

# $p$ -adic modular forms

Mini-satellite conference to Journées Arithmétiques XXXI

July 6-8, 2019 (Istanbul)



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## Conference Programme

	<u>July 6 (Saturday)</u>	<u>July 7 (Sunday)</u>	<u>July 8 (Monday)</u>
09:00 - 10:00	Denis Benois	Matteo Longo	Fabian Januszewski
10:00 - 10:15	Tea Break	Tea Break	Tea Break
10:15 - 11:15	Francesca Gatti	Alice Pozzi	Oscar Rivero
11:15 - 11:30	Tea Break	Tea Break	Tea Break
11:30 - 12:30	Joaquin Rodrigues	Marco Seveso	Lennart Gehrman
12:30 - 13:30	Lunch Break	Free Afternoon in Old Town	Lunch Break
13:30 - 14:30	Santiago Molina		Antonio Lei
14:30 - 14:45	Tea Break		
14:45 - 15:45	Stefano Vigni		
18:00 -		Conference Dinner	

All talks will take place in the IMBM (Istanbul Centre for Mathematical Sciences) building. The Turkish name reads “Istanbul Matematiksel Bilimler Merkezi”.



# Abstracts

## On extra zeros of $p$ -adic $L$ -functions

Denish Benois

Institut de Mathématiques de Bordeaux

We discuss extra-zeros of motives having good reduction at  $p$ . An archetypical example is provided by the Kubota–Leopoldt  $L$ -function associated to a character  $\chi$  such that  $\chi(p) = 1$  and the theorem of Ferrero and Greenberg. Other interesting examples arise from some modular forms of odd weight. In this situation, the special value of the  $p$ -adic  $L$ -function can be expressed in terms of an  $\mathcal{L}$ -invariant defined using  $p$ -adic Hodge theory.

In this talk, we are mainly interested in the non-critical case. The basic example we have in mind is provided by the Rankin–Selberg convolution of two modular forms of the same weight (joint work with S. Horte).

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## A special case of triple product $p$ -adic $L$ -function and non-cristalline Kato classes

Francesca Gatti

Universitat Politècnica de Catalunya

I will describe a joint work (in progress) with X. Guitart, M. Masdeu and V. Rotger, where we study special values of a triple product  $p$ -adic  $L$ -function. More precisely, Let  $F, G, H$  be three Hida families. The triple product  $p$ -adic  $L$ -function  $\mathcal{L}_p^g(F, G, H)$  interpolates the central complex  $L$ -values  $L(F_k, G_\ell, H_m, (k + \ell + m)/2)$  for classical weights  $(k, \ell, m)$  such that  $\ell \geq k + m$ . The point  $(2, 1, 1)$  lies outside the region of classical interpolation and  $L(F_2 \otimes G_1 \otimes H_1, s) = L(E \otimes \rho, s)$ , where  $E$  is an elliptic curve over  $\mathbb{Q}$  and  $\rho$  is an Artin representation. Assume this function does not vanish at  $s = 1$  and that the Selmer group attached to  $(E, \rho)$  is trivial. In this setting, we describe the value  $\mathcal{L}_p^g(F, G, H)(2, 1, 1)$  in terms of a non-cristalline cohomology class which lies in the  $p$ -relaxed Selmer group attached to  $(E, \rho)$ .

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## Automorphic $\mathcal{L}$ -invariants for reductive groups

Lennart Gehrmann

Universität Duisburg-Essen

Automorphic  $\mathcal{L}$ -invariants were first constructed by Henri Darmon for modular forms of weight 2. Darmon's construction was generalized by various authors to modular forms of higher weight, Hilbert and Bianchi modular forms. In this talk I define automorphic  $\mathcal{L}$ -invariants for cohomological, cuspidal automorphic representations of arbitrary reductive groups. If time permits, I will discuss their connection to completed cohomology.

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## $p$ -adic $L$ -functions for $GL(2n)$

Fabian Januszewski

Karlsruhe Institute of Technology

I will report on joint work with Dimitrov and Raghuram on the construction of  $p$ -adic  $L$ -functions for  $GL(2n)$  over totally real fields.

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## Codimension two cycles and tensor products of Hida families

Antonio Lei

Université Laval

In a recent work of Bleher, Chinburg, Greenberg, Kakde, Pappas, Sharifi and Taylor, certain codimension two cycles are defined to study pseudo-null Iwasawa modules. In particular, they relate a pair of Katz  $p$ -adic  $L$ -functions to a pseudo-null Selmer group. We show that their result can be extended to tensor products of Hida families. Given two Hida families, there are two  $p$ -adic  $L$ -functions attached to their rank-Selberg convolution, depending on which family is “dominant”. We show that if these two  $p$ -adic  $L$ -functions are coprime, then we may relate them to a pseudo-null Selmer group. If time permits, we will also talk about generalizations of this result for triple products. This is joint work with Bharath Palvannan.

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# Generalized Heegner cycles on Mumford curves

Matteo Longo

University of Padova

We study generalised Heegner cycles, originally introduced by Bertolini-Darmon-Prasanna for modular curves, in the context of Mumford curves. The main result of this paper relates generalized Heegner cycles with a two variable anticyclotomic  $p$ -adic  $L$ -function attached to a Coleman family and an imaginary quadratic field. In particular, when the restriction of this function to (non necessarily central) critical lines is zero, we express its derivative as combination of generalized Heegner cycles. This is a joint work, in progress, with M. R. Pati.

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## Triple product $p$ -adic $L$ -functions over totally real number fields

Santiago Molina

IUniversitat Politècnica de Catalunya

During the nineties Kato obtained deep results on the Birch and Swinnerton-Dyer conjecture in rank 0 for twists of elliptic curves over  $\mathbb{Q}$  by Dirichlet characters. More recently, Bertolini- Darmon-Rotger and Darmon-Rotger developed analogous methods to treat twists by certain Artin representations of dimension 2 and 4. The main tool of these methods is the construction of triple product  $p$ -adic  $L$ -functions attached to modular forms over  $\mathbb{Q}$ .

The aim of this talk is to explain how to construct triple product  $p$ -adic  $L$ -functions attached to automorphic forms over totally real fields by exploiting the techniques of Andreatta and Iovita. Moreover, we will introduce the main ideas of joint ongoing work with Barrera and Rotger which aims to generalize the results used of Kato, BDR and DR to totally real number fields.

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## Rigid meromorphic cocycles at real multiplication points

Alice Pozzi

University College London

A rigid meromorphic cocycle is a class in the first cohomology of the group  $SL_2(\mathbb{Z}[1/p])$  acting on the non-zero rigid meromorphic functions on the Drinfeld  $p$ -adic upper half plane by Möbius transformation. Rigid meromorphic cocycles can be evaluated at points of real multiplication, and their RM values conjecturally lie in the ring class field of real quadratic fields, suggesting a striking analogy with the classical theory of complex multiplication.

In this talk, we discuss a special case of the conjecture, relating the RM value of the “Eisenstein” Dedekind-Rademacher cocycle to a Gross-Stark unit. We explain the connection with certain deformations of Hilbert Eisenstein series of weight one. This is work in progress with Henri Darmon and Jan Vonk.

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## **The Euler system of Beilinson-Flach elements and Gross-Stark units.**

Oscar Rivero Salgado

Universitat Politècnica de Catalunya

Kings, Lei, Loeffler and Zerbes constructed an Euler system attached to two Hida families and proved an explicit reciprocity law in that case, connecting Beilinson-Flach elements with the so-called Hida-Rankin  $p$ -adic  $L$ -function. In this talk, we focus on the self-dual case, where an exceptional zero phenomenon emerges and the connection with the Hida-Rankin  $p$ -adic  $L$ -function is now via certain derived Beilinson-Flach elements. At the same time, we see how the weight one specialization of this derived class encodes arithmetic information concerning the group of units of the associated number fields, proving a conjecture of Darmon, Lauder and Rotger around  $p$ -adic iterated integrals.

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## **Norm-compatible cohomology classes in Siegel varieties**

Joaquin Rodrigues Jacinto

Université Aix-Marseille

We will explain how to construct towers of interesting classes in the cohomology of Siegel six-folds. We will study their complex regulator and we will give an application to Iwasawa theory. This is joint work with Antonio Cauchi and Francesco Lemma.

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## **On the $p$ -adic deformation of algebraic cycles**

Marco Seveso

University of Milan

I will report on a work in progress with M. Bertolini and R. Venerucci about  $p$ -adic interpolation of algebraic cycles in quite general frameworks, such as those predicted by the Gan-Gross-Prasad conjectures. I will illustrate this general picture in a couple of examples.

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# On Kolyvagin's conjecture and the Bloch-Kato formula for modular forms

Stefano Vigni

Università degli Studi di Genova

A few years ago, Wei Zhang proved (under certain assumptions) Kolyvagin's conjecture on the non-triviality of his system of cohomology classes built out of the Euler system of Heegner points on a rational elliptic curve. He also proved the  $p$ -part of the Birch and Swinnerton-Dyer formula in analytic rank one. In this talk I will describe an analogue of Kolyvagin's conjecture for Heegner cycles on Kuga-Sato varieties and state the  $p$ -part of the Bloch-Kato formula for higher (even) weight modular forms in analytic rank one. Time permitting, I will briefly sketch our strategy of proof of these results, with a special focus on Kolyvagin's conjecture. This is joint work (in progress) with Matteo Longo and Daniele Masoero.

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