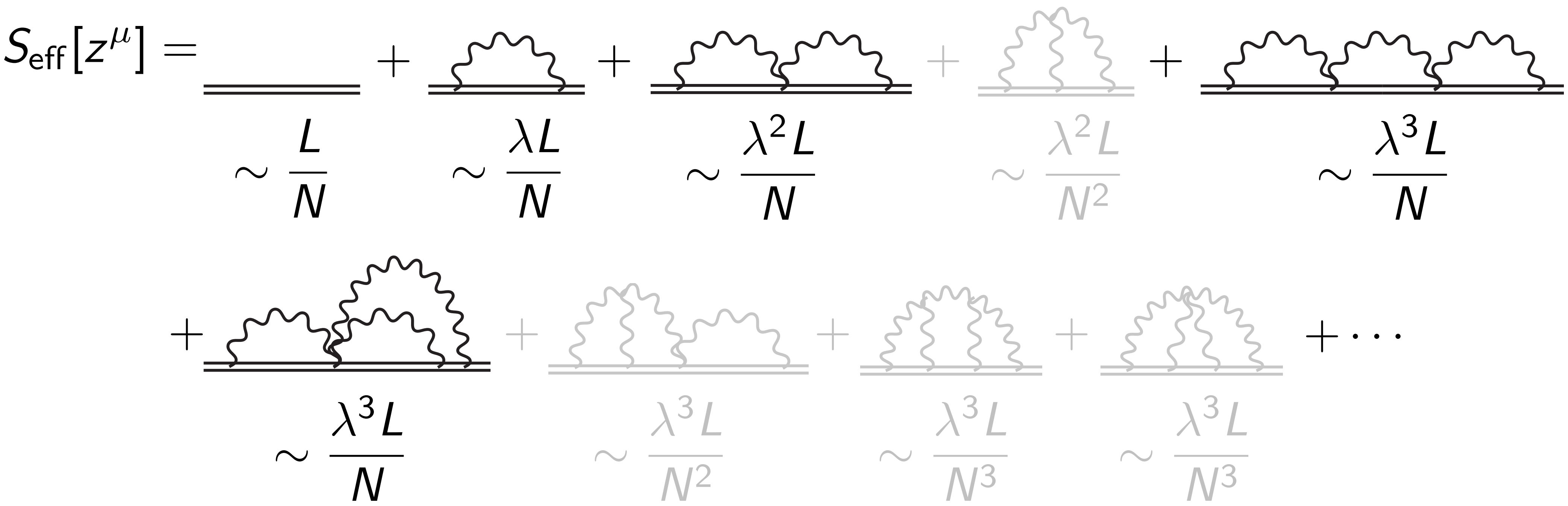

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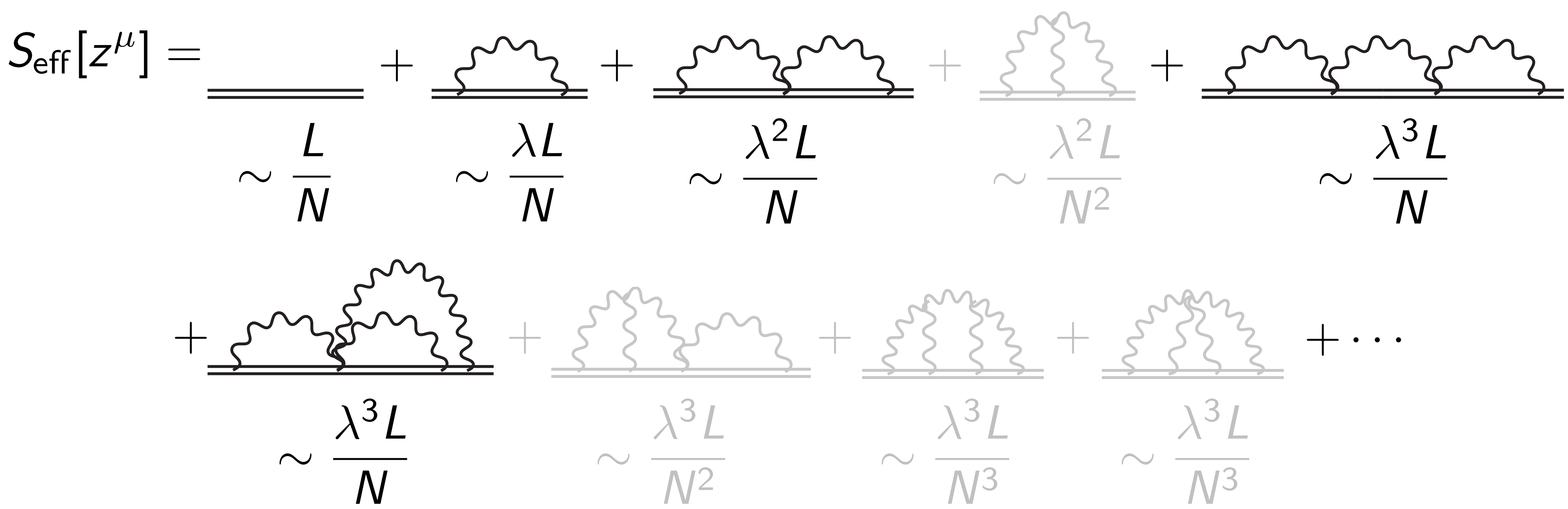

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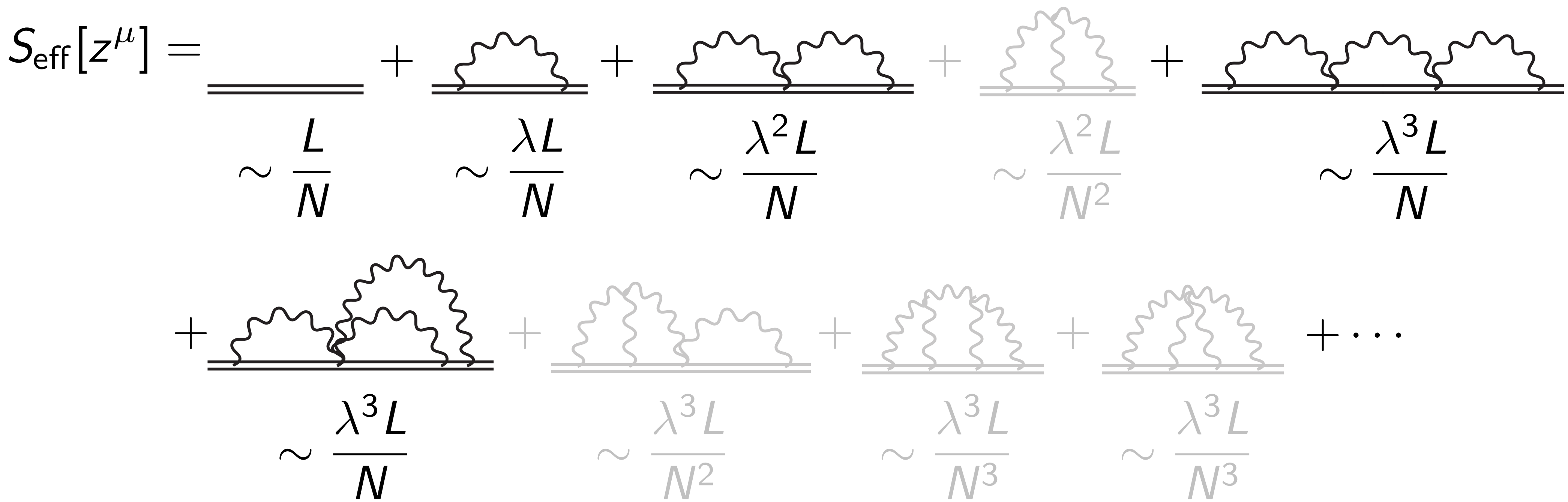

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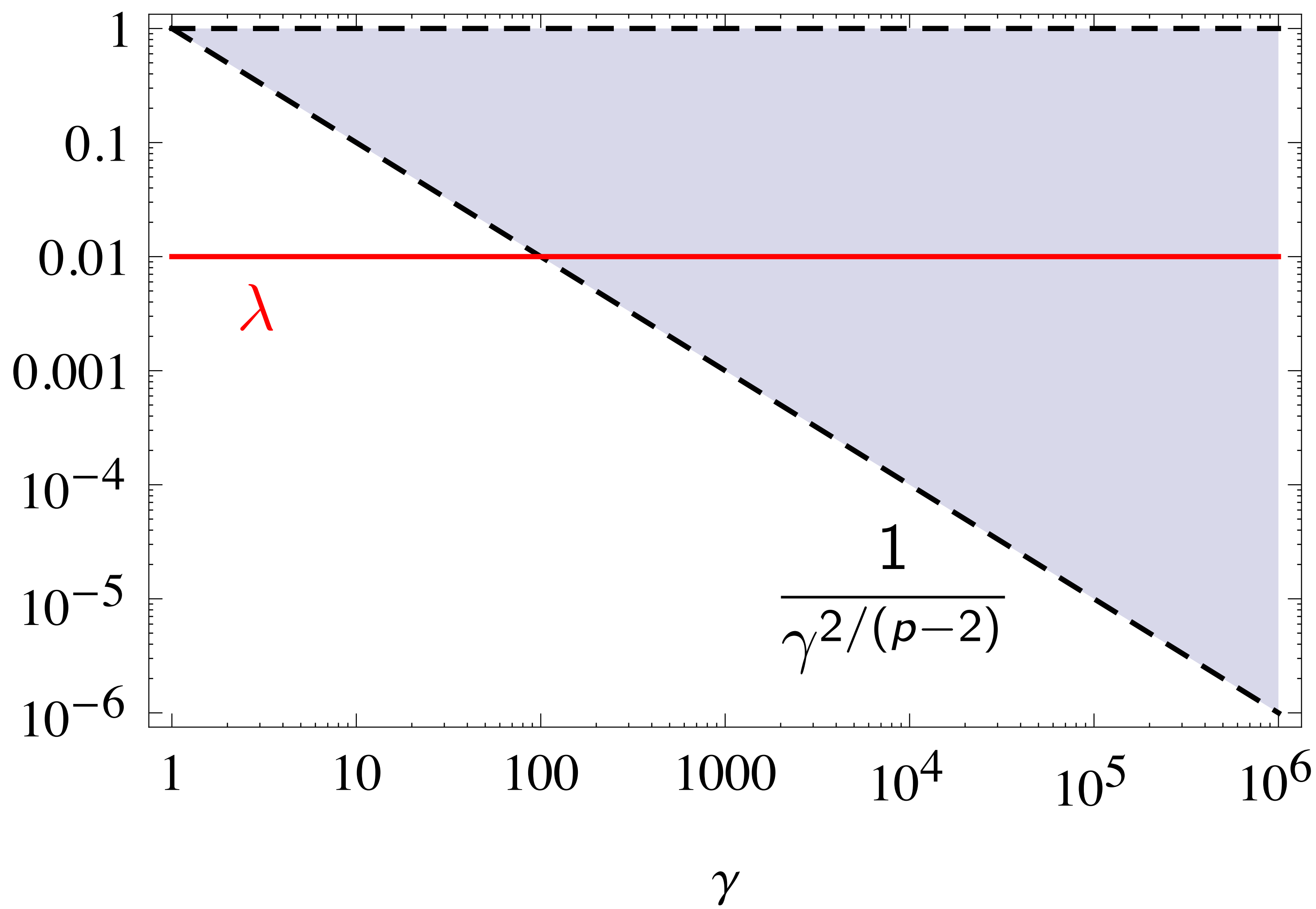
$$\frac{1}{\gamma^{2/(p-2)}} \ll \lambda \ll 1$$

For example, ignoring $1/N^2$ terms and truncating after 4th order:



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$$q \lesssim \frac{10^{-2}}{(10^2)^3} = 10^{-8}$$

$$q \lesssim \frac{10^{-2}}{(10^3)^3} = 10^{-11}$$

Gravitational perturbations & self-force

Easier to compute gravitational perturbations first in a theory with only worldline couplings -- a *master source* *CRG (2011)*

$$h_{\mu\nu}(x) = \int d\tau' \, G^{\rm ret}_{\mu\nu\alpha'\beta'}(x, z^{\mu'}) \, \mathcal{S}_R^{\alpha'\beta'}(z^{\mu'})$$

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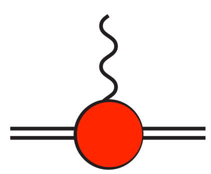
$=$ 

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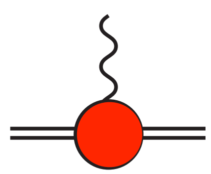
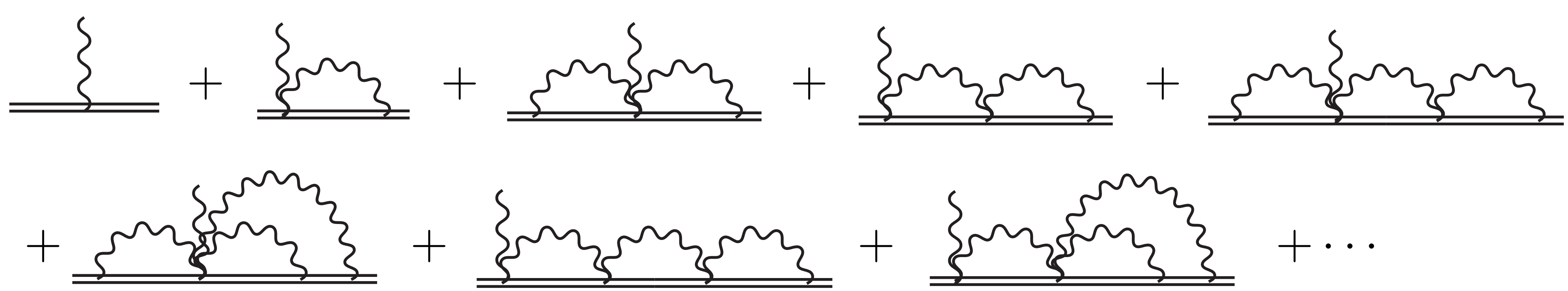
$=$ 

$=$ 

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To derive the master source:

- Use an action that accommodates outgoing boundary conditions
  CRG & Tiglio (2009); CRG (2010); CRG (2013) [PRL Editors' Highlight]

PRL **110,** 174301 (2013) PHYSICAL REVIEW LETTERS week ending 26 APRIL 2013

Classical Mechanics of Nonconservative Systems

Chad R. Galley*

Jet Propulsion Laboratory, California Institute of Technology, Pasadena, California 91109 and Theoretical Astrophysics, California Institute of Technology, Pasadena, California 91125, USA

(Received 1 September 2012; published 22 April 2013)

Hamilton's principle of stationary action lies at the foundation of theoretical physics and is applied in many other disciplines from pure mathematics to economics. Despite its utility, Hamilton's principle has a subtle pitfall that often goes unnoticed in physics: it is formulated as a boundary value problem in time but is used to derive equations of motion that are solved with initial data. This subtlety can have undesirable effects. I present a formulation of Hamilton's principle that is compatible with initial value problems. Remarkably, this leads to a natural formulation for the Lagrangian and Hamiltonian dynamics of generic nonconservative systems, thereby filling a long-standing gap in classical mechanics. Thus, dissipative effects, for example, can be studied with new tools that may have applications in a variety of disciplines. The new formalism is demonstrated by two examples of nonconservative systems: an object moving in a fluid with viscous drag forces and a harmonic oscillator coupled to a dissipative environment.

DOI: 10.1103/PhysRevLett.110.174301 PACS numbers: 45.20.−d, 02.30.Xx, 05.20.−y, 47.10.−g

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  CRG & Tiglio (2009); CRG (2010); CRG (2013) [PRL Editors' Highlight]

Features of the Causal Action Principle

CRG (2013) [PRL Editors' Highlight]; CRG & Stein (in prep)

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To derive the master source:

- Use an action that accommodates outgoing boundary conditions
  CRG & Tiglio (2009); CRG (2010); CRG (2013) [PRL Editors' Highlight]

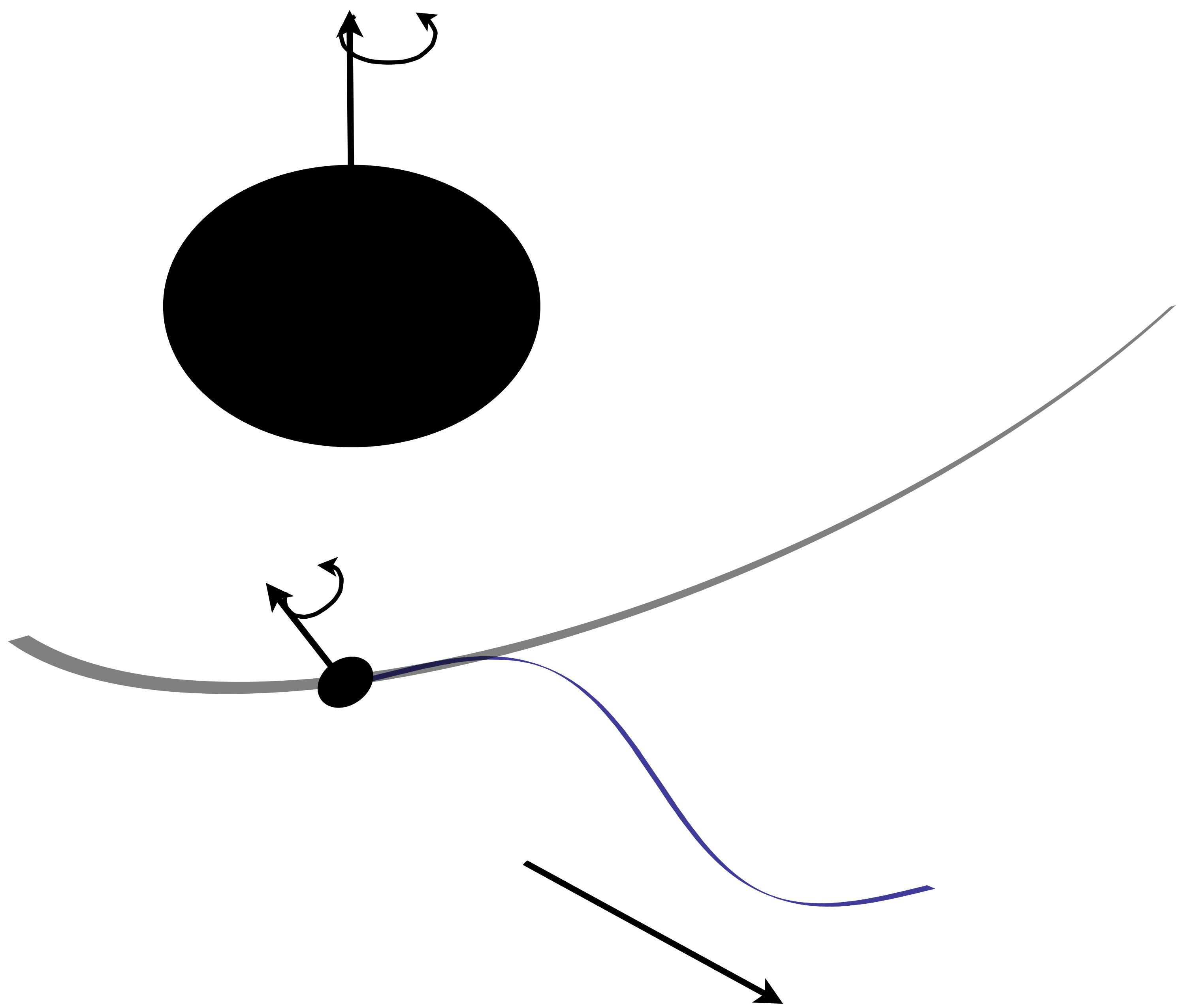
Features of the Causal Action Principle

CRG (2013) [PRL Editors' Highlight]; CRG & Stein (in prep)

- Applicable to any generic system, including non-conservative ones
- A variational principle consistent with initial data
- Naturally gives definitions of action, Lagrangian, & Hamiltonian for general non-conservative (e.g., dissipative) systems
- Applies to "non-Hamiltonian" systems
- Can handle non-holonomic constraints
- Consistent quantization of open systems *CRG & Chen (in progress)*
- Gives a powerful framework to tackle any real-world problem with similar tools learned in courses

Applications of Causal Action Principle

Successfully applied in black hole binaries...



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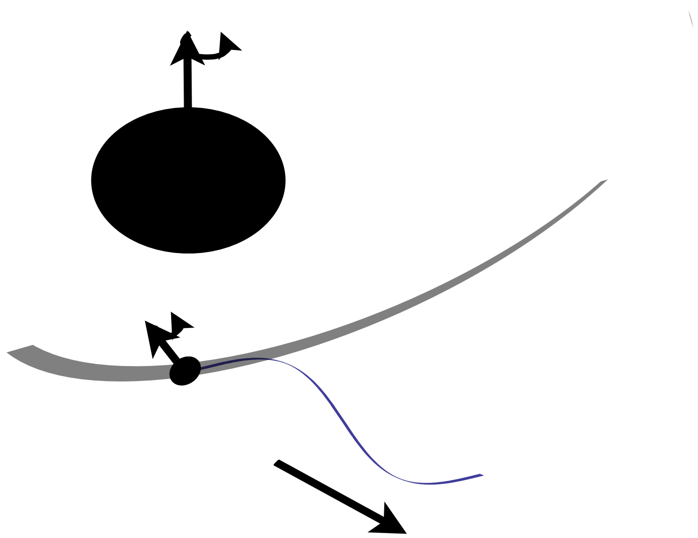

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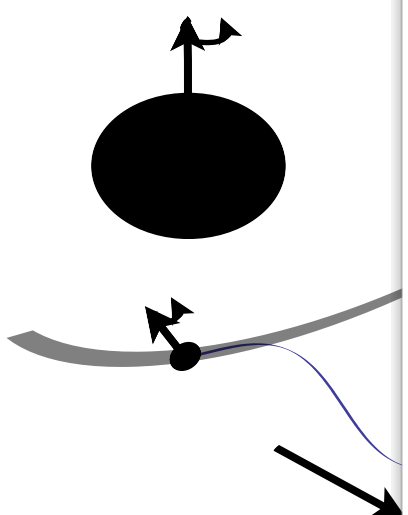

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Chad R Galley

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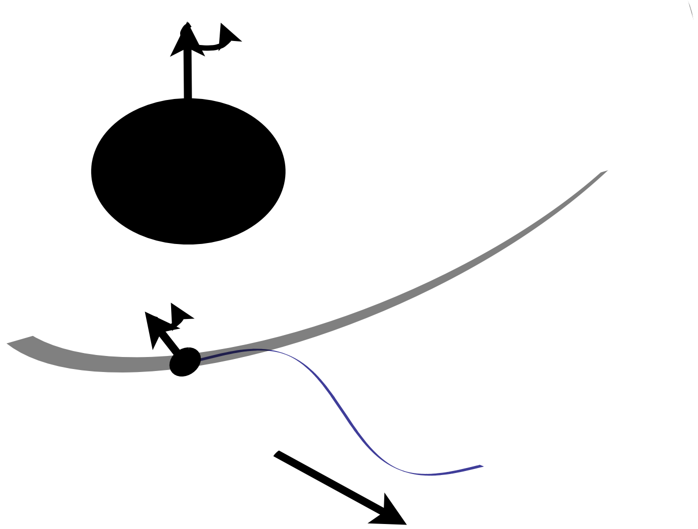

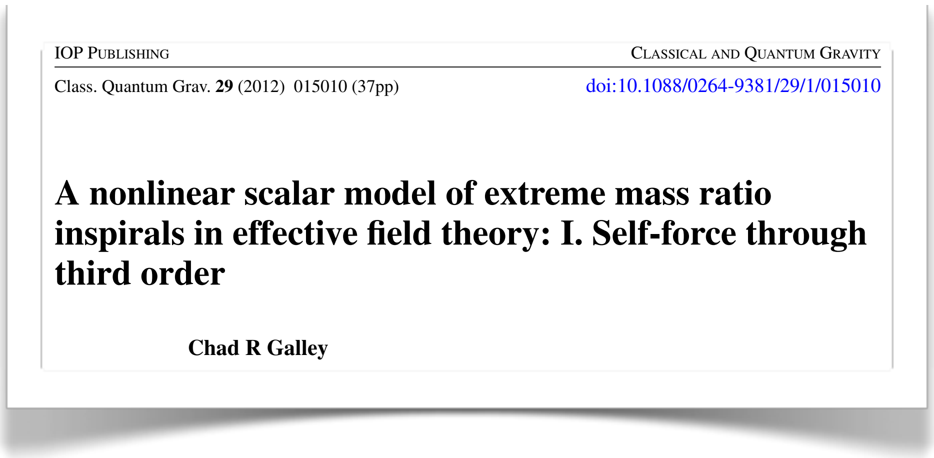

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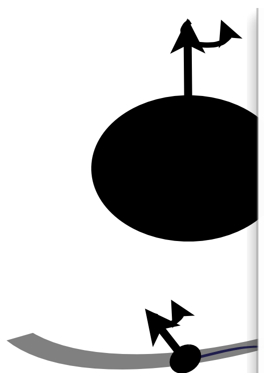

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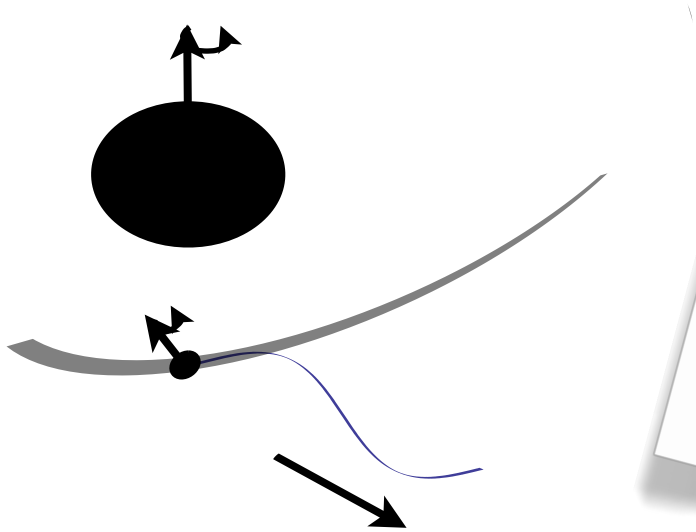

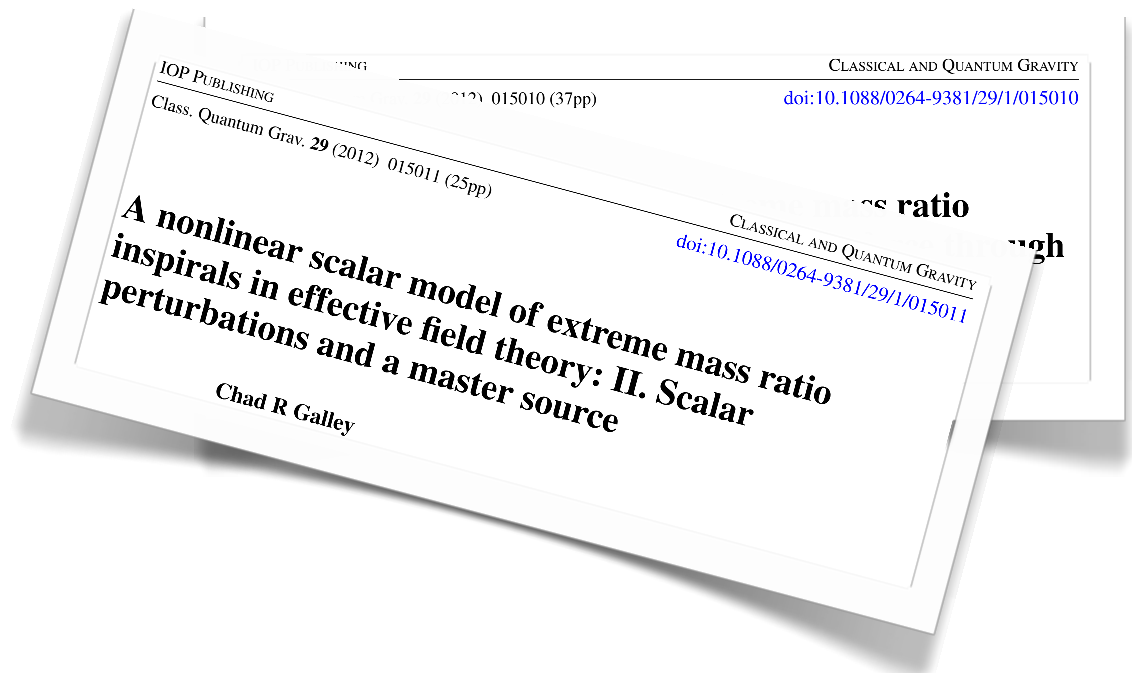

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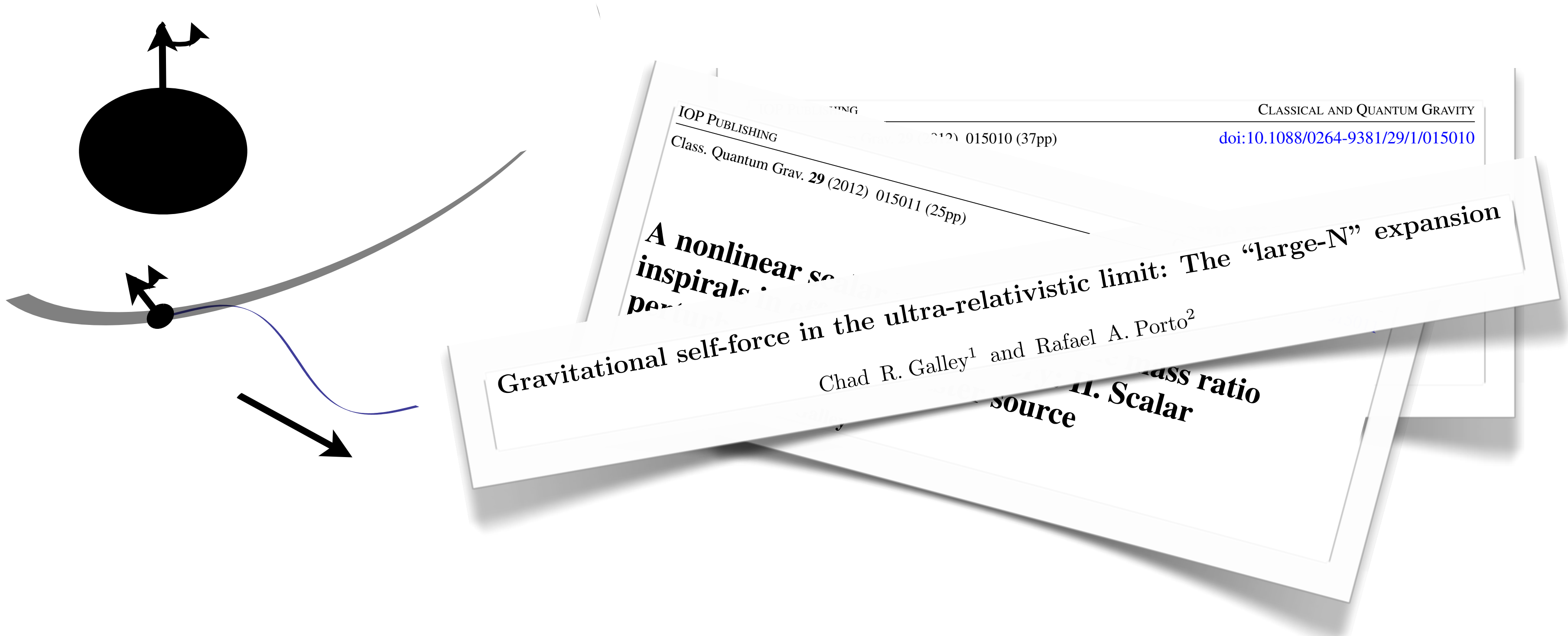

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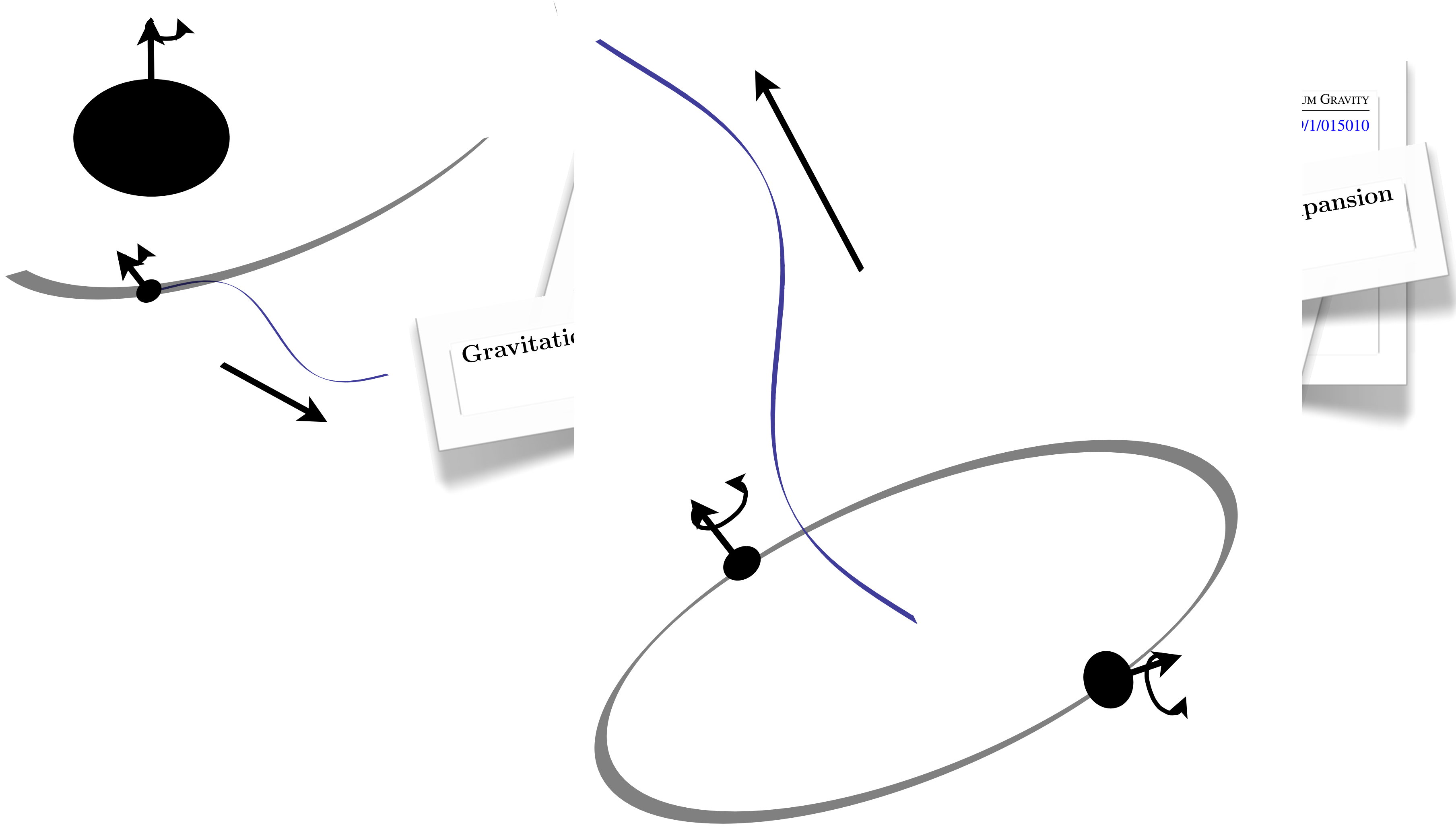

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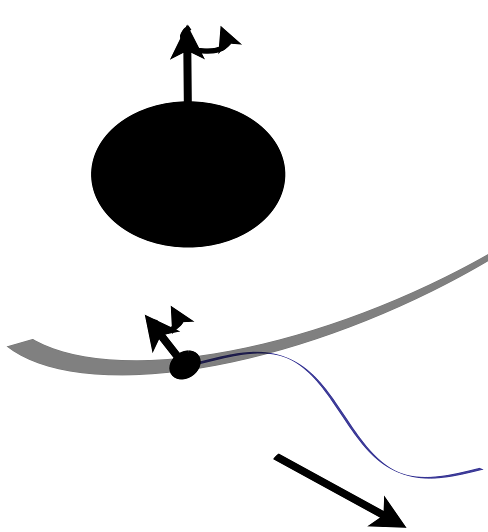

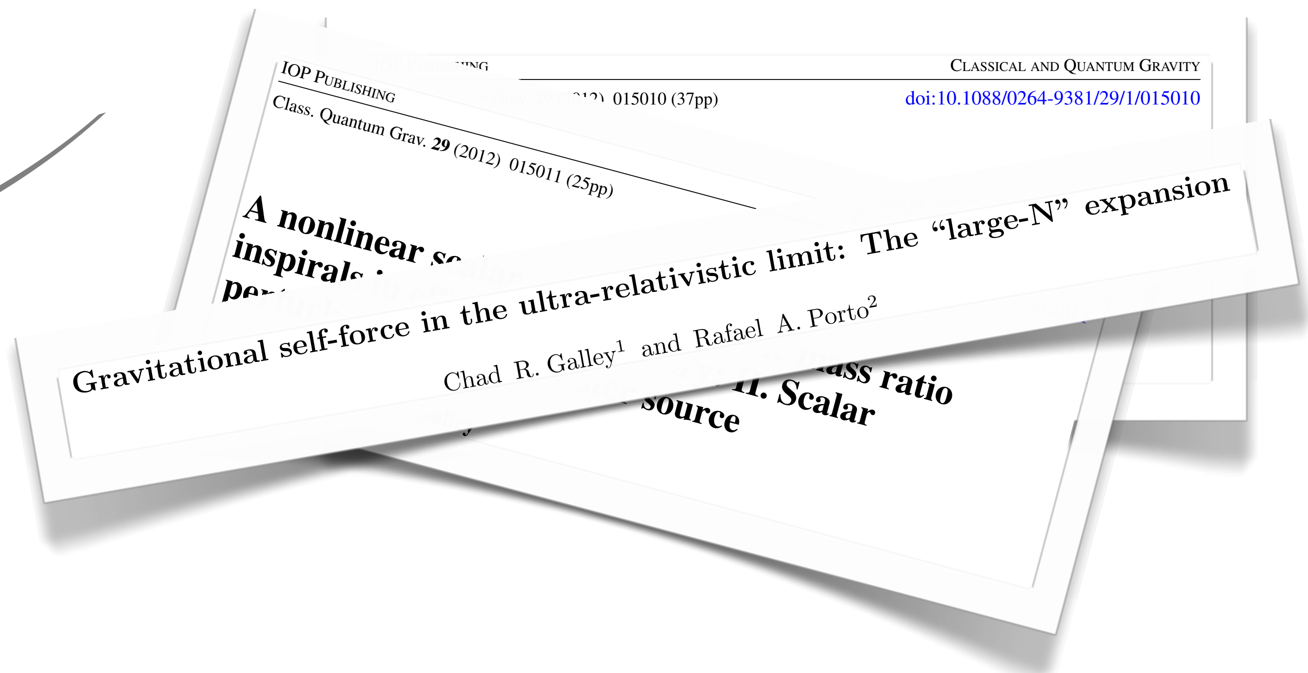
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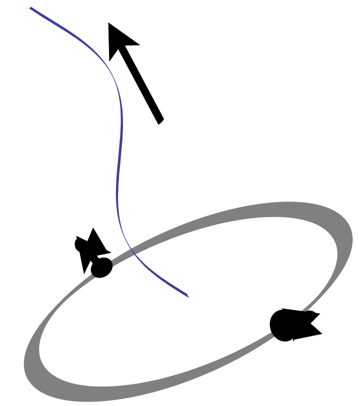

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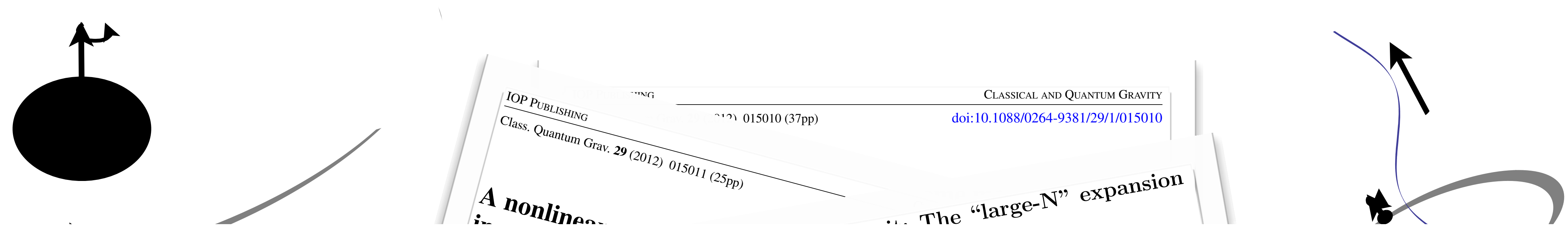

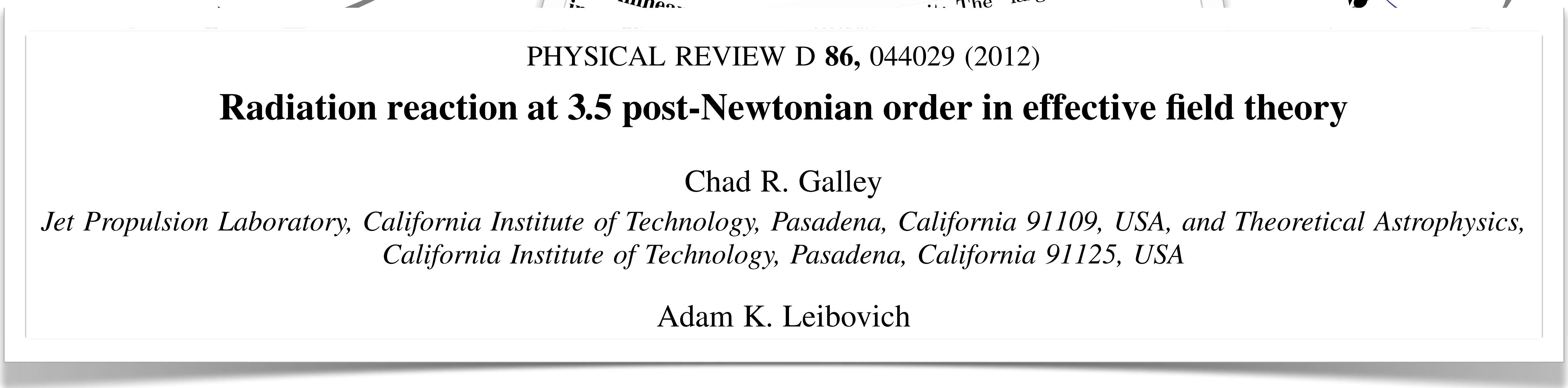
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Radiation reaction at 3.5 post-Newtonian order in effective field theory

Chad R. Galley
Jet Propulsion Laboratory, California Institute of Technology, Pasadena, California 91109, USA, and Theoretical Astrophysics, California Institute of Technology, Pasadena, California 91125, USA

Adam K. Leibovich

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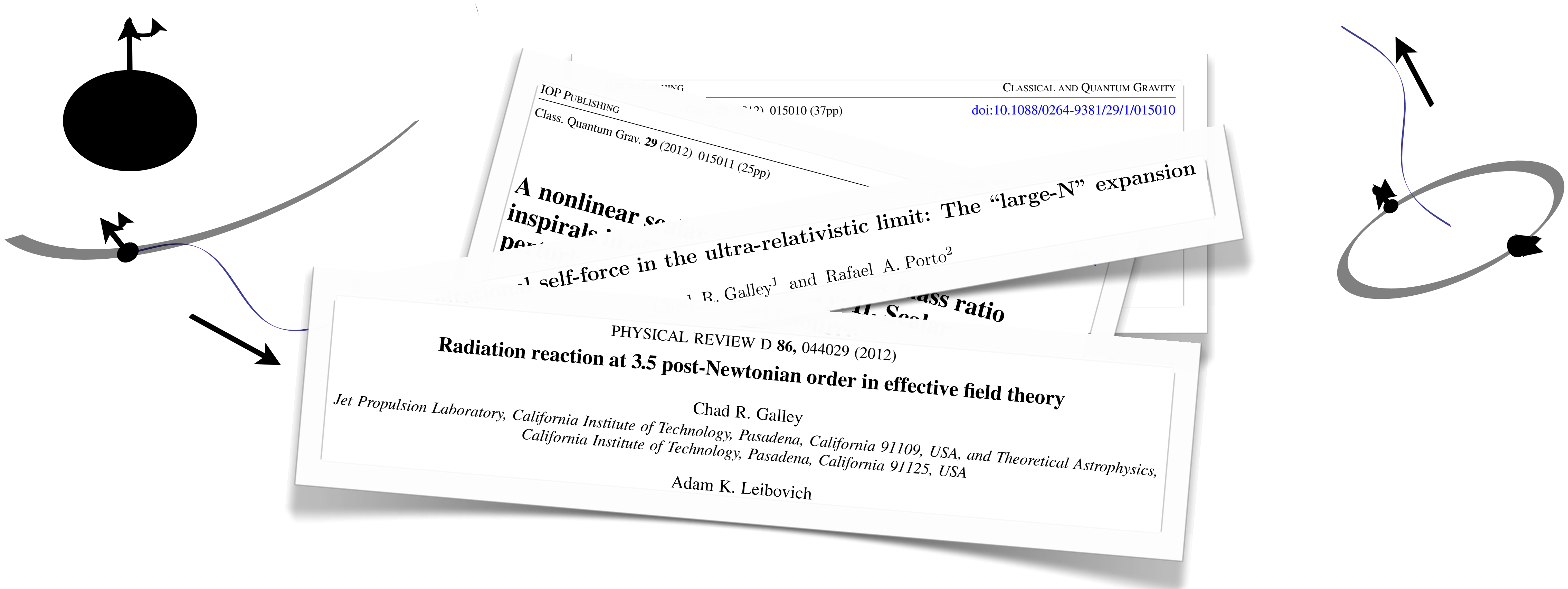

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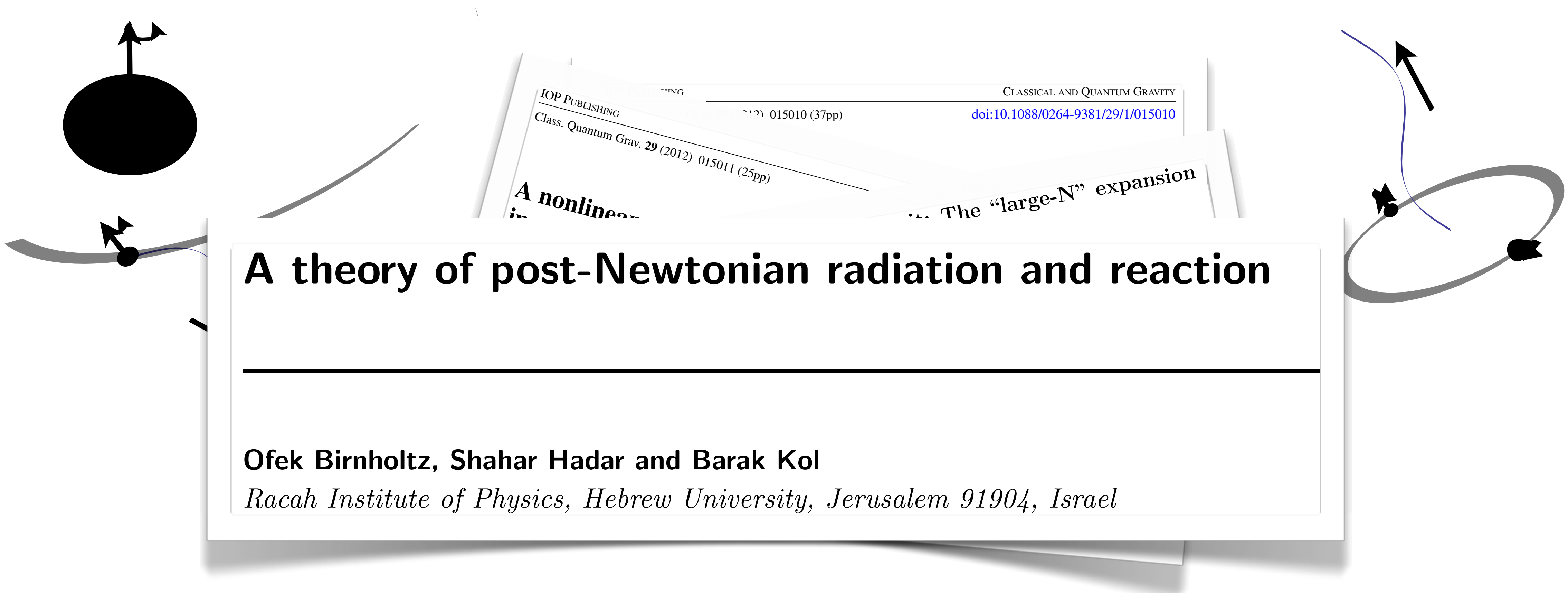

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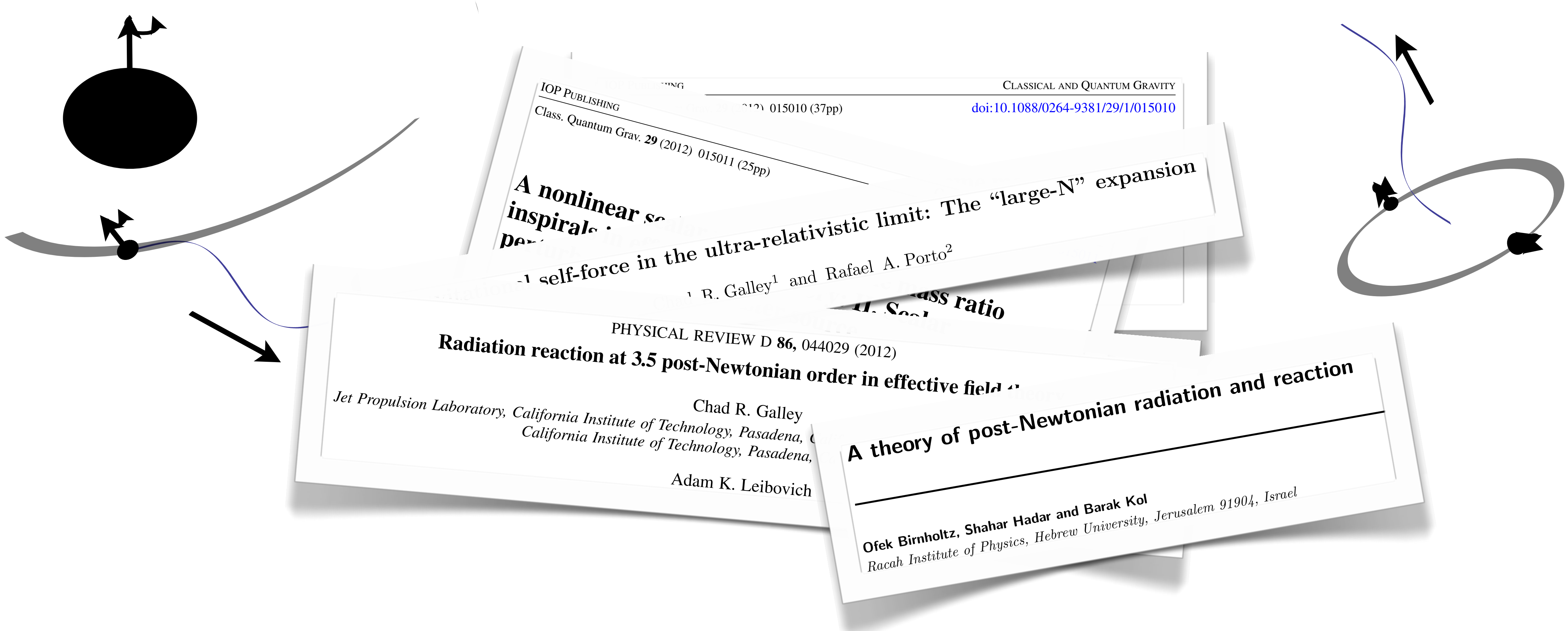

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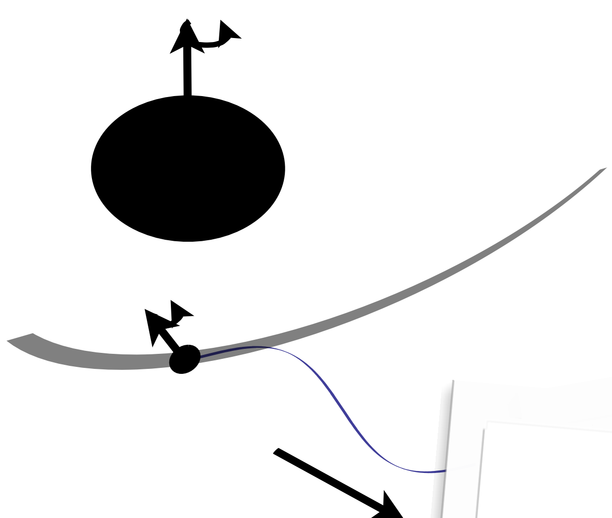

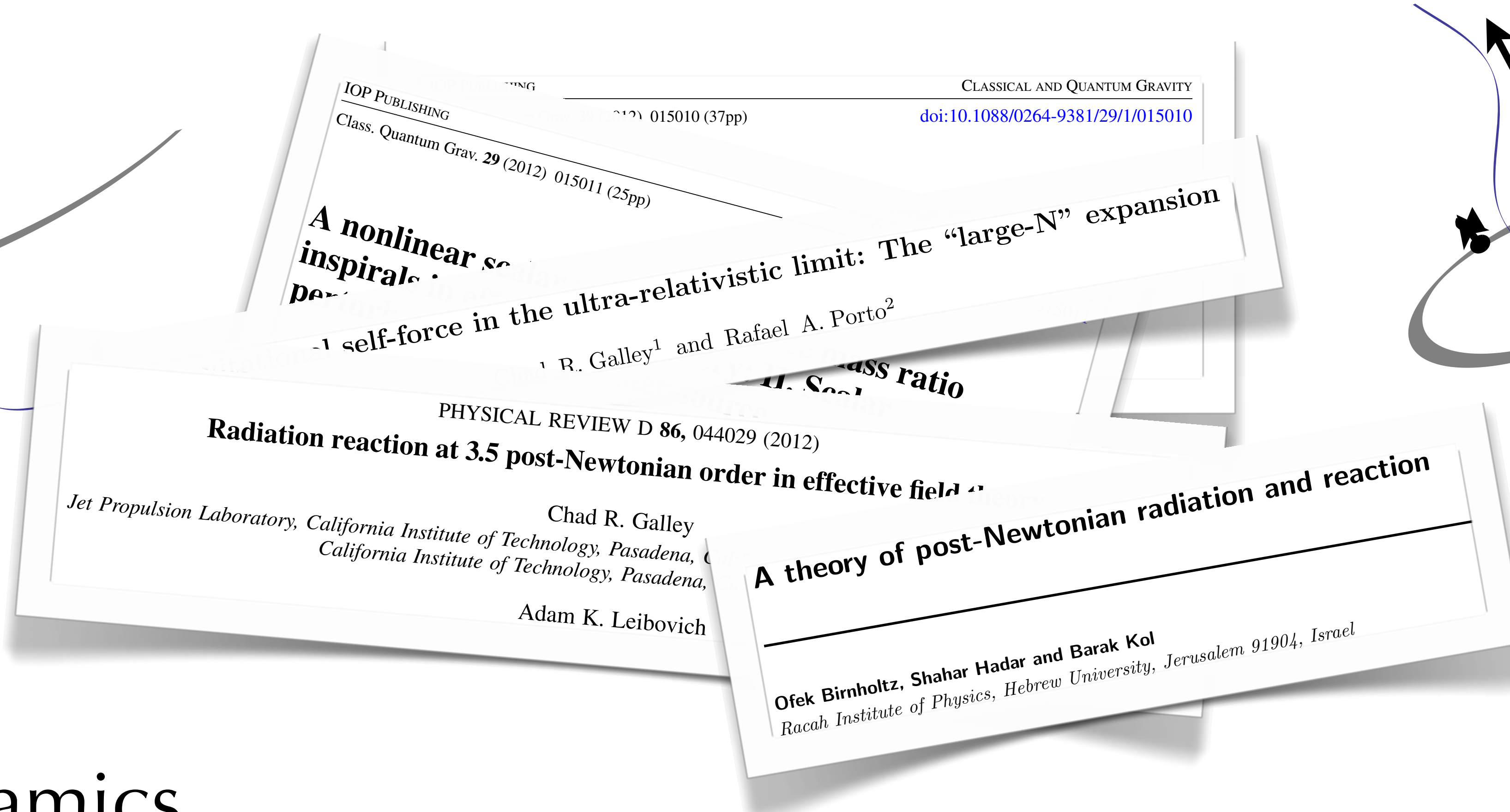

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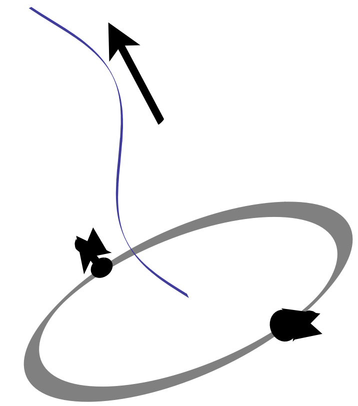

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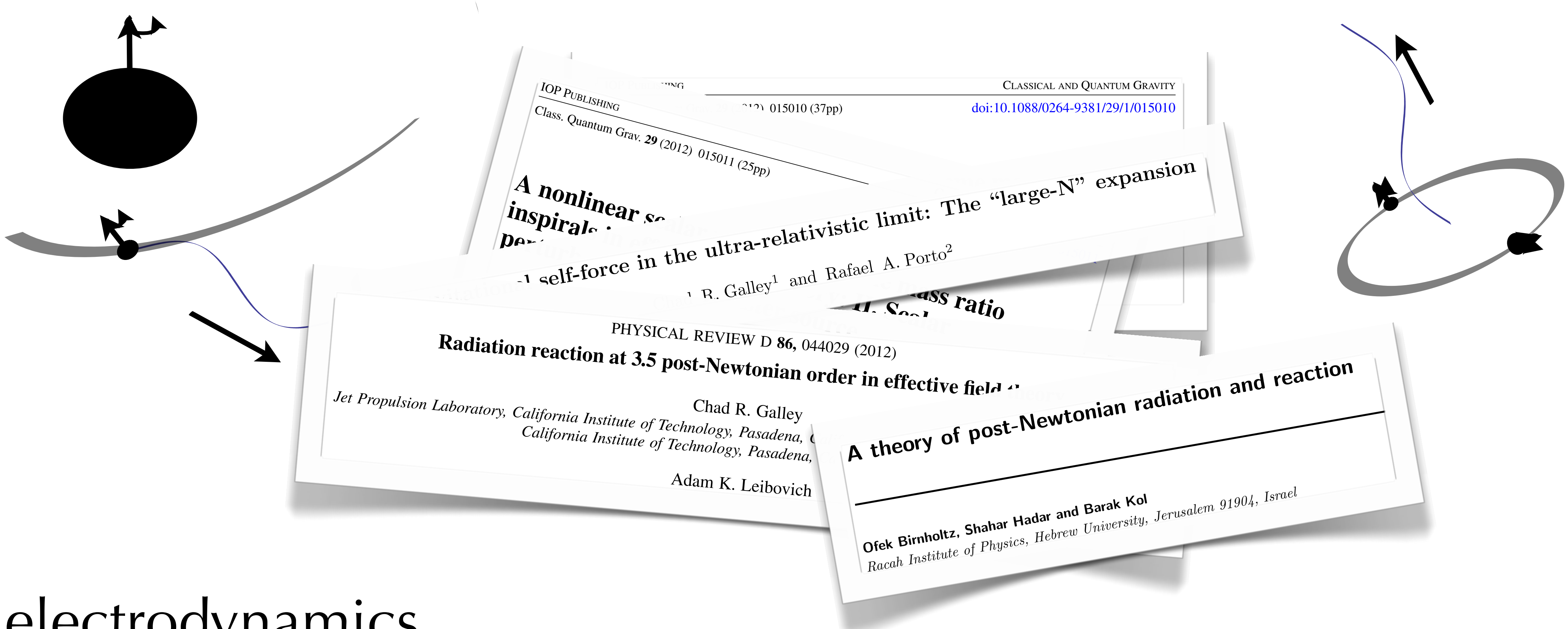

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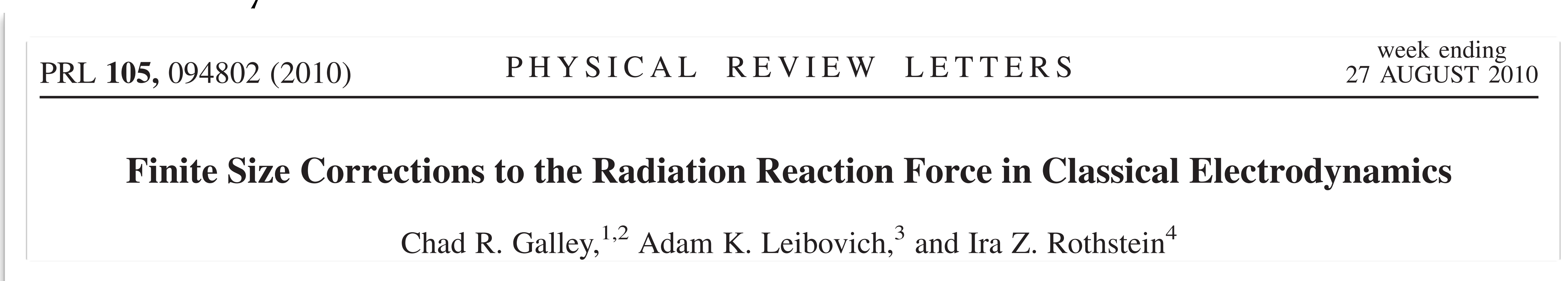

PRL 105, 094802 (2010) PHYSICAL REVIEW LETTERS week ending 27 AUGUST 2010

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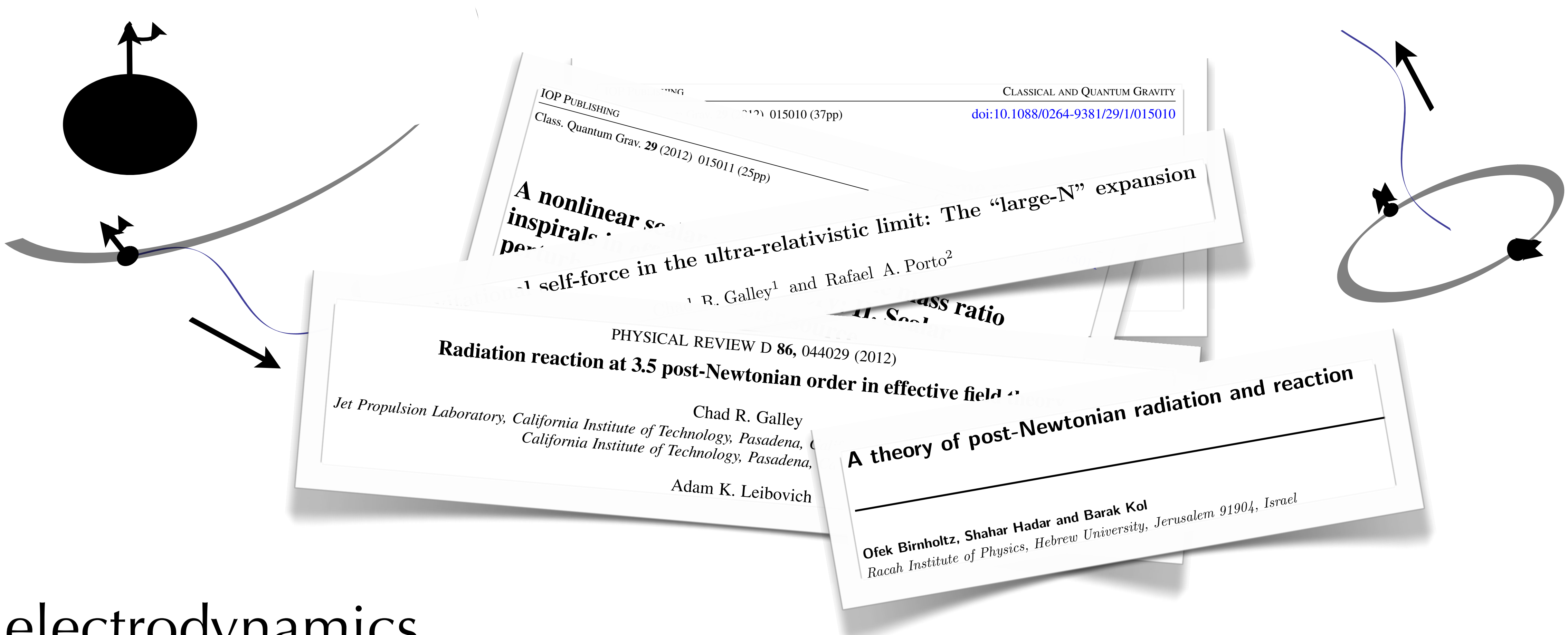

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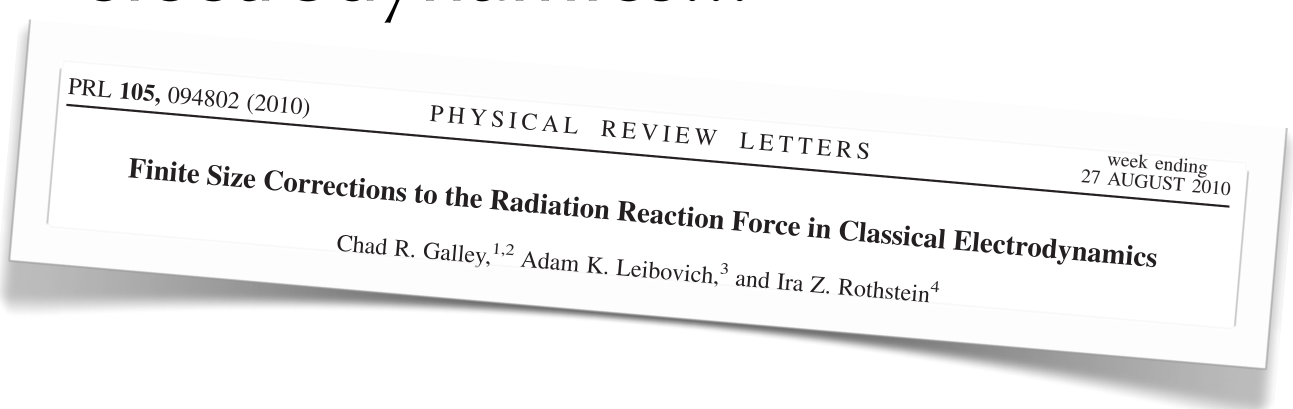

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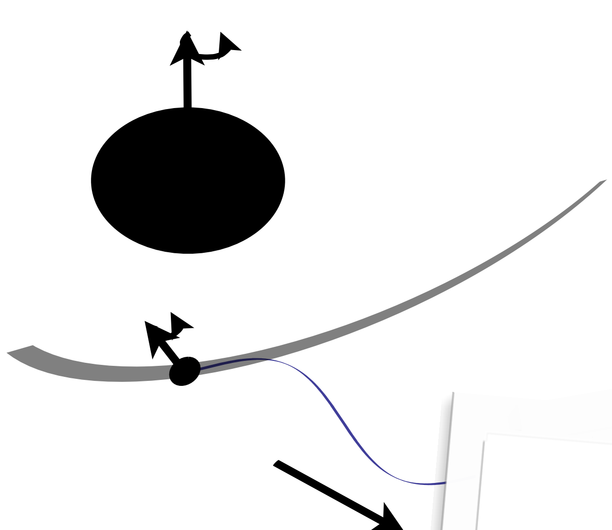

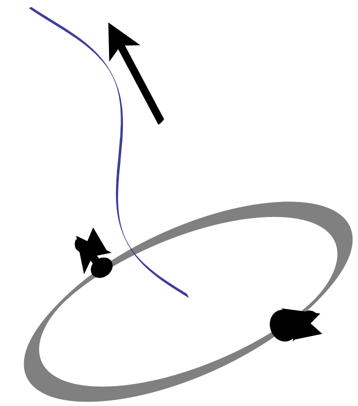

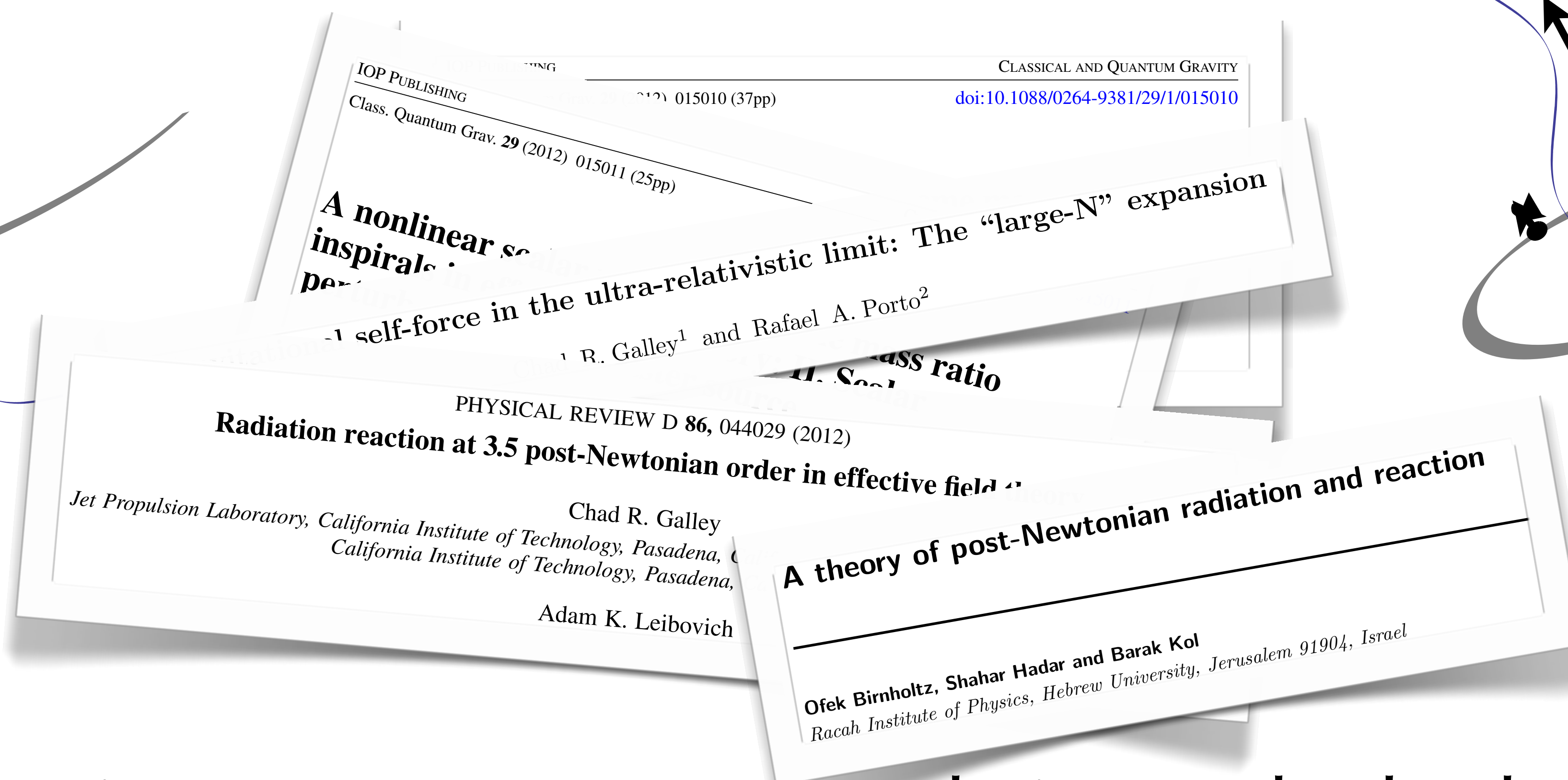

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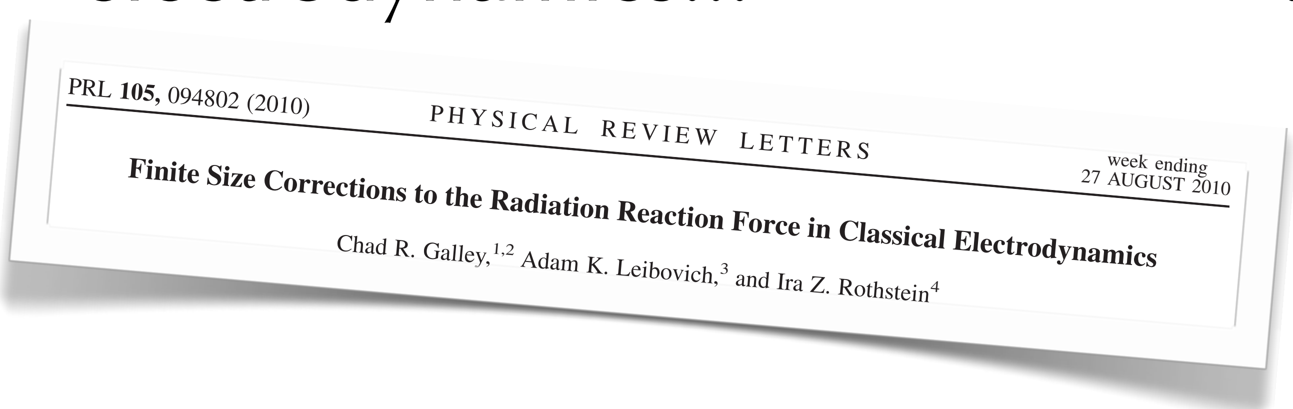

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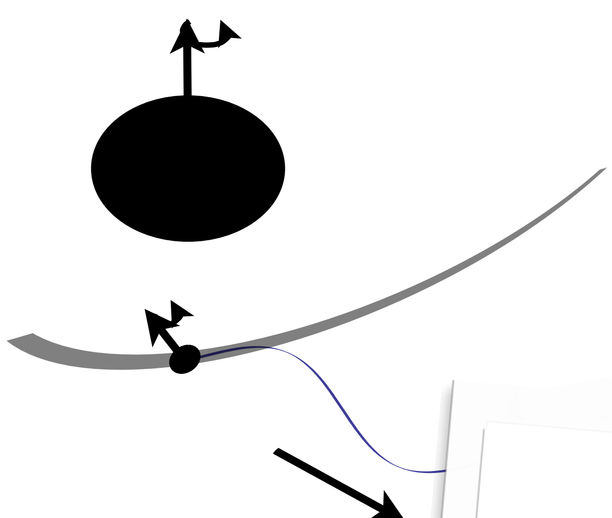

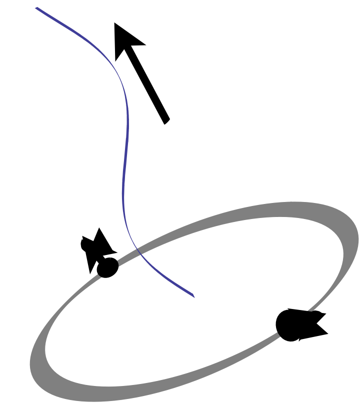

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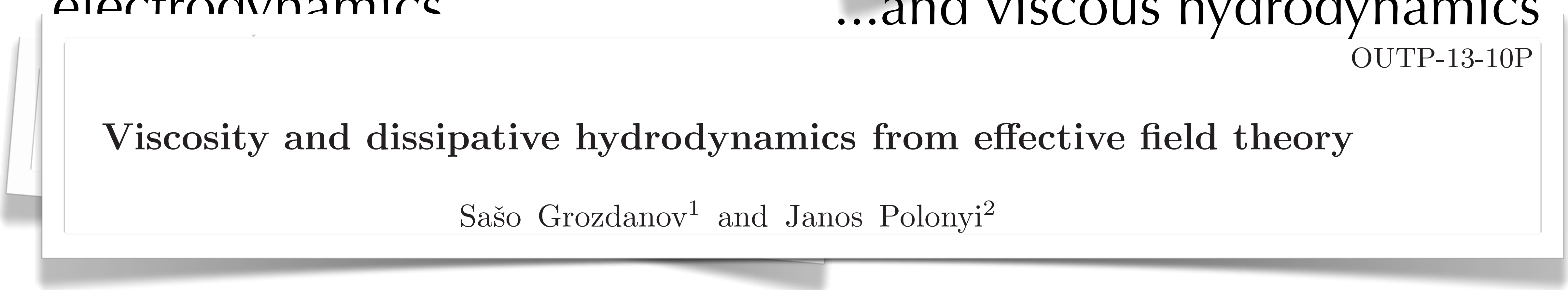

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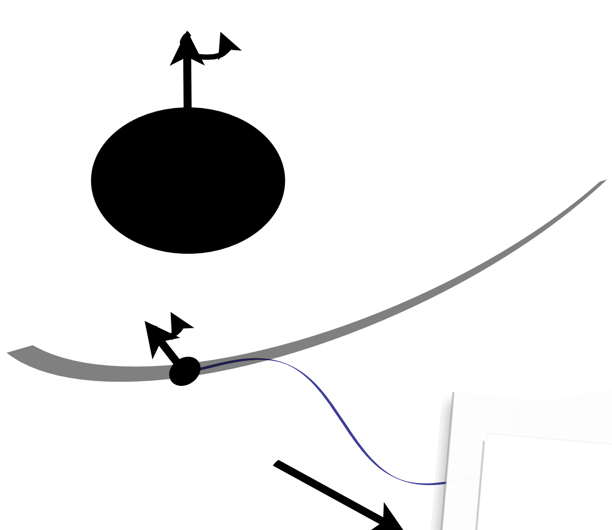

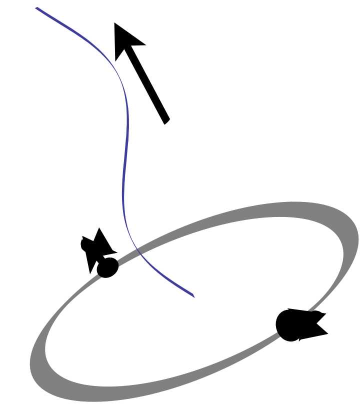

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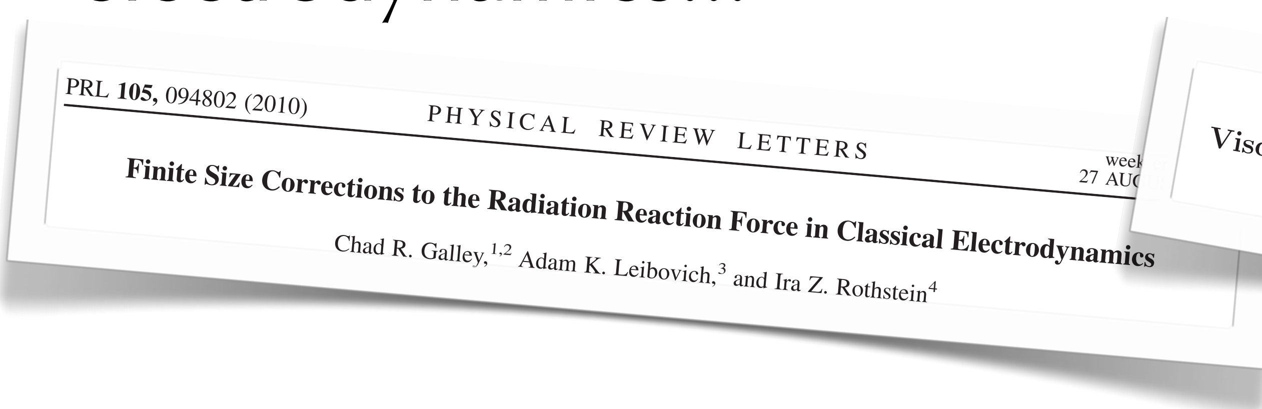

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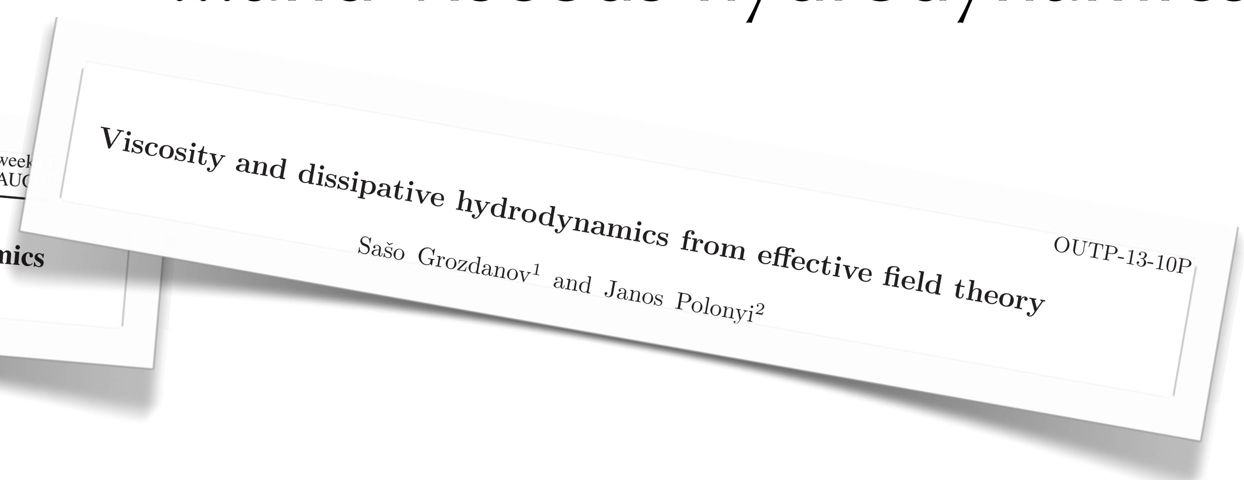

OUTP-13-10P
Viscosity and dissipative hydrodynamics from effective field theory
Sašo Grozdanov[1] and Janos Polonyi[2]

To derive the master source:

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- Use Detweiler-Whiting decomposition for $G_{ret}(x,x')$
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$$
\begin{aligned}
\mathcal{S}_R^{\alpha'\beta'}(z^{\mu'}) = \frac{m}{2} u^{\alpha'} u^{\beta'} \Big\{ & 1 + \frac{m}{4} I_R(z^{\mu'}) + \frac{3m^2}{32} I_R^2(z^{\mu'}) + \frac{m^2}{16} u^{\gamma'} u^{\delta'} \int d\tau'' D^R_{\gamma'\delta'\epsilon''\eta''} u^{\epsilon''} u^{\eta''} I_R(z^{\mu''}) \\
& + \frac{3m^3}{128} u^{\gamma'} u^{\delta'} \int d\tau'' D^R_{\gamma'\delta'\epsilon''\eta''} u^{\epsilon''} u^{\eta''} I_R^2(z^{\mu''}) + \frac{5m^3}{128} I_R^3(z^{\mu'}) \\
& + \frac{3m^3}{64} I_R(z^{\mu'}) u^{\gamma'} u^{\delta'} \int d\tau'' D^R_{\gamma'\delta'\epsilon''\eta''} u^{\epsilon''} u^{\eta''} I_R(z^{\mu''}) \\
& + \frac{m^3}{64} u^{\gamma'} u^{\delta'} \int d\tau'' D^R_{\gamma'\delta'\epsilon''\eta''} u^{\epsilon''} u^{\eta''} u^{\rho''} u^{\lambda''} \int d\tau''' D^R_{\rho''\lambda''\tau'''\sigma'''} u^{\tau'''} u^{\sigma'''} I_R(z^{\mu'''}) \\
& + \mathcal{O}(\lambda^4) \Big\} + \cdots
\end{aligned}
$$

$$
I_R(z^{\mu}) \equiv u^{\alpha} u^{\beta} \int d\tau' \, D^R_{\alpha\beta\gamma'\delta'}(z^{\mu}, z^{\mu'}) u^{\gamma'} u^{\delta'}
$$

CRG & Porto (2013)

To derive the self-force: *CRG (2011)*

- Use master source to construct regular field on worldline

$$h^R_{\alpha\beta}(x) = \int d\tau' \, D^R_{\alpha\beta\gamma'\delta'}(x, z^{\mu'}) \, \mathcal{S}_R^{\gamma'\delta}(z^{\mu'})$$

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- Use regular field in full worldline equations of motion (in ultra-rel limit)

$$S_{\mathrm{pp}}[z^\mu] = -m \int d\lambda \sqrt{-\big(g_{\alpha\beta}(z^\mu) + h^R_{\alpha\beta}(z^\mu)\big) u^\alpha u^\beta} + \text{ finite size corrections}$$

$$\left(g_{\mu\nu} + h^R_{\mu\nu} + \frac{g_{\mu\alpha} + h^R_{\mu\alpha}}{1 - h^R_{\gamma\delta} u^\gamma u^\delta} \, u^\alpha h^R_{\nu\beta} u^\beta \right) a^\nu = \left(\frac{1}{2} u^\alpha u^\beta g_\mu{}^\nu - (g_\mu{}^\alpha u^\beta + g_\mu{}^\beta u^\alpha) u^\nu - \frac{1}{2} \frac{g_{\mu\gamma} + h^R_{\mu\gamma}}{1 - h^R_{\epsilon\eta} u^\epsilon u^\eta} \, u^\alpha u^\beta u^\gamma u^\nu \right) \nabla_\nu h^R_{\alpha\beta}$$

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$$\left(g_{\mu\nu} + h^R_{\mu\nu} + \frac{g_{\mu\alpha} + h^R_{\mu\alpha}}{1 - h^R_{\gamma\delta} u^\gamma u^\delta} \, u^\alpha h^R_{\nu\beta} u^\beta \right) a^\nu = \left(\frac{1}{2} u^\alpha u^\beta g_\mu{}^\nu - (g_\mu{}^\alpha u^\beta + g_\mu{}^\beta u^\alpha) u^\nu \right.$$
$$\left. - \frac{1}{2} \frac{g_{\mu\gamma} + h^R_{\mu\gamma}}{1 - h^R_{\epsilon\eta} u^\epsilon u^\eta} \, u^\alpha u^\beta u^\gamma u^\nu \right) \nabla_\nu h^R_{\alpha\beta}$$

- Expand to desired order in $\lambda = \gamma^3 m/M$

Example: Circular orbits in Schwarzschild

For ultra-relativistic circular orbits in a Schwarzschild background
I_R is a constant on a given orbital radius
Barack & Sago (2007); Detweiler (2008)

$$\mathcal{S}_R^{\alpha'\beta'}(z_o^\mu) = \frac{m}{2} u_o^{\alpha'} u_o^{\beta'} \left[1 + \frac{m}{4} I_R(z_o^\mu) + \frac{5m^2}{32} I_R^2(z_o^\mu) + \frac{m^3}{8} I_R^3(z_o^\mu) + \mathcal{O}(\lambda^4) \right] + \cdots$$

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Circular orbits possess certain
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$$\Omega \ , \quad \mathcal{E} \ , \quad z$$

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Need to compute I_R numerically in ultra-relativistic regime to make predictions

Akcay + (2013)

Self-force on "photon"

Effective action in strict ultra-relativistic limit is degenerate...

$$\gamma \to \infty \qquad m \to 0$$

$$\lambda = \gamma^3 \frac{m}{M} = \text{constant}$$

$$S_{\text{eff}}[z^\mu] \sim \frac{\lambda M^2}{\gamma^4}\left(1 + \lambda + \lambda^2 + \frac{\lambda^2}{\gamma^2} + \cdots\right)$$

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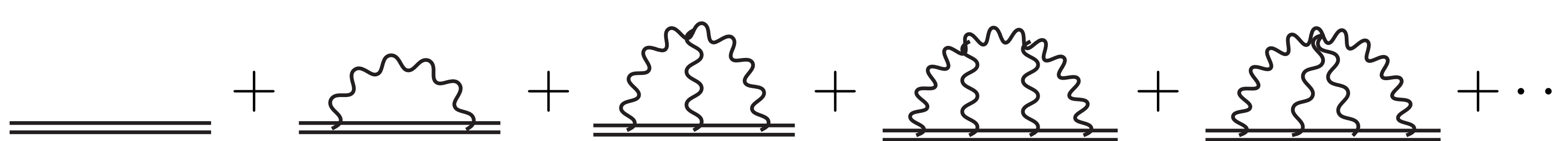
$S_{\text{eff}}[z^\mu] =$ 

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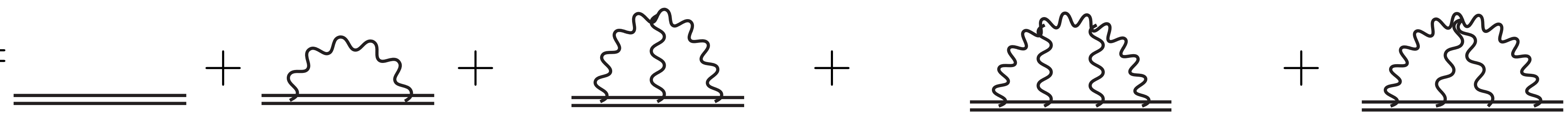
$S_{\text{eff}}[z^\mu] =$  $+ \cdots$

Does the massless limit of a massive particle give same answers as massless case?

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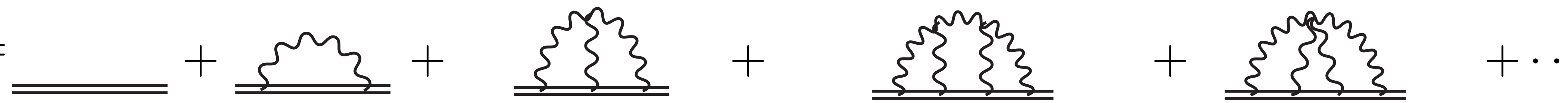
$S_{\text{eff}}[z^{\mu}] =$ 

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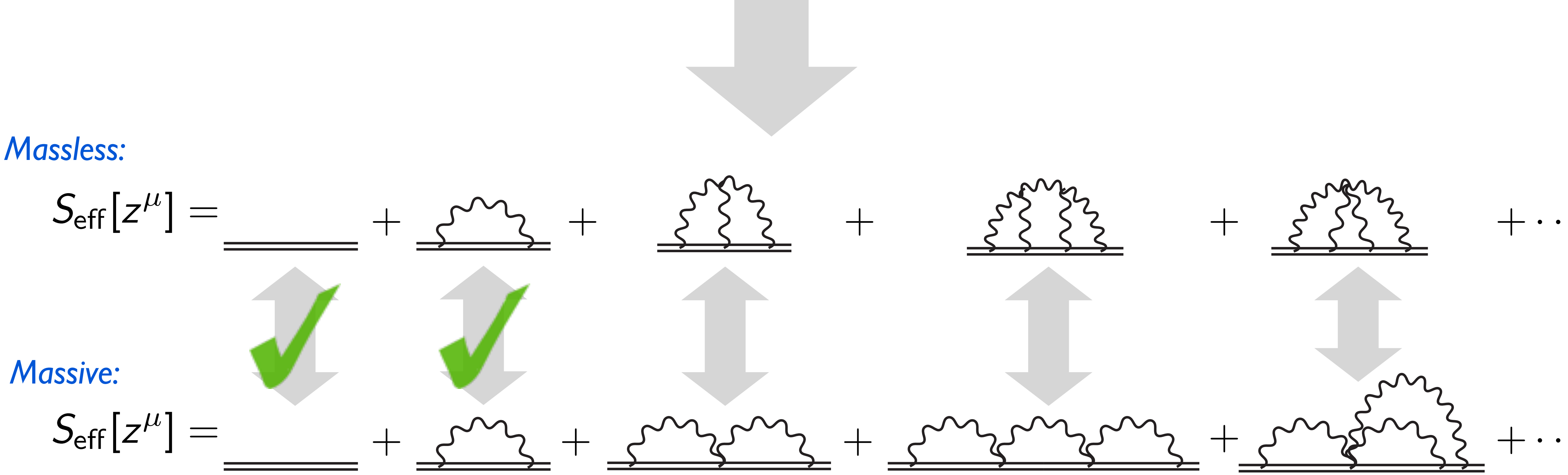
Suggests a duality between certain bulk and worldline diagrams



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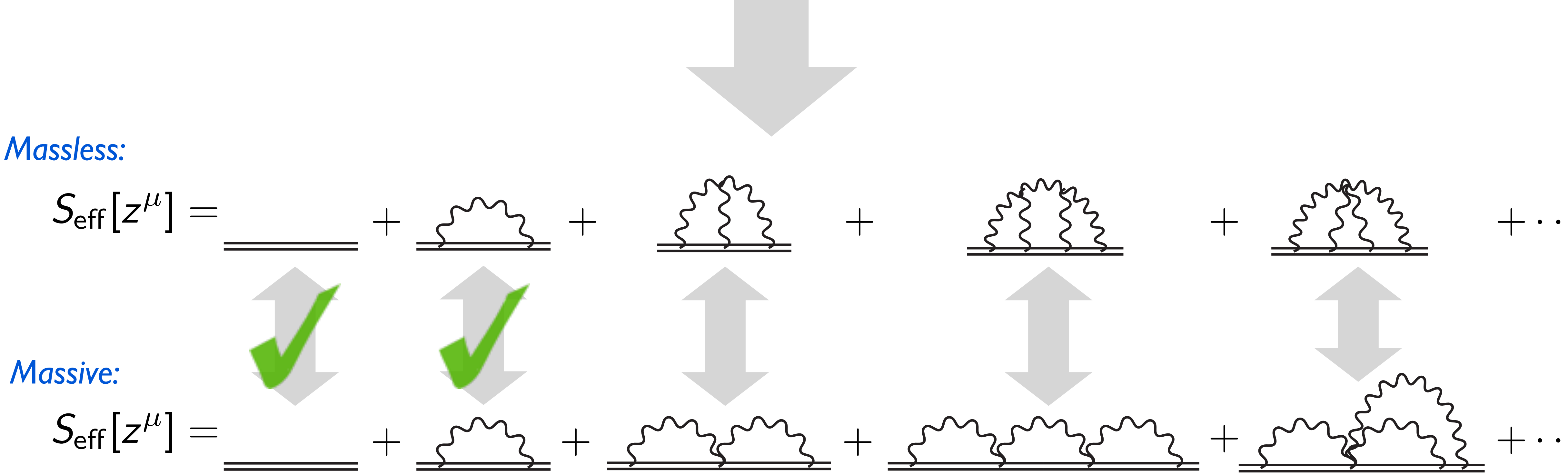

Does the massless limit of a massive particle give same answers as massless case?

<u>YES</u>

Suggests a duality between certain bulk and worldline diagrams

<u>NO</u>

Why is the limit discontinuous?

Conclusion & Outlook

In the ultra-relativistic regime: $\gamma \gg 1 \qquad \lambda = \gamma^3 m/M = \text{constant}$

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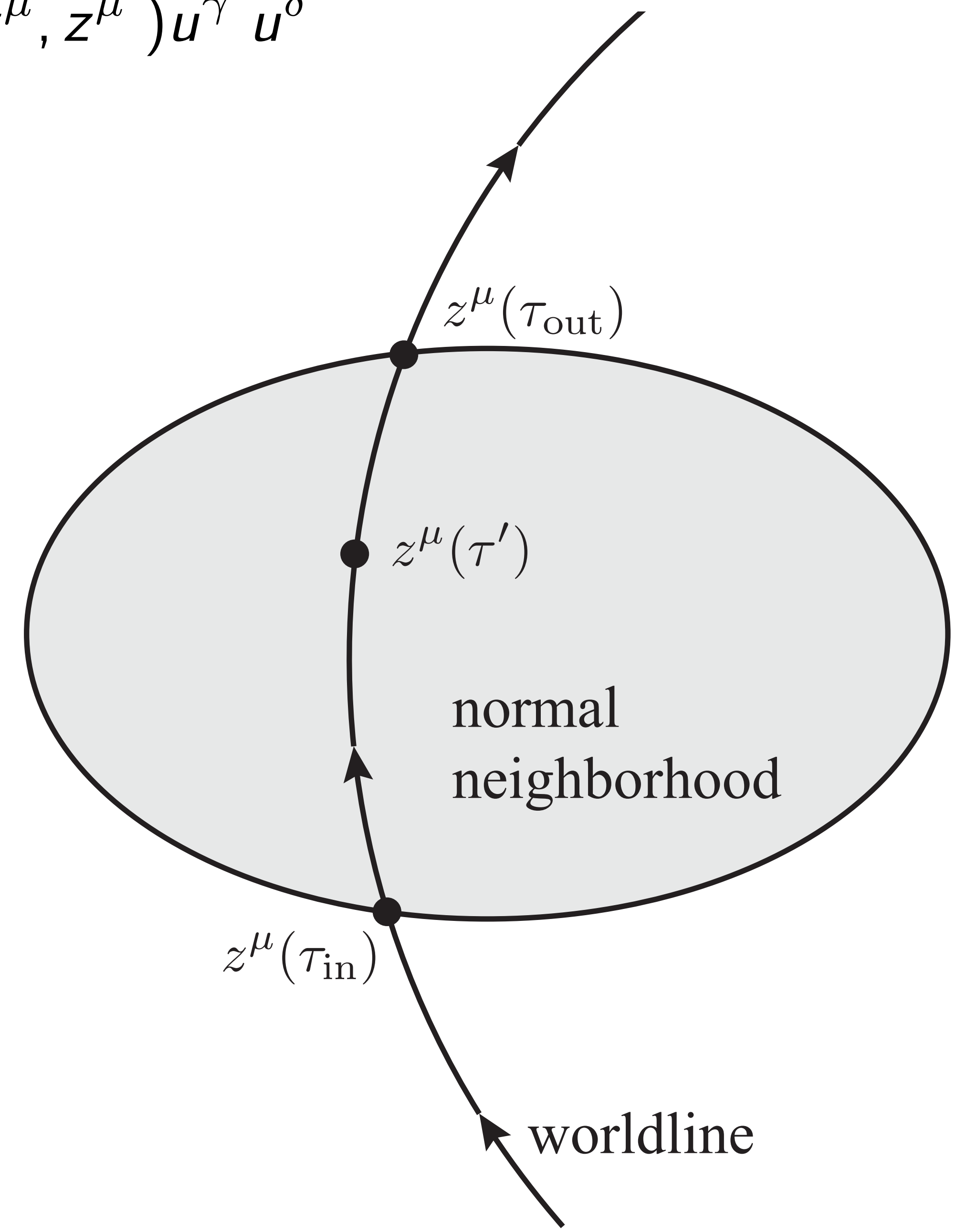
Are the massless limit and massless case equal???

- If so, worldline diagrams are somehow dual to certain bulk diagrams (would aid self-force calculations for EMRIs, etc.)

Extra slides

Normal neighborhood decomposition

$$I(z^\mu) = u^\alpha u^\beta \left(\int_{-\infty}^{\tau_{\rm in}} + \int_{\tau_{\rm in}}^{\tau_{\rm out}} + \int_{\tau_{\rm out}}^{\infty} \right) d\tau' \, G^S_{\alpha\beta\gamma'\delta'}(z^\mu, z^{\mu'}) u^{\gamma'} u^{\delta'}$$



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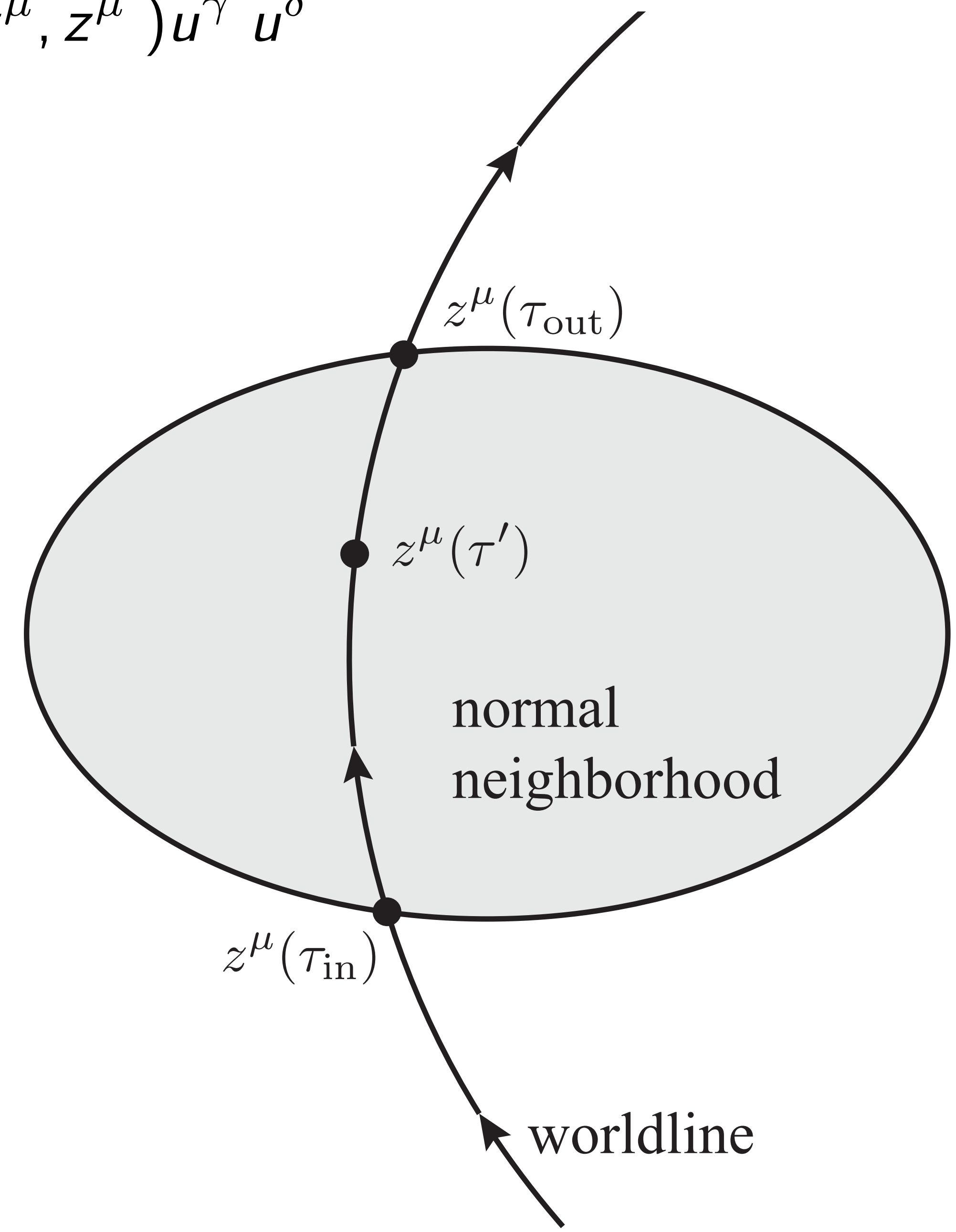

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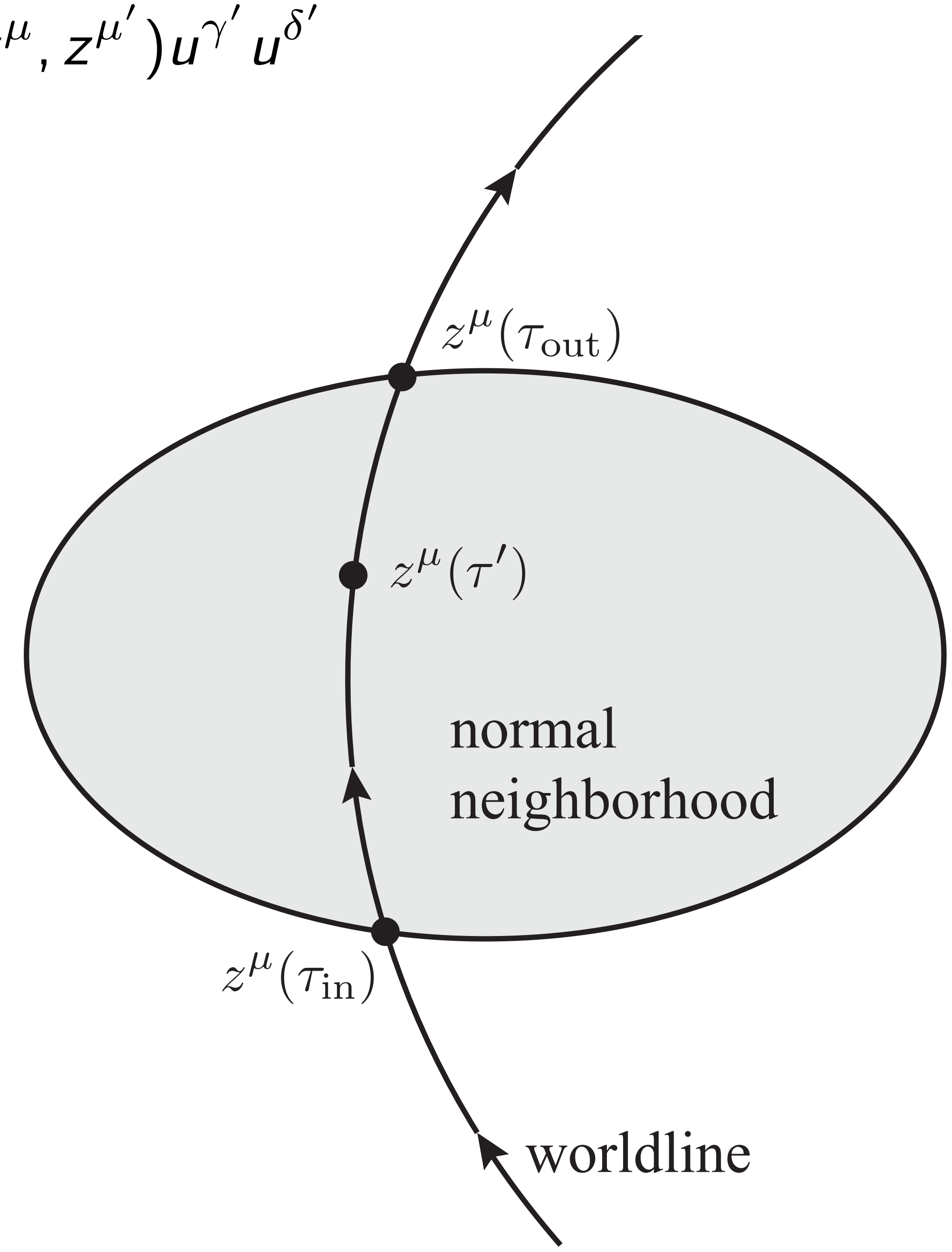

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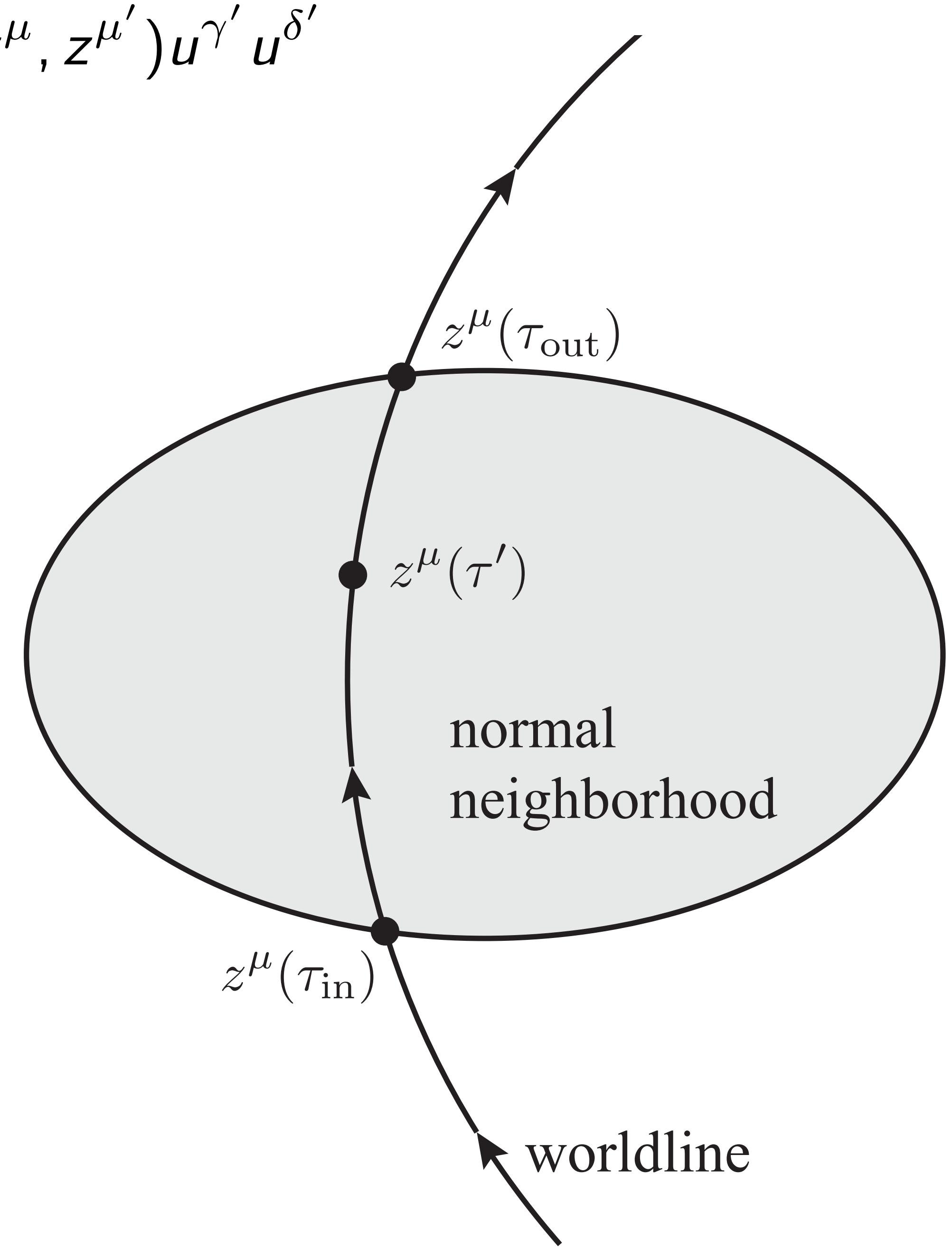

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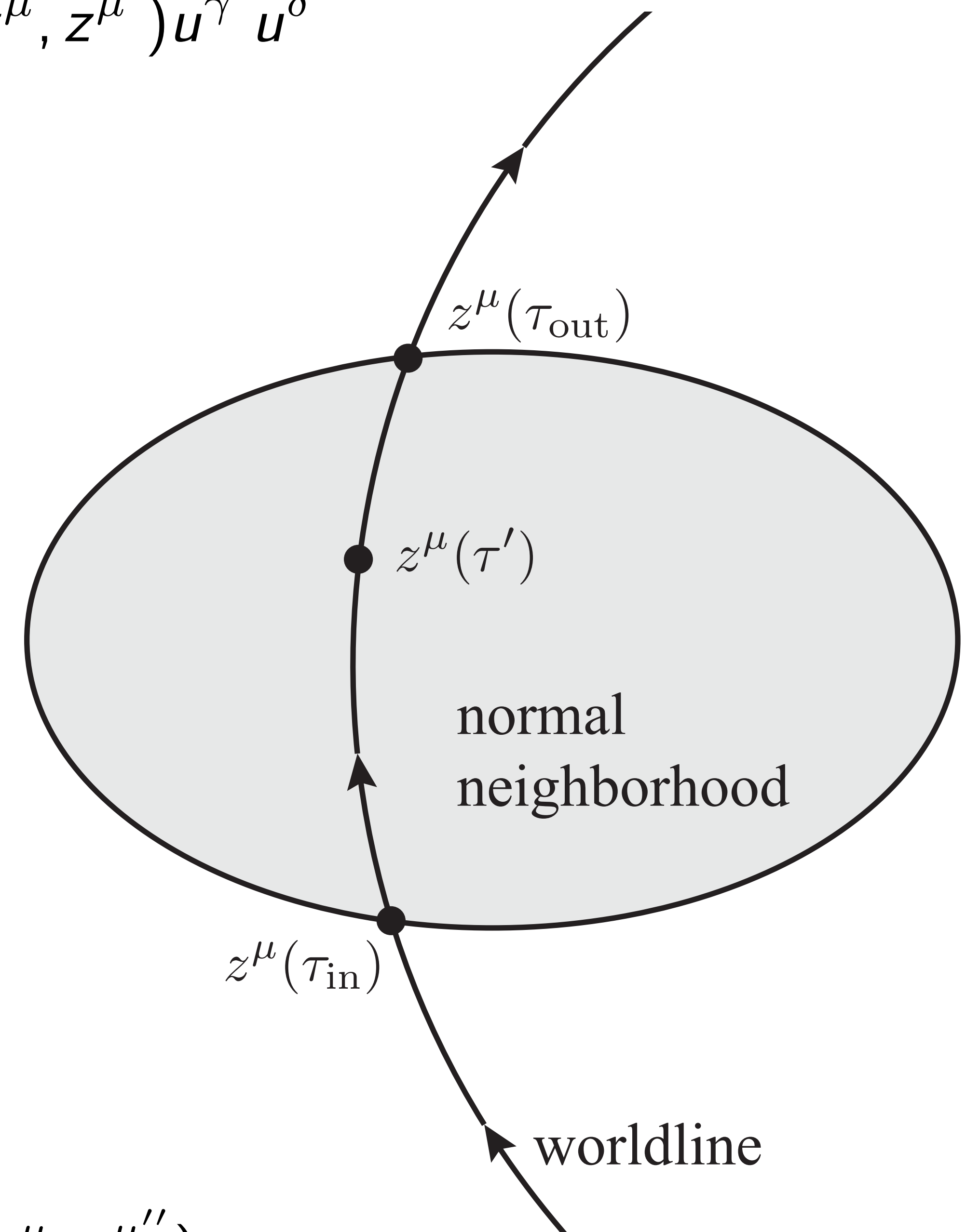

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$$\begin{aligned} D^R_{\alpha\beta\gamma'\delta'}(z^\mu, z^{\mu'}) \equiv{} & \Theta(\tau' - \tau_{\rm out}\Theta(\tau_{\rm in} - \tau') G^{\rm ret}_{\alpha\beta\gamma'\delta'}(z^\mu, z^{\mu''}) \\ & + \Theta(\tau_{\rm out} - \tau')\Theta(\tau' - \tau_{\rm in}) G^R_{\alpha\beta\gamma'\delta'}(z^\mu, z^{\mu'}) \end{aligned}$$

Momentum space representation

$$G^S_{\alpha\beta\gamma'\delta'}(x,x') = 4P_{\alpha\beta\gamma'\delta'}(x,x')\Delta^{1/2}(x,x')\delta(\sigma) - 4V_{\alpha\beta\gamma'\delta'}(x,x')\Theta(\sigma)$$

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Finally, in d spacetime dimensions (for dimensional regularization):

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