Integrable and non-integrable nonlinear PDEs with peakon solutions

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In this talk we introduce the concept for the singular (peakon) solution of a PDE. The word peakon is an abreviation from "peaked soliton". In the case of integrable systems this is a soliton with discontinuous first derivative; the wave profile is the graph of exp(-|x|). The best known examples of non-linear partial differential equations with (multi-)peakon solutions are the Camassa–Holm (CH) equation and the Degasperis–Procesi equation. The concept was introduced in 1993 by Camassa and Holm in the short but much cited paper where they derived their integrable equation in the context of shallow water waves. The singular solutions however are observed not only in integrable systems and we stydy several instructive examples in nonintegrable nonlinear equations like the multidimentional and multicomponent analogs of CH. In some of the examples a new type of nonlinear behavior is observed - the so called 'waltzing peakons'. These are peakons moving together and oscillating around their common center. Compactly supported waves (compactons) are another example of singular solutions which will be discussed.