M.Sc. in Meteorology UCD

Numerical Weather Prediction

Prof Peter Lynch

Meteorology & Climate Cehtre
School of Mathematical Sciences
University College Dublin

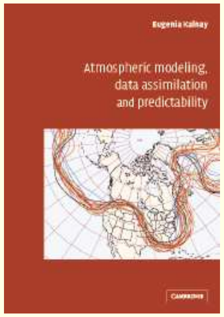
Second Semester, 2005–2006.

Text for the Course

The lectures will be based closely on the text

Atmospheric Modeling, Data Assimilation and Predictability
by
Eugenia Kalnay

published by Cambridge University Press (2002).



Data Assimilation (Kalnay, Ch. 5)

- NWP is an initial/boundary value problem
- Given
  - an estimate of the present state of the atmosphere (initial conditions)
  - appropriate surface and lateral boundary conditions

  the model simulates or forecasts the evolution of the atmosphere.
- The more accurate the estimate of the initial conditions, the better the quality of the forecasts.
- Operational NWP centers produce initial conditions through a statistical combination of observations and short-range forecasts.
- This approach is called data assimilation

Data Assimilation

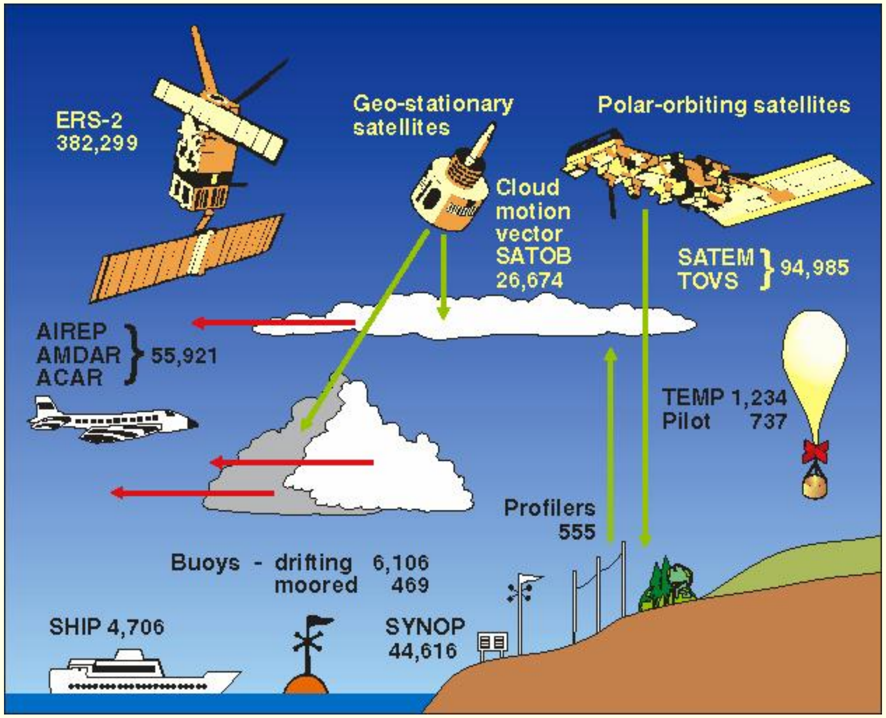
The model integrates the equations forward in time, starting from the initial conditions.

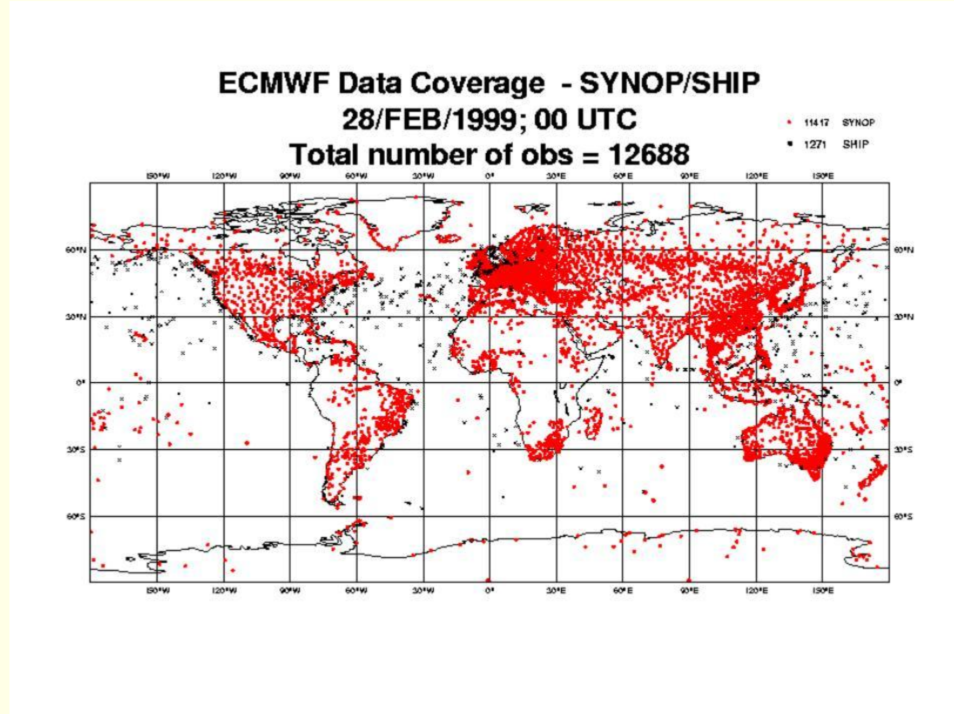
In the early NWP experiments, hand interpolations of the observations to grid points were performed.

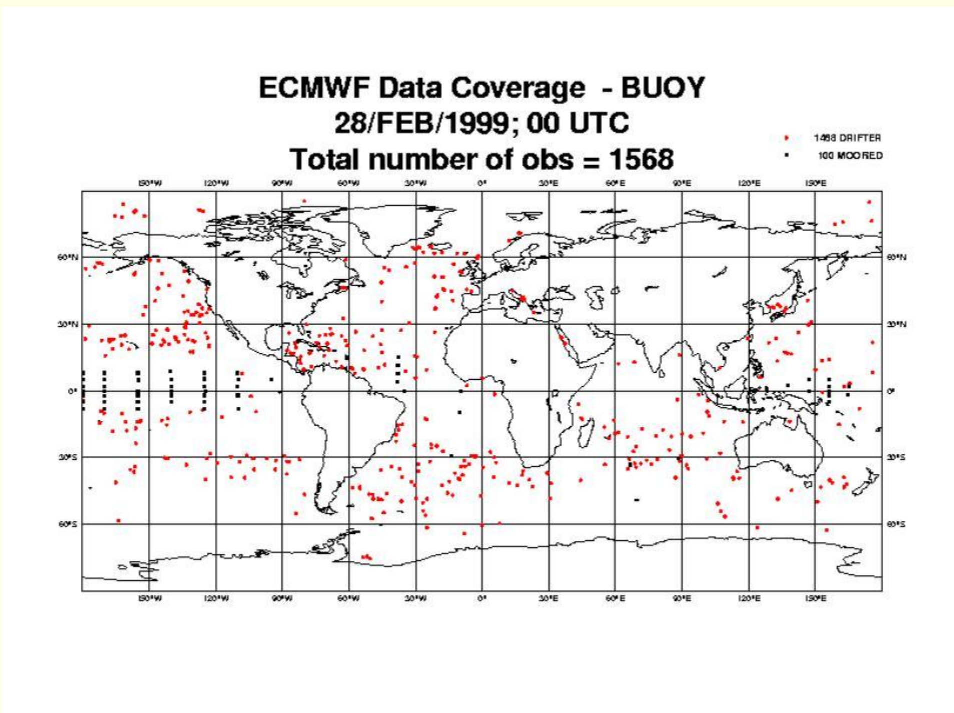
These fields of initial conditions were manually digitized.

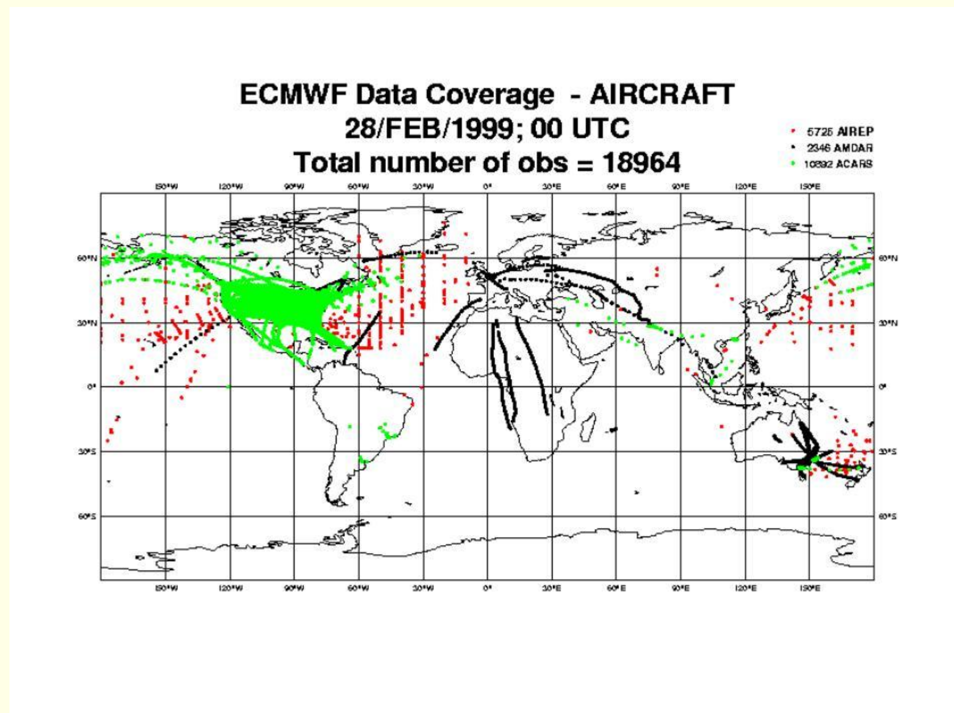
The need for an automatic "objective analysis" quickly became apparent.

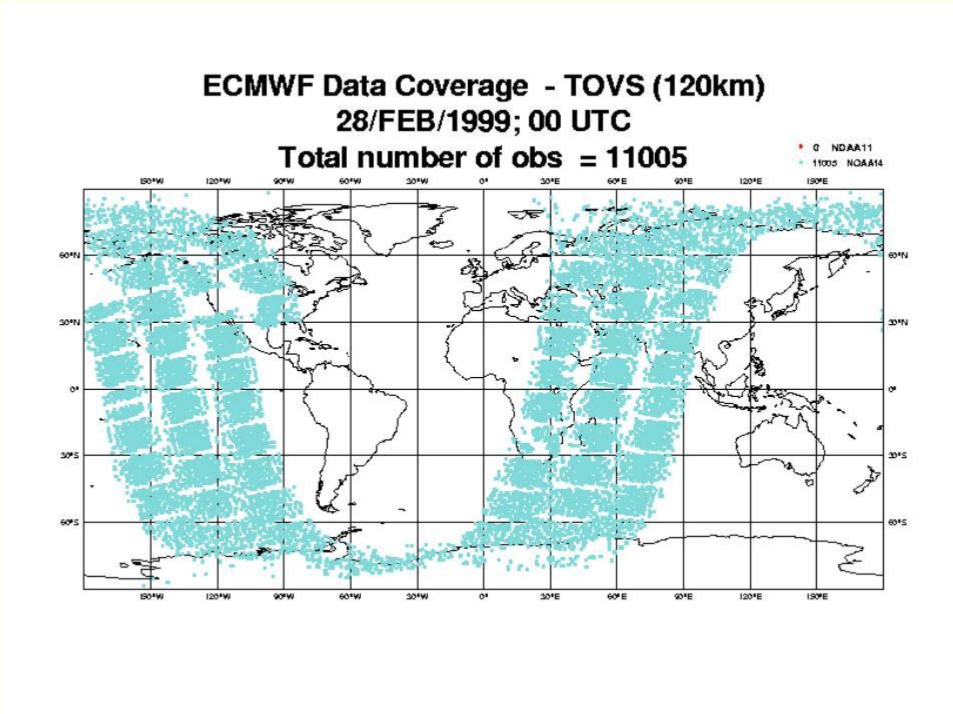
The first objective analysis systems were developed (independently) in Sweden and in USA in the 1950s.

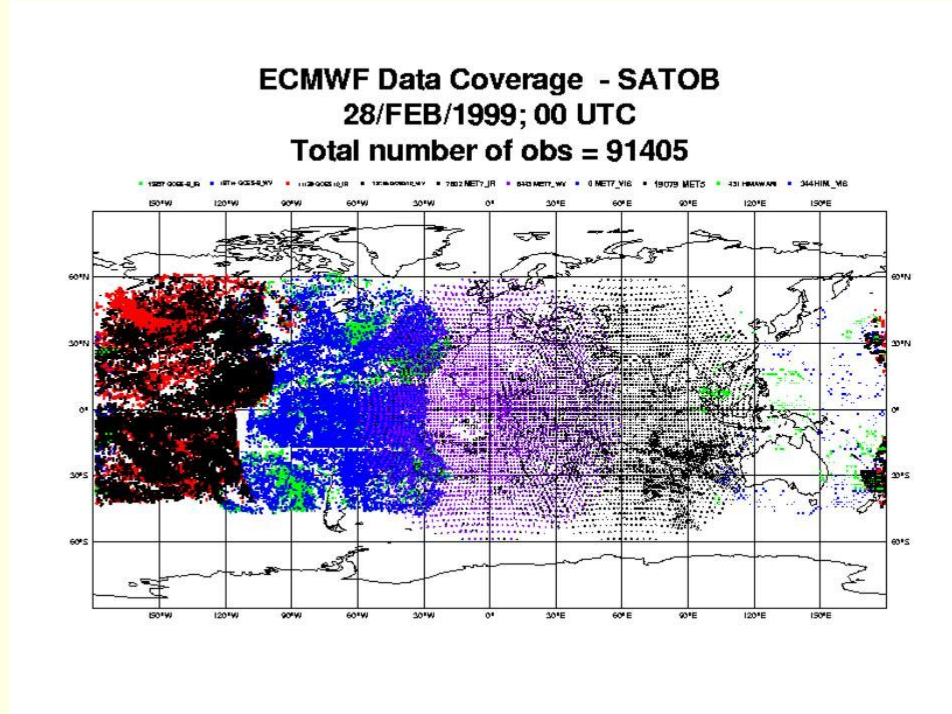

ERS-2
382,299
Geo-stationary
satellites
Polar-orbiting satellites
Cloud
motion
vector
SATOB
26,674
SATEM
TOVS
94,985
AIREP
AMDAR
ACAR
55,921
TEMP 1,234
Pilot 737
Profilers
555
Buoys - drifting 6,106
moored 469
SHIP 4,706
SYNOP
44,616

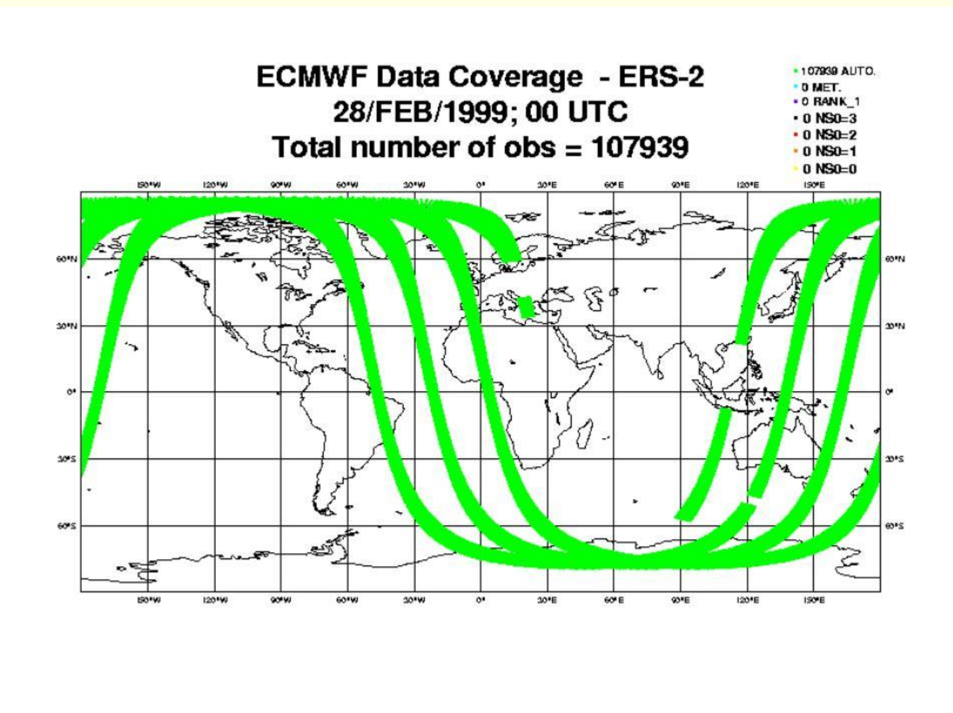

ECMWF Data Coverage - SYNOP/SHIP
28/FEB/1999; 00 UTC
Total number of obs = 12688
11417 SYNOP
1271 SHIP


ECMWF Data Coverage - BUOY
28/FEB/1999; 00 UTC
Total number of obs = 1568
1468 DRIFTER
100 MOORED


ECMWF Data Coverage - AIRCRAFT
28/FEB/1999; 00 UTC
Total number of obs = 18964
5726 AIREP
2346 AMDAR
10892 ACARS







Insufficiency of Data Coverage

Modern primitive equations models have a number of degrees of freedom of the order of 10^7.

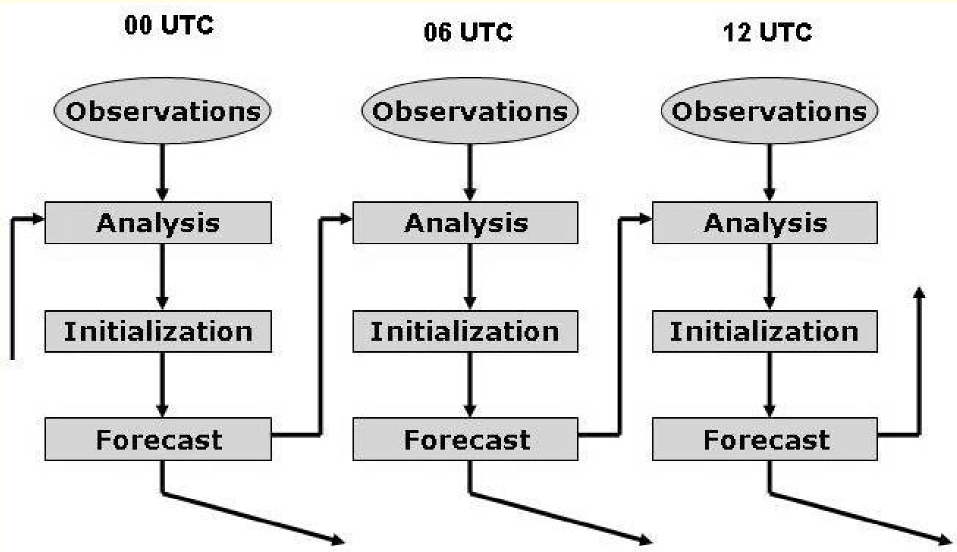
For a time window of ± 3 hours, there are typically 10 to 100 thousand observations of the atmosphere, two orders of magnitude less than the number of degrees of freedom of the model.

Moreover, they are distributed nonuniformly in space and time.

It is necessary to use additional information, called the *background field*, *first guess* or *prior information*.

A short-range forecast is used as the first guess in operational data assimilation systems.

Present-day operational systems typically use a 6-h cycle performed four times a day.



Typical 6-hour analysis cycle.

Suppose the background field is a model 6-h forecast:

$$\mathbf{x}_\mathrm{b}$$

To obtain the background or first guess "observations", the model forecast is interpolated to the observation location

If the observed quantities are not the same as the model variables, the model variables are converted to observed variables $\mathbf{y}_\mathrm{o}$.

The first guess of the observations is denoted

$$\mathbf{H}(\mathbf{x}_\mathrm{b})$$

where $\mathbf{H}$ is called the observation operator.

The difference between the observations and the background,

$$\mathbf{y}_\mathrm{o} - \mathbf{H}(\mathbf{x}_\mathrm{b}),$$

is called the observational increment or innovation.

The analysis $\mathbf{x}_\mathrm{a}$ is obtained by adding the innovations to the background field with weights $\mathbf{W}$ that are determined based on the estimated statistical error covariances of the forecast and the observations:

$$\mathbf{x}_a = \mathbf{x}_b + \mathbf{W}[\mathbf{y}_o - \mathbf{H}(\mathbf{x}_b)]$$

Different analysis schemes (SCM, OI, 3D-Var, and KF) are based on this equation, but differ by the approach taken to combine the background and the observations to produce the analysis.

Earlier methods such as the SCM used weights which were determined empirically.

The weights were a function of the distance between the observation and the grid point, and the analysis wass iterated several times.

In Optimal Interpolation (OI), the matrix of weights $\mathbf{W}$ is determined from the minimization of the analysis errors at each grid point.

In the 3D-Var approach one defines a cost function proportional to the square of the distance between the analysis and both the background and the observations.

This cost function is minimized to obtain the analysis.

Lorenc (1986) showed that OI and the 3D-Var approach are equivalent if the cost function is defined as:

$$J = \frac{1}{2}\left\{[\mathbf{y}_o - H(\mathbf{x})]^T\mathbf{R}^{-1}[\mathbf{y}_o - H(\mathbf{x})] + (\mathbf{x}-\mathbf{x}_b)^T\mathbf{B}^{-1}(\mathbf{x}-\mathbf{x}_b)\right\}$$

The cost function J measures:

- The distance of a field $\mathbf{x}$ to the observations (first term)
- The distance to the background $\mathbf{x}_\mathrm{b}$ (second term).

The distances are scaled by the observation error covariance R and by the background error covariance B respectively.

The minimum of the cost function is obtained for $\mathrm{x} = \mathrm{x_a}$, which is defined as the analysis.

The analysis obtained by OI and 3DVar is the same if the weight matrix is given by

$$\mathbf{W} = \mathbf{B}\mathbf{H}^T(\mathbf{H}\mathbf{B}\mathbf{H}^T + \mathbf{R}^{-1})^{-1}$$

The difference between OI and the 3D-Var approach is in the method of solution:

- In OI, the weights W are obtained for each grid point or grid volume, using suitable simplifications.
- In 3D-Var, the minimization of J is performed directly, allowing for additional flexibility and a simultaneous global use of the data.

Recently, the variational approach has been extended to four dimensions, by including within the cost function the distance to observations over a time interval (assimilation window).

This is called four-dimensional variational assimilation (4DVar)

In the analysis cycle, the importance of the model cannot be overemphasized:

- It transports information from data-rich to data-poor regions
- It provides a complete estimation of the four-dimensional state of the atmosphere.

The introduction of 4DVar at ECMWF has resulted in marked improvements in the quality of medium-range forecasts.

End of Introduction

Richardson (1922) and Charney *et al.* (1950) performed hand interpolations of the observations to a regular grid.

These fields of initial conditions were then manually digitized, which was a very time consuming procedure.

The need for an automatic "objective analysis" became quickly apparent.

Interpolation methods fitting observations to a regular grid were soon developed.

Panofsky (1949) developed the first objective analysis algorithm.

It was based on two-dimensional polynomial interpolation, a global procedure (the same function is used to fit all the observations).

However, why should an observation in New Zealand be used to determine the pressure pattern in Ireland?

Gilchrist and Cressman (1954) developed a local polynomial interpolation scheme for the geopotential height.

A quadratic in x and y was defined at each grid point:

$$z(x,y) = a_{00} + a_{10}x + a_{01}y + a_{20}x^2 + a_{11}xy + a_{02}y^2$$

The coefficients were determined by minimizing the mean square difference

$$\min_{a_{ij}} E = \min_{a_{ij}} \Bigg[\sum_{k=1}^{K_z} p_k\big(z_k^o - z(x_k,y_k)\big)^2 + \sum_{k=1}^{K_v} q_k \Big\{ \big[u_k^o - u_g(x_k,y_k)\big]^2 + \big[v_k^o - v_g(x_k,y_k)\big]^2 \Big\} \Bigg]$$

Here p_k, q_k are empirical weighting coefficients and K is the total number of observations within the radius of influence.

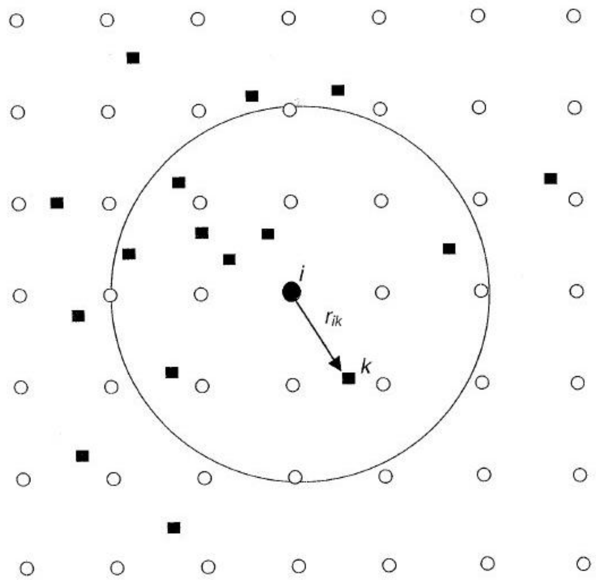


Figure 5.1.1: Schematic of grid points (circles), irregularly distributed observations (squares), and a radius of influence around a grid point i marked with a black circle. In 4DDA, the grid-point analysis is a combination of the forecast at the grid point (first guess) and the observational increments (observation minus first guess) computed at the observational points k. In certain analysis schemes, like SCM, only observations within the radius of influence, indicated by a circle, affect the analysis at the black grid point.

Note that although the geopotential height field is being analysed, the wind observations are also used:

The winds provide information about the gradient of z.

This is called multi-variate analysis.

When only heights are used to analyse heights, and winds to analyse winds, we have a uni-variate analysis.

⋆ ⋆ ⋆

Exercise: Consider the Gilchrist and Cressman scheme. What does the analysis look like if there is (i) a single pressure observation; (ii) two pressure observations close together; (iii) two pressure obs. far apart?

Background Field

For operational models, it is not enough to perform spatial interpolation of observations into regular grids:

There are not enough data available to define the initial state.

The number of degrees of freedom in a modern NWP model is of the order of 10^7.

The total number of conventional observations is of the order of 10^4–10^5.

There are many new types of data, such as satellite and radar observations, but:

- they <u>don't measure the variables</u> used in the models
- their distribution in space and time is <u>very nonuniform</u>.

Background Field

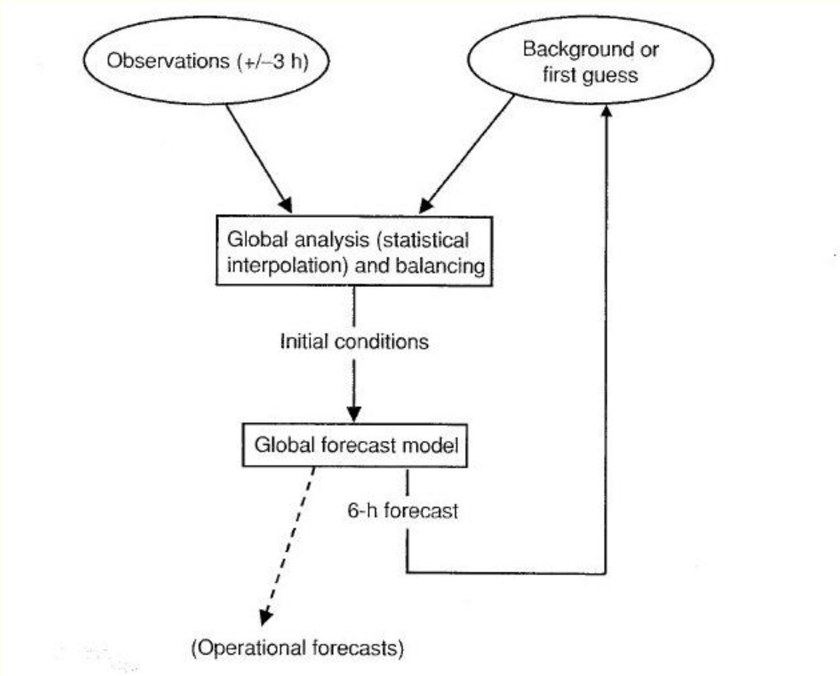
In addition to observations, it is necessary to use a first guess estimate of the state of the atmosphere at the grid points.

The first guess (also known as background field or prior information) is our best estimate of the state of the atmosphere *prior to* the use of the observations.

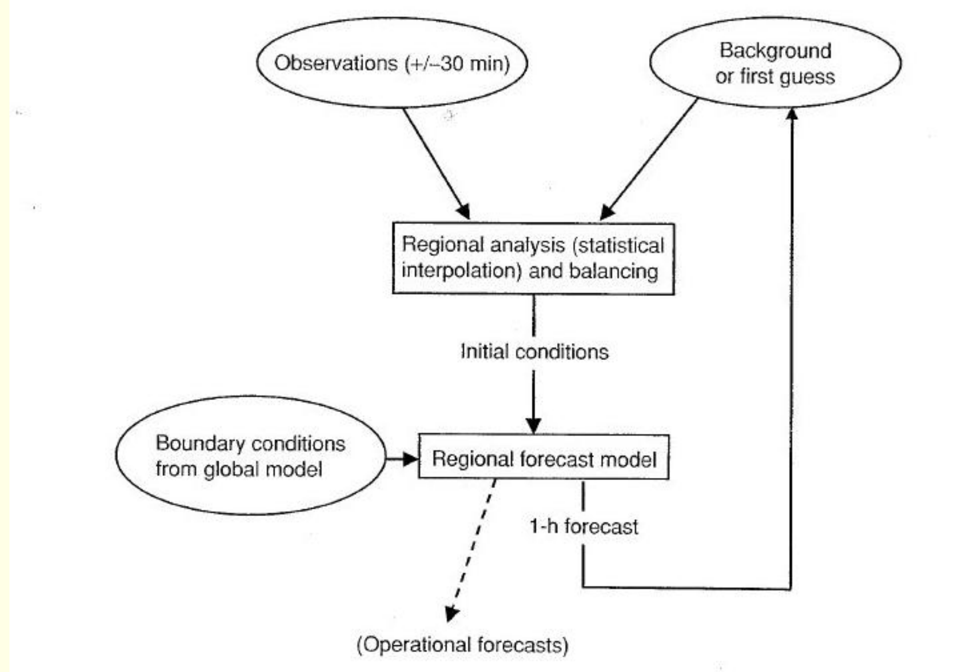
A short-range forecast is normally used as a first guess in operational systems in what is called an analysis cycle.

If a forecast is unavailable (e.g., if the cycle is broken), we may have to use climatological fields . . .

. . . but they are normally a poor estimate of the initial state.



Global 6-h analysis cycle (00, 06, 12, and 18 UTC).



Regional analysis cycle, performed (perhaps) every hour.

Intermittent data assimilation is used in most global operational systems, typically with a 6-h cycle performed four times a day.

The model forecast plays a very important role:

- Over data-rich regions, the analysis is dominated by the information contained in the observations.
- In data-poor regions, the forecast benefits from the information upstream.

For example, 6-h forecasts over the North Atlantic Ocean are relatively good, because of the information coming from North America.

The model is able to transport information from data-rich to data-poor areas.

Exercise: Simple chart analysis.