Numerical Weather Prediction

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Meteorology & Climate Cehtre
School of Mathematical Sciences
University College Dublin

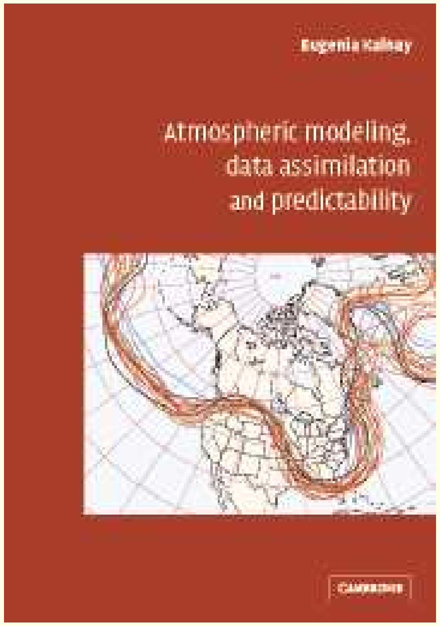
Second Semester, 2005–2006.

Text for the Course

The lectures will be based closely on the text

Atmospheric Modeling, Data Assimilation and Predictability
by
Eugenia Kalnay

published by Cambridge University Press (2002).



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- The more accurate the estimate of the initial conditions, the better the quality of the forecasts.
- Operational NWP centers produce initial conditions through a statistical combination of observations and short-range forecasts.
- This approach is called data assimilation

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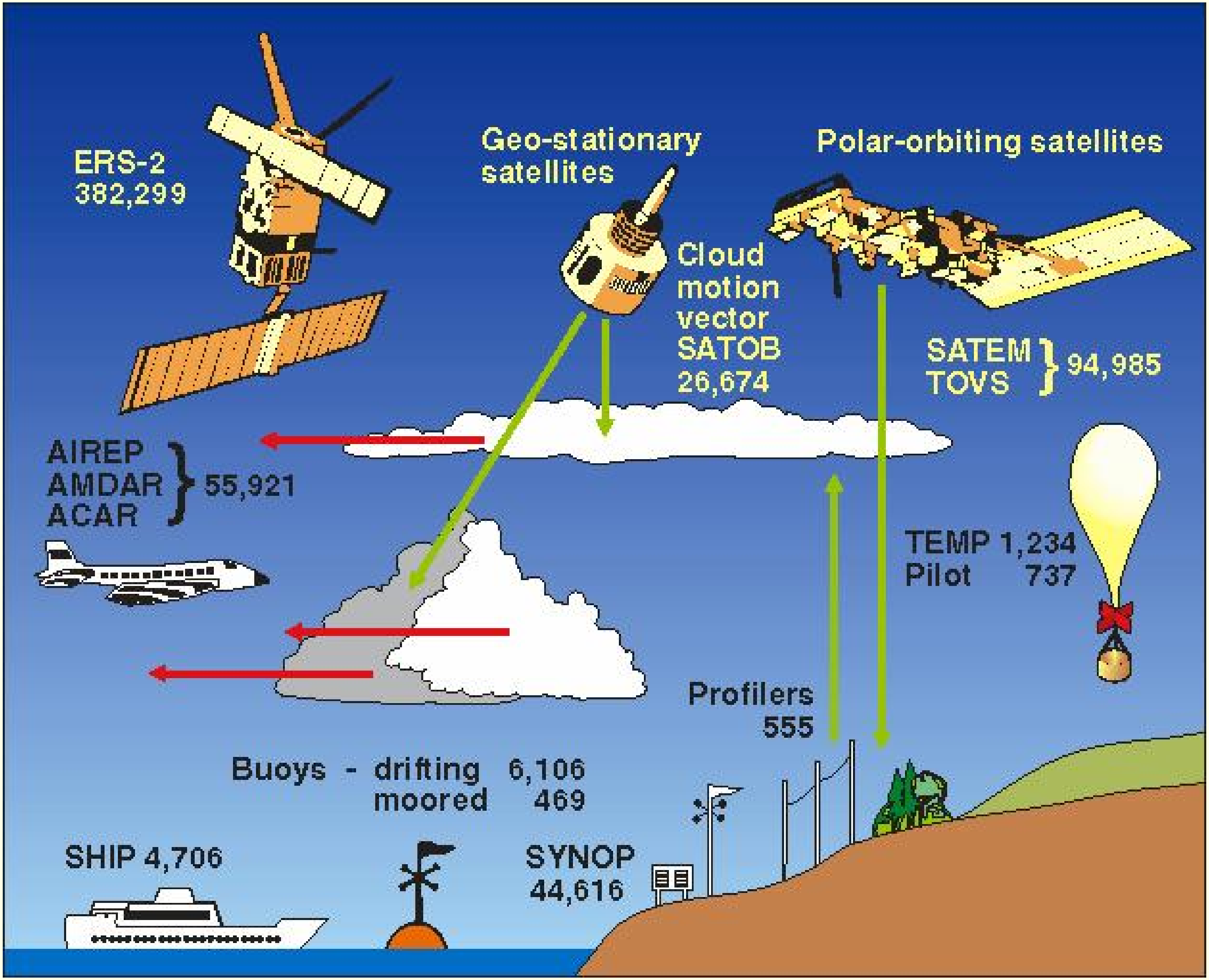
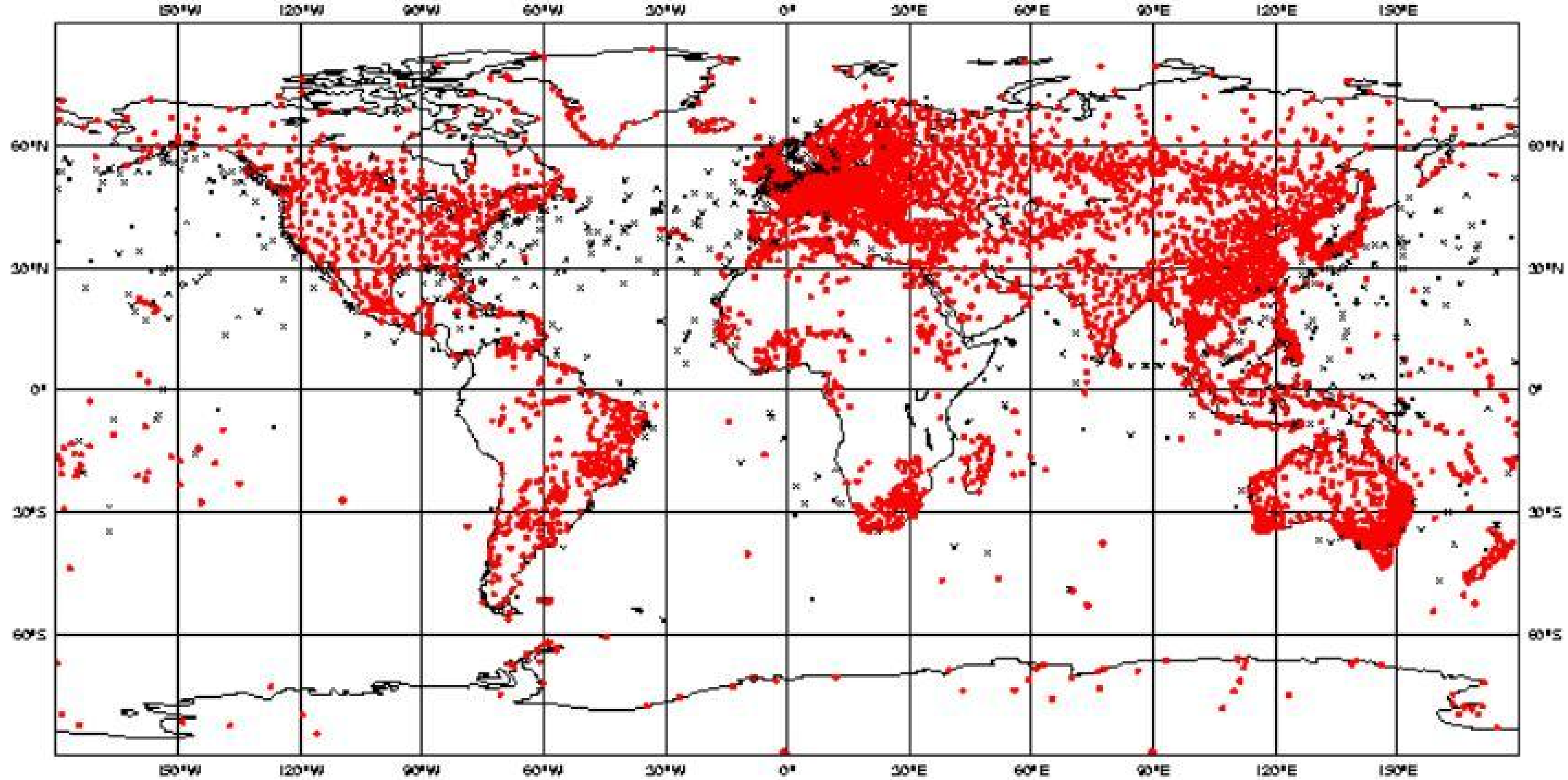
The first objective analysis systems were developed (independently) in Sweden and in USA in the 1950s.

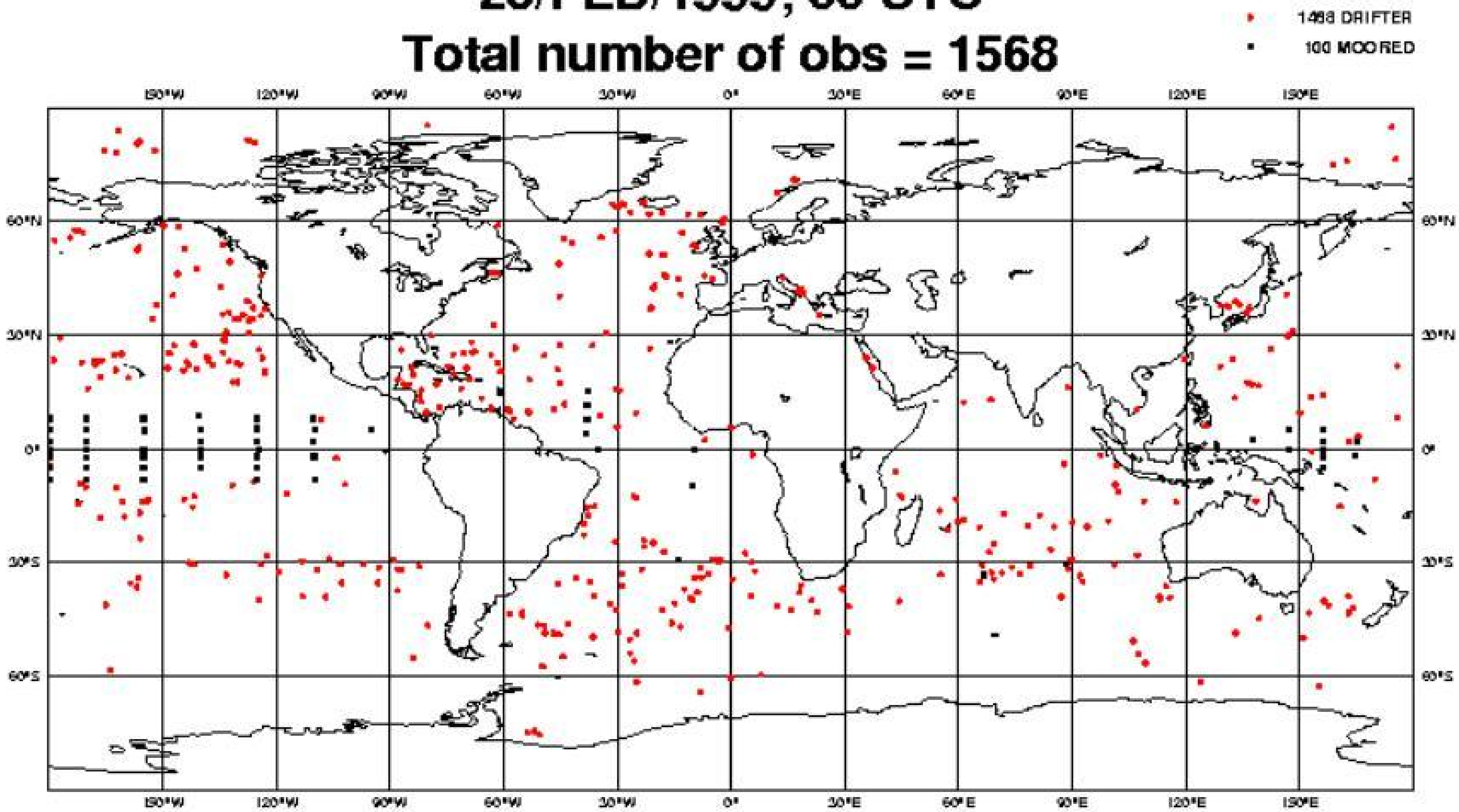

ERS-2
382,299
Geo-stationary
satellites
Polar-orbiting satellites
Cloud
motion
vector
SATOB
26,674
SATEM
TOVS
94,985
AIREP
AMDAR
ACAR
55,921
TEMP 1,234
Pilot 737
Profilers
555
Buoys - drifting 6,106
moored 469
SHIP 4,706
SYNOP
44,616

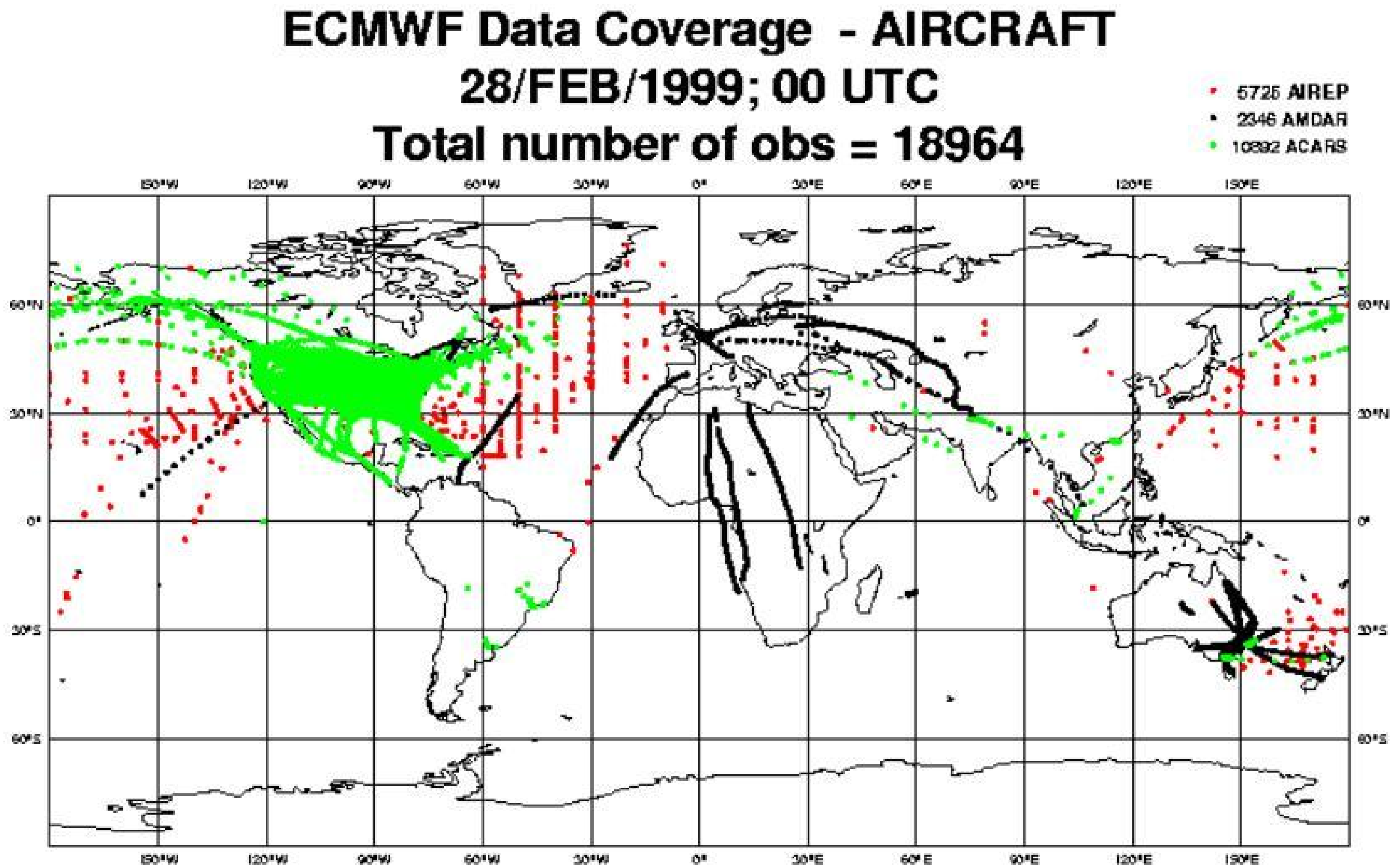
ECMWF Data Coverage - SYNOP/SHIP
28/FEB/1999; 00 UTC
Total number of obs = 12688

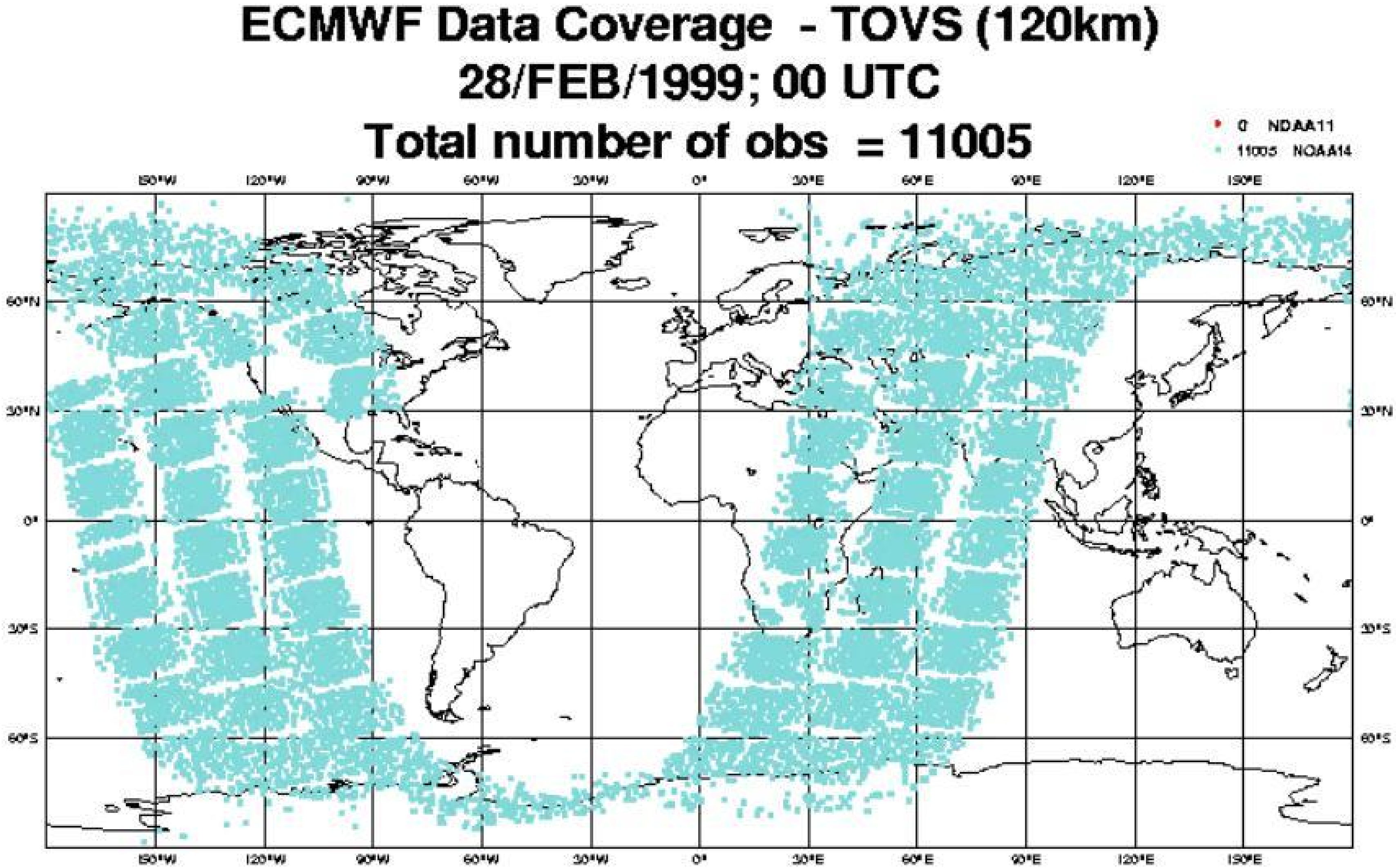
• 11417 SYNOP
• 1271 SHIP



ECMWF Data Coverage - BUOY
28/FEB/1999; 00 UTC
Total number of obs = 1568

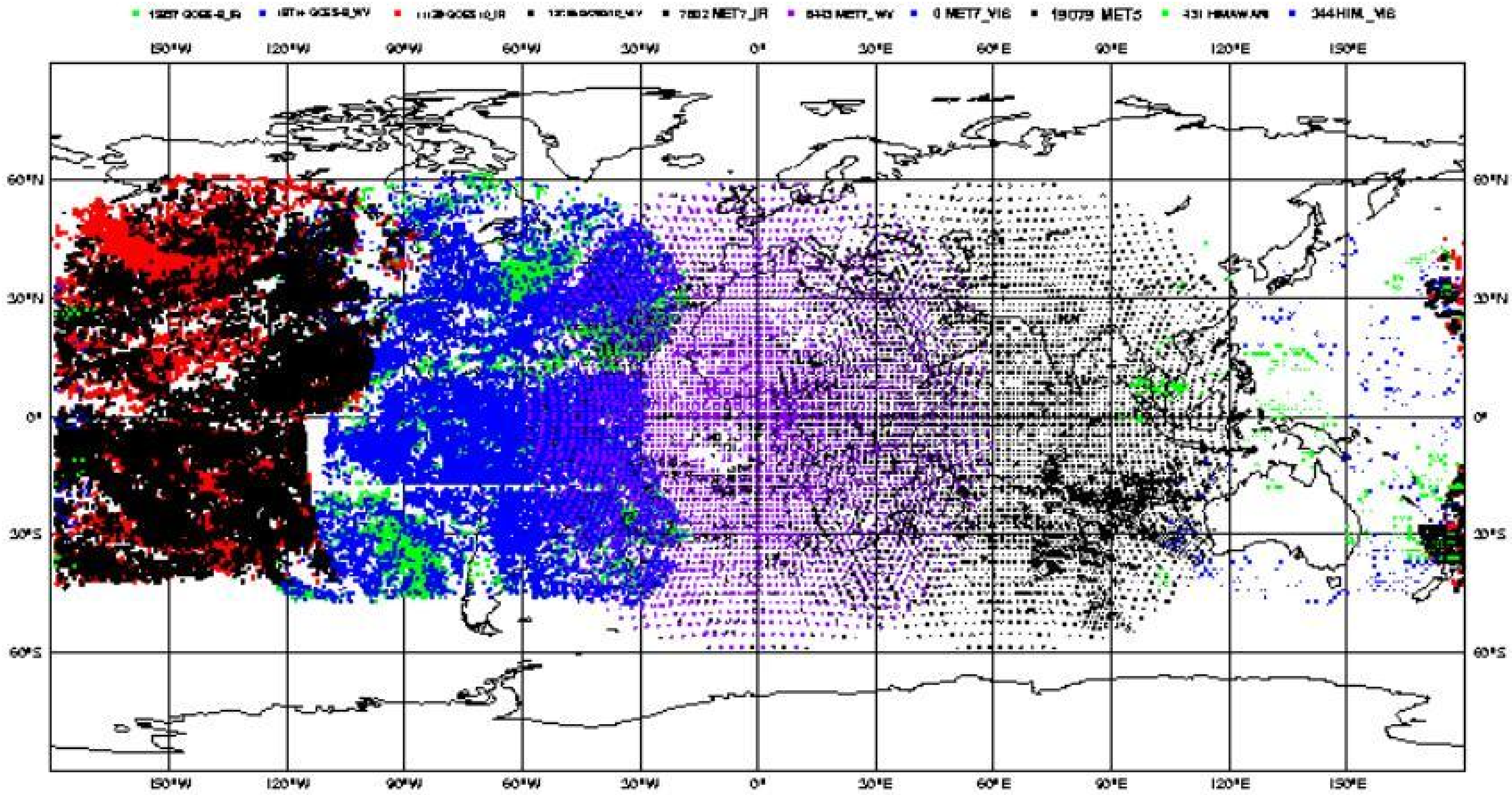


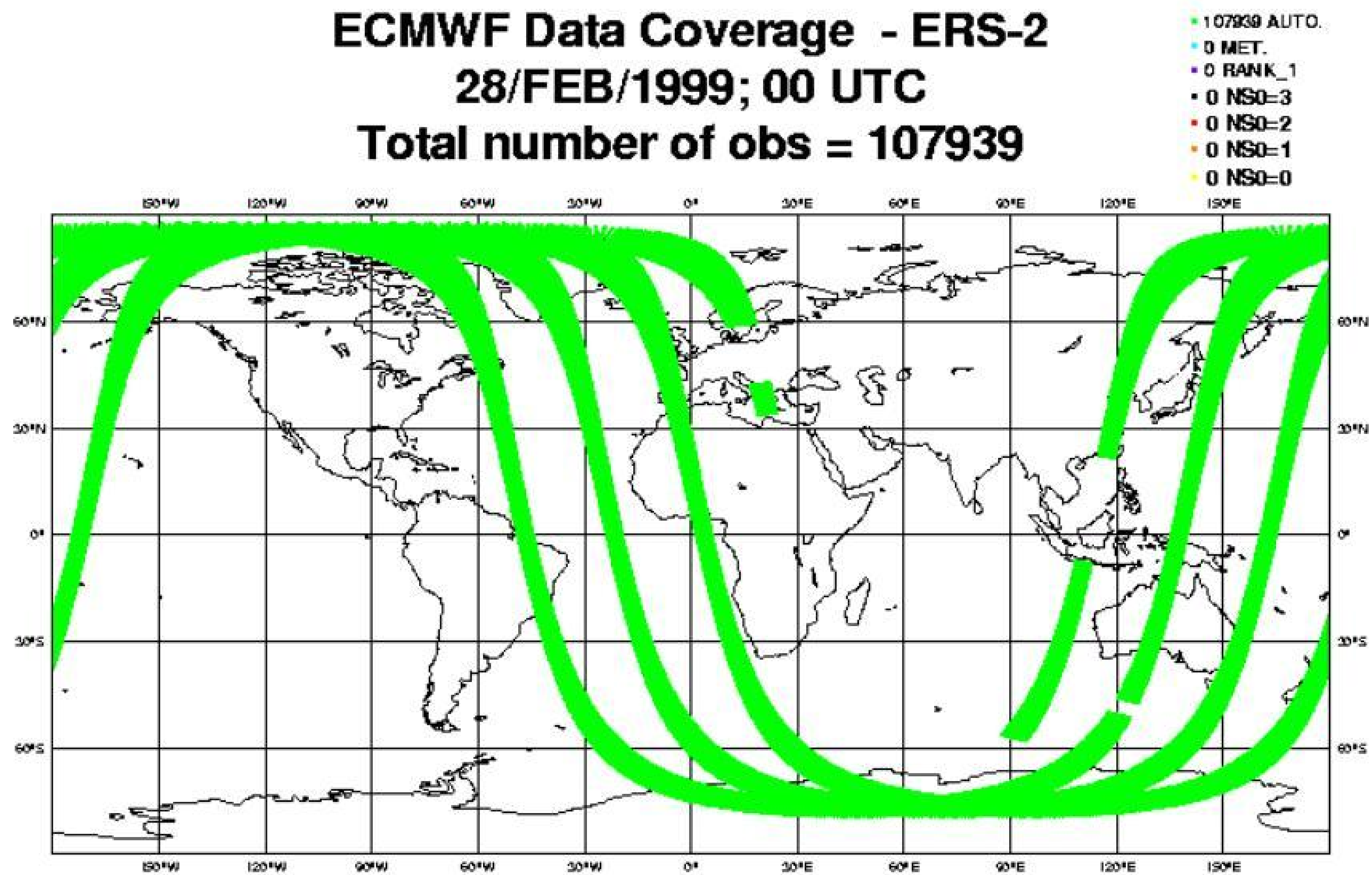

ECMWF Data Coverage - AIRCRAFT
28/FEB/1999; 00 UTC
Total number of obs = 18964
5725 AIREP
2346 AMDAR
10892 ACARS
150°W
120°W
90°W
60°W
30°W
0°
30°E
60°E
90°E
120°E
150°E
60°N
30°N
0°
30°S
60°S



ECMWF Data Coverage - TOVS (120km)
28/FEB/1999; 00 UTC
Total number of obs = 11005
0 NOAA11
11005 NOAA14
150°W
120°W
90°W
60°W
30°W
0°
30°E
60°E
90°E
120°E
150°E
60°N
30°N
0°
30°S
60°S

ECMWF Data Coverage - SATOB
28/FEB/1999; 00 UTC
Total number of obs = 91405




ECMWF Data Coverage - ERS-2
28/FEB/1999; 00 UTC
Total number of obs = 107939
107939 AUTO.
0 MET.
0 RANK_1
0 NSO=3
0 NSO=2
0 NSO=1
0 NSO=0
150°W
120°W
90°W
60°W
30°W
0°
30°E
60°E
90°E
120°E
150°E
60°N
30°N
0°
30°S
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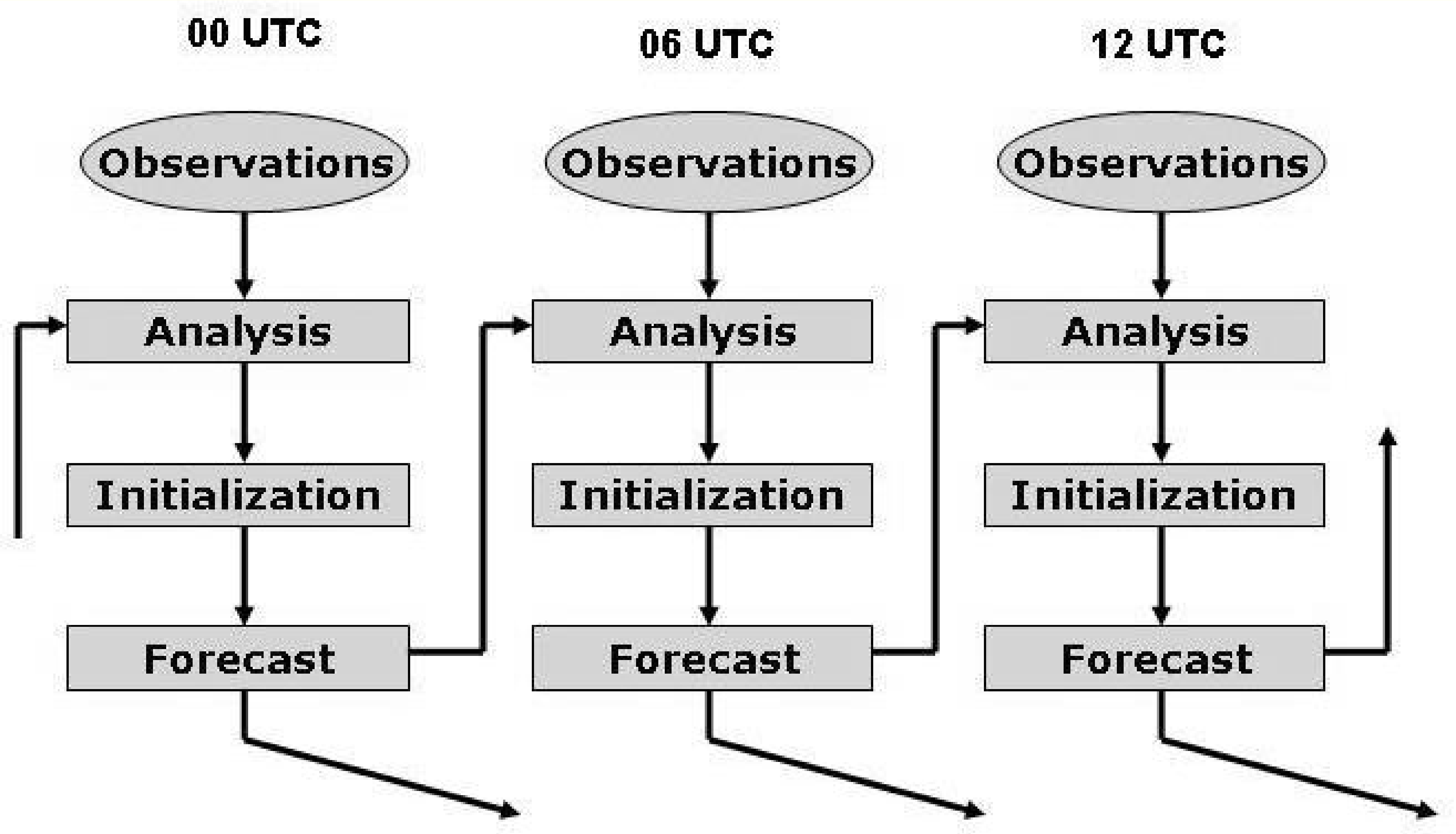
Present-day operational systems typically use a 6-h cycle performed four times a day.



Typical 6-hour analysis cycle.

Suppose the background field is a model 6-h forecast:

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The difference between the observations and the background,

$$\mathbf{y_o - H(x_b)}\,,$$

is called the observational increment or innovation.

The analysis $\mathbf{x_a}$ is obtained by adding the innovations to the background field with weights $\mathbf{W}$ that are determined based on the estimated statistical error covariances of the forecast and the observations:

$$\mathbf{x}_a = \mathbf{x}_b + \mathbf{W}[\mathbf{y}_o - \mathbf{H}(\mathbf{x}_b)]$$

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Earlier methods such as the SCM used weights which were determined empirically.

The weights were a function of the distance between the observation and the grid point, and the analysis wass iterated several times.

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Lorenc (1986) showed that OI and the 3D-Var approach are equivalent if the cost function is defined as:

$$J = \frac{1}{2}\Big\{[\mathbf{y}_o - H(\mathbf{x})]^T\mathbf{R}^{-1}[\mathbf{y}_o - H(\mathbf{x})] + (\mathbf{x} - \mathbf{x}_b)^T\mathbf{B}^{-1}(\mathbf{x} - \mathbf{x}_b)\Big\}$$

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The cost function J measures:

- **The distance of a field x to the observations (first term)**
- **The distance to the background $\mathbf{x_b}$ (second term).**

The distances are scaled by the observation error covariance $\mathbf{R}$ and by the background error covariance $\mathbf{B}$ respectively.

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- **In OI, the weights W are obtained for each grid point or grid volume, using suitable simplifications.**
- **In 3D-Var, the minimization of J is performed directly, allowing for additional flexibility and a simultaneous global use of the data.**

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However, why should an observation in New Zealand be used to determine the pressure pattern in Ireland?

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The coefficients were determined by minimizing the mean square difference

$$\min_{a_{ij}} E = \min_{a_{ij}} \Bigg[\sum_{k=1}^{K_z} p_k \big(z_k^o - z(x_k, y_k)\big)^2 + \sum_{k=1}^{K_v} q_k \Big\{ \big[u_k^o - u_g(x_k, y_k)\big]^2 + \big[v_k^o - v_g(x_k, y_k)\big]^2 \Big\} \Bigg]$$

Here p_k, q_k are empirical weighting coefficients and K is the total number of observations within the radius of influence.

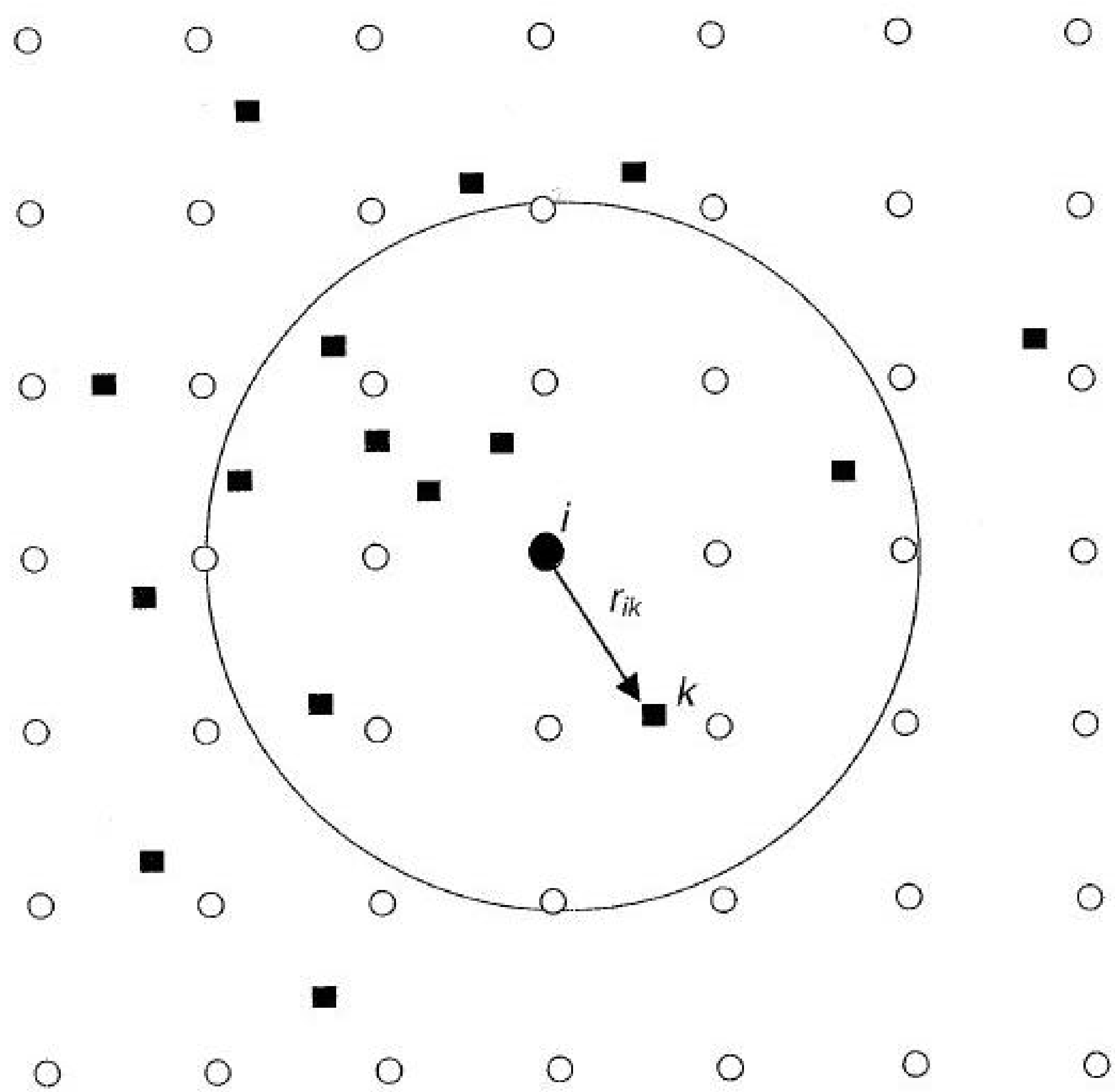

Figure 5.1.1: Schematic of grid points (circles), irregularly distributed observations (squares), and a radius of influence around a grid point i marked with a black circle. In 4DDA, the grid-point analysis is a combination of the forecast at the grid point (first guess) and the observational increments (observation minus first guess) computed at the observational points k. In certain analysis schemes, like SCM, only observations within the radius of influence, indicated by a circle, affect the analysis at the black grid point.

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$\star \quad \star \quad \star$

Exercise: Consider the Gilchrist and Cressman scheme. What does the analysis look like if there is (i) a single pressure observation; (ii) two pressure observations close together; (iii) two pressure obs. far apart?

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There are many new types of data, such as satellite and radar observations, but:

- they don't measure the variables used in the models
- their distribution in space and time is very nonuniform.

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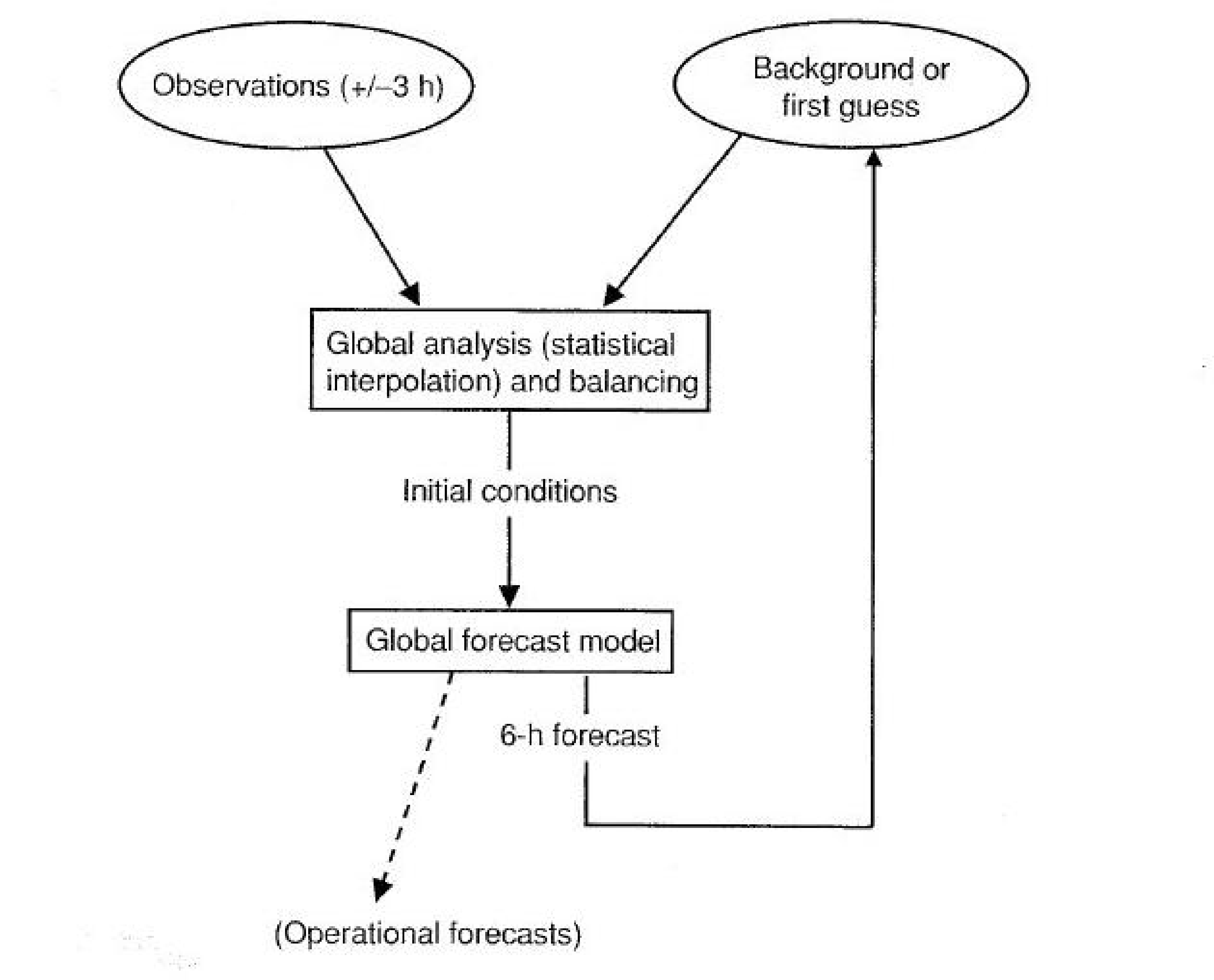
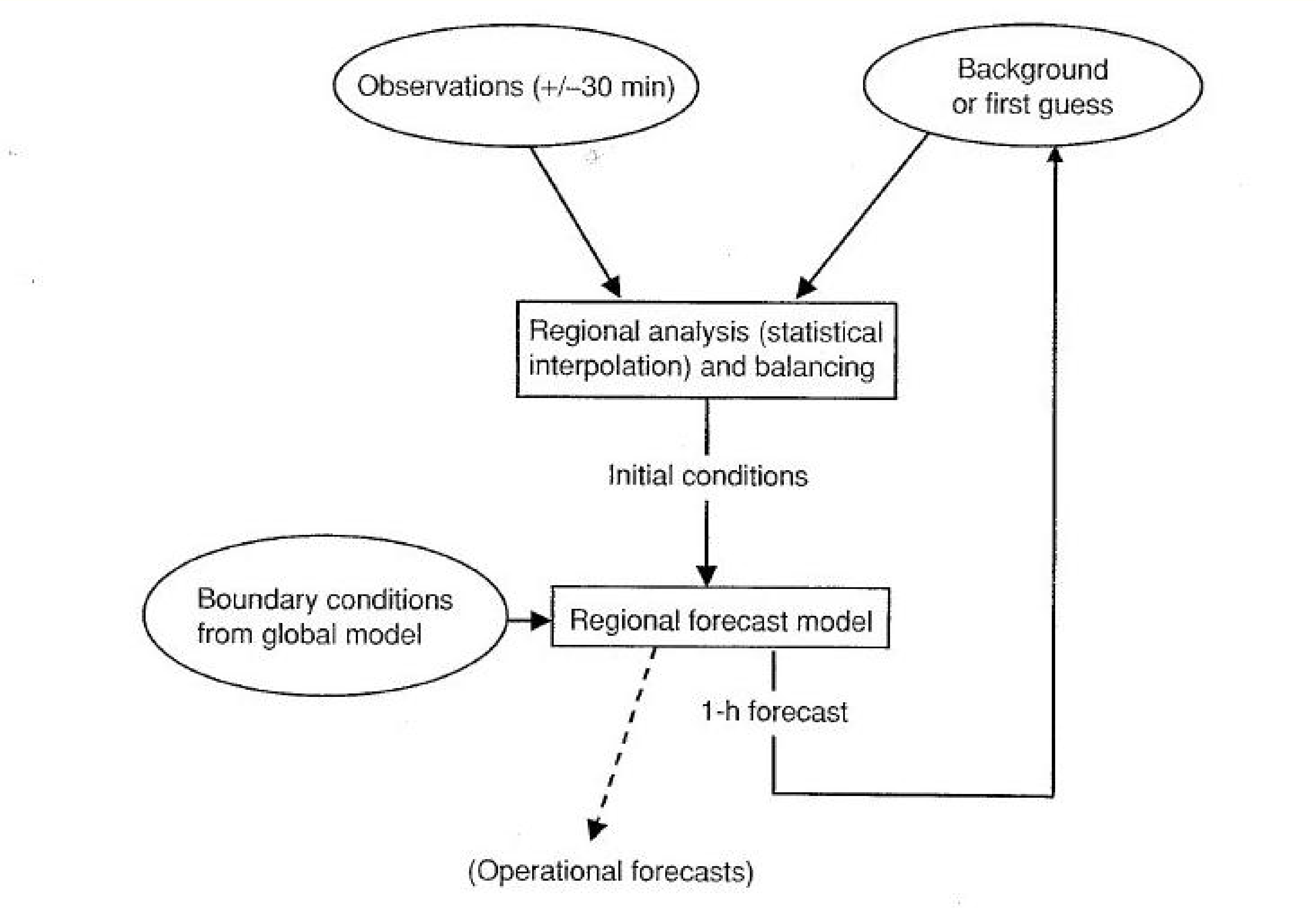
If a forecast is unavailable (e.g., if the cycle is broken), we may have to use climatological fields . . .

. . . but they are normally a poor estimate of the initial state.



Global 6-h analysis cycle (00, 06, 12, and 18 UTC).



Regional analysis cycle, performed (perhaps) every hour.

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For example, 6-h forecasts over the North Atlantic Ocean are relatively good, because of the information coming from North America.

Intermittent data assimilation is used in most global operational systems, typically with a 6-h cycle performed four times a day.

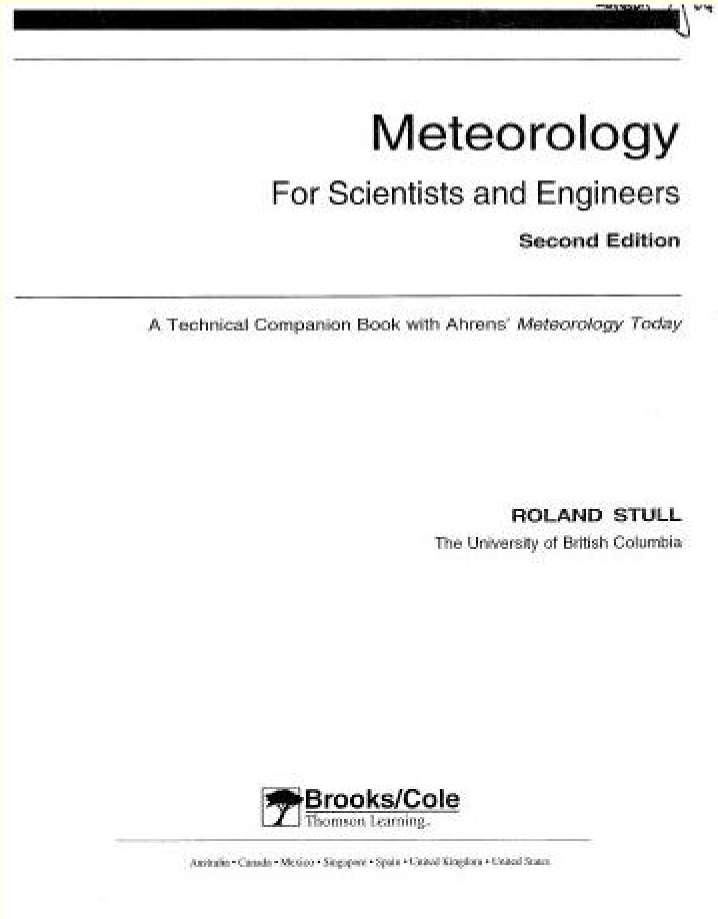
The model forecast plays a very important role:

- **Over data-rich regions, the analysis is dominated by the information contained in the observations.**
- **In data-poor regions, the forecast benefits from the information upstream.**

For example, 6-h forecasts over the North Atlantic Ocean are relatively good, because of the information coming from North America.

The model is able to transport information from data-rich to data-poor areas.

Exercise: Simple chart analysis.



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