Finite dimensional minimax filters for Euler equations with uncertain forcing

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Abstract.
The talk presents a new deterministic state estimation algorithm for 2D incompressible Euler equations with periodic boundary conditions and uncertain but bounded inputs and initial conditions. Deterministic state estimators assume that errors have bounded energy and belong to a given bounding set. The state estimate is then defined as a minimax center of the reachability set, a set of all states of the physical model which are reachable from the given set of initial conditions and are compatible with observations. Dynamics of the minimax center is described by a minimax filter. The latter is constructed by using dynamic programming, i.e., the set V < 1, where V is the so-called value function V solving a Hamilton-Jacobi-Bellman (HJB) equation, coincides with the reachability set. Statistically, the uncertainty description in the form of a bounding set represents the case of uniformly distributed bounded errors in contrast to stochastic filtering, where all the errors are usually assumed to be in the form of "white noise". In many cases (e.g. linear dynamics and ellipsoidal uncertainty description) exp{-V} coincides with the conditional density of the states of a Markov diffusion process, which solves the so-called Kushner-Stratonovich (KS) equation, a backbone of the stochastic filtering theory.
The proposed algorithm converges to the minimax filter given incomplete and noisy observations. The results are illustrated for the case of Euler equation with Kolmogorov forcing.