Empirical Schemes (Kalnay, §5.2)

We consider two empirical schemes:

- The Successive Corrections Method (SCM)
- The Nudging Method

They are mainly of historical interest.

SCM was used in the 1950s and 1960s operational NWP systems. It is no longer popular (but still worth studying!).

Nudging is still used for mesoscale analysis, for example, using radar data.

Successive corrections method

One of the first analysis methods used in operational NWP was the SCM, developed by Bergthorsson and Döös (1955) in Sweden and by Cressman, US Weather Service (1959).

SCM is an iterative scheme.

The first estimate at grid-point i is given by the background (or *first guess*) field:

$$f_i^0 = f_i^b$$

where f_i^0 is the *zeroth iteration* estimate of the gridded field.

Then, observations are used to 'correct' the analysis, in an iterative procedure.

Large scales are analysed first, then smaller scales.

Refresher on Statistics

Suppose we have the noon-day pressure p_i and temperature T_i at Belfield, every day for a year. Let $n = 365$.

The mean pressure, $\bar{p}$, is defined to be

$$\bar{p} = \frac{1}{n}\sum_{i=1}^{n} p_i$$

and similarly for mean temperature $\bar{T}$.

The variance of pressure, σ_p^2,

$$\sigma_p^2 = \frac{1}{n}\sum_{i=1}^{n}(p_i - \bar{p})^2$$

and similarly for temperature variance σ_T^2.

The standard deviations, σ_p and σ_T are the square roots of the variances. They measure the root mean square deviation from the mean.

The covariance of p and T is defined as

$$\mu_{pT} = \frac{1}{n}\sum_{i=1}^{n}(p_i - \bar{p})(T_i - \bar{T})$$

Note that it is defined using deviations from the means.

The correlation between p and T is the normalized covariance:

$$\rho_{pT} = \frac{\mu_{pT}}{\sigma_p \sigma_T}$$

It is dimensionless, and bound between -1 and $+1$.

If p and T tend to be greater than their mean values at the same time, and less than their mean values at the same time, they are positively correlated and $\rho_{pT} > 0$.

If p tends to be greater than its mean value when T is less than its mean, and vice-versa, then p and T are negatively correlated and $\rho_{pT} < 0$.

Exercise: Consider the pressure and temperature at Belfield. Would you expect them to be correlated?

If so, is ρ_{pt} positive or negative?

What about the correlation between pressure and density?

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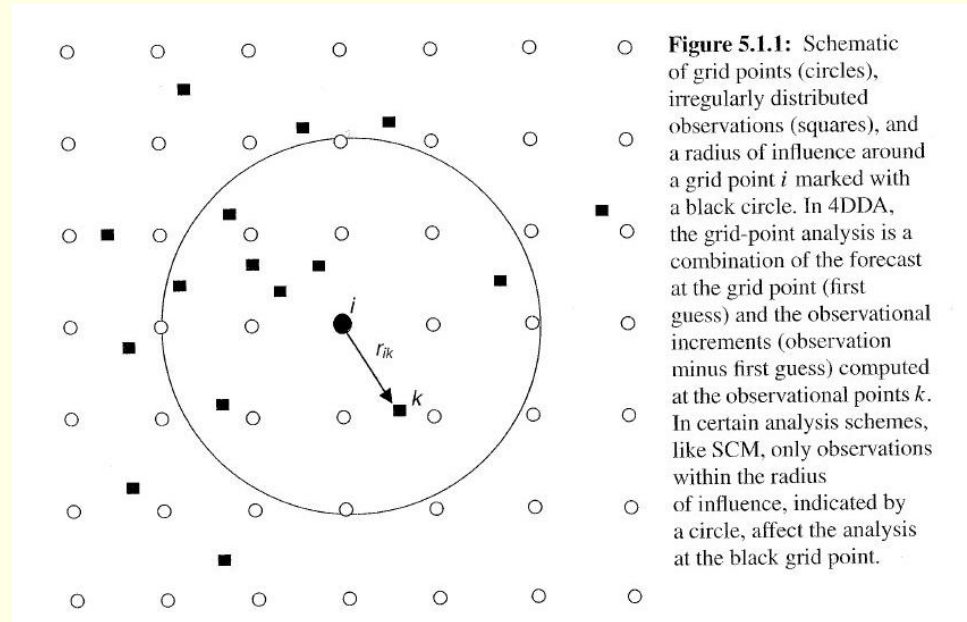The **Normal** or **Gaussian** distribution of a variable x is

$$P(x) = \frac{1}{\sqrt{2\pi}}\frac{1}{\sigma}\exp\left[-\frac{(x-\bar{x})^2}{2\sigma^2}\right]$$

The mean is $\bar{x}$ and the standard deviation σ. The constant term is included so that $\int_{-\infty}^{+\infty} P(x)\,dx = 1$.

Many variables have distributions which can be accurately modelled by the normal distribution.

Back to the SCM Analysis

Figure 5.1.1: Schematic of grid points (circles), irregularly distributed observations (squares), and a radius of influence around a grid point i marked with a black circle. In 4DDA, the grid-point analysis is a combination of the forecast at the grid point (first guess) and the observational increments (observation minus first guess) computed at the observational points k. In certain analysis schemes, like SCM, only observations within the radius of influence, indicated by a circle, affect the analysis at the black grid point.

The following iterations are obtained by <u>successive corrections</u>:

$$f_i^{n+1} = f_i^n + \left[\frac{\displaystyle\sum_{k=1}^{K_i^n} w_{ik}^n\left(f_k^O - f_k^n\right)}{\displaystyle\sum_{k=1}^{K_i^n} w_{ik}^n + \varepsilon^2}\right]$$

i.e.
New Value
= Old Value
+Correction

The notation is as follows:

- f_i^n is the nth iteration estimation at the grid point i

- f_k^O is the kth observation surrounding the grid point i

- f_k^n is the value of the nth field estimate evaluated at the observation point k

- K_i^n is the number of observations within a distance R_n of the grid point i

- ε^2 is an estimate of the ratio of the observation error variance to the background error variance.

Interpolation

To calculate the difference $f_k^O - f_k^n$, we have to interpolate the background field to the observation point.
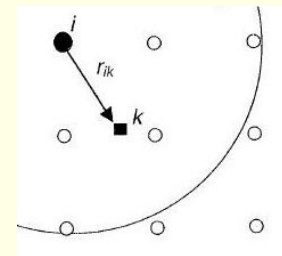
For the multi-dimensional case, we can do this one dimension at a time.

We interpolate first in the x-direction, and then interpolate the results in the y-direction.

Usually, **bi-quadratic** or **bi-cubic** interpolation is used.

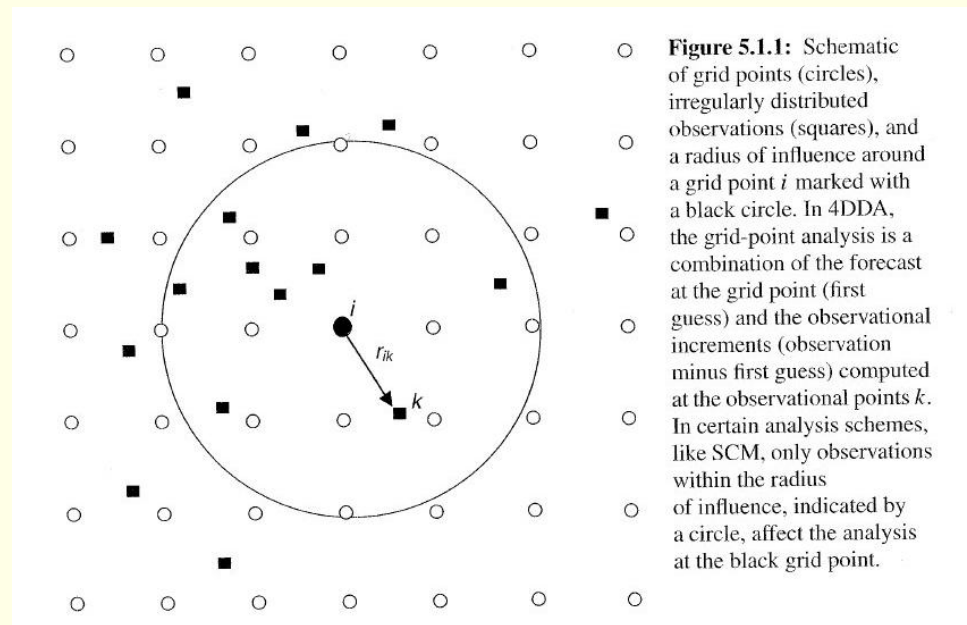Cressman (1959) defined the weights w_{ik}^n in the SCM as

$$w_{ik}^n = \frac{R_n^2 - r_{ik}^2}{R_n^2 + r_{ik}^2} \qquad \text{for} \qquad r_{ik}^2 \leq R_n^2$$

$$w_{ik}^n = 0 \qquad \text{for} \qquad r_{ik}^2 > R_n^2$$

where r_{ik}^2 is the square of the distance between an observation point $\mathbf{r}_k$ and a grid point at $\mathbf{r}_i$.

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Exercise: Consider a single observation at a gridpoint. If the first guess is a uniform field, what does the SCM analysis look like? (Assume $\varepsilon = 0$).

Exercise: Draw a series of weight curves using `MatLab`.

Figure 5.1.1: Schematic of grid points (circles), irregularly distributed observations (squares), and a radius of influence around a grid point i marked with a black circle. In 4DDA, the grid-point analysis is a combination of the forecast at the grid point (first guess) and the observational increments (observation minus first guess) computed at the observational points k. In certain analysis schemes, like SCM, only observations within the radius of influence, indicated by a circle, affect the analysis at the black grid point.

There are eight observations within a distance R from grid point i.

The **radius of influence** R_n may vary with the iteration.

For example, the Swedish operational system (in 1980s) used

- $R_1 = 1500$ km, $R_2 = 900$ km for upper air analyses
- $R_1 = 1500$ km, $R_2 = 1200$ km, $R_3 = 750$ km, $R_4 = 300$ km for the surface pressure analysis

The field reflects the **large scales** after the first iteration, and converges towards the **smaller scales** during later iterations.

Error estimate ε^2

In the original SCM, the coefficient ε^2 was set to zero.

This results in a "credulous" analysis that faithfully reflects the observations: the analysis **converges to the observation values** if the observations are located at the grid points.

If the data are **noisy** (e.g., if an observation has gross errors) this can lead to "bull's eyes" (many isolines around an unrealistic value).

Including $\varepsilon^2 > 0$ assumes that the observations have errors, and gives some **weight to the background field**.

Exercise: Consider the effect of a single observation, located at a greid point (a) for $\varepsilon^2 = 0$, (b) for $\varepsilon^2 = 0.5$.

Nudging

Another empirical and fairly widely used method for data assimilation is *Newtonian relaxation* or **nudging**.

[References are Hoke and Anthes (1976) and Kistler (1974)]

We add to the prognostic equations a term that <u>nudges the solution</u> towards the observations.

For example, for a primitive equation model, the zonal velocity forecast equation is written as

$$\frac{\partial u}{\partial t} = -\mathbf{v} \cdot \nabla u + fv - \frac{\partial \phi}{\partial x} + \left[\frac{u_{obs} - u}{\tau_u} \right]$$

Similar terms are added to the other equations.

The relaxation time scale, τ, is chosen based on empirical considerations.

Example: Solve for u if the *only* right-hand term is the nudging term.

Solution: The equation is

$$\frac{\partial u}{\partial t} = \left[\frac{u_{obs} - u}{\tau} \right]$$

we write this as

$$\frac{\partial u}{\partial t} + \frac{u}{\tau} = \frac{u_{obs}}{\tau}$$

Recall how to solve this o.d.e. using an integrating factor. Multiply by $\exp\left(\int dt/\tau\right) = \exp(t/\tau)$:

$$\frac{d}{dt}\left[\exp\left(\frac{t}{\tau}\right) u \right] = \exp\left(\frac{t}{\tau}\right) \frac{u_{obs}}{\tau}$$

The solution is found by integration:

$$u(t) = u_{obs} + [u(0) - u_{obs}] \exp(-t/\tau)$$

So, u approaches u_{obs} exponentially, with time-scale τ.

If τ is very small, the solution converges towards the observations too fast, and the dynamics cannot adjust in time.

If τ is too large, the errors in the model can grow too much before the nudging becomes effective.

Hoke and Anthes indicated that τ should be chosen so that the last term is similar in magnitude to the less dominant terms.

They used a very short time scale, about 20 minutes, in their experiments.

Kaas *et al.* (1999) performed an interesting experiment, nudging a model towards a 15-y reanalysis from the ECMWF.

By averaging the mean forcing due to nudging, they empirically determined corrections to reduce model deficiencies.

Question: How is the observational information *spread out in space* in the nudging analysis scheme?

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The nudging method is not generally used for large-scale assimilation.

Some groups use it for assimilating small-scale observations (e.g., **radar observations**).

It is especially useful when there are **no available statistics** to perform a statistical interpolation.

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Exercise: Assuming that the **nudging term** is comparable in size to the Coriolis term, estimate the relaxation time τ.