Sparse grid finite element methods in two and three dimensions

Niall Madden, NUI Galway

Abstract

Classical finite element methods suffer from the infamous "curse of dimensionality": for fixed accuracy, the computational cost grows exponentially with the number of dimensions. Sparse grid methods are a family of schemes for which the complexity grows only slowly with the number of dimensions. These methods have been discovered, and rediscovered, over the past 30 years or so, with many applications including function approximation, quadrature, and numerical solution of PDEs.

For PDEs, popular sparse grid methods include the celebrated hierarchical basis approach of Zenger (1991), and the so-called two-scale method developed by Zhou and co-authors. The two-scale method is, arguably, the simplest sparse grid method, in both theory and practice. The goal of this talk will be to establish the links between it and the hierarchical basis approach (and multigrid methods), in an effort to combine the simplicity of one, with the optimal efficiency of the other.

The underlying motivation for our approach is to study the numerical solution of problems whose solutions feature boundary and interior layers. However, the focus of this talk will be on the general applicability of the schemes, rather than technicalities associated wit layer-resolving techniques.

This is joint work with Stephen Russell.