# Numerical Weather Prediction

## Prof Peter Lynch

*Meteorology & Climate Centre*
*School of Mathematical Sciences*
*University College Dublin*

*Second Semester, 2005–2006.*

In this section we consider the Initialization of the analysed fields.

# Text for the Course

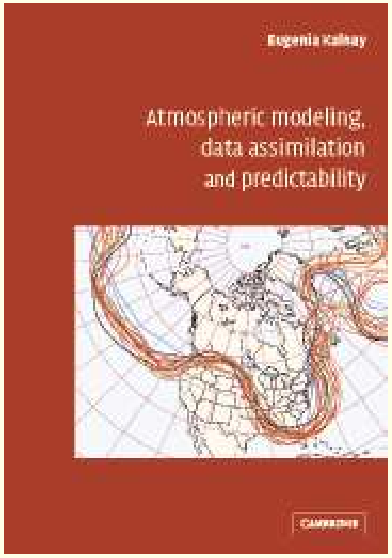
**The lectures will be based closely on the text**

**Atmospheric Modeling, Data Assimilation and Predictability**

**by**

**Eugenia Kalnay**

**published by Cambridge University Press (2002).**




See also Lynch, Peter, 2003: Introduction to Initialization. Pp. 97-111 in *Data Assimilation for the Earth System.*

Eds. R. Swinbank, V. Shutyaev and W. Lahoz, 378pp. [http://maths.ucd.ie/∼plynch/Publications.html]

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- ***The spectrum of atmospheric motions is vast, encompassing phenomena having periods ranging from seconds to millennia.***
- ***The motions of primary interest have timescales greater than a day.***
- ***The mathematical models used for numerical prediction describe a broader span of dynamical features than those of direct concern.***

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- *The elimination of this noise is achieved by adjustment of the initial fields, a process called* ***initialization****.*

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- *It is illustrated by application to a simple mechanical system, the swinging spring.*

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- ***His results implied a change in surface pressure of 145 hPa in 6 hours.***
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- *He ascribed the unrealistic value of pressure tendency to errors in the winds.*
- *This is only a partial explanation of the problem.*

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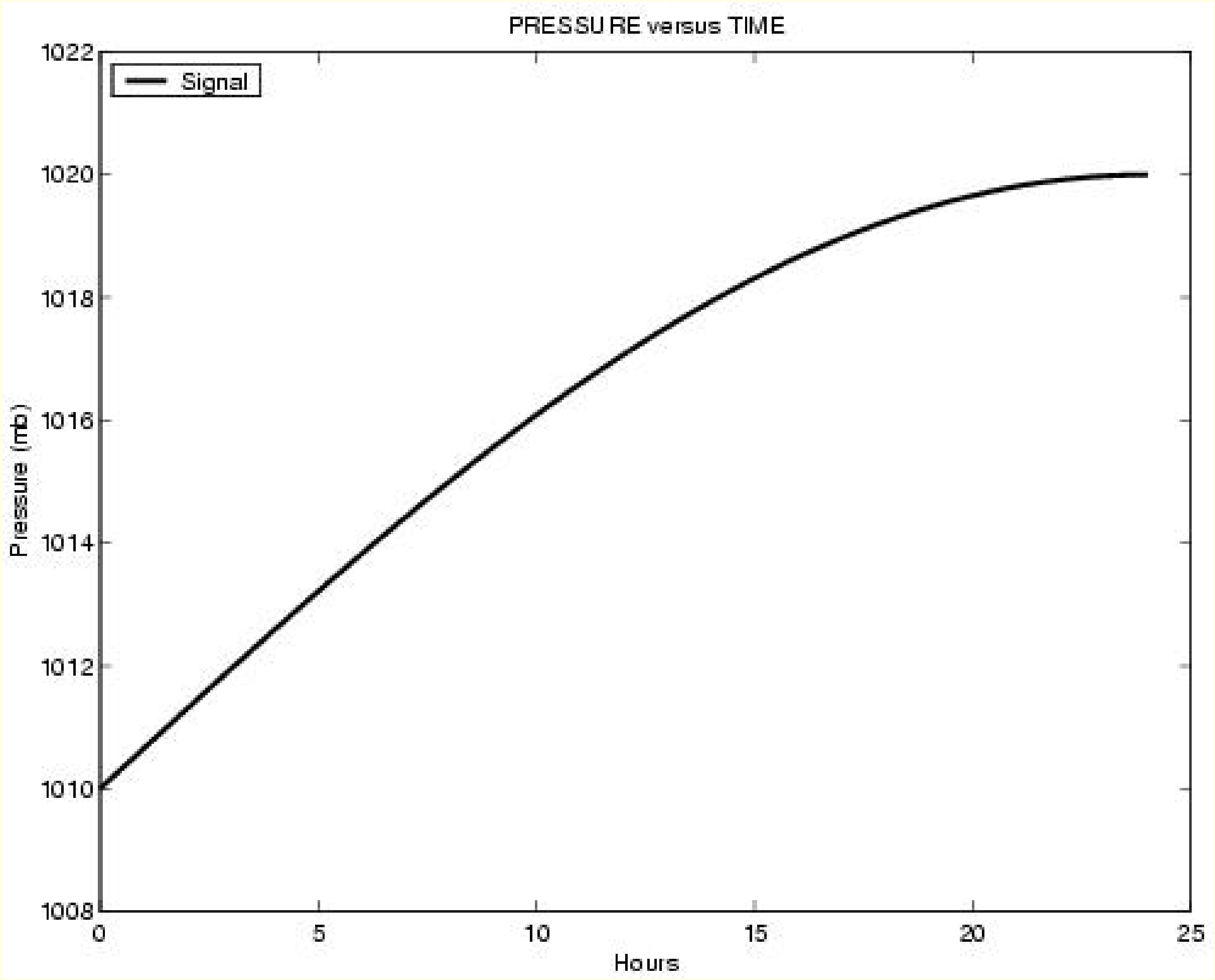
**. . . but overall they are of minor importance and may be regarded as undesirable noise.**

# Smooth Evolution of Pressure



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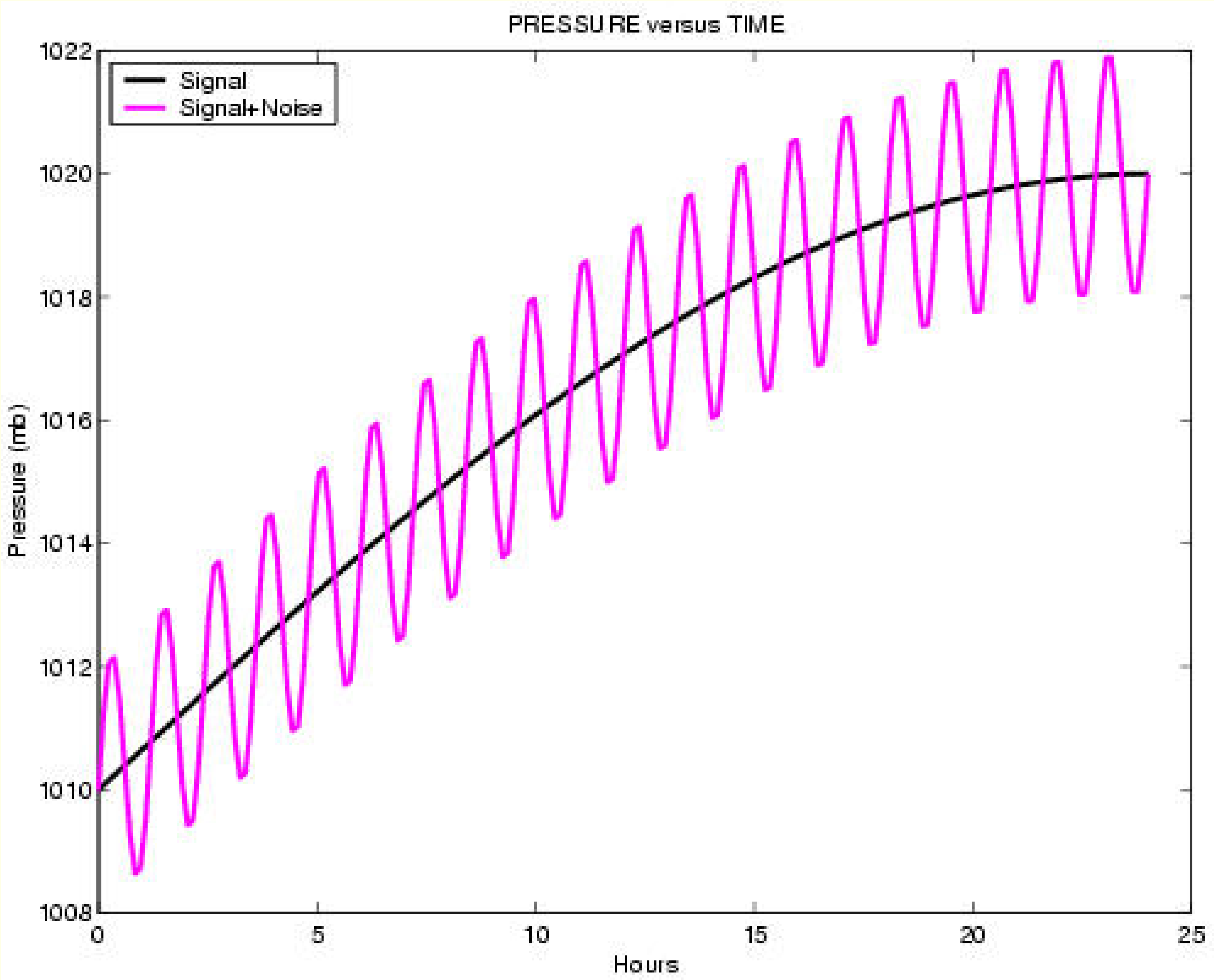

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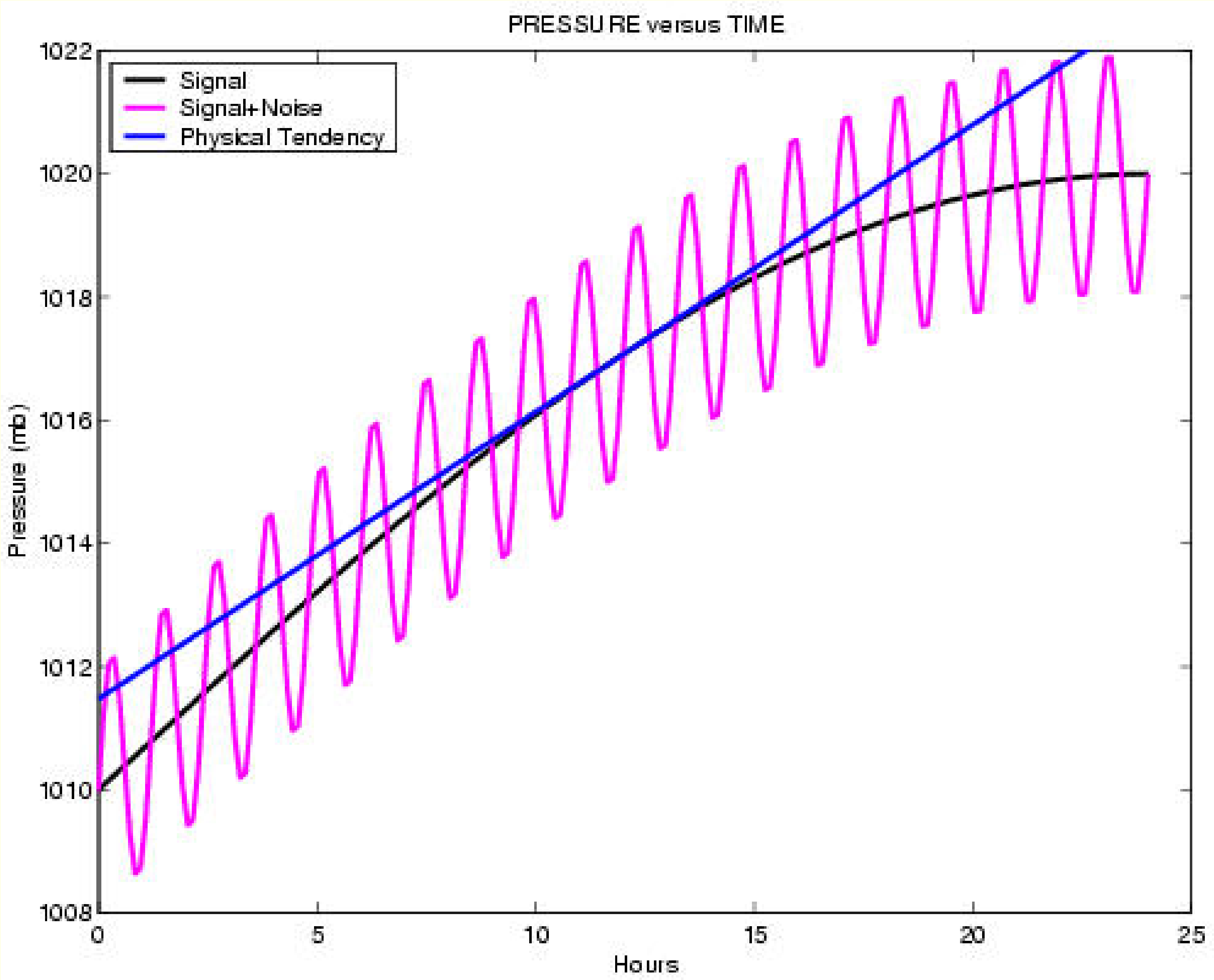

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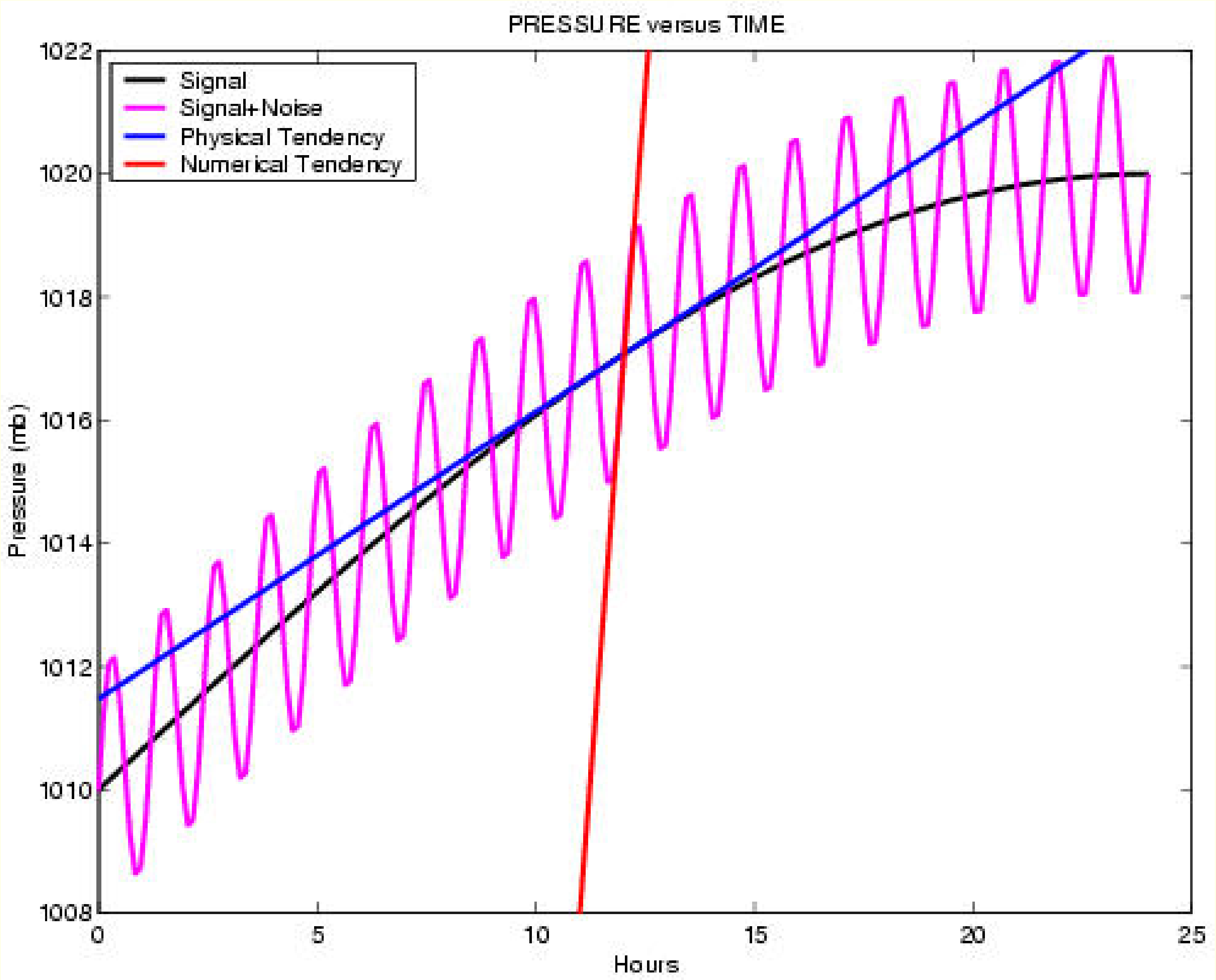

# A Reading from

# The Book of Limerick

**Young Richardson wanted to know**
**How quickly the pressure would grow.**
**But, what a surprise, 'cos**
**The six-hourly rise was,**
**In Pascals, One Four Five Oh Oh!**


**A Freak Wave?**

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**The Forty-foot, Sandycove.**

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**The same is true of the atmosphere!**

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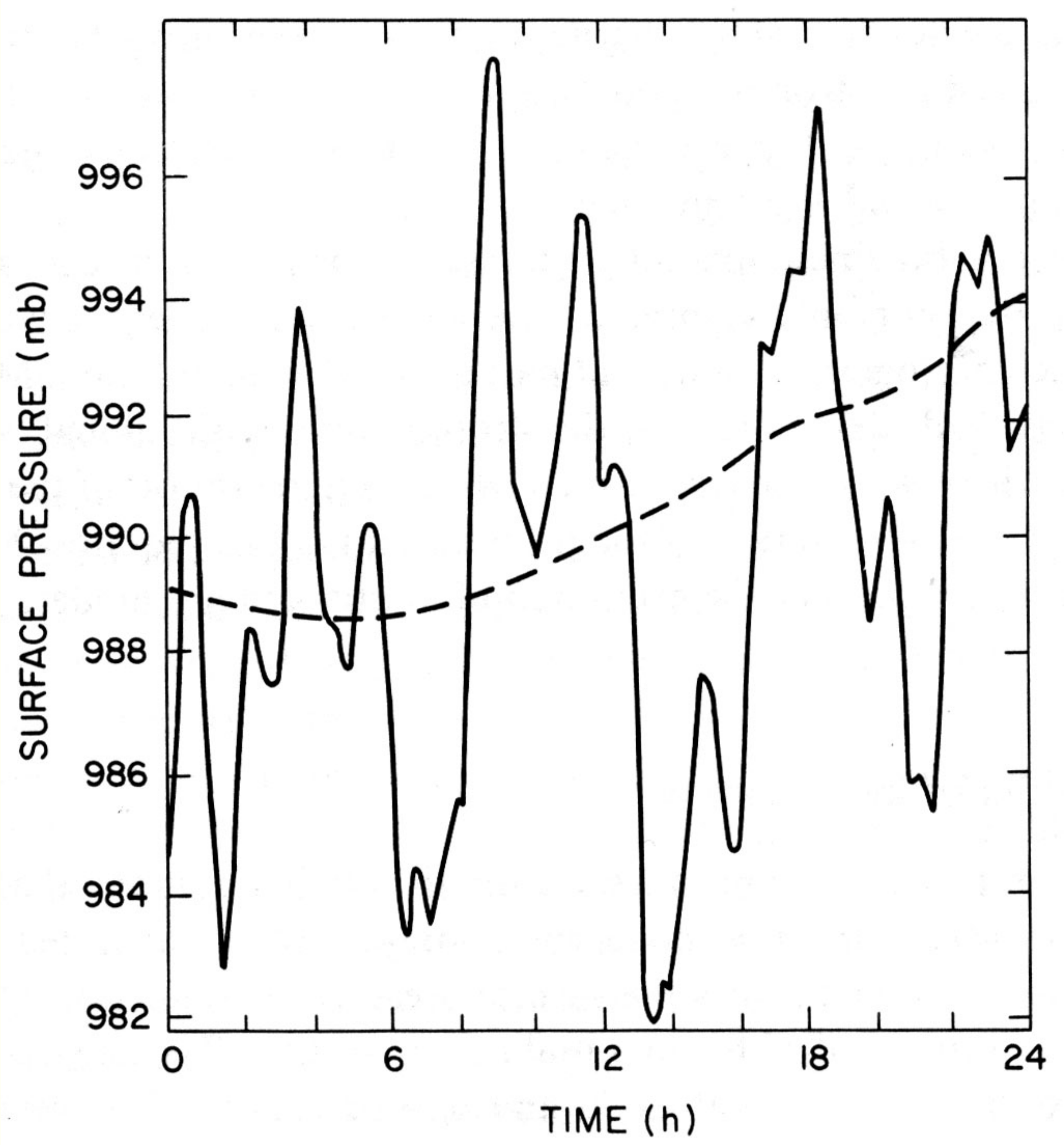
The existence of this geostrophic balance is a perennial source of interest

It is a consequence of the forcing mechanisms and dominant modes of hydrodynamic instability and of the manner in which energy is dispersed and dissipated in the atmosphere.

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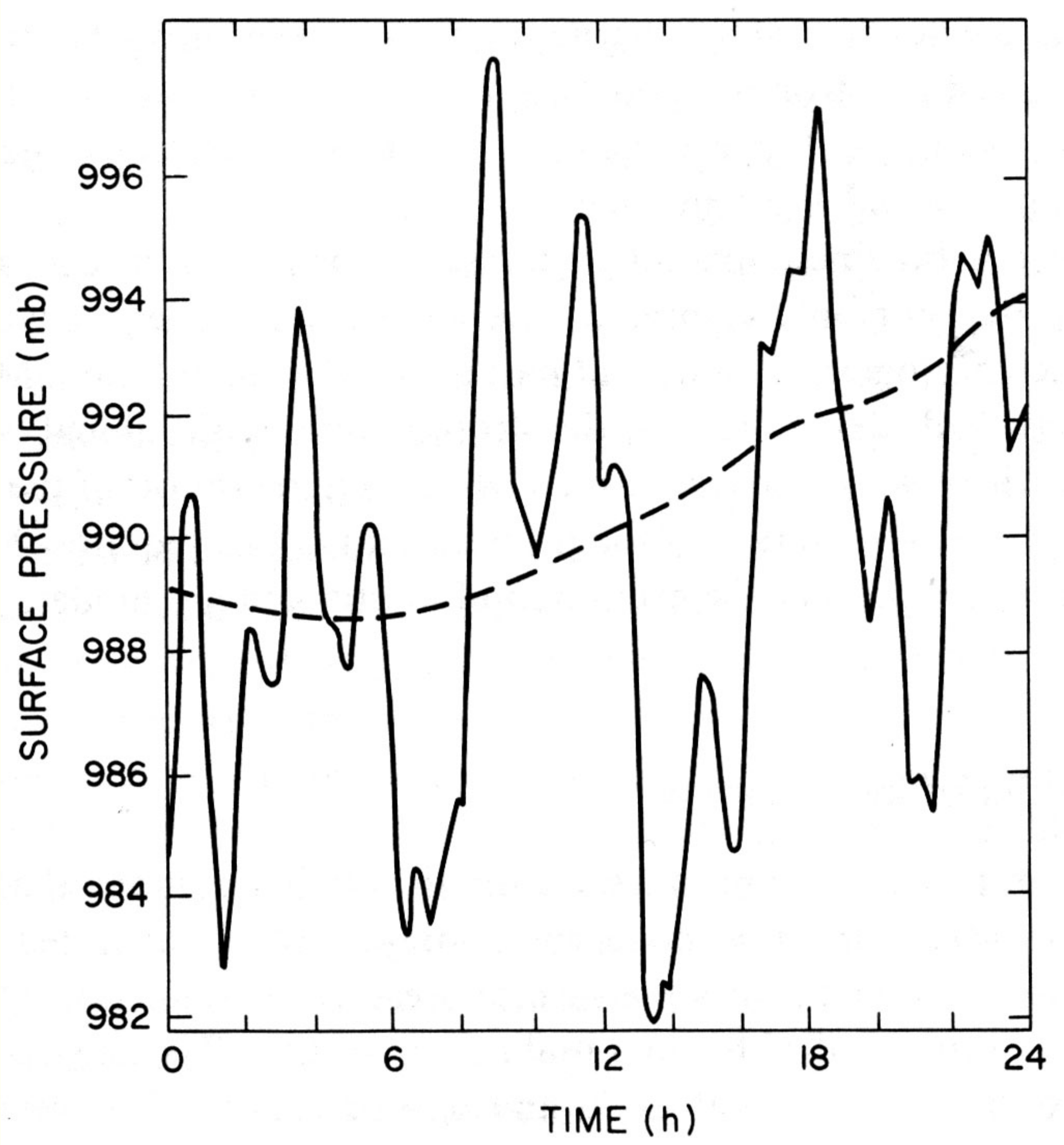
**The problems associated with high frequency motions are overcome by the process known as *initialization*.**

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**Specific Requirements for Initialization:**

- **Essential for satisfactory data assimilation**
- **Noisy forecasts have unrealistic vertical velocity**
- **Hopelessly inaccurate short-range rainfall patterns**
- **Spin-up of the humidity/water fields.**
- **Imbalance can lead to numerical instabilities.**

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**For simplicity, we may assume $\rho_0 \equiv 1$, though this is not essential.**

**The linear rotational shallow water equations are:**

$$\underbrace{\frac{\partial u}{\partial t}}_{\mathsf{V^2/L}} - \underbrace{fv}_{\mathsf{f\,V}} + \underbrace{\frac{1}{\rho_0}\frac{\partial p}{\partial x}}_{\mathsf{P/L}} = 0$$

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**If there is approximate balance between the Coriolis and pressure gradient terms, we must have**

$$\frac{\mathsf{P}}{\mathsf{L}} = f\mathsf{V} \qquad \textbf{or} \qquad \mathsf{P} = f\mathsf{LV} = 10^3\,\mathbf{Pa}$$

**The ratio of the velocity tendencies to the Coriolis terms is the Rossby number**

$$\mathsf{Ro} \equiv \frac{\mathsf{V}}{f\mathsf{L}} = \frac{10}{10^{-4}\cdot 10^{6}} = 10^{-1}\,,$$

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**To the lowest order of approximation, the tendency terms are negligible; there is geostrophic balance between the Coriolis and pressure terms.**

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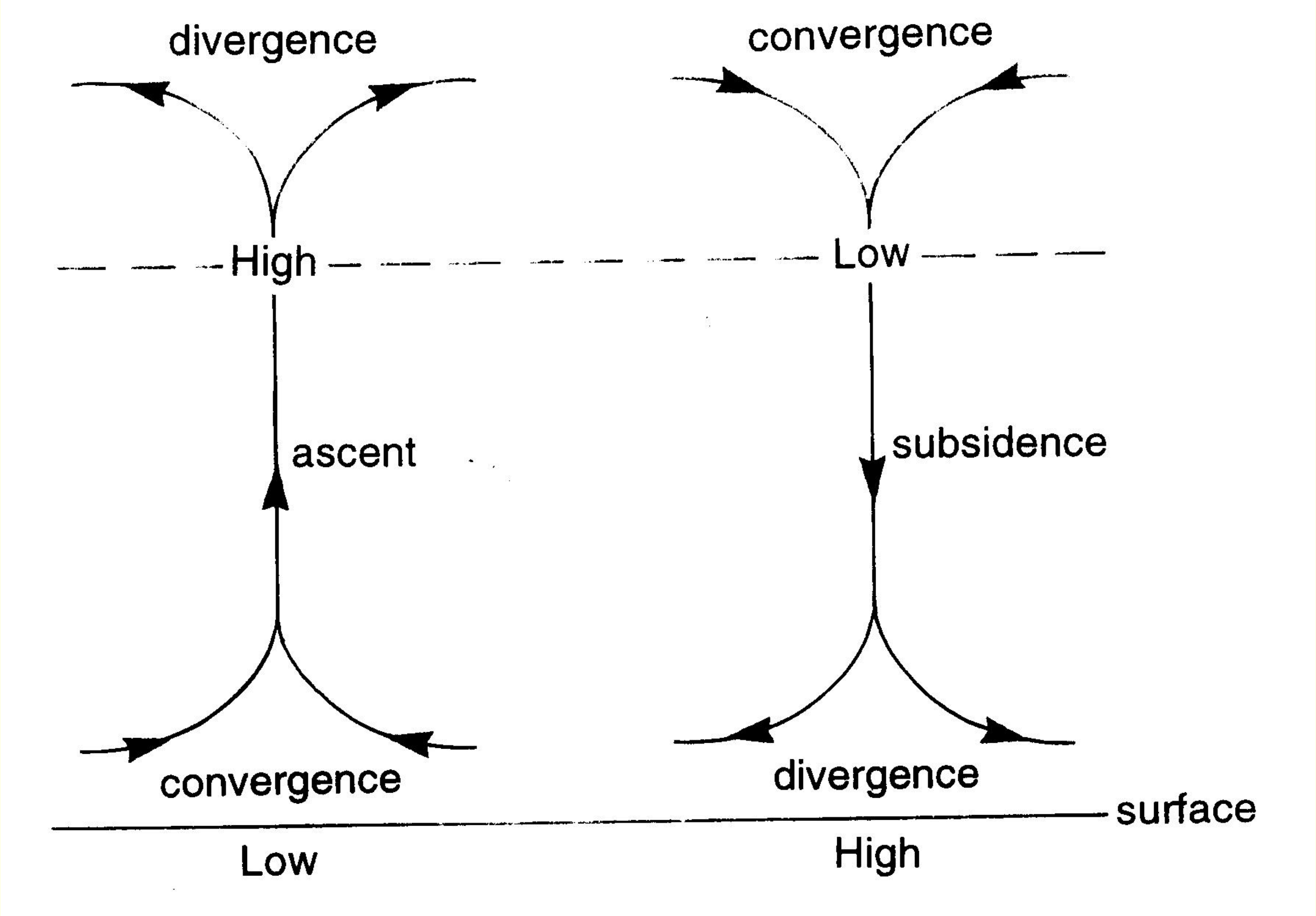
***Impossible*: there is nothing to balance the second term.**




**Dines Compensation mechanism:**

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**So,** $\partial p / \partial t \sim 10^{-2}\,\mathbf{Pa\,s}^{-1}$**, which is about 1 hPa per 3 hours.**

**Break here**

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**The forecast may be qualitatively reasonable, but it will be quantitatively invalid.**

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$$\frac{\partial p}{\partial x} \sim \frac{\mathsf{P}}{\mathsf{L}}, \qquad \text{but} \qquad \Delta\frac{\partial p}{\partial x} \sim \frac{\Delta p}{\Delta x} \sim \frac{\mathsf{P}}{\mathsf{L}} \sim \frac{\partial p}{\partial x},$$

**Thus, that the error in the wind tendency is now**

$$\Delta\frac{\partial u}{\partial t} \sim \frac{1}{\rho_0}\frac{\partial p}{\partial x} \sim 10^{-3} \gg \frac{\partial u}{\partial t}.$$

**The forecast will be qualitatively incorrect (i.e., useless!).**

**Now consider the continuity equation.**

**The pressure tendency has scale**

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**Worse still, if the wind error is of small spatial scale, the divergence error is correspondingly greater:**

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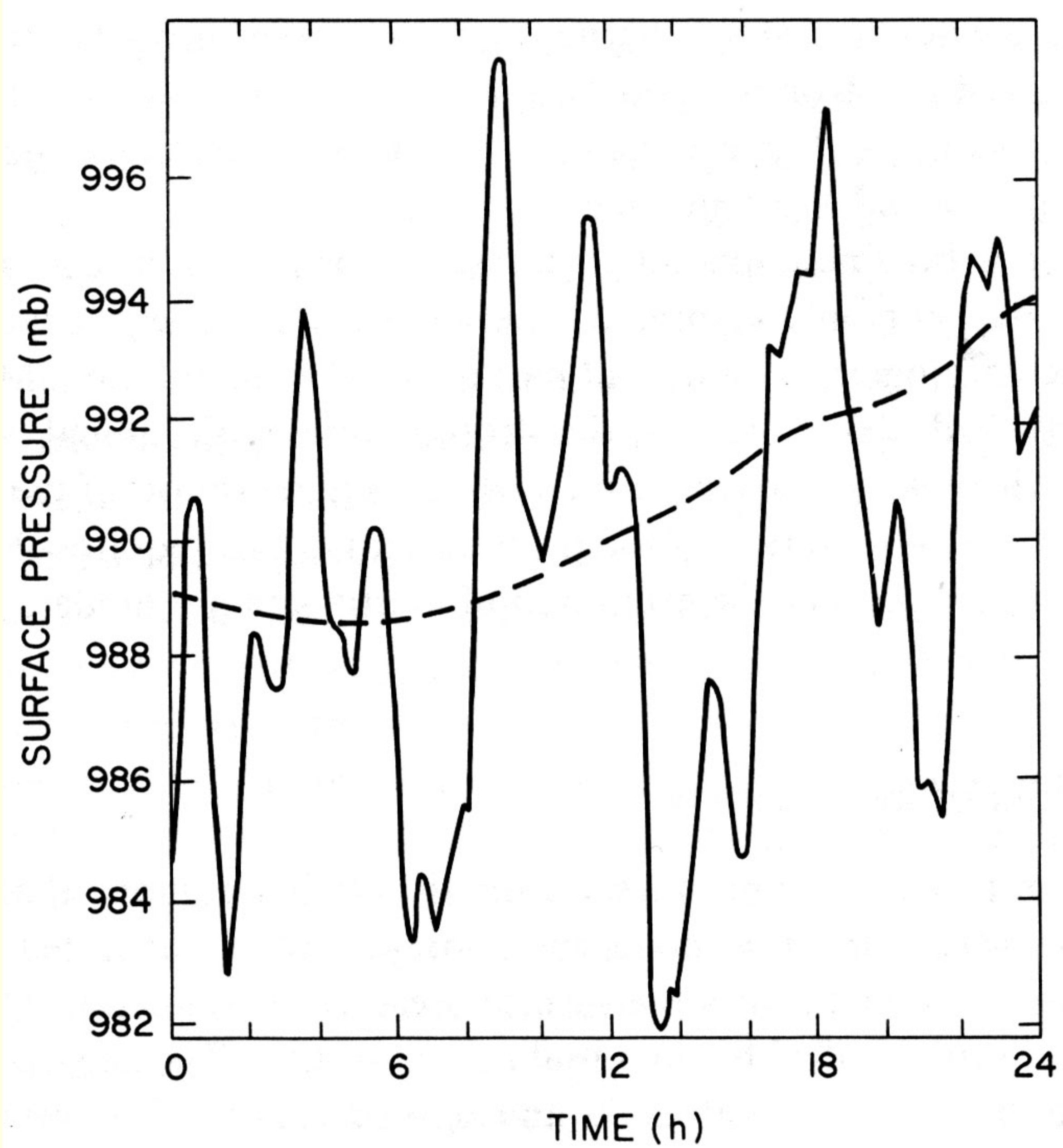
**Instead of the value** $\partial p/\partial t \sim 1$ **hPa in 3 hours we get a change of order 100 hPa in 3 hours (like Richardson's result).**




**Evolution of surface pressure before and after NNMI.**

**(Williamson and Temperton, 1981)**

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- ***2. Static Initialization***
- ***3. Dynamic Initialization***
- ***4. Variational Initialization***

# 1. The Filtered Equations

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**The basic filtered system is the quasi-geostrophic equations.**

**The *barotropic, quasi-geostrophic potential vorticity equation* (the QGPV Equation) is**

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**This is a *single equation* for a *single variable*, $\psi$.**

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**A more accurate filtering of the primitive equations leads to the balance equations.**

**This system is more complicated to solve than the quasi-geostrophic system, and has not been widely used.**

**However one diagnostic component has been used for initialization. We discuss this presently.**

# 2. Static Initialization

**Hinkelmann (1951) investigated the problem of noise in numerical integrations of the primitive equations.**

**He concluded that if the initial winds were geostrophic:**

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**This can easily be solved for $\psi$ if $\Phi$ is given, or for $\Phi$ if $\psi$ is given.**

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**This equation is a diagnostic relationship between the pressure and wind fields.**

$$\nabla^2\Phi - \nabla\cdot f\nabla\psi + 2\left[\left(\frac{\partial^2\psi}{\partial x\partial y}\right)^2 - \frac{\partial^2\psi}{\partial x^2}\frac{\partial^2\psi}{\partial y^2}\right] = 0$$

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**This is a Poisson equation for $\Phi$ when $\psi$ is given. However, it is nonlinear in $\psi$ and much harder to solve for $\psi$ when $\Phi$ is given. This is the usual case.**

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**Each of these steps represented some progress, but the noise problem still remained essentially unsolved.**

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**. . . however, the procedure is expensive in computer time.**

**Moreover, it damps the meteorologically significant motions as well as the gravity waves so it is no longer popular.**

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**If time permits, we will return to DFI later.**

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**Recall that, in variational assimilation, we minimize a cost function, $J$, which is normally a sum of two terms**

$$J = J_B + J_O$$

**Here, $J_B$ is the distance between the analysis and the background field**

$$J_B = \frac{1}{2}(\mathbf{x} - \mathbf{x}_b)^T \mathbf{B}^{-1}(\mathbf{x} - \mathbf{x}_b)$$

**and $J_O$ is the distance to the observations**

$$J_O = \frac{1}{2}(\mathbf{y}_o - H(\mathbf{x}))^T \mathbf{R}^{-1}(\mathbf{y}_o - H(\mathbf{x}_b))$$

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**The constrained variational assimilation finds the minimum of the cost function**

$$J = J_B + J_O + J_C$$

**End of §4.1**