17. A reasonable estimate for $y$ at $t = 2.5$ is between 18 and 19. No reliable estimate is possible at $t = 3$ from the specified data.
18. (b) $2.37 < \alpha_0 < 2.38$
19. (b) $0.67 < \alpha_0 < 0.68$

**Section 2.8, page 113**

1. $dw/ds = (s+1)^2 + (w+2)^2, \quad w(0) = 0$
2. $dw/ds = 1 - (w+3)^3, \quad w(0) = 0$
3. (a) $\phi_n(t) = \sum_{k=1}^{n} \frac{2^k t^k}{k!}$ (c) $\lim_{n\to\infty} \phi_n(t) = e^{2t} - 1$
4. (a) $\phi_n(t) = \sum_{k=1}^{n} \frac{(-1)^k t^k}{k!}$ (c) $\lim_{n\to\infty} \phi_n(t) = e^{-t} - 1$
5. (a) $\phi_n(t) = \sum_{k=1}^{n} (-1)^{k+1} t^{k+1}/(k+1)!2^{k-1}$
   (c) $\lim_{n\to\infty} \phi_n(t) = 4e^{-t/2} + 2t - 4$
6. (a) $\phi_n(t) = t - \frac{t^{n+1}}{(n+1)!}$
   (c) $\lim_{n\to\infty} \phi_n(t) = t$
7. (a) $\phi_n(t) = \sum_{k=1}^{n} \frac{t^{2k-1}}{1\cdot 3\cdot 5\cdots(2k-1)}$
8. (a) $\phi_n(t) = -\sum_{k=1}^{n} \frac{t^{3k-1}}{2\cdot 5\cdot 8\cdots(3k-1)}$
9. (a) $\phi_1(t) = \frac{t^3}{3}; \quad \phi_2(t) = \frac{t^3}{3} + \frac{t^7}{7\cdot 9}; \quad \phi_3(t) = \frac{t^3}{3} + \frac{t^7}{7\cdot 9} + \frac{2t^{11}}{3\cdot 7\cdot 9\cdot 11} + \frac{t^{15}}{(7\cdot 9)^2\cdot}$
10. (a) $\phi_1(t) = t; \quad \phi_2(t) = t - \frac{t^4}{4}; \quad \phi_3(t) = t - \frac{t^4}{4} + \frac{3t^7}{4\cdot 7} - \frac{3t^{10}}{16\cdot 10} + \frac{t^{13}}{64\cdot 13}$
11. (a) $\phi_1(t) = t, \quad \phi_2(t) = t - \frac{t^2}{2!} + \frac{t^4}{4!} - \frac{t^6}{6!} + O(t^7),$

$$\phi_3(t) = t - \frac{t^2}{2!} + \frac{t^3}{3!} + \frac{t^4}{4!} - \frac{7t^5}{5!} + \frac{14t^6}{6!} + O(t^7),$$

$$\phi_4(t) = t - \frac{t^2}{2!} + \frac{t^3}{3!} - \frac{7t^5}{5!} + \frac{31t^6}{6!} + O(t^7)$$

12. (a) $\phi_1(t) = -t - t^2 - \frac{t^3}{2},$

$$\phi_2(t) = -t - \frac{t^2}{2} + \frac{t^3}{6} + \frac{t^4}{4} - \frac{t^5}{5} - \frac{t^6}{24} + O(t^7),$$

$$\phi_3(t) = -t - \frac{t^2}{2} + \frac{t^4}{12} - \frac{3t^5}{20} + \frac{4t^6}{45} + O(t^7),$$

$$\phi_4(t) = -t - \frac{t^2}{2} + \frac{t^4}{8} - \frac{7t^5}{60} + \frac{t^6}{15} + O(t^7)$$

**Section 2.9, page 124**

1. $y_n = (-1)^n (0.9)^n y_0; \quad y_n \to 0$ as $n \to \infty$
2. $y_n = y_0/(n+1); \quad y_n \to 0$ as $n \to \infty$
3. $y_n = y_0\sqrt{(n+2)(n+1)/2}; \quad y_n \to \infty$ as $n \to \infty$
4. $y_n = \begin{cases} y_0, & \text{if } n = 4k \text{ or } n = 4k-1; \\ -y_0, & \text{if } n = 4k-2 \text{ or } n = 4k-3; \end{cases}$ $y_n$ has no limit as $n \to \infty$
5. $y_n = (0.5)^n (y_0 - 12) + 12; \quad y_n \to 12$ as $n \to \infty$
6. $y_n = (-1)^n (0.5)^n (y_0 - 4) + 4; \quad y_n \to 4$ as $n \to \infty$
7. 7.25%
8. \$2283.63
9. \$258.14

10. (a) \$804.62 (b) \$877.57 (c) \$1028.61
11. 30 years: \$804.62/month; \$289,663.20 total 20 years: \$899.73/month; \$215,935.20 total
12. \$103,624.62 13. 9.73%
16. (b) $u_n \to -\infty$ as $n \to \infty$
19. (a) 4.7263 (b) 1.223% (c) 3.5643 (e) 3.5699

**Miscellaneous Problems, page 126**

1. $y = (c/x^2) + (x^3/5)$
2. $\arctan(y/x) - \ln\sqrt{x^2+y^2} = c$
3. $x^2 + xy - 3y - y^3 = 0$
4. $x = ce^y + ye^y$
5. $x^2y + xy^2 + x = c$
6. $y = x^{-1}(1 - e^{1-x})$
7. $(x^2 + y^2 + 1)e^{-y^2} = c$
8. $y = (4 + \cos 2 - \cos x)/x^2$
9. $x^2y + x + y^2 = c$
10. $(y^2/x^3) + (y/x^2) = c$
11. $x^3/3 + xy + e^y = c$
12. $y = ce^{-x} + e^{-x}\ln(1 + e^x)$
13. $2(y/x)^{1/2} - \ln|x| = c$; also $y = 0$
14. $x^2 + 2xy + 2y^2 = 34$
15. $y = c/\cosh^2(x/2)$
16. $(2/\sqrt{3})\arctan[(2y - x)/\sqrt{3}x] - \ln|x| = c$
17. $y = ce^{3x} - e^{2x}$
18. $y = cx^{-2} - x$
19. $3y - 2xy^3 - 10x = 0$
20. $e^x + e^{-y} = c$
21. $e^{-y/x} + \ln|x| = c$
22. $y^3 + 3y - x^3 + 3x = 2$
23. $\dfrac{1}{y} = -x\displaystyle\int \frac{e^{2x}}{x^2}\,dx + cx$; also $y = 0$
24. $\sin^2 x \sin y = c$
25. $x^2/y + \arctan(y/x) = c$
26. $x^2 + 2x^2y - y^2 = c$
27. $\sin x \cos 2y - \frac{1}{2}\sin^2 x = c$
28. $2xy + xy^3 - x^3 = c$
29. $\arcsin(y/x) - \ln|x| = c$; also $y = x$ and $y = -x$
30. $xy^2 - \ln|y| = 0$
31. $x + \ln|x| + x^{-1} + y - 2\ln|y| = c$; also $y = 0$
32. $x^3y^2 + xy^3 = -4$

## CHAPTER 3

**Section 3.1, page 136**

1. $y = c_1e^t + c_2e^{-3t}$
2. $y = c_1e^{-t} + c_2e^{-2t}$
3. $y = c_1e^{t/2} + c_2e^{-t/3}$
4. $y = c_1e^{t/2} + c_2e^t$
5. $y = c_1 + c_2e^{-5t}$
6. $y = c_1e^{3t/2} + c_2e^{-3t/2}$
7. $y = c_1\exp[(9 + 3\sqrt{5})t/2] + c_2\exp[(9 - 3\sqrt{5})t/2]$
8. $y = c_1\exp[(1 + \sqrt{3})t] + c_2\exp[(1 - \sqrt{3})t]$
9. $y = e^t$; $y \to \infty$ as $t \to \infty$
10. $y = \frac{5}{2}e^{-t} - \frac{1}{2}e^{-3t}$; $y \to 0$ as $t \to \infty$
11. $y = 12e^{t/3} - 8e^{t/2}$; $y \to -\infty$ as $t \to \infty$
12. $y = -1 - e^{-3t}$; $y \to -1$ as $t \to \infty$
13. $y = \frac{1}{26}(13 + 5\sqrt{13})\exp[(-5 + \sqrt{13})t/2] + \frac{1}{26}(13 - 5\sqrt{13})\exp[(-5 - \sqrt{13})t/2]$; $y \to 0$ as $t \to \infty$
14. $y = (2/\sqrt{33})\exp[(-1 + \sqrt{33})t/4] - (2/\sqrt{33})\exp[(-1 - \sqrt{33})t/4]$; $y \to \infty$ as $t \to \infty$
15. $y = \frac{1}{10}e^{-9(t-1)} + \frac{9}{10}e^{t-1}$; $y \to \infty$ as $t \to \infty$
16. $y = -\frac{1}{2}e^{(t+2)/2} + \frac{3}{2}e^{-(t+2)/2}$; $y \to -\infty$ as $t \to \infty$
17. $y'' + y' - 6y = 0$
18. $2y'' + 5y' + 2y = 0$
19. $y = \frac{1}{4}e^t + e^{-t}$; minimum is $y = 1$ at $t = \ln 2$
20. $y = -e^t + 3e^{t/2}$; maximum is $y = \frac{9}{4}$ at $t = \ln(9/4)$, $y = 0$ at $t = \ln 9$
21. $\alpha = -2$
22. $\beta = -1$