M.Sc. in Meteorology UCD

Numerical Weather Prediction

Prof Peter Lynch

Meteorology & Climate Centre
School of Mathematical Sciences
University College Dublin

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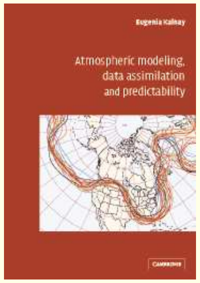
In this section we consider the Initialization of the analysed fields.

Text for the Course

The lectures will be based closely on the text

Atmospheric Modeling, Data Assimilation and Predictability
by
Eugenia Kalnay

published by Cambridge University Press (2002).



See also Lynch, Peter, 2003: Introduction to Initialization. Pp. 97-111 in *Data Assimilation for the Earth System.* Eds. R. Swinbank, V. Shutyaev and W. Lahoz, 378pp. [http://maths.ucd.ie/~plynch/Publications.html]

§4.1. Introduction to Initialization

- *The spectrum of atmospheric motions is vast, encompassing phenomena having periods ranging from seconds to millennia.*
- *The motions of primary interest have timescales greater than a day.*
- *The mathematical models used for numerical prediction describe a broader span of dynamical features than those of direct concern.*
- *For many purposes these higher frequency components can be regarded as noise contaminating the motions of meteorological interest.*
- *The elimination of this noise is achieved by adjustment of the initial fields, a process called initialization.*

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- ***We examine the fundamental equations and elucidate the causes of spurious oscillations.***
- ***The history of methods of eliminating high-frequency noise is recounted.***
- ***Various initialization methods are described.***
- ***The normal mode initialization method is described.***
- ***It is illustrated by application to a simple mechanical system, the swinging spring.***

Richardson's Forecast

The story of Lewis Fry Richardson's forecast is well known.

- ***Richardson forecast the change in surface pressure at a point in central Europe, using the mathematical equations.***
- ***His results implied a change in surface pressure of 145 hPa in 6 hours.***
- ***As Sir Napier Shaw remarked, "the wildest guess . . . would not have been wider of the mark . . . ".***

- ***Yet, Richardson claimed that his forecast was ". . . a fairly correct deduction from a somewhat unnatural initial distribution".***
- ***He ascribed the unrealistic value of pressure tendency to errors in the winds.***
- ***This is only a partial explanation of the problem.***

Spectrum of Atmospheric Motions

The natural oscillations of the atmosphere fall into two groups

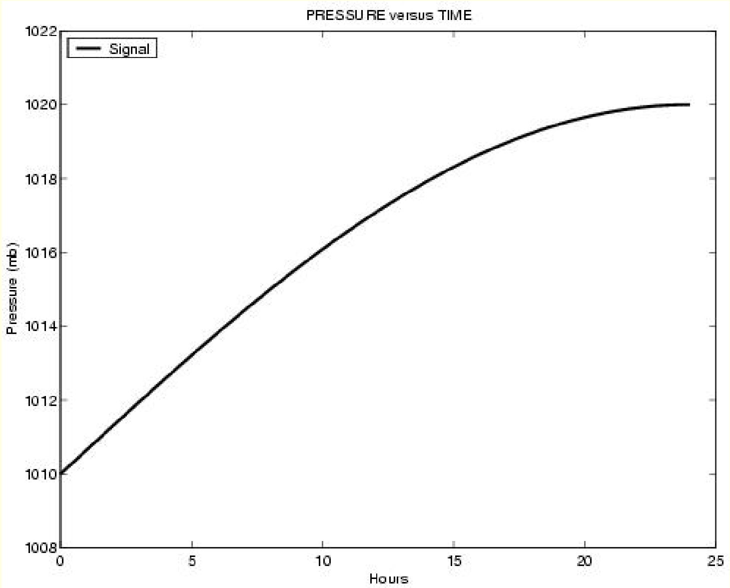
- **Rotational or vortical modes (Rossby-Haurwitz waves)**
- **Gravity-inertia wave oscillations**

For typical conditions of large scale atmospheric flow the two types of motion are clearly separated and interactions between them are weak.

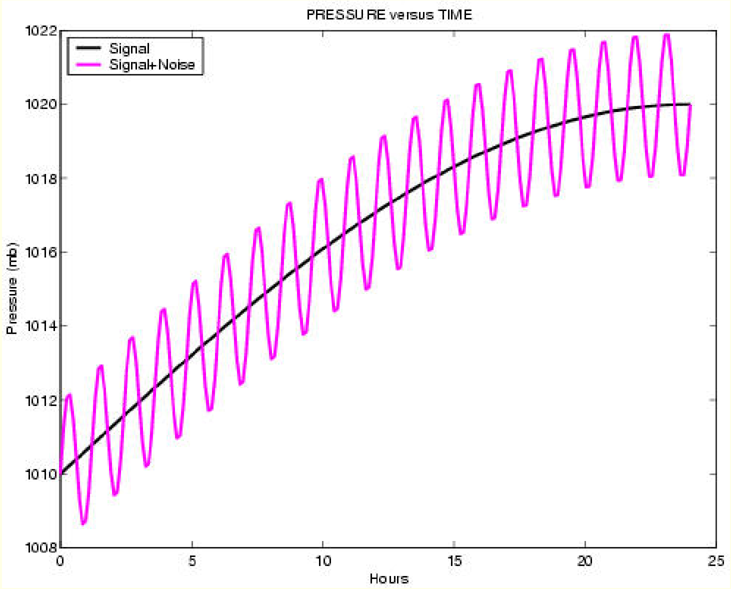
The high frequency gravity-inertia waves may be locally significant in the vicinity of steep orography, where there is strong thermal forcing or where very rapid changes are occurring . . .

. . . but overall they are of minor importance and may be regarded as undesirable noise.

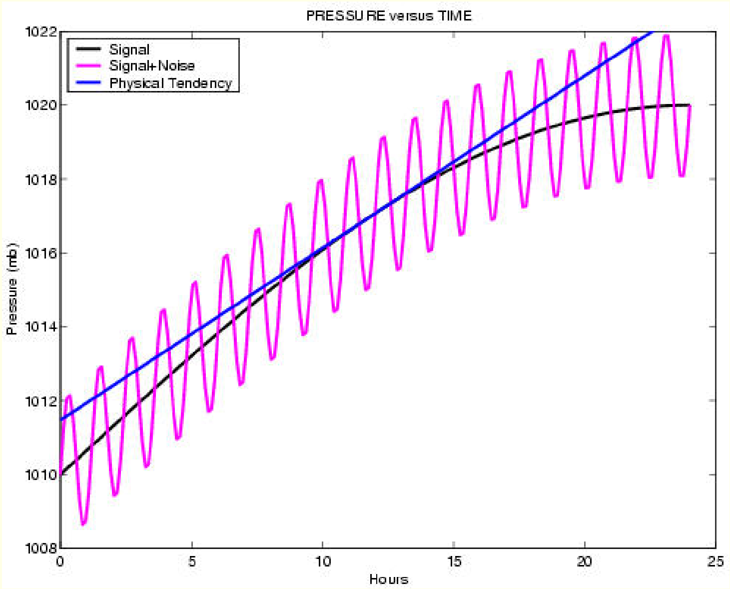
Smooth Evolution of Pressure



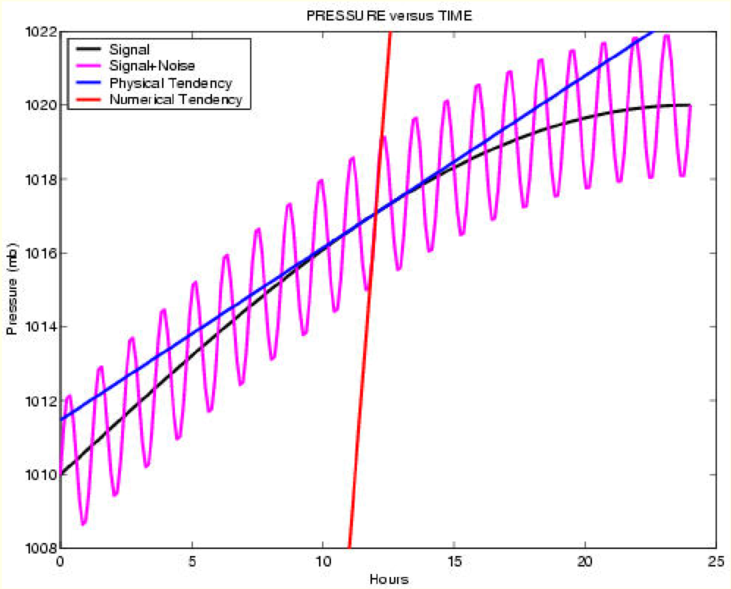
Noisy Evolution of Pressure



Tendency of a Smooth Signal



Tendency of a Noisy Signal



A Reading from

The Book of Limerick

Young Richardson wanted to know
How quickly the pressure would grow.
    But, what a surprise, 'cos
    The six-hourly rise was,
In Pascals, One Four Five Oh Oh!

A Freak Wave?

A Freak Wave?

The Forty-foot, Sandycove.

Simple Example

As a simple example of a system with multiple timescales, consider the water level at the Forty-foot on a stormy day.

The tidal variation, the slow changes between low and high water, has a period of about twelve hours.

Water level changes due to sea and swell have periods of less than a minute.

Clearly, the instantaneous value of water level cannot be used for tidal analysis.

If the vertical velocity observed at an instant is used to predict the long-term movement of the water, a nonsensical forecast is obtained.

The instantaneous rate-of-change is no a guide to the long-term evolution.

The same is true of the atmosphere!

The Problem of Initialization.

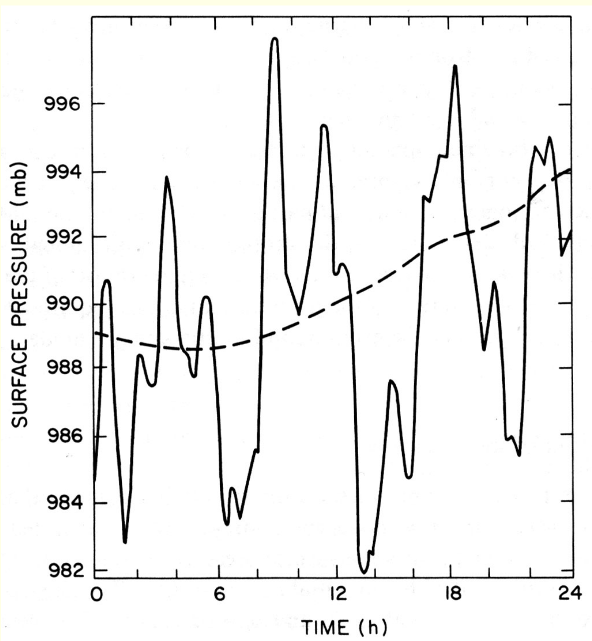
A subtle and delicate state of balance exists in the atmosphere between the wind and pressure fields.

This ensuring that the fast gravity waves have much smaller amplitude than the slow rotational part of the flow.

The pressure and wind fields in regions not too near the equator are close to a state of geostrophic balance and the flow is quasi-nondivergent.

The existence of this geostrophic balance is a perennial source of interest

It is a consequence of the forcing mechanisms and dominant modes of hydrodynamic instability and of the manner in which energy is dispersed and dissipated in the atmosphere.



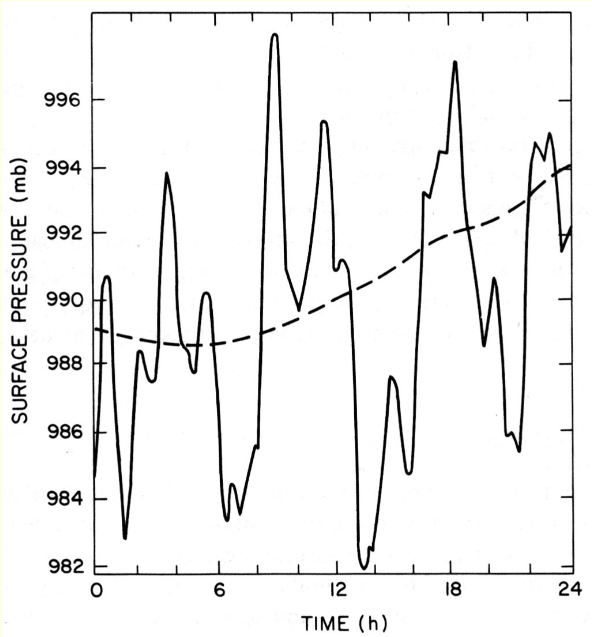
Evolution of surface pressure before and after NNMI.
(Williamson and Temperton, 1981)

When the primitive equations are used for numerical prediction the forecast may contain spurious large amplitude high frequency oscillations.

These result from anomalously large gravity-inertia wave components, arising from in the observations and analysis.

It was the presence of such imbalance in the initial fields which gave rise to the totally unrealistic pressure tendency of 145 hPa/6h obtained by Lewis Fry Richardson.

The problems associated with high frequency motions are overcome by the process known as *initialization*.



Evolution of surface pressure before and after NNMI.
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Need for Initialization

The principal aim of initialization is to define the initial fields so that the gravity inertia waves remain small throughout the forecast.

Specific Requirements for Initialization:

- Essential for satisfactory data assimilation
- Noisy forecasts have unrealistic vertical velocity
- Hopelessly inaccurate short-range rainfall patterns
- Spin-up of the humidity/water fields.
- Imbalance can lead to numerical instabilities.

Scale-analysis of the SWE

We introduce characteristic scales for the dependent variables, and examine the relative sizes of the terms in the equations.

- Length scale: $\mathsf{L} = 10^6\,\mathrm{m}$
- Velocity scale: $\mathsf{V} = 10\,\mathrm{m\,s^{-1}}$
- Advective time scale: $\mathsf{T} = \mathsf{L}/\mathsf{V} = 10^5\,\mathrm{s}$
- Pressure variation scale: P
- Scale height: $\mathsf{H} = 10^4\,\mathrm{m}$
- Acceleration of gravity: $g = 10\,\mathrm{m\,s^{-2}}$
- Coriolis parameter: $f = 10^{-4}\,\mathrm{s^{-1}}$
- Density: $\rho_0 = 1\,\mathrm{kg\,m^{-3}}$

For simplicity, we may assume $\rho_0 \equiv 1$, though this is not essential.

The linear rotational shallow water equations are:

$$\underbrace{\frac{\partial u}{\partial t}}_{\mathsf{V^2/L}} - \underbrace{fv}_{\mathsf{fV}} + \underbrace{\frac{1}{\rho_0}\frac{\partial p}{\partial x}}_{\mathsf{P/L}} = 0$$

$$\underbrace{\frac{\partial v}{\partial t}}_{\mathsf{V^2/L}} + \underbrace{fu}_{\mathsf{fV}} + \underbrace{\frac{1}{\rho_0}\frac{\partial p}{\partial y}}_{\mathsf{P/L}} = 0$$

$$\underbrace{\frac{1}{\rho_0}\frac{\partial p}{\partial t}}_{\mathsf{PV/L}} + \underbrace{gH\left[\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right]}_{\mathsf{gHV/L}} = 0$$

The scale of each term in the equations is indicated.

If there is approximate balance between the Coriolis and pressure gradient terms, we must have

$$\frac{\mathsf{P}}{\mathsf{L}} = f\mathsf{V} \qquad \text{or} \qquad \mathsf{P} = f\mathsf{LV} = 10^3\,\mathrm{Pa}$$

The ratio of the velocity tendencies to the Coriolis terms is the Rossby number

$$\mathsf{Ro} \equiv \frac{\mathsf{V}}{f\mathsf{L}} = \frac{10}{10^{-4}\cdot 10^{6}} = 10^{-1},$$

a small parameter.

The scales of the terms in the momentum equations are

$$\underbrace{\frac{\partial u}{\partial t}}_{10^{-4}} - \underbrace{fv}_{10^{-3}} + \underbrace{\frac{1}{\rho_0}\frac{\partial p}{\partial x}}_{10^{-3}} = 0$$

$$\underbrace{\frac{\partial v}{\partial t}}_{10^{-4}} + \underbrace{fu}_{10^{-3}} + \underbrace{\frac{1}{\rho_0}\frac{\partial p}{\partial y}}_{10^{-3}} = 0\,.$$

To the lowest order of approximation, the tendency terms are negligible; there is geostrophic balance between the Coriolis and pressure terms.

Scaling the Divergence

The vorticity is the same scale as each of its components:

$$\zeta = \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) \sim \frac{\mathsf{V}}{\mathsf{L}} = 10^{-5}\,\mathrm{s}^{-1}\,.$$

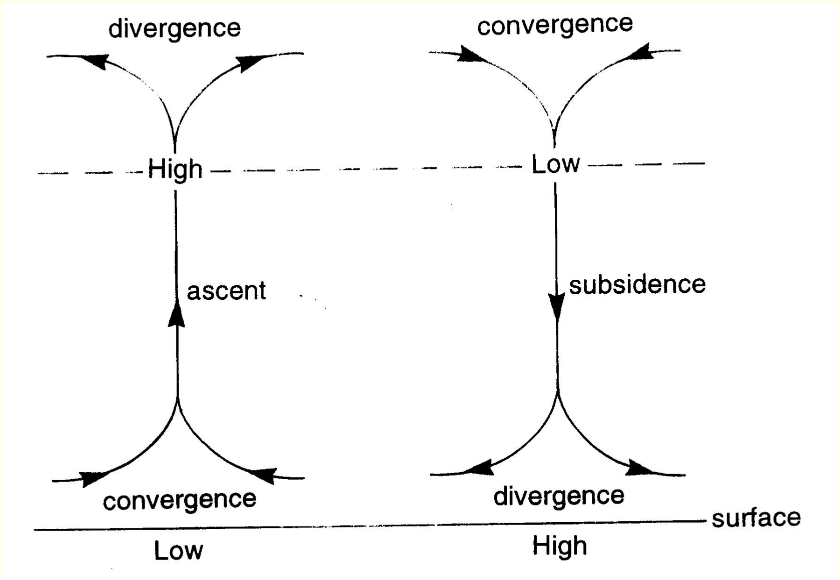
Due to the cancellation between the two terms in the divergence, one might expect it to scale an order of magnitude smaller than each of its terms:

$$\delta = \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) \sim \mathsf{Ro}\frac{\mathsf{V}}{\mathsf{L}} = 10^{-6}\,\mathrm{s}^{-1} \qquad (?)$$

If we assume this magnitude for the divergence, and take $g = 10\,\mathrm{m\,s^{-2}}$ and $\mathsf{H} = 10^4\,\mathrm{m}$ the continuity equation scales as

$$\underbrace{\frac{1}{\rho_0}\frac{\partial p}{\partial t}}_{10^{-2}} + \underbrace{g\mathsf{H}\left[\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right]}_{10^{-1}} = 0 \quad ???$$

Impossible: there is nothing to balance the second term.



Dines Compensation mechanism:
Cancellation of converegence and divergence.

We recall that the divergence term

$$g\int \delta\,dz \quad \approx \quad g\mathsf{H}\left[\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right].$$

arises through vertical integration.

There is a tendency for cancellation between convergence at low levels and divergence at higher levels and vice-versa.

This is called the Dines compensation mechanism.

(Illustrate the Dines compensation mechanism for a cyclone.)

Thus, we assume

$$\int \delta\,dz \sim \mathrm{Ro}\,\delta\mathsf{H}\,, \qquad \text{so that} \qquad g\int \delta\,dz \sim \mathrm{Ro}^2 g\mathsf{H}\frac{\mathsf{V}}{\mathsf{L}} = 10^{-2}\,.$$

The terms of the continuity equation are now in balance:

$$\underbrace{\frac{1}{\rho_0}\frac{\partial p}{\partial t}}_{10^{-2}} + \underbrace{g\mathsf{H}\left[\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right]}_{10^{-2}} = 0$$

So, $\partial p/\partial t \sim 10^{-2}\,\mathrm{Pa\,s^{-1}}$, which is about 1 hPa per 3 hours.

Break here

The Effect of Data Errors

Suppose there is a 10% error Δv in the v-component of the wind observation at a point.

The scales of the terms are as before:

$$\underbrace{\frac{\partial u}{\partial t}}_{10^{-4}} \quad - \quad \underbrace{f(v+\Delta v)}_{10^{-3}} \quad + \quad \underbrace{\frac{1}{\rho_0}\frac{\partial p}{\partial x}}_{10^{-3}} \quad = \quad 0$$

However, the error in the tendency is $\Delta(\partial u/\partial t) \sim f\Delta v \sim 10^{-4}$, comparable in size to the tendency itself.

The signal-to-noise ratio is 1.

The forecast may be qualitatively reasonable, but it will be quantitatively invalid.

A similar conclusion is reached for a 10% error in the pressure gradient.

However, if the spatial scale Δx of the pressure error is small (say, $\Delta x \sim \mathsf{L}/10$) the error in its gradient is correspondingly large:

$$\frac{\partial p}{\partial x} \sim \frac{\mathsf{P}}{\mathsf{L}}, \qquad \text{but} \qquad \Delta\frac{\partial p}{\partial x} \sim \frac{\Delta p}{\Delta x} \sim \frac{\mathsf{P}}{\mathsf{L}} \sim \frac{\partial p}{\partial x},$$

Thus, that the error in the wind tendency is now

$$\Delta\frac{\partial u}{\partial t} \sim \frac{1}{\rho_0}\frac{\partial p}{\partial x} \sim 10^{-3} \gg \frac{\partial u}{\partial t}.$$

The forecast will be qualitatively incorrect (i.e., useless!).

Now consider the continuity equation.

The pressure tendency has scale

$$\frac{\partial p}{\partial t} \sim 10^{-2}\,\mathrm{Pa\,s^{-1}} \approx 1\,\mathrm{hPa} \text{ in 3 hours}.$$

If there is a 10% error in the wind, the resulting error in divergence is $\Delta\delta \sim \Delta v/\mathsf{L} \sim 10^{-6}$.

The error is larger than the divergence itself!

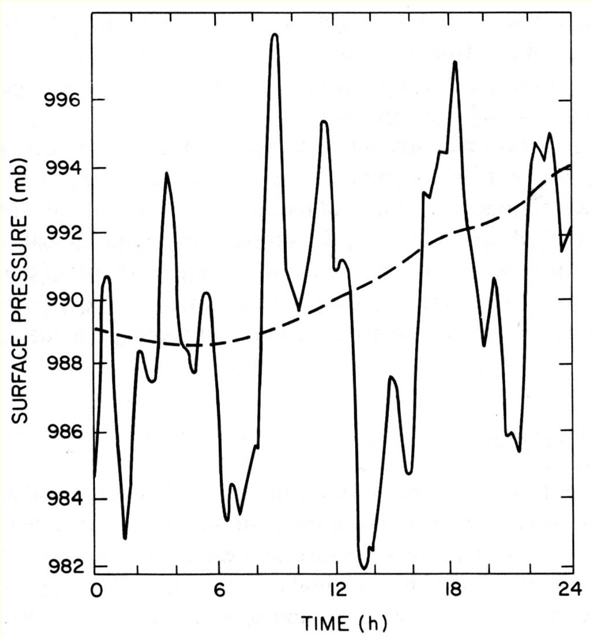
Thus, the pressure tendency is unrealistic.

Worse still, if the wind error is of small spatial scale, the divergence error is correspondingly greater:

$$\Delta\delta \sim \Delta\frac{\partial v}{\partial x} \sim \frac{\Delta v}{\Delta x} \sim \frac{\mathsf{V}}{\mathsf{L}} \sim 10^{-5} \sim 10^2\delta.$$

This implies a pressure tendency *two orders of magnitude larger* than the correct value.

Instead of the value $\partial p/\partial t \sim 1$ hPa in 3 hours we get a change of order 100 hPa in 3 hours (like Richardson's result).



Evolution of surface pressure before and after NNMI.

(Williamson and Temperton, 1981)

Early Initialization Methods

We will describe, in outline, a number of methods which have been used to overcome the problems of noise in numerical integrations.

- ***1. The Filtered Equations***
- ***2. Static Initialization***
- ***3. Dynamic Initialization***
- ***4. Variational Initialization***

1. The Filtered Equations

The first computer forecast was made in 1950 by Charney, Fjørtoft and Von Neumann. They used the equation

$$\frac{d}{dt}(\zeta + f) = 0$$

which has no gravity wave components.

Systems like this are called Filtered Equations.

The basic filtered system is the quasi-geostrophic equations.

The *barotropic, quasi-geostrophic potential vorticity equation* (the QGPV Equation) is

$$\frac{\partial}{\partial t}\left(\nabla^2\psi - F\psi\right) + J(\psi, \nabla^2\psi) + \beta\frac{\partial\psi}{\partial x} = 0\,.$$

This is a *single equation* for a *single variable*, ψ.

The simplifying assumptions have the effect of eliminating high-frequency gravity wave solutions, so that only the slow Rossby wave solutions remain.

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A more accurate filtering of the primitive equations leads to the balance equations.

This system is more complicated to solve than the quasi-geostrophic system, and has not been widely used.

However one diagnostic component has been used for initialization. We discuss this presently.

2. Static Initialization

Hinkelmann (1951) investigated the problem of noise in numerical integrations of the primitive equations.

He concluded that if the initial winds were geostrophic:

$$\mathbf{V} = \frac{1}{f}\mathbf{k} \times \nabla\Phi$$

high frequency oscillations would occur but would remain small in amplitude.

If we express the wind in terms of a stream function $\mathbf{V} = \mathbf{k} \times \nabla\psi$, we can write

$$f\nabla\psi = \nabla\Phi$$

The divergence of this is the linear balance equation:

$$\nabla\cdot f\nabla\psi = \nabla^2\Phi$$

This can easily be solved for ψ if Φ is given,
or for Φ if ψ is given.

Forecasts made with the primitive equations were soon shown to be clearly superior to those using the quasi-geostrophic system . . .

. . . however, the use of geostrophic initial winds has a huge disadvantage:

Observations of the wind field are completely ignored.

Charney (1955) proposed that a better estimate of the wind could be obtained from the nonlinear balance equation.

This equation is a diagnostic relationship between the pressure and wind fields.

$$\nabla^2\Phi - \nabla\cdot f\nabla\psi + 2\left[\left(\frac{\partial^2\psi}{\partial x\partial y}\right)^2 - \frac{\partial^2\psi}{\partial x^2}\frac{\partial^2\psi}{\partial y^2}\right] = 0$$

This is a Poisson equation for Φ when ψ is given. However, it is nonlinear in ψ and much harder to solve for ψ when Φ is given. This is the usual case.

When ψ is obtained from the nonlinear balance equation, a non-divergent wind is constructed: $\mathbf{V} = \mathbf{k} \times \nabla\psi$.

Phillips (1960) argued that, in addition to getting ψ from the nonlinear balance equation, a divergent component of the wind should be included.

He proposed that a further improvement would result if the divergence of the initial field were set equal to that implied by quasi-geostrophic theory.

This can be sone by solving the QG omega equation.

Each of these steps represented some progress, but the noise problem still remained essentially unsolved.

3. Dynamic Initialization

Another approach, called dynamic initialization, uses the forecast model itself to define the initial fields.

The dissipative processes in the model can damp out high frequency noise as the forecast procedes.

We integrate the model first forward and then backward in time, keeping the dissipation active all the time.

We repeat this forward-backward cycle many times until we finally obtain fields, valid at the initial time, from which the high frequency components have been damped out.

The forecast starting from these fields is noise-free . . .

. . . however, the procedure is expensive in computer time.

Moreover, it damps the meteorologically significant motions as well as the gravity waves so it is no longer popular.

Digital filtering initialization (DFI) is essentially a refinement of dynamic initialization.

Because it used a highly selective filtering technique, is is computationally more efficient than the older method.

If time permits, we will return to DFI later.

4. Variational Initialization

An elegant initialization method based on the calculus of variations was introduced by Sasaki (1958).

Although the method was not widely used, the variational method is now at the centre of modern data assimilation practice.

Recall that, in variational assimilation, we minimize a cost function, J, which is normally a sum of two terms

$$J = J_B + J_O$$

Here, J_B is the distance between the analysis and the background field

$$J_B = \tfrac{1}{2}(\mathbf{x} - \mathbf{x}_b)^T \mathbf{B}^{-1} (\mathbf{x} - \mathbf{x}_b)$$

and J_O is the distance to the observations

$$J_O = \tfrac{1}{2}(\mathbf{y}_o - H(\mathbf{x}))^T \mathbf{R}^{-1} (\mathbf{y}_o - H(\mathbf{x}_b))$$

The variational problem can be be modified to include as the minimization of an integral representing the deviation of the resulting fields from balance.

We add a constraint which requires the analysis to be close to geostrophic balance:

$$J_C = \tfrac{1}{2}\alpha \sum_{ij} \left[\left(fu + \frac{\partial \Phi}{\partial y} \right)^2_{ij} + \left(fv - \frac{\partial \Phi}{\partial x} \right)^2_{ij} \right]$$

This term J_C is large if the analysis is far from geostrophic balance. It vanishes for perfect geostrophic balance.

The weight α is chosen to give the constraint an appropriate weight. This is known as a weak constraint. It is not satisfied exactly, only approximately.

The constrained variational assimilation finds the minimum of the cost function

$$J = J_B + J_O + J_C$$

End of §4.1