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Nudging is still used for mesoscale analysis, for example, using radar data.

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Large scales are analysed first, then smaller scales.

Refresher on Statistics

Suppose we have the noon-day pressure p_i and temperature T_i at Belfield, every day for a year. Let $n = 365$.

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The standard deviations, σ_p and σ_T are the square roots of the variances. They measure the root mean square deviation from the mean.

The covariance of p and T is defined as

$$\mu_{pT} = \frac{1}{n}\sum_{i=1}^{n}(p_i - \bar{p})(T_i - \bar{T})$$

Note that it is defined using <u>deviations</u> from the means.

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If p tends to be greater than its mean value when T is less than its mean, and vice-versa, then p and T are negatively correlated and $\rho_{pT} < 0$.

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Many variables have distributions which can be accurately modelled by the normal distribution.

Back to the SCM Analysis

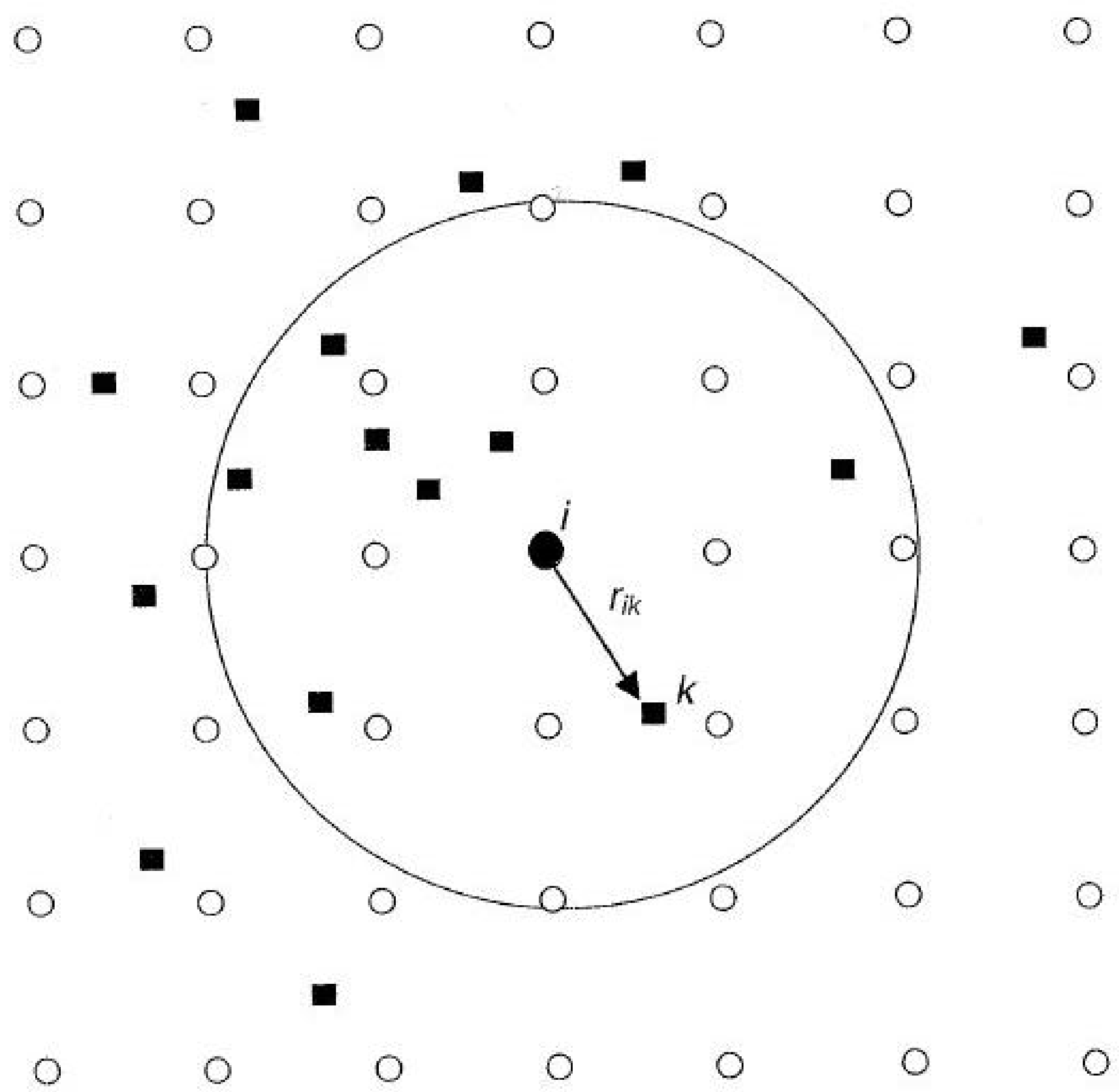

Figure 5.1.1: Schematic of grid points (circles), irregularly distributed observations (squares), and a radius of influence around a grid point i marked with a black circle. In 4DDA, the grid-point analysis is a combination of the forecast at the grid point (first guess) and the observational increments (observation minus first guess) computed at the observational points k. In certain analysis schemes, like SCM, only observations within the radius of influence, indicated by a circle, affect the analysis at the black grid point.

The following iterations are obtained by successive corrections:

$$f_i^{n+1} = f_i^n + \left[\frac{\sum_{k=1}^{K_i^n} w_{ik}^n (f_k^O - f_k^n)}{\sum_{k=1}^{K_i^n} w_{ik}^n + \varepsilon^2} \right]$$

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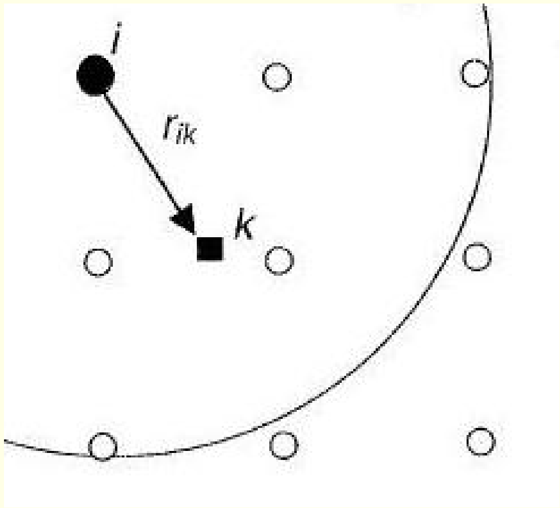
- ε^2 is an estimate of the ratio of the observation error variance to the background error variance.

Interpolation

To calculate the difference $f_k^O - f_k^n$, we have to interpolate the background field to the observation point.



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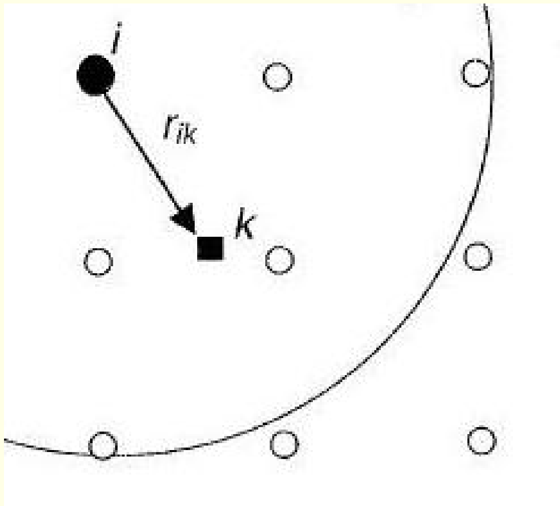

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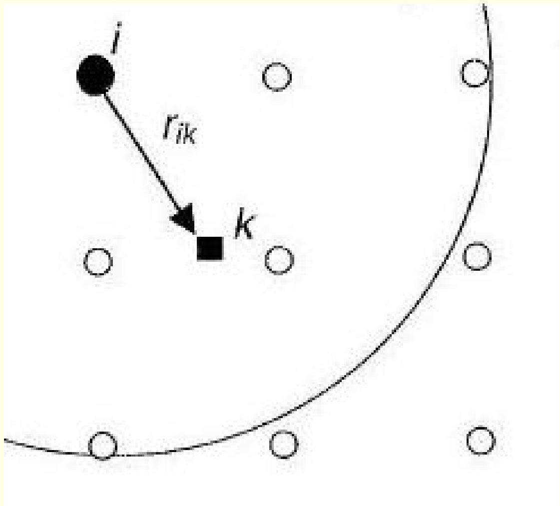

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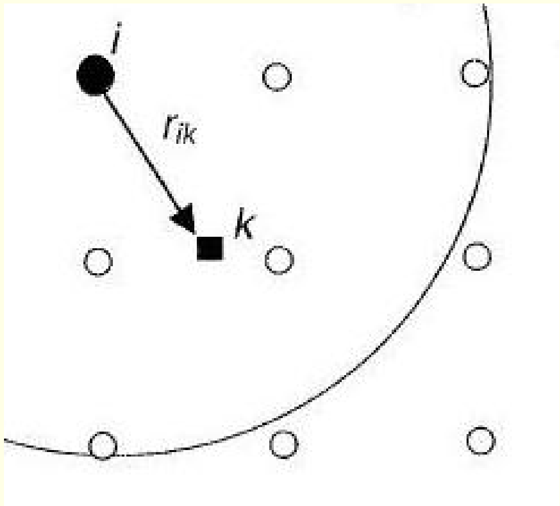

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Usually, bi-quadratic or bi-cubic interpolation is used.

Cressman (1959) defined the weights w_{ik}^n in the SCM as

$$\left.\begin{aligned} w_{ik}^n &= \frac{R_n^2 - r_{ik}^2}{R_n^2 + r_{ik}^2} \qquad \textbf{for} \qquad r_{ik}^2 \le R_n^2 \\ w_{ik}^n &= 0 \qquad\qquad\qquad \textbf{for} \qquad r_{ik}^2 > R_n^2 \end{aligned}\right\}$$

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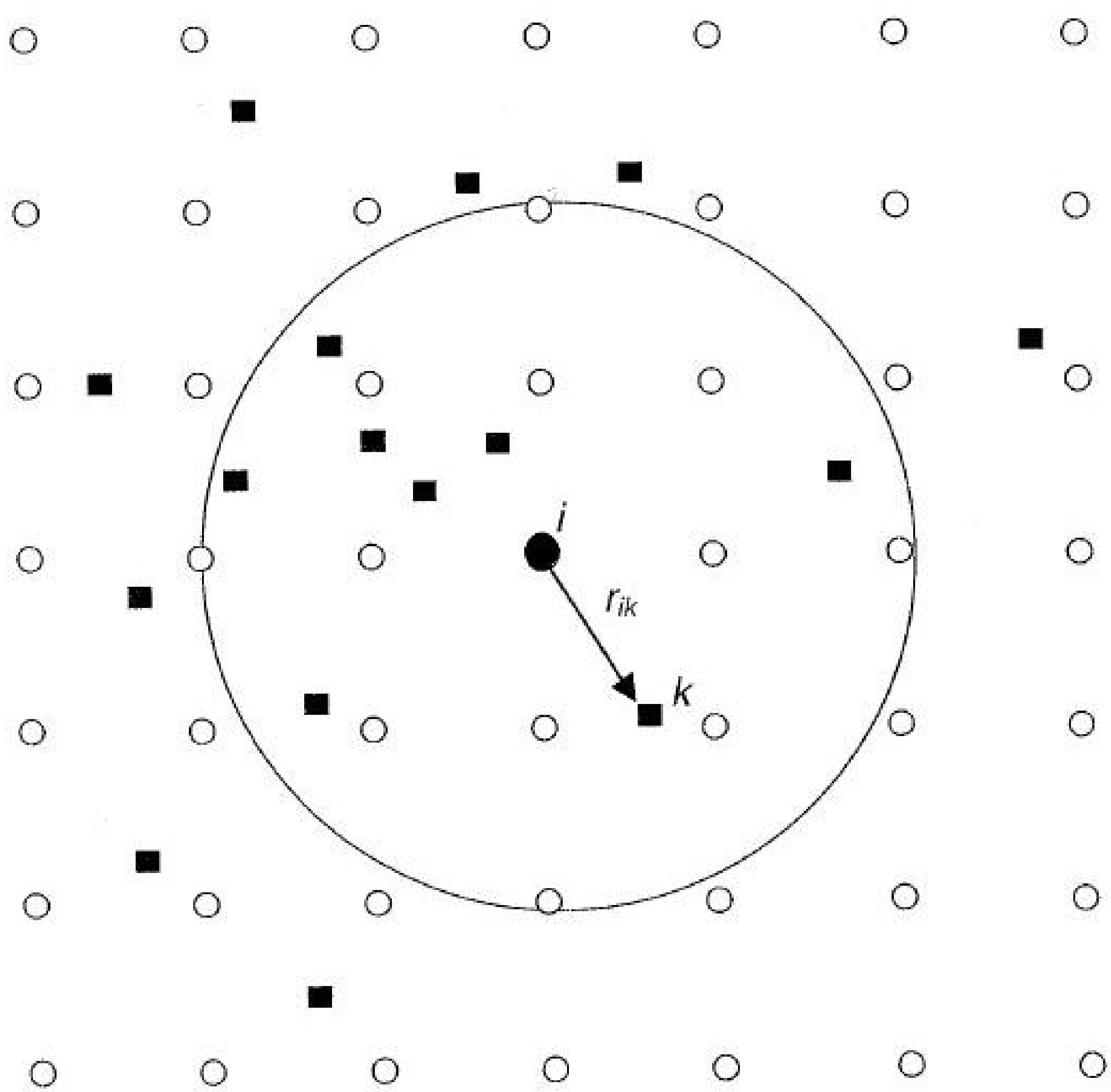
Exercise: **Draw a series of weight curves using `MatLab`.**



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There are eight observations within a distance R from grid point i.

The radius of influence R_n may vary with the iteration.

For example, the Swedish operational system (in 1980s) used

- $R_1 = 1500$ **km,** $R_2 = 900$ **km for upper air analyses**
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The field reflects the large scales after the first iteration, and converges towards the smaller scales during later iterations.

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Exercise: **Consider the effect of a single observation, located at a greid point (a) for $\varepsilon^2 = 0$, (b) for $\varepsilon^2 = 0.5$.**

Nudging

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For example, for a primitive equation model, the zonal velocity forecast equation is written as

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The relaxation time scale, τ, is chosen based on empirical considerations.

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Kaas *et al.* (1999) performed an interesting experiment, nudging a model towards a 15-y reanalysis from the ECMWF.

By averaging the mean forcing due to nudging, they empirically determined corrections to reduce model deficiencies.

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Exercise: Assuming that the nudging term is comparable in size to the Coriolis term, estimate the relaxation time τ.