Approximate solution methods for one-dimensional Stefan problems

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Stefan problems, which describe the melting or solidification of a material, occur in a wide variety of natural and industrial applications. Mathematically, these problems represent a particular kind of boundary value problem where the phase boundary moves with time, and its location is not known a priori. The first part of the talk concerns the study of heat balance integral methods (HBIMs) applied to a variety of phase-change problems. We give an overview of the development of this method, which was originally used for analysing boundary layers, and show how it can be applied to Stefan problems, where it has made most impact since very few exact solutions exist. The HBIM has two main drawbacks and we discuss refinements which address both of these issues.

The second part of the talk considers the numerical solution of Stefan problems using finite difference methods with increased accuracy and correct initialisation. Although the numerical solution of phase-change problems is well documented, there are still unresolved issues regarding the start-up of a computation for a region that initially has zero thickness, as well as how to determine the position of the moving boundary thereafter. A combined analytical and numerical approach is described, which eliminates completely the ad-hoc treatment of the starting solution often used.

These methods are demonstrated using the classic problem of melting a semi-infinite material at the solidus but we also discuss extensions to applications such as the continuous casting of metals, removal of mass from an object by vaporization (known as ablation) and solvent diffusion in glassy polymers.