List 1: Classification for char$k \neq 2$

$g(C_0)$	d, n	C_0	Hyper/Nonhyper	$\#C_0$
1	$d=2, n=2$	$y^2 = (x-\alpha)g(x)$	Hyper	$\Theta(q^2)$
	$d=5, n=4$	$y^2 = (x-\alpha)(x-\alpha^q)(x-\alpha^{q^2})(x-\alpha^{q^3})$	Non-hyper	$\Theta(q^2)$
	$d=3, n=2$	$y^2 = (x-\alpha)(x-\alpha^q)(x-\beta^q)(x-\beta^{q^2})$ either $\alpha, \beta \in k_3 \setminus k$ or $\alpha \in k_6 \setminus (k_2 \cup k_3), \quad \beta = \alpha^{q^3}$ C_0:Hyper $\iff \exists A \in GL_2(k), \beta = A \cdot \alpha, Tr(A) = 0$	 Hyper	$\Theta(q^3)$ $\Theta(q^2)$
	$d=3, n=3$	$y^2 = (x-\alpha)(x-\alpha^q)(x-\beta)(x-\beta^q)$ $\beta = A\alpha, \ \exists A \in GL_2(k), \ Tr(A) = 0$	Hyper	$\Theta(q^2)$
	$d=7, n=3$	$y^2 = (x-\alpha)(x-\alpha^q)(x-\alpha^{q^2})(x-\alpha^{q^r}), \quad r = 4, 5$	Nonhyper	$\Theta(q^4)$?
2	$d=2, n=2$	$y^2 = (x-\alpha)g(x)$	Hyper	$\Theta(q^4)$
	$d=3, n=2$	$y^2 = (x-\alpha)(x-\alpha^q)(x-\beta)(x-\beta^q)(x-\gamma)(x-\gamma^q)$ either $\alpha \in k_9 \setminus k_3, \beta = \alpha^{q^3}, \gamma = \alpha^{q^6}$ or $\alpha \in k_6 \setminus (k_2 \cup k_3), \beta = \alpha^{q^3}, \gamma = k_3 \setminus k$ or $\alpha, \beta, \gamma \in k_3 \setminus k$	Nonhyper	$\Theta(q^6)$?
3	$d=2, n=2$	$y^2 = (x-\alpha)g(x)$	Hyper	$\Theta(q^6)$
	$d=3, n=2$	$y^2 = (x-\alpha)(x-\alpha^q)(x-\beta)(x-\beta^q)(x-\gamma)(x-\gamma^q)$ $\times(x-\delta)(x-\delta^q)$ Either $\alpha \in k_{12} \setminus (k_6 \cup k_4), \beta = \alpha^{q^3}, \gamma = \alpha^{q^6}, \delta = \alpha^{q^9}$ or $\alpha \in k_9 \setminus k_3, \beta = \alpha^{q^3}, \gamma = \alpha^{q^6}, \delta \in k_3 \setminus k$ or $\alpha \in k_6 \setminus (k_2 \cup k_3), \beta = \alpha^{q^3}, \gamma \in k_6 \setminus (k_2 \cup k_3), \delta = \alpha^{q^3}$ or $\alpha \in k_6 \setminus (k_2 \cup k_3), \beta = \alpha^{q^3}, \gamma, \delta \in k_3 \setminus k$ or $\alpha, \beta, \gamma, \delta \in k_3 \setminus k$	Nonhyper	$\Theta(q^9)$?
	$d=7, n=3$	$y^2 = (x-\alpha)(x-\alpha^q)(x-\alpha^{q^2})(x-\alpha^{q^r})$ $\times(x-\beta)(x-\beta^q)(x-\beta^{q^2})(x-\beta^{q^r}), \quad r = 4, 5$ either $\alpha \in k_{14} \setminus (k_2 \cup k_7), \beta = \alpha^{q^7}$ or $\alpha, \beta \in k_7 \setminus k$	Nonhyper	$\Theta(q^{11})$?
	$d=15, n=4$	$y^2 = (x-\alpha)(x-\alpha^q)(x-\alpha^{q^2})(x-\alpha^{q^3})$ $\times(x-\alpha^{q^7})(x-\alpha^{q^{10}})(x-\alpha^{q^{11}})(x-\alpha^{q^{13}})$ or $y^2 = (x-\alpha)(x-\alpha^q)(x-\alpha^{q^2})(x-\alpha^{q^3})$ $\times(x-\alpha^{q^5})(x-\alpha^{q^7})(x-\alpha^{q^8})(x-\alpha^{q^{11}})$ $\alpha \in k_{15} \setminus k$	Nonhyper	$\Theta(q^{12})$?

List 2 : Classification for char$(k) = 2$

$g(C_0)$	d, n	Ordinary?	C_0	Hyper?	$\#C_0$
1	$d=2$ $n=2$	ordinary	$y^2+xy=x^3+ax^2+bx$	Hyper	$\Theta(q^2)$
		non-ordin	$y^2+y=ax^3+bx^2+cx+d$ $a^q=a\neq 0, b^q+b\neq 0$ or $c^q+c\neq 0$	Hyper	
	$d=4$ $n=3$	ordinary	$y^2+xy=x^3+cx$ $c\in k_4\setminus k_2, Tr(c)=0$	Hyper	$\Theta(q^3)$
		non-ordin	$y^2+y=ax^3+bx^2+cx+d$ $a^q=a\neq 0, Tr(b)=Tr(c)=Tr(d)=0$ b or $c\in k_4\setminus k_2$	Hyper	
	$d=2^n-1$ $n\geq 2$		(1) ${}^\sigma g(x)=g(x), n\geq 2$ $y^2+g(x)y=f(x), \quad L(f)=0$		$\Theta(q^{2n-1})$
		ordinary	The same as above e.g. $n=2$ $y^2+xy=x^3+ax^2+bx$ $a\in k, Tr(b)=0$	Hyper	$\Theta(q^n)$? $\Theta(q^2)$
		ordinary	(2) ${}^\sigma g(x)\neq g(x), d=3, n=2$ $g(x)=(x+\alpha^q)(x+\alpha^{q^2}), \alpha\in k_3\setminus k$ $Tr((x+\alpha)^2 f)=0$		$\Theta(q^3)$?
	$d=(2^{n_1}-1)(2^{n_2}-1)$ $n=n_1+n_2, 2\leq n_1, n_2$ $(2^{n_1}-1, 2^{n_2}-1)=1$	ordinary		Nonhyper	$\Theta(q^{n_1+n_2-1})$?
2	$d=2$ $n=2$		$y^2+g(x)y=f(x)$ $\deg f(x)=5, \deg_k g(x)\leq 2$ ${}^\sigma f=f+g^2 l, l\in k[x], \deg l=1,2$	Hyper	$\Theta(q^4)$
	$d=4$ $n=3$		$y^2+g(x)y=f(x)$ $\deg f(x)=5, \deg_k g(x)\leq 2$ ${}^\sigma f=f+g^2 l, l\in k_2[x]$, $\deg l=1,2, \deg(l+^\sigma l=1,2)$	Hyper	$\Theta(q^5)$
	$d=2^n-1$ $n\geq 2$		${}^\sigma g(x)=g(x)$ $y^2+g(x)y=f(x), \ L(f)=0$	Nonhyper	$\Theta(q^{3n})$
3	$d=2, n=2$		$y^2+g(x)y=f(x)$ (*1)	Hyper	$\Theta(q^6)$
	$d=4, n=3$		$y^2+g(x)y=f(x)$ (*2)	Hyper	$\Theta(q^7)$?
	$d=2^n-1$		(1) ${}^\sigma g(x)=g(x)$ $y^2+g(x)y=f(x), \ L(f)=0$	Nonhyper	$\Theta(q^{4n+1})$
	$d=3$		(2) ${}^\sigma g(x)\neq g(x)$ Either $g=g_1(x)(x+\alpha^q)(x+\alpha^{q^2}), \alpha\in k_3\setminus k$ $g_1\in k[x], \deg g_1\leq 2, L((x+\alpha)^2 f)=0$ Or $g=(x+\alpha^q)^2(x+\alpha^{q^2})^2$, $\alpha\in k_3\setminus k, L((x+\alpha)^4 f)=0$	Nonhyper	
	$d=7$	ordinary	$g=(x+\alpha^q)(x+\alpha^{q^2})(x+\alpha^{q^3})(x+\alpha^{q^r})$, $r=4,5, \quad \alpha\in k_7\setminus k$, $L((x+\alpha^{q^3})^2(x+\alpha^{q^5})^2(x+\alpha^{q^7})^2 f)\equiv 0$ (*3)	Nonhyper	
	$d=15$	ordinary	$g=(x+\alpha^q)(x+\alpha^{q^2})(x+\alpha^{q^3})(x+\alpha^{q^4})$, $\alpha\in k_5\setminus k, L((x+\alpha^{q^4})^2 f)\equiv 0$ (*3)	Nonhyper	

(*1) With the same conditions as $g_0=2, d=n=2$.

(*2) With the same conditions as $g_0=2, d=4, n=3$

(*3) Here "$\equiv$" means $\equiv 0 \bmod L(\ell^2+\hat{g}\ell)$.

Note: Ordinary nonhyper curves also exist for $g_0=1, d=(2^{n_1}-1)(2^{n_2-1})$,

$2\leq n_1, n_2, (2^{n_1}-1, 2^{n_2}-1)=1$