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However, it has a number of limitations. In particular, it is not straightforward to apply NNMI in limited geographical domains.

Recently, an alternative method of initialization, called digital filter initialization (DFI), was introduced.

In this lecture we review DFI, and describe how the method is applied in operational NWP.

The Notion of Filtering

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A filter is any device or contrivance designed to carry out such a selection.

It may be represented by a simple system diagram, having an input with both desired and undesired components, and an output comprising only the former.

$$Good/Bad/Ugly \quad \Longrightarrow \quad \boxed{\textbf{Filter}} \quad \longrightarrow \quad Good$$

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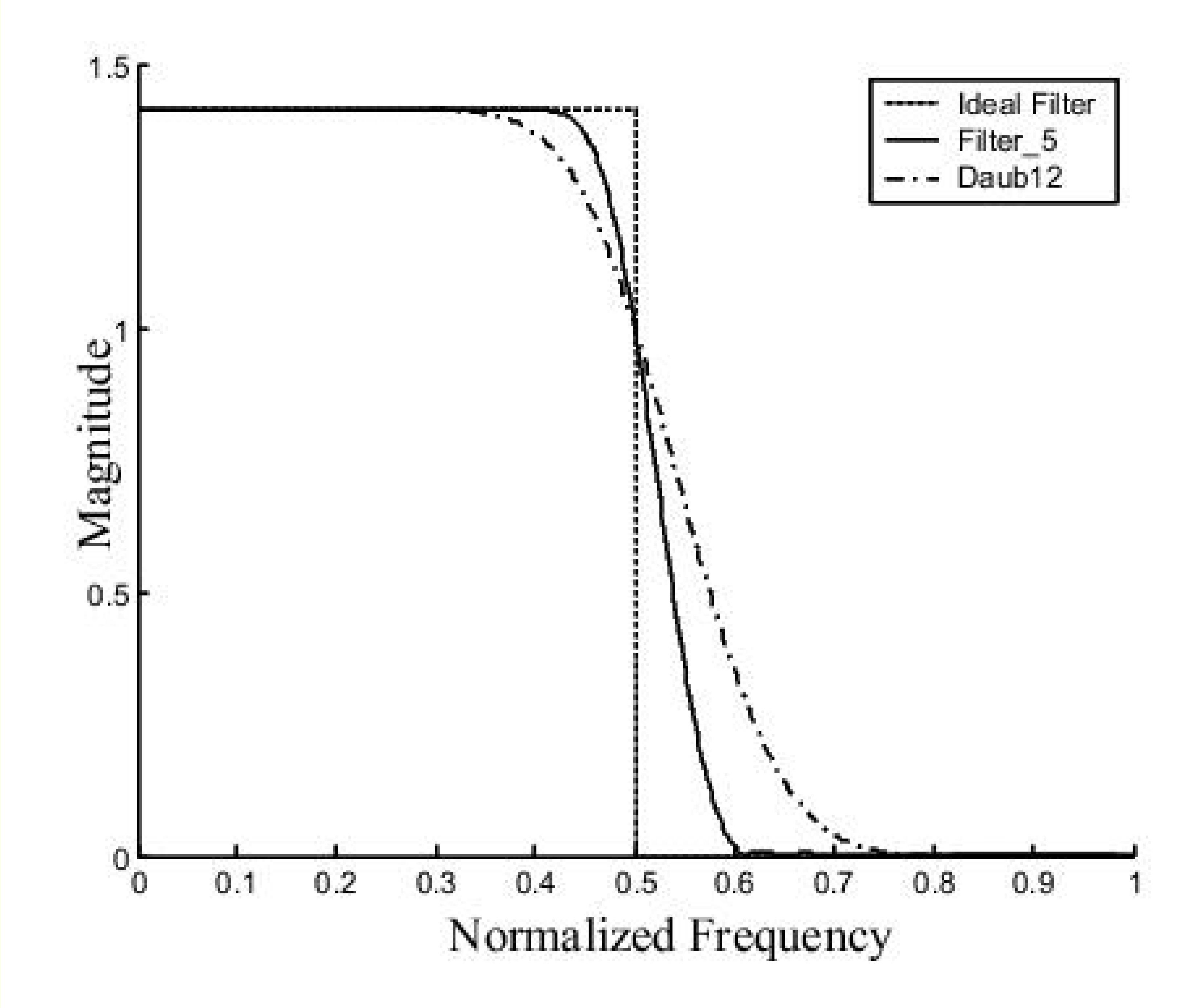
Other ideal filters can be discussed:

- **High-pass filters**
- **Band-pass filters**
- **Notch filters**

But the Low-Pass Filter is the one needed for initialization.



Frequency response of ideal low-pass filter.

Nonrecursive and Recursive Filters

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where L and N are positive integers. Usually, $K = 0$.

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To find the frequency response of a recursive filter, let

$$x_n = \exp(in\theta)$$

and assume an output of the form

$$y_n = H(\theta)\exp(in\theta)$$

Substitute $y_n = H(\theta)\exp(in\theta)$ into the defining formula

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$$H(\theta) = \frac{\sum_{k=K}^{N} a_k e^{-ik\theta}}{1 - \sum_{k=1}^{L} b_k e^{-ik\theta}}.$$

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For nonrecursive filters the denominator reduces to unity:

$$H(\theta) = \sum_{k=-N}^{N} a_k e^{-ik\theta}$$

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The entire area of filter design is concerned with finding filters haveing desired properties.

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Typically, $H_c(\omega)$ is a step function

$$H_c(\omega) = \begin{cases} 1, & |\omega| \leq |\omega_c|; \\ 0, & |\omega| > |\omega_c|, \end{cases}$$

where ω_c is a cutoff frequency.

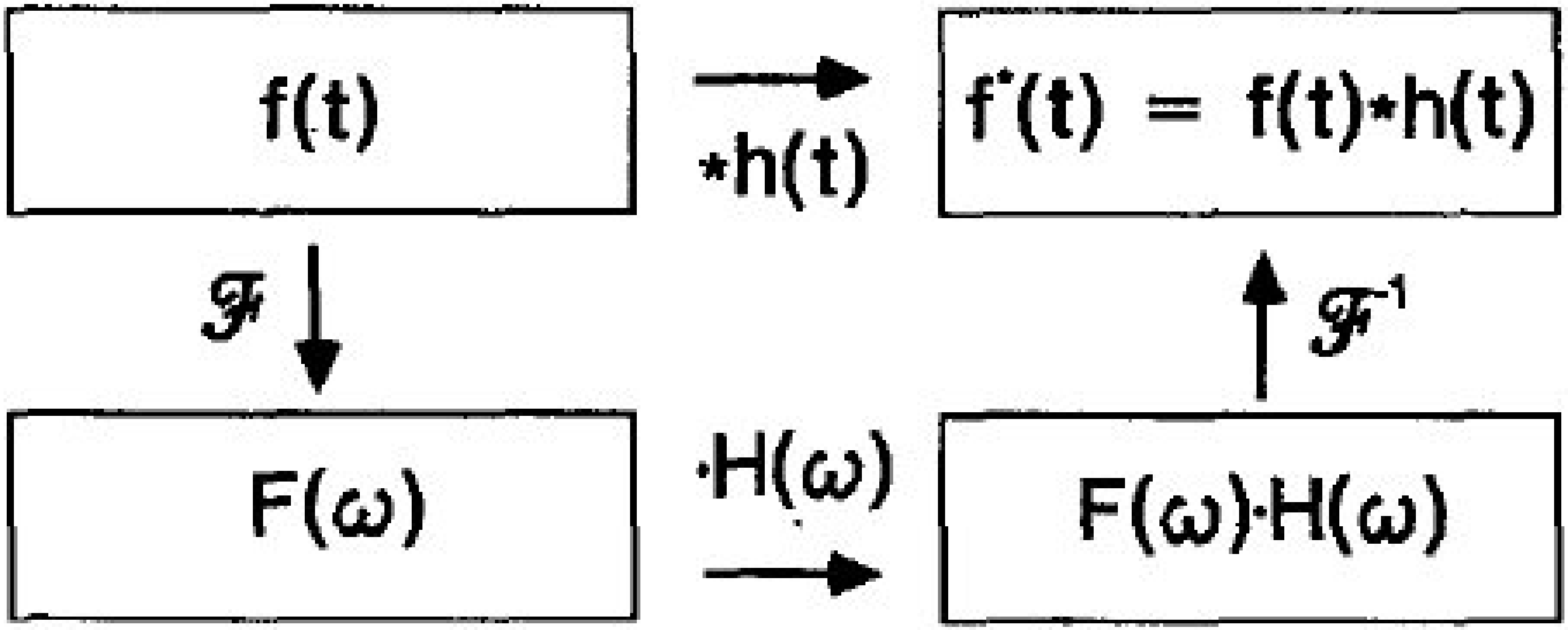

FIG. 1. Schematic representation of the equivalence between convolution and filtering in Fourier space.

Equivalence of filtering and convolution.

$$(h * f)(t) = \mathcal{F}^{-1}\{\mathcal{F}\{h\} \cdot \mathcal{F}\{f\}\}$$

These three steps are equivalent to a convolution of $f(t)$ with the inverse Fourier transform of $H_c(\omega)$ ($h(t) = \sin(\omega_c t)/\pi t$).

This follows from the convolution theorem

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$$f^\star(t) = (h * f)(t) = \int_{-\infty}^{+\infty} h(\tau) f(t - \tau) d\tau.$$

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For example, f_n could be the value of some model variable at a particular grid point at time t_n.

The shortest period component which can be represented with a time step Δt is $\tau_{\mathrm{Ny}} = 2\Delta t$, corresponding to a maximum frequency, the so-called Nyquist frequency, $\omega_{\mathrm{Ny}} = \pi/\Delta t$.

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The sequence $\{f_n\}$ may be regarded as the Fourier coefficients of a function $F(\theta)$:

$$F(\theta) = \sum_{n=-\infty}^{\infty} f_n e^{-in\theta},$$

where $\theta = \omega\Delta t$ is the digital frequency and $F(\theta)$ is periodic with period 2π: $F(\theta) = F(\theta + 2\pi)$. [Note: $\theta_{\mathrm{Ny}} = \omega_{\mathrm{Ny}}\Delta t = \pi$]

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$$H_d(\theta) = \begin{cases} 1, & |\theta| \le |\theta_c|; \\ 0, & |\theta| > |\theta_c|, \end{cases}$$

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The cutoff frequency $\theta_c = \omega_c\Delta t$ is assumed to fall in the Nyquist range $(-\pi, \pi)$ and $H_d(\theta)$ has period 2π.

The function $H_d(\theta)$ may be expanded:

$$H_d(\theta) = \sum_{n=-\infty}^{\infty} h_n e^{-in\theta} \qquad ; \qquad h_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(\theta) e^{in\theta} d\theta.$$

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Clearly,

$$H_d(\theta) \cdot F(\theta) = \sum_{n=-\infty}^{\infty} f_n^\star e^{-in\theta}.$$

The convolution theorem now implies that $H_d(\theta) \cdot F(\theta)$ is the transform of the convolution of $\{h_n\}$ with $\{f_n\}$:

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In practice the summation must be **truncated**.

Thus, an approximation to the LF part of $\{f_n\}$ is given by

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We see that the finite approximation to the discrete convolution is identical to a nonrecursive digital filter.

Gibbs oscillations

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These may be greatly reduced by means of an appropriately defined "window" function.

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Exercise: Write a MatLab program to compute the FFT of a step function with various truncations. Thus investigate the Gibbs phenomenon.

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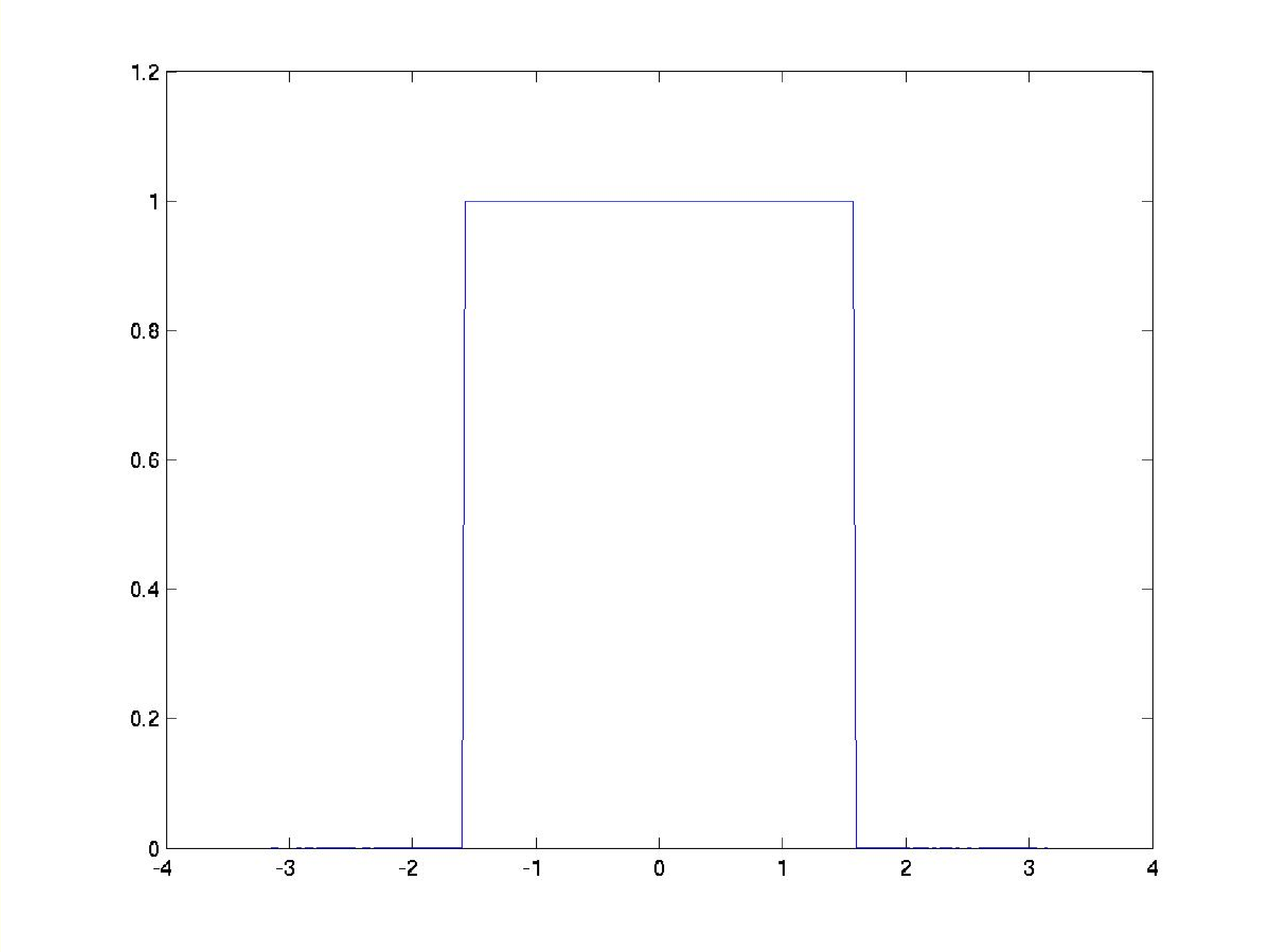
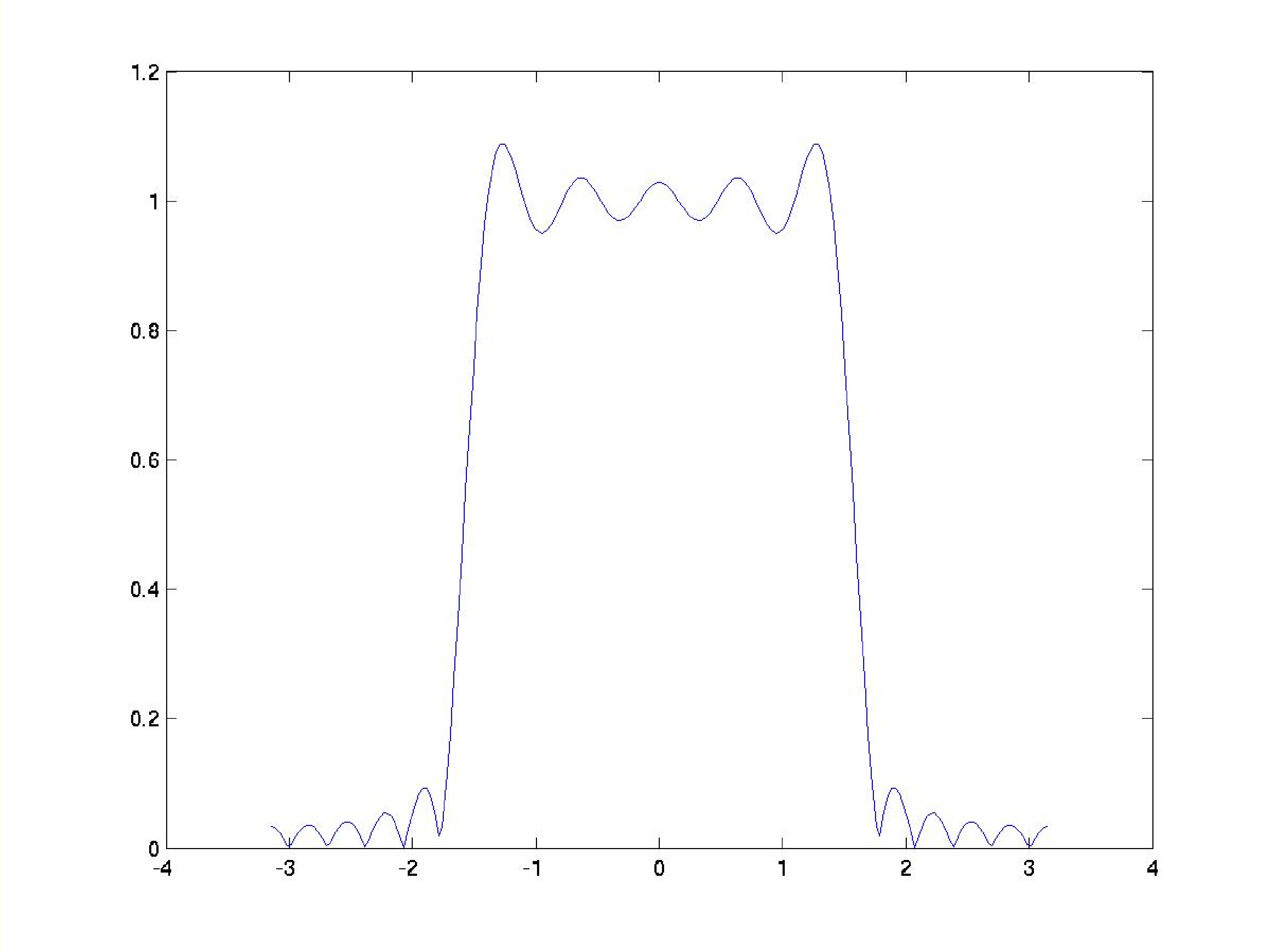
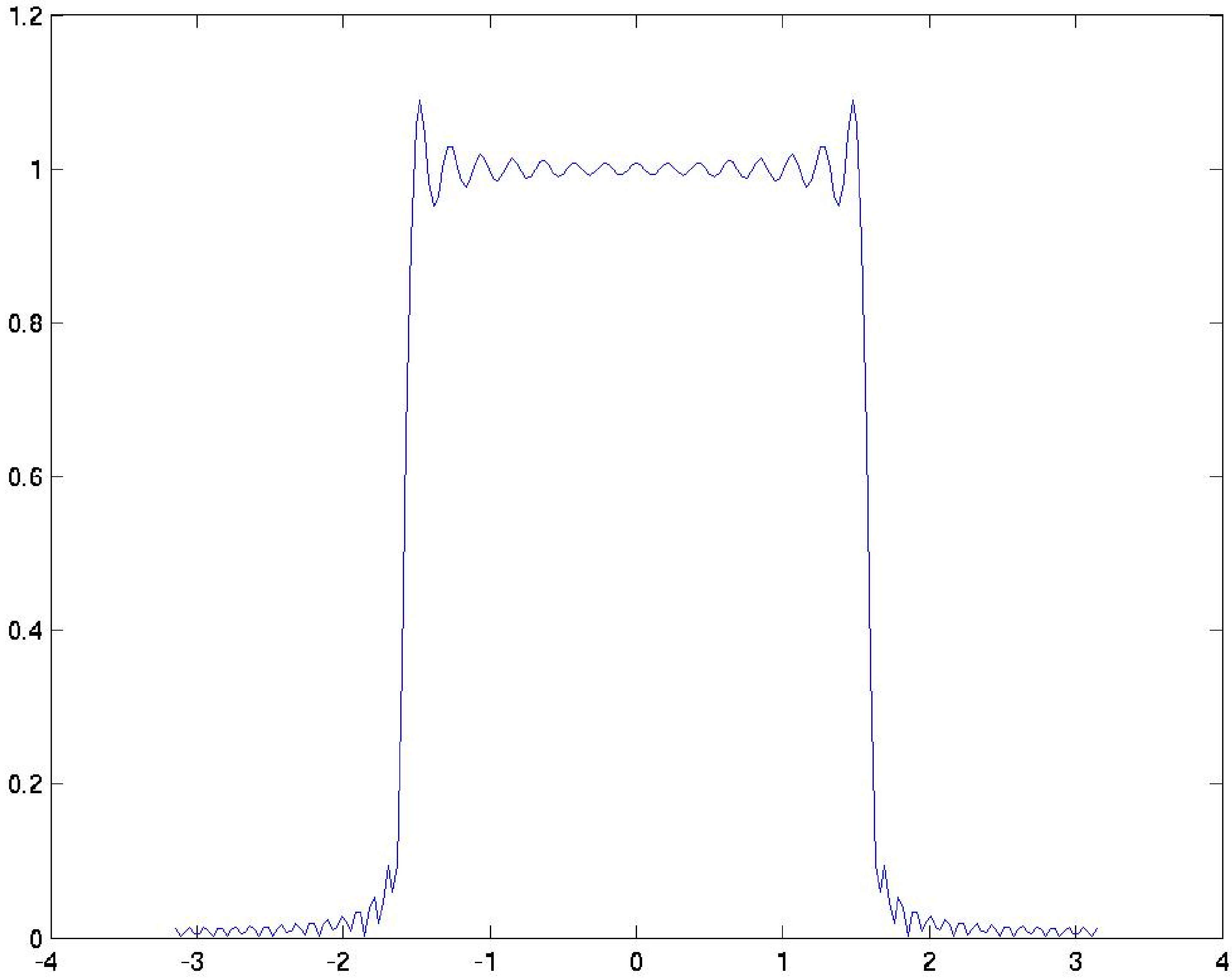
The truncated Fourier analysis of a square wave is shown in the following figures.



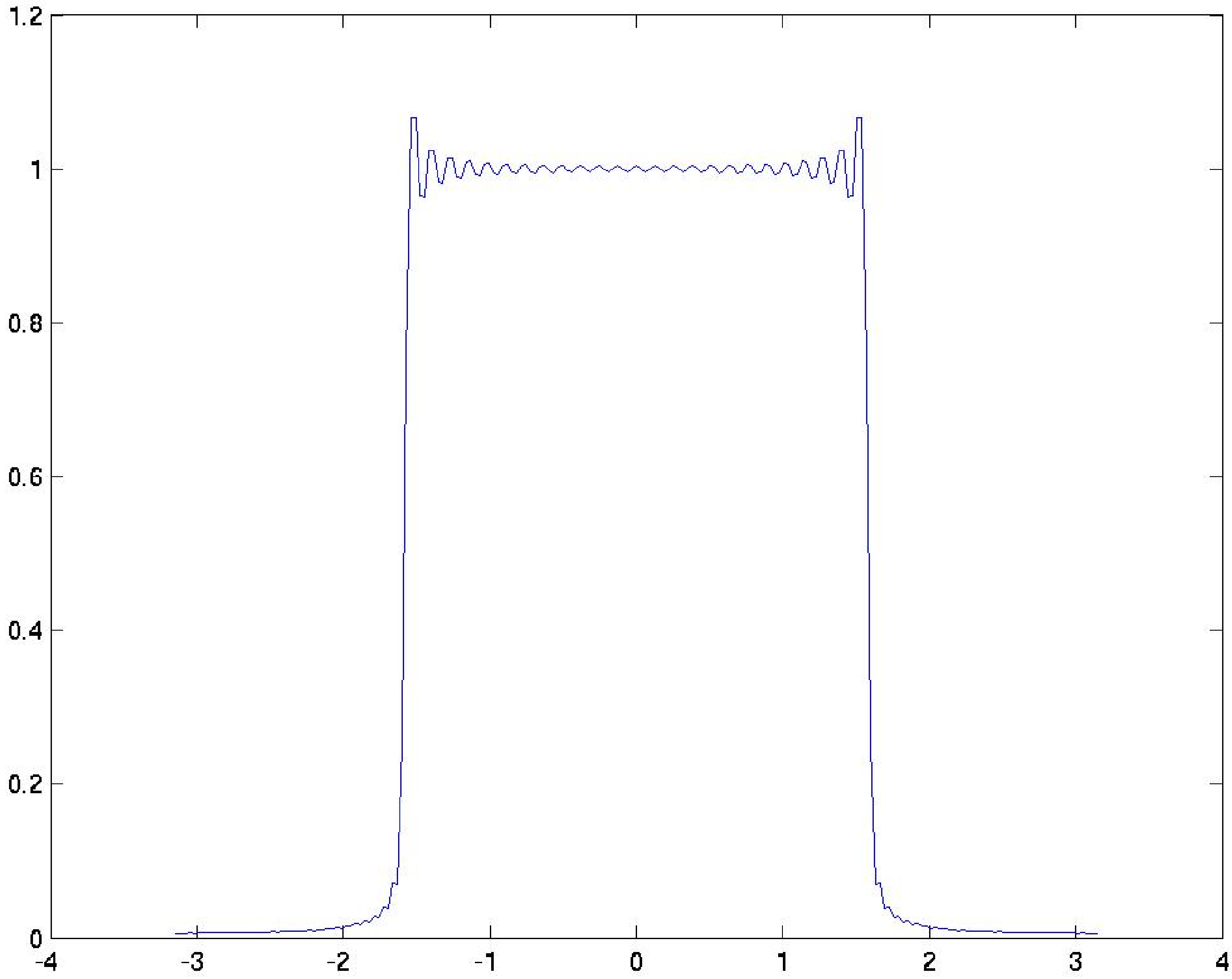
Original Square wave function.



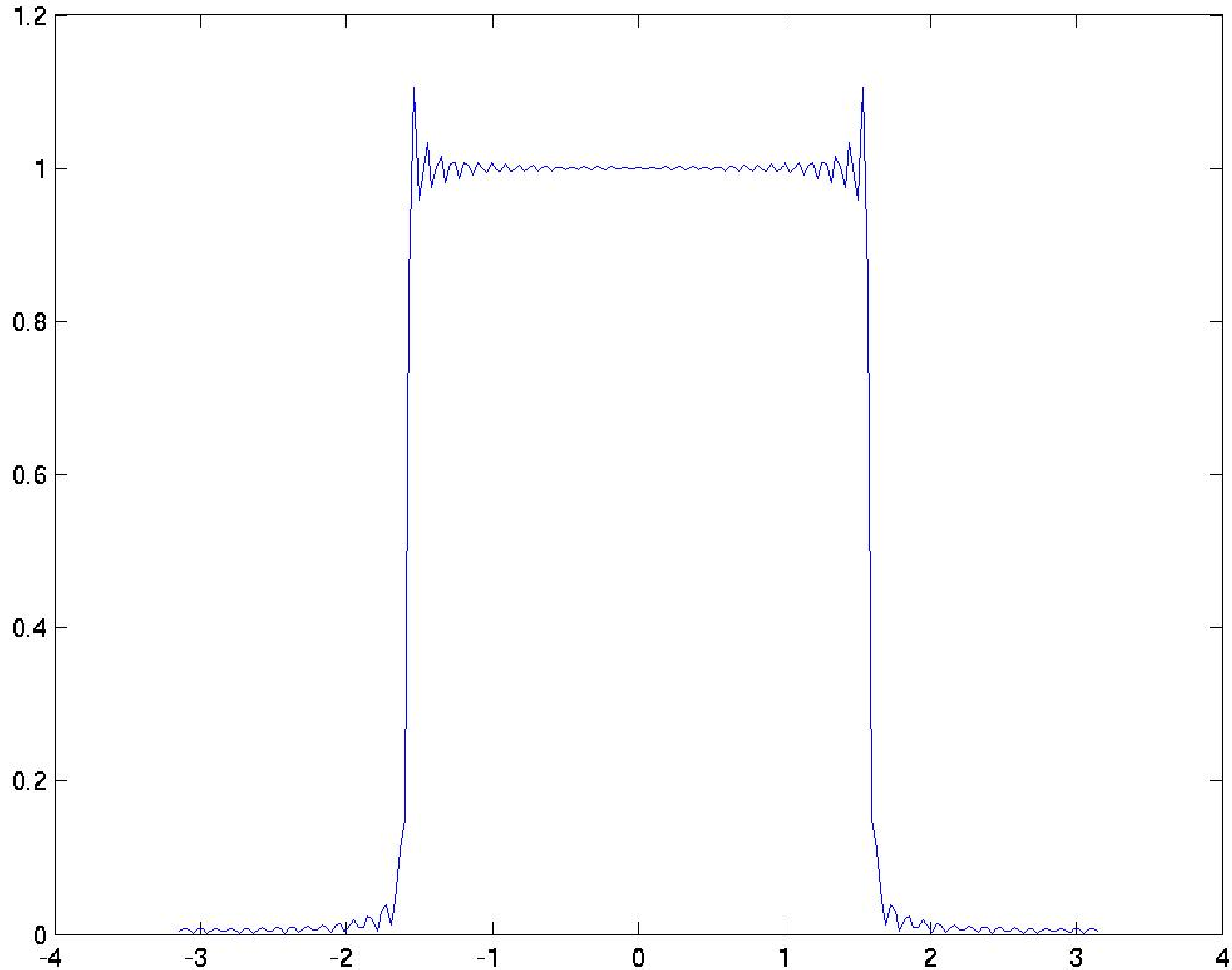
Truncation: $N = 11\,(N_{\max} = 50)$



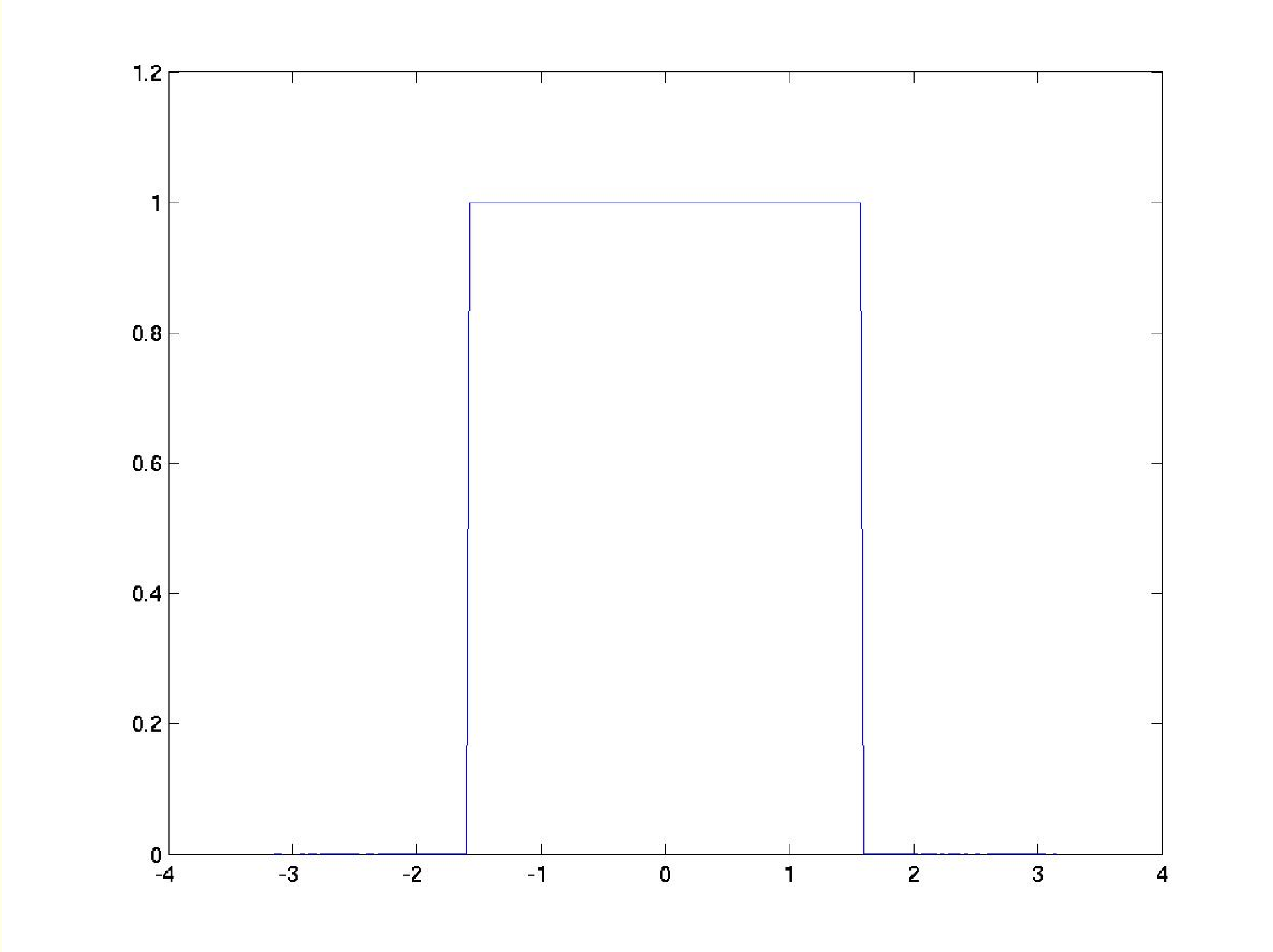
Truncation: $N = 21\,(N_{\max} = 50)$



Truncation: $N = 31\,(N_{\max} = 50)$



Truncation: $N = 41\,(N_{\max} = 50)$



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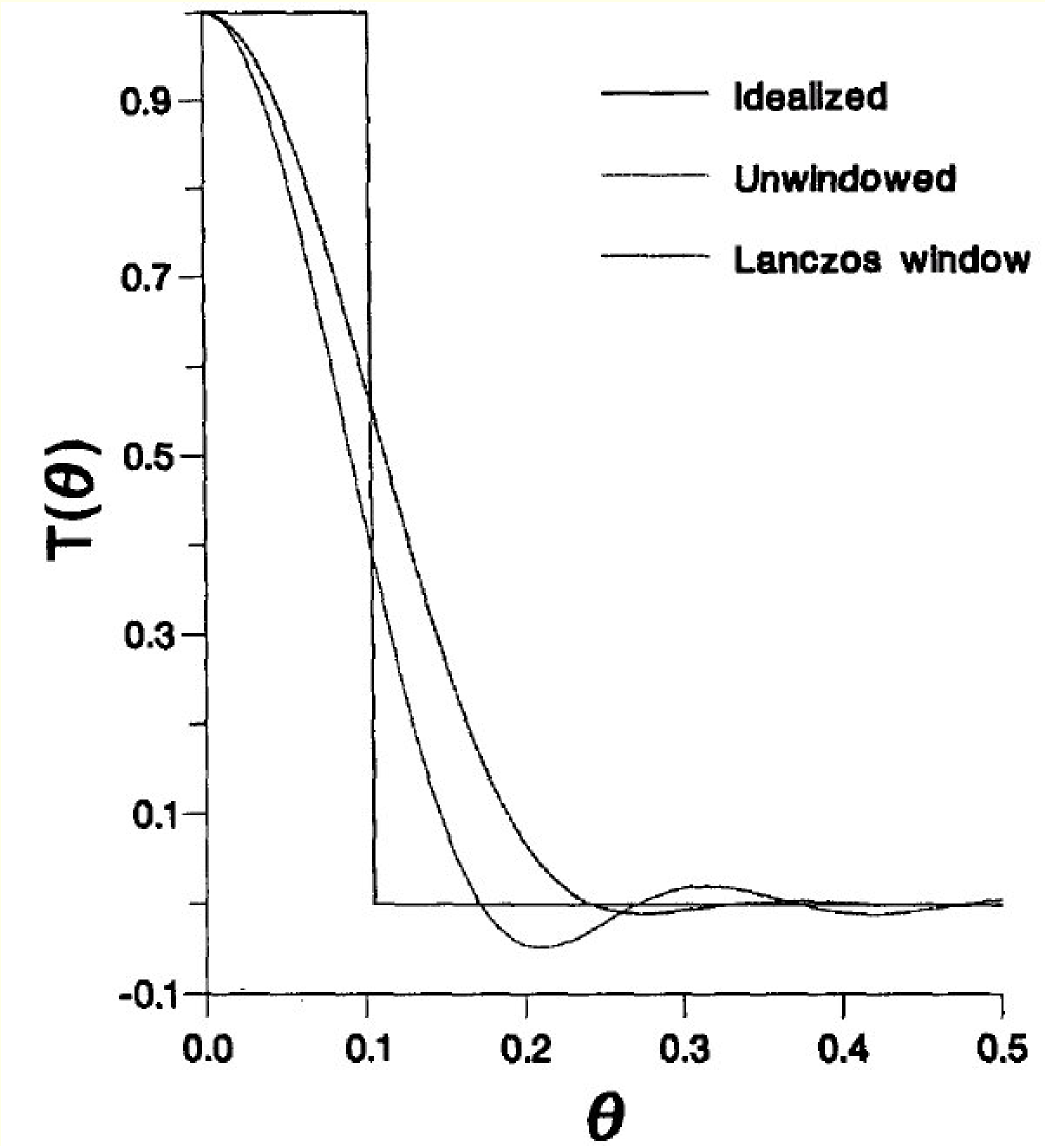
The coefficients were derived by Fourier expansion of a step-function, truncated at $N = 30$**, with a Lanczos window:**

$$h_n = \left[\frac{\sin\big(n\pi/(N+1)\big)}{n\pi/(N+1)}\right]\left(\frac{\sin(n\theta_c)}{n\pi}\right) .$$



The use of the window decreases the Gibbs oscillations in the stop-band $|\theta| > |\theta_c|$.

However, it also has the effect of widening the pass-band beyond the nominal cutoff.

For a fuller discussion of windowing see *e.g.* Hamming (1989) or Oppenheim and Schafer (1989).

The central lobe of the coefficient function spans a period of six hours, from $t = -3\,\mathbf{h}$ to $t = +3\,\mathbf{h}$: $T_{\rm Span} = 6\,\mathbf{hours}$.

The filter summation was calculated over this range, with the coefficients normalized to have unit sum over the span.

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These fields correspond to the application of the digital filter to the original data. They are the filtered data.

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It is salutary to recall that phase-errors are amongst the most prevalent and pernicious problems in forecasting.

Break here

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By means of the definition of $T_n(x)$ and basic trigonometric identities, $H(\theta)$ can be written as a finite expansion

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The weights $\{h_n : -M \leq n \leq +M\}$ define the Dolph-Chebyshev or, for short, Dolph filter.

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The Dolph filter has minimum ripple-ratio for a given main-lobe width and filter order.

Example of Dolph Filter

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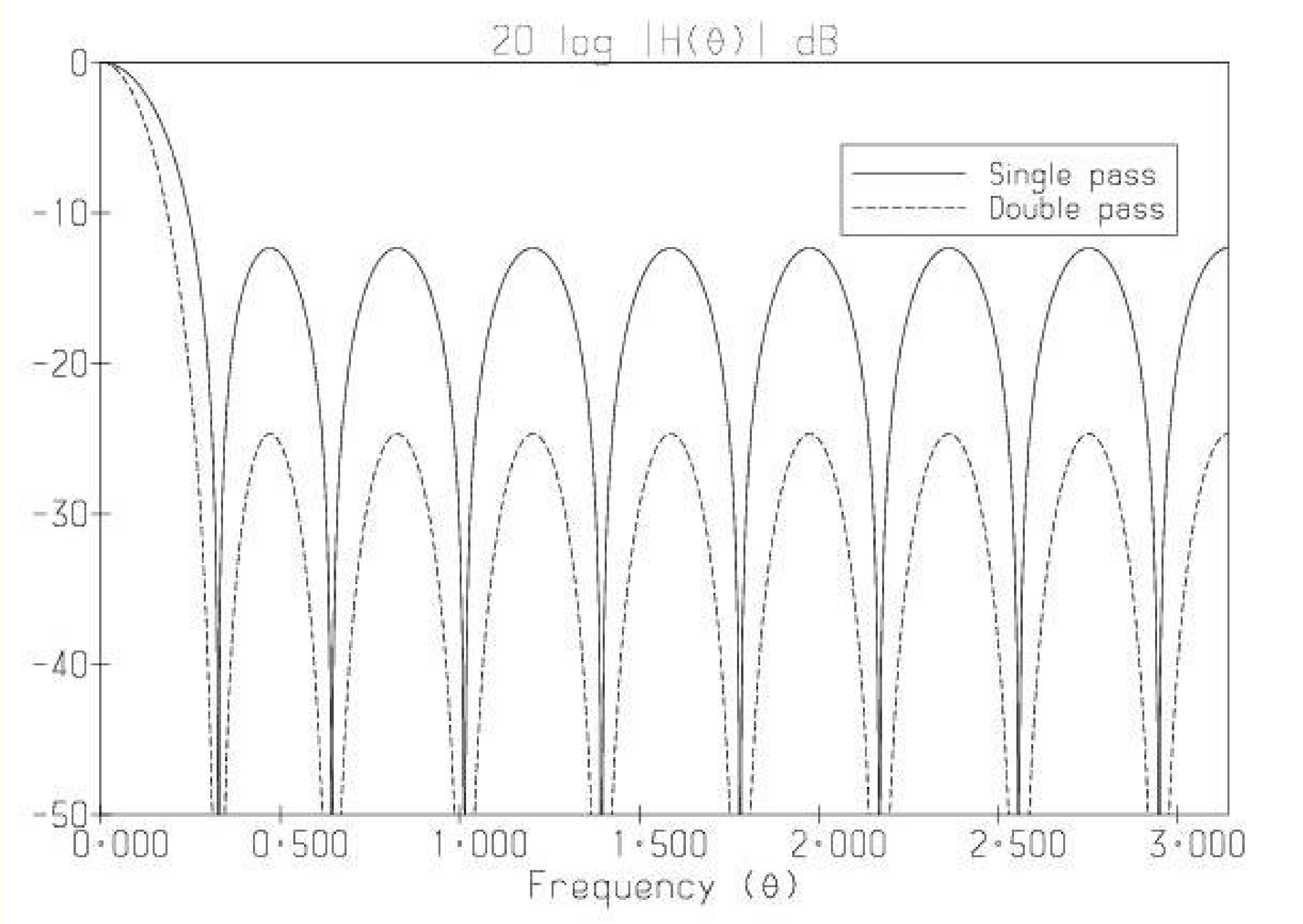
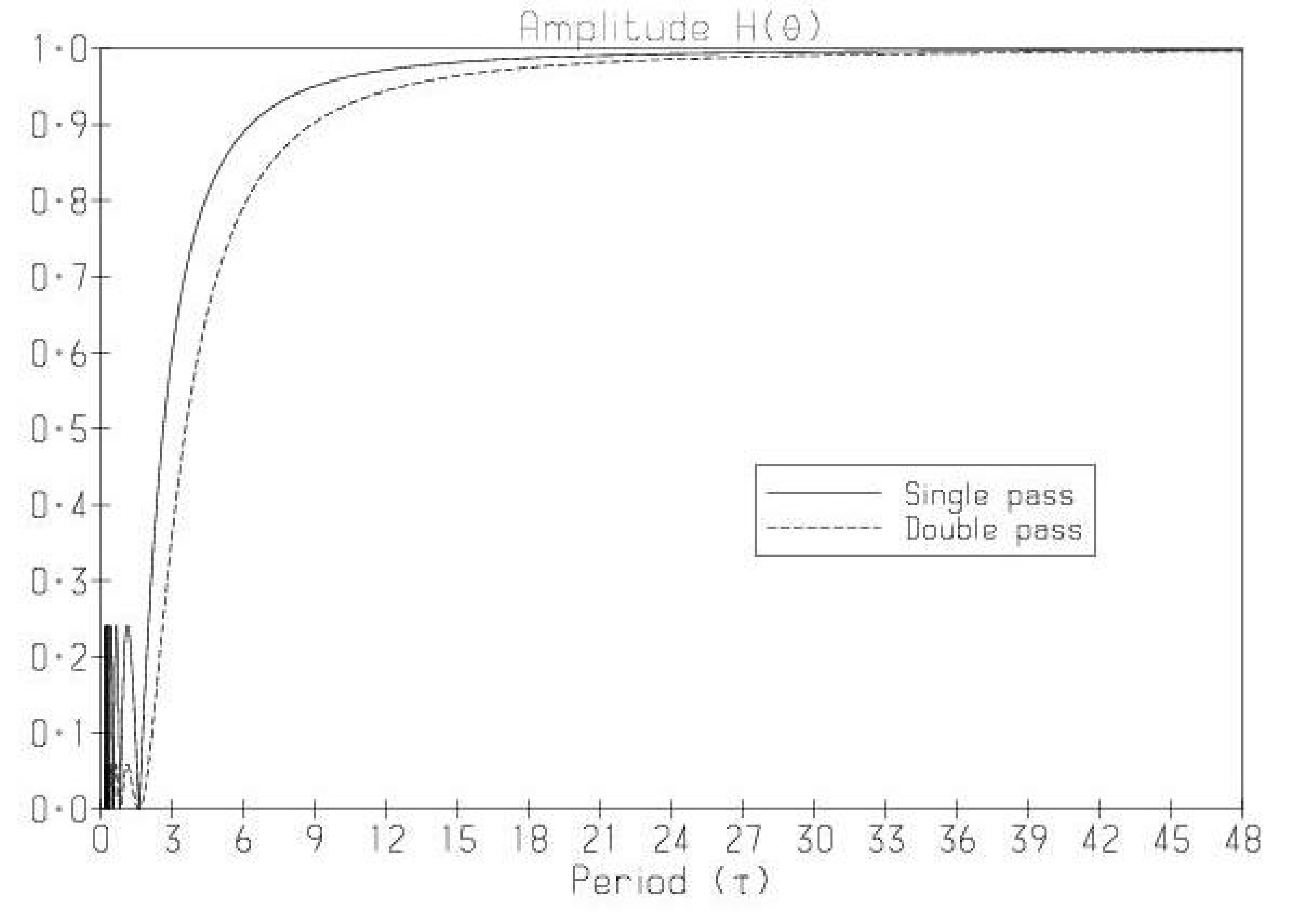
The DFI procedure employed in the HIRLAM model involves a double application of the filter.

We examine the frequency response $H(\theta)$ **and its square,** $H(\theta)^2$ **(a second pass squares the frequency response).**



Frequency response for Dolph filter with span $T_S = 2\text{h}$, order $N = 2M + 1 = 17$ and cut-off $\tau_s = 3\text{h}$. Results for single and double application are shown.

Logarithmic response (dB) as a function of frequency.



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Amplitude response as a function of period.

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It can be proved (Lynch, 1997) that the Dolph window is an optimal filter whose pass-band edge, θ_p, is the solution of the equation $H(\theta) = 1 - r$.

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Again, the filter is applied by accumulating sums formally identical to those of the first stage.

The output of the second stage is valid at the centre of the interval $[-\frac{1}{2}T_{\mathrm{S}}, +\frac{1}{2}T_{\mathrm{S}}]$, *i.e.*, at $t = 0$.

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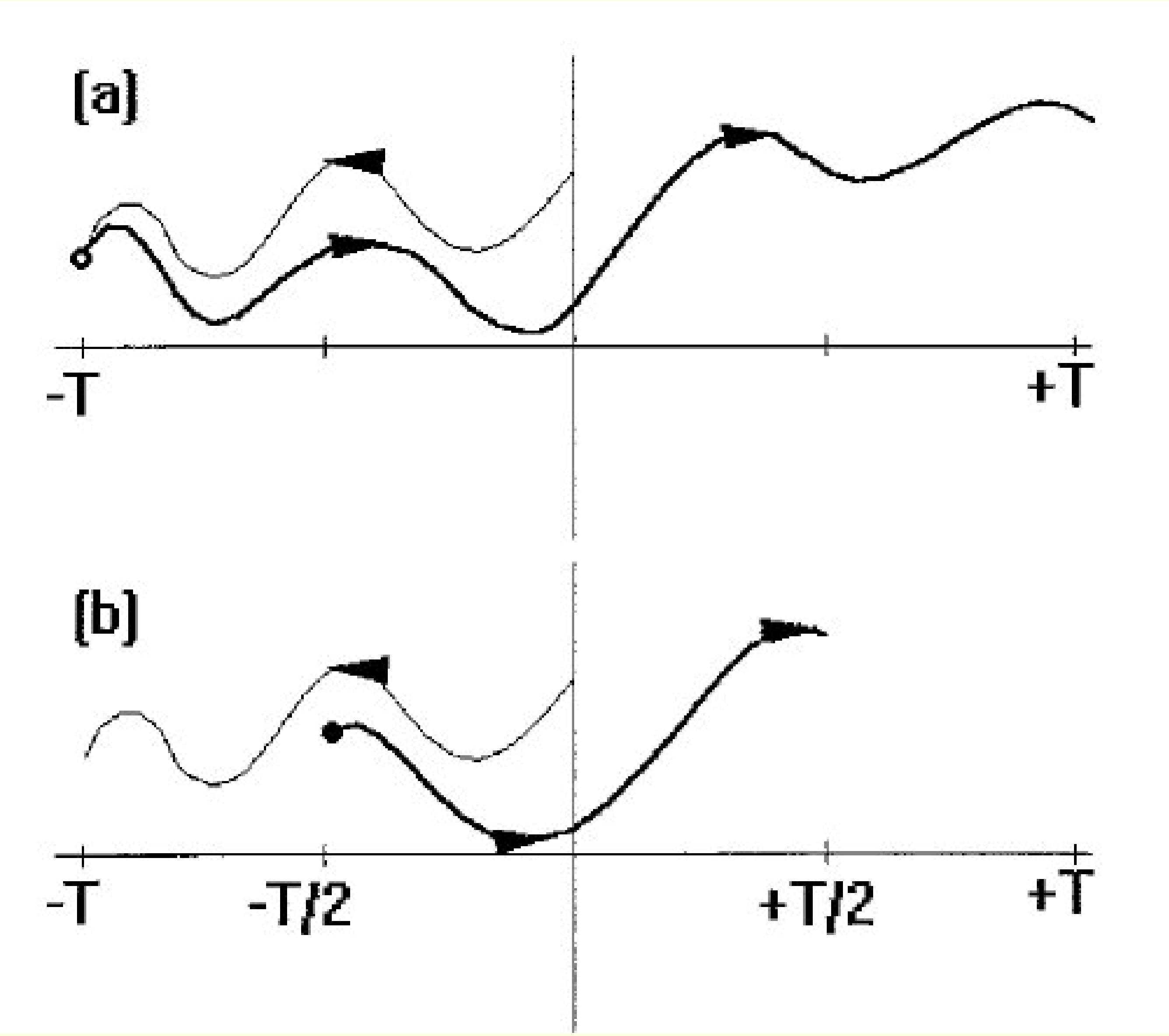
The output of the second pass is the initialized data.



DFI: Sample Results

The basic measure of noise is the mean absolute value of the surface pressure tendency

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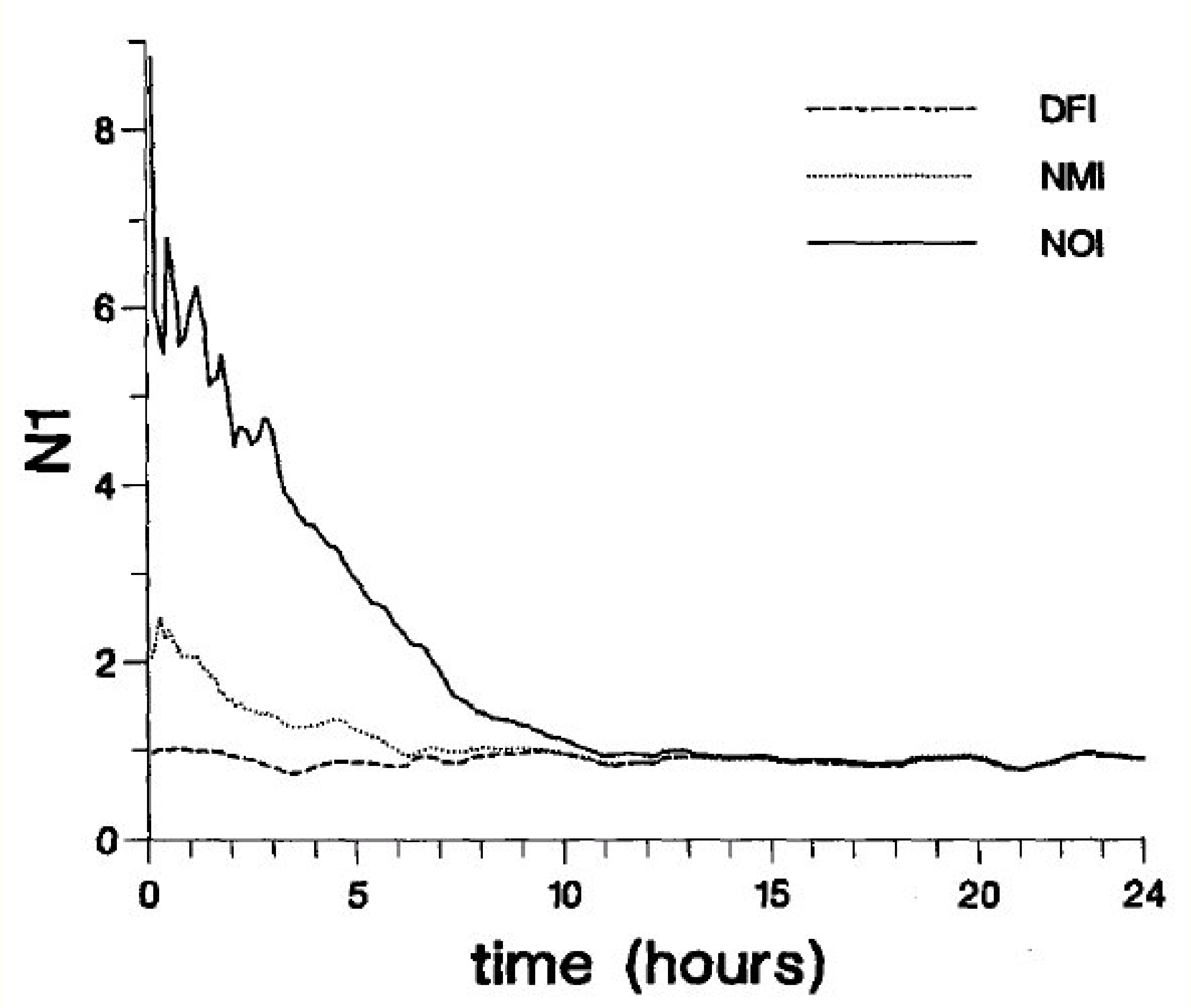
In the following figure, we plot the value of N_1 for three forecasts.



Mean absolute surface pressure tendency for three forecasts. Forecast with no initialization (NIL); normal mode initialization (NMI); digital filter initialization (DFI). Units are hPa/3 hours.

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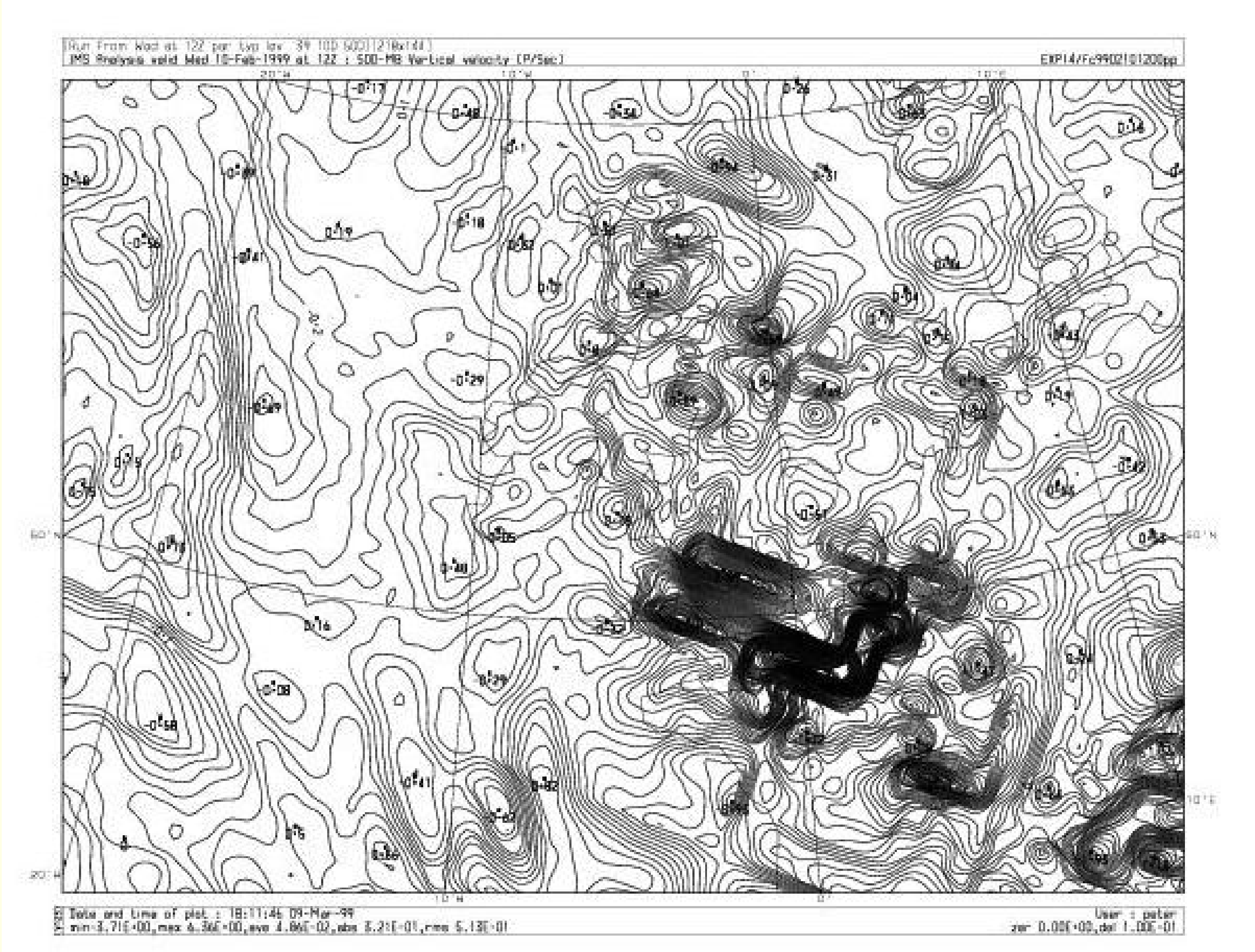
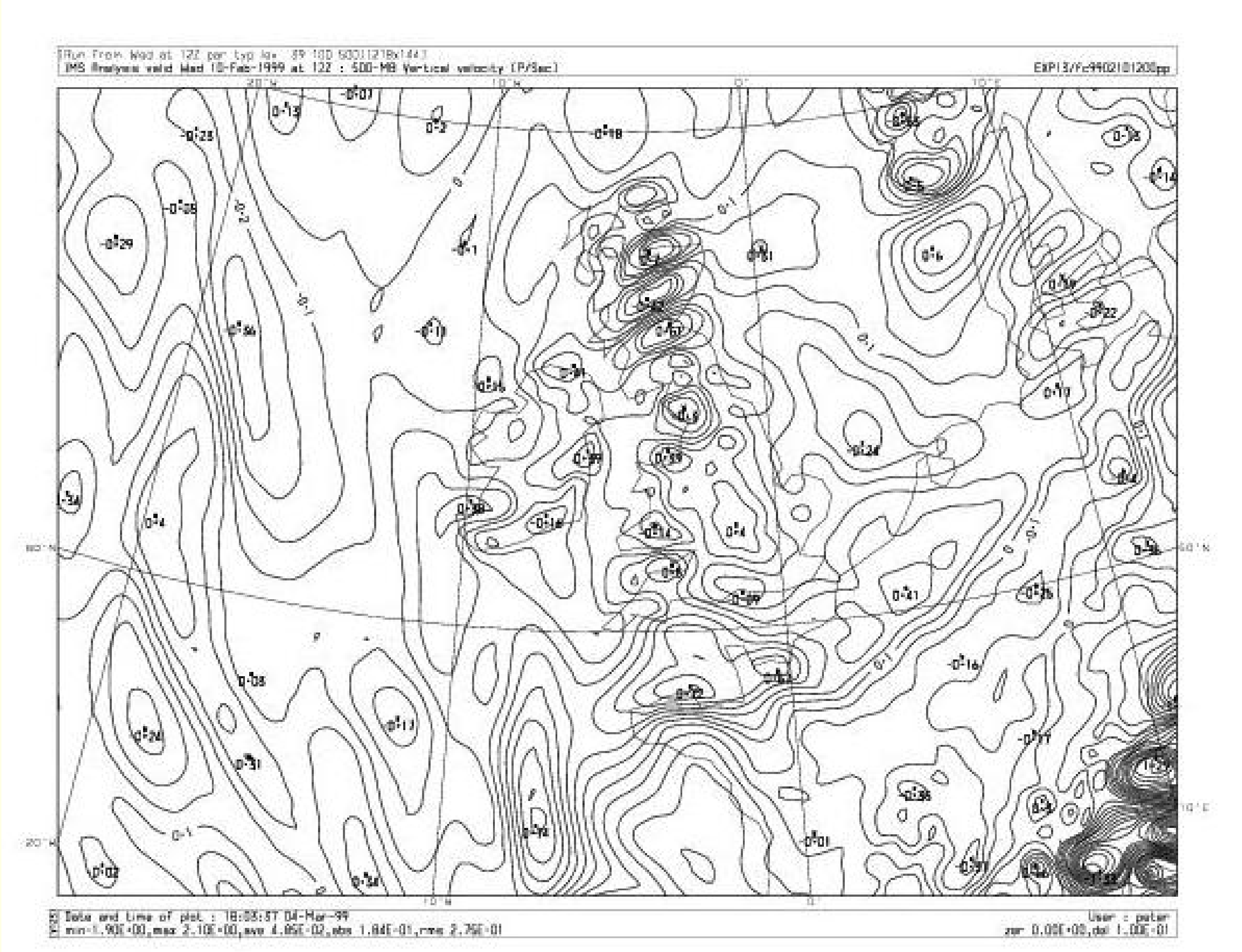
The uninitialized vertical velocity field is physically quite unrealistic.

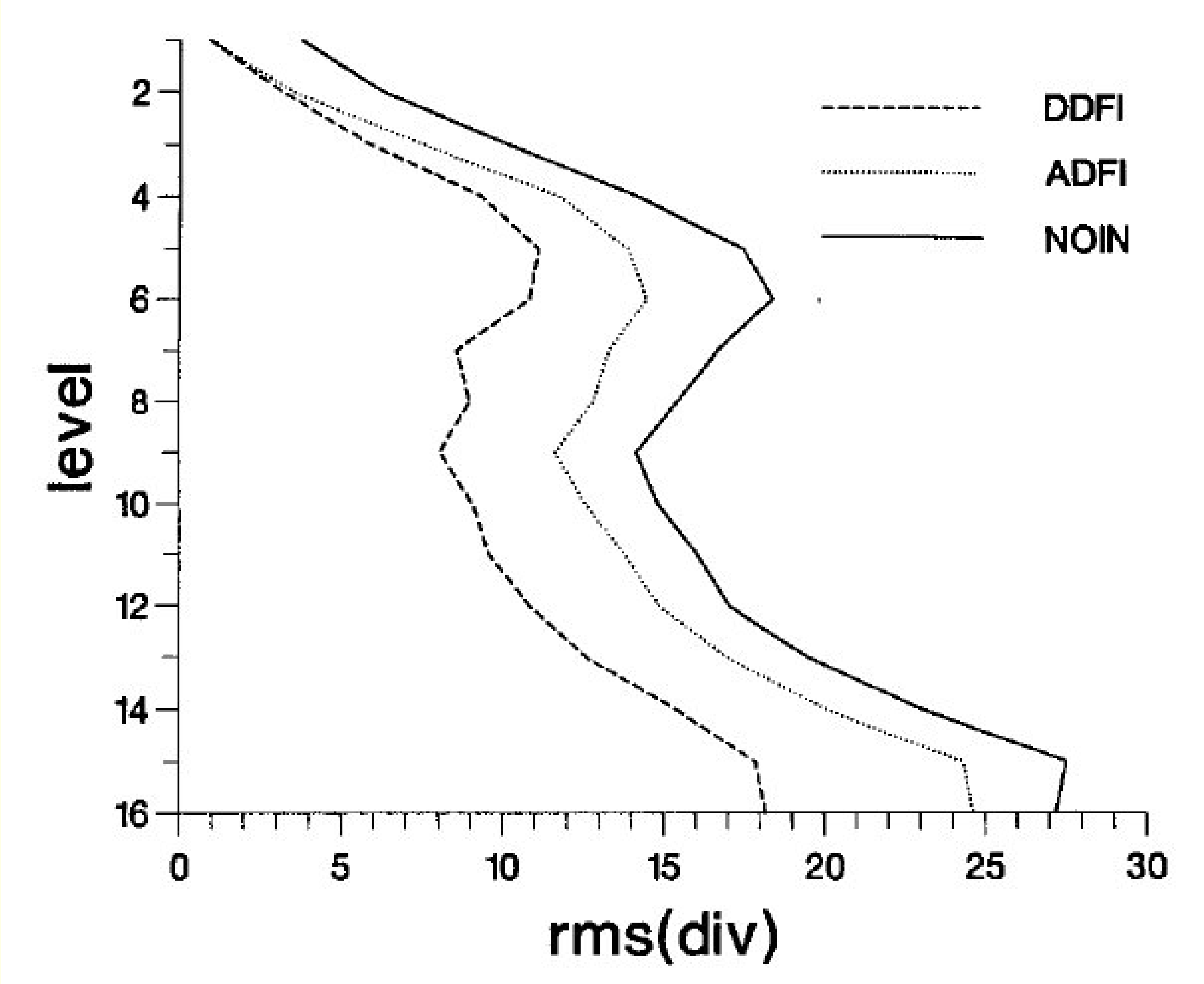
The DFI vertical velocity is much smoother, and much more realistic.



Vertical velocity at 500 hPa for uninitialized analysis (NIL).



Vertical velocity at 500 hPa after digital filtering (DFI).



Root mean square divergence at each model level.

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5. **No iterative numerical procedure which may diverge;**
6. **Ease of implementation and maintenance;**
7. **Applicable to all prognostic model variables;**
8. **Applicable to non-hydrostatic models.**

End of §4.3