24. $y \to -\infty$ or is asymptotic to $\sqrt{2t-1}$ depending on the initial value of y
25. $y \to 0$ and then fails to exist after some $t_f \geq 0$
26. $y \to \infty$ or $-\infty$ depending on the initial value of y

Section 1.2, page 14

1. (a) $y = 5 + (y_0 - 5)e^{-t}$ (b) $y = (5/2) + [y_0 - (5/2)]e^{-2t}$
   (c) $y = 5 + (y_0 - 5)e^{-2t}$
   Equilibrium solution is $y = 5$ in (a) and (c), $y = 5/2$ in (b); solution approaches equilibrium faster in (b) and (c) than in (a).
2. (a) $y = 5 + (y_0 - 5)e^{t}$ (b) $y = (5/2) + [y_0 - (5/2)]e^{2t}$
   (c) $y = 5 + (y_0 - 5)e^{2t}$
   Equilibrium solution is $y = 5$ in (a) and (c), $y = 5/2$ in (b); solution diverges from equilibrium faster in (b) and (c) than in (a).
3. (a) $y = ce^{-at} + (b/a)$
   (c) (i) Equilibrium is lower and is approached more rapidly. (ii) Equilibrium is higher. (iii) Equilibrium remains the same and is approached more rapidly.
4. (a) $y_1(t) = ce^{at}$ (b) $y = ce^{at} + (b/a)$
5. $y = ce^{-at} + (b/a)$
6. (a) $T = 2\ln 18 \cong 5.78$ months (b) $T = 2\ln[900/(900 - p_0)]$ months
   (c) $p_0 = 900(1 - e^{-6}) \cong 897.8$
7. (a) $r = (\ln 2)/30$ day^{-1} (b) $r = (\ln 2)/N$ day^{-1}
8. (a) $T = 5\ln 50 \cong 19.56$ sec (b) 718.34 m
9. (a) $dv/dt = 9.8, \quad v(0) = 0$ (b) $T = \sqrt{300/4.9} \cong 7.82$ sec
   (c) $v \cong 76.68$ m/sec
10. (a) $r \cong 0.02828$ day^{-1} (b) $Q(t) = 100e^{-0.02828t}$ (c) $T \cong 24.5$ days
12. $1620\ln(4/3)/\ln 2 \cong 672.4$ years
13. (a) $Q(t) = CV(1 - e^{-t/RC})$ (b) $Q(t) \to CV = Q_L$
    (c) $Q(t) = CV\exp[-(t - t_1)/RC]$
14. (a) $Q' = 3(1 - 10^{-4}Q), \quad Q(0) = 0$
    (b) $Q(t) = 10^4(1 - e^{-3t/10^4})$, t in hrs; after 1 year $Q \cong 9277.77$ g
    (c) $Q' = -3Q/10^4, \quad Q(0) = 9277.77$
    (d) $Q(t) = 9277.77e^{-3t/10^4}$, t in hrs; after 1 year $Q \cong 670.07$ g
    (e) $T \cong 2.60$ years
15. (a) $q' = -q/300, \quad q(0) = 5000$ g (b) $q(t) = 5000e^{-t/300}$ (c) no
    (d) $T = 300\ln(25/6) \cong 7.136$ hr
    (e) $r = 250\ln(25/6) \cong 256.78$ gal/min

Section 1.3, page 22

1. Second order, linear
2. Second order, nonlinear
3. Fourth order, linear
4. First order, nonlinear
5. Second order, nonlinear
6. Third order, linear
15. $r = -2$
16. $r = \pm 1$
17. $r = 2, -3$
18. $r = 0, 1, 2$
19. $r = -1, -2$
20. $r = 1, 4$
21. Second order, linear
22. Second order, nonlinear
23. Fourth order, linear
24. Second order, nonlinear

CHAPTER 2

Section 2.1, page 38

1. (c) $y = ce^{-3t} + (t/3) - (1/9) + e^{-2t}$; y is asymptotic to $t/3 - 1/9$ as $t \to \infty$
2. (c) $y = ce^{2t} + t^3e^{2t}/3$; $y \to \infty$ as $t \to \infty$
3. (c) $y = ce^{-t} + 1 + t^2e^{-t}/2$; $y \to 1$ as $t \to \infty$

4. (c) $y = (c/t) + (3\cos 2t)/4t + (3\sin 2t)/2$; y is asymptotic to $(3\sin 2t)/2$ as $t \to \infty$
5. (c) $y = ce^{2t} - 3e^{t}$; $y \to \infty$ or $-\infty$ as $t \to \infty$
6. (c) $y = (c - t\cos t + \sin t)/t^2$; $y \to 0$ as $t \to \infty$
7. (c) $y = t^2e^{-t^2} + ce^{-t^2}$; $y \to 0$ as $t \to \infty$
8. (c) $y = (\arctan t + c)/(1 + t^2)^2$; $y \to 0$ as $t \to \infty$
9. (c) $y = ce^{-t/2} + 3t - 6$; y is asymptotic to $3t - 6$ as $t \to \infty$
10. (c) $y = -te^{-t} + ct$; $y \to \infty, 0$, or $-\infty$ as $t \to \infty$
11. (c) $y = ce^{-t} + \sin 2t - 2\cos 2t$; y is asymptotic to $\sin 2t - 2\cos 2t$ as $t \to \infty$
12. (c) $y = ce^{-t/2} + 3t^2 - 12t + 24$; y is asymptotic to $3t^2 - 12t + 24$ as $t \to \infty$
13. $y = 3e^{t} + 2(t - 1)e^{2t}$
14. $y = (t^2 - 1)e^{-2t}/2$
15. $y = (3t^4 - 4t^3 + 6t^2 + 1)/12t^2$
16. $y = (\sin t)/t^2$
17. $y = (t + 2)e^{2t}$
18. $y = t^{-2}[(\pi^2/4) - 1 - t\cos t + \sin t]$
19. $y = -(1 + t)e^{-t}/t^4, \quad t \neq 0$
20. $y = (t - 1 + 2e^{-t})/t, \quad t \neq 0$
21. (b) $y = -\frac{4}{5}\cos t + \frac{8}{5}\sin t + (a + \frac{4}{5})e^{t/2}$; $a_0 = -\frac{4}{5}$
    (c) y oscillates for $a = a_0$
22. (b) $y = -3e^{t/3} + (a + 3)e^{t/2}$; $a_0 = -3$
    (c) $y \to -\infty$ for $a = a_0$
23. (b) $y = te^{-t} + (ea - 1)e^{-t}/t$; $a_0 = 1/e$
    (c) $y \to 0$ as $t \to 0$ for $a = a_0$
24. (b) $y = -\cos t/t^2 + \pi^2 a/4t^2$; $a_0 = 4/\pi^2$
    (c) $y \to \frac{1}{2}$ as $t \to 0$ for $a = a_0$
25. $(t, y) = (1.364312, 0.820082)$
26. $y_0 = -1.642876$
27. (a) $y = 12 + \frac{8}{65}\cos 2t + \frac{64}{65}\sin 2t - \frac{788}{65}e^{-t/4}$; y oscillates about 12 as $t \to \infty$
    (b) $t = 10.065778$
28. $y_0 = -5/2$
29. $y_0 = -16/3$; $y \to -\infty$ as $t \to \infty$ for $y_0 = -16/3$
36. See Problem 2.
37. See Problem 4.

Section 2.2, page 45

1. $3y^2 - 2x^3 = c; \quad y \neq 0$
2. $3y^2 - 2\ln|1 + x^3| = c; \quad x \neq -1, y \neq 0$
3. $y^{-1} + \cos x = c$ if $y \neq 0$; also $y = 0$; everywhere
4. $3y + y^2 - x^3 + x = c; \quad y \neq -3/2$
5. $2\tan 2y - 2x - \sin 2x = c$ if $\cos 2y \neq 0$; also $y = \pm(2n + 1)\pi/4$ for any integer n; everywhere
6. $y = \sin[\ln|x| + c]$ if $x \neq 0$ and $|y| < 1$; also $y = \pm 1$
7. $y^2 - x^2 + 2(e^{y} - e^{-x}) = c; \quad y + e^{y} \neq 0$
8. $3y + y^3 - x^3 = c$; everywhere
9. (a) $y = 1/(x^2 - x - 6)$
   (c) $-2 < x < 3$
10. (a) $y = -\sqrt{2x - 2x^2 + 4}$
    (c) $-1 < x < 2$
11. (a) $y = [2(1 - x)e^{x} - 1]^{1/2}$
    (c) $-1.68 < x < 0.77$ approximately
12. (a) $r = 2/(1 - 2\ln\theta)$
    (c) $0 < \theta < \sqrt{e}$
13. (a) $y = -[2\ln(1 + x^2) + 4]^{1/2}$
    (c) $-\infty < x < \infty$
14. (a) $y = \left[3 - 2\sqrt{1 + x^2}\right]^{-1/2}$
    (c) $|x| < \frac{1}{2}\sqrt{5}$
15. (a) $y = -\frac{1}{2} + \frac{1}{2}\sqrt{4x^2 - 15}$
    (c) $x > \frac{1}{2}\sqrt{15}$
16. (a) $y = -\sqrt{(x^2 + 1)/2}$
    (c) $-\infty < x < \infty$
17. (a) $y = 5/2 - \sqrt{x^3 - e^{x} + 13/4}$
    (c) $-1.4445 < x < 4.6297$ approximately
18. (a) $y = -\frac{3}{4} + \frac{1}{4}\sqrt{65 - 8e^{x} - 8e^{-x}}$
    (c) $|x| < 2.0794$ approximately
19. (a) $y = [\pi - \arcsin(3\cos^2 x)]/3$
    (c) $|x - \pi/2| < 0.6155$
20. (a) $y = \left[\frac{3}{2}(\arcsin x)^2\right]^{1/3}$
    (c) $-1 < x < 1$

21. $y^3 - 3y^2 - x - x^3 + 2 = 0, \quad |x| < 1$
22. $y^3 - 4y - x^3 = -1, \quad |x^3 - 1| < 16/3\sqrt{3}$ or $-1.28 < x < 1.60$
23. $y = -1/(x^2/2 + 2x - 1); \quad x = -2$
24. $y = -3/2 + \sqrt{2x - e^x + 13/4}; \quad x = \ln 2$
25. $y = -3/2 + \sqrt{\sin 2x + 1/4}; \quad x = \pi/4$
26. $y = \tan(x^2 + 2x); \quad x = -1$
27. (a) $y \to 4$ if $y_0 > 0$; $\quad y = 0$ if $y_0 = 0$; $\quad y \to -\infty$ if $y_0 < 0$
    (b) $T = 3.29527$
28. (a) $y \to 4$ as $t \to \infty$
    (b) $T = 2.84367$
    (c) $3.6622 < y_0 < 4.4042$
29. $x = \dfrac{c}{a}y + \dfrac{ad - bc}{a^2} \ln|ay + b| + k; \quad a \neq 0, \; ay + b \neq 0$
30. (e) $|y + 2x|^3 |y - 2x| = c$
31. (b) $\arctan(y/x) - \ln|x| = c$
32. (b) $x^2 + y^2 - cx^3 = 0$
33. (b) $|y - x| = c|y + 3x|^5$; $\quad$ also $y = -3x$
34. (b) $|y + x|\,|y + 4x|^2 = c$
35. (b) $2x/(x + y) + \ln|x + y| = c$; $\quad$ also $y = -x$
36. (b) $x/(x + y) + \ln|x| = c$; $\quad$ also $y = -x$
37. (b) $|x|^3 |x^2 - 5y^2| = c$
38. (b) $c|x|^3 = |y^2 - x^2|$

Section 2.3, page 57

1. $t = 100 \ln 100$ min $\cong 460.5$ min
2. $Q(t) = 120\gamma[1 - \exp(-t/60)]; \quad 120\gamma$
3. $Q = 50e^{-0.2}(1 - e^{-0.2})$ lb $\cong 7.42$ lb
4. $Q(t) = 200 + t - [100(200)^2/(200 + t)^2]$ lb, $\quad t < 300; \quad c = 121/125$ lb/gal; $\lim_{t \to \infty} c = 1$ lb/gal
5. (a) $Q(t) = \frac{63{,}150}{2501}e^{-t/50} + 25 - \frac{625}{2501}\cos t + \frac{25}{5002}\sin t$
   (c) level $= 25$; amplitude $= 25\sqrt{2501}/5002 \cong 0.24995$
6. (a) $(\ln 2)/r$ years $\quad$ (b) 9.90 years $\quad$ (c) 8.66%
7. (a) $k(e^{rt} - 1)/r \quad$ (b) $k \cong \$3930 \quad$ (c) 9.77%
8. (a) A: \$337,733.85; $\quad$ B: \$250,579.41
   (b) A: $2000e^{30r}(e^{10r} - 1)/r$; $\quad$ B: $2000(e^{30r} - 1)/r$
   (d) $r \cong 0.0609$
9. $k = \$3086.64$/year; $\quad \$1259.92$
10. (a) \$89,034.79 $\quad$ (b) \$102,965.21
11. (a) \$99,498.08 $\quad$ (b) \$188,501.92
12. (a) $t \cong 135.36$ months
    (b) \$152,698.56
13. (a) $(k/r) + [S_0 - (k/r)]e^{rt} \quad$ (b) $rS_0 \quad$ (c) $(1/r)\ln[k/(k - k_0)]$ years
    (d) $T \cong 8.66$ years $\quad$ (e) $rS_0e^{rT}/(e^{rT} - 1) \quad$ (f) \$119,716
14. (a) 0.00012097 year$^{-1} \quad$ (b) $Q_0 \exp(-0.00012097t)$, $\quad t$ in years
    (c) 13,305 years
15. $P = 201{,}977.31 - 1977.31e^{(\ln 2)t}, \quad 0 \leq t \leq t_f \cong 6.6745$ (weeks)
16. (a) $\tau \cong 2.9632$; no
    (b) $\tau = 10 \ln 2 \cong 6.9315$
    (c) $\tau = 6.3805$
17. (b) $y_c \cong 0.83$
18. $t = \ln \frac{13}{8} / \ln \frac{13}{12}$ min $\cong 6.07$ min