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We saw this clearly in the analysis of the six-point Crank-Nicholson scheme.

For this reason, implicit schemes are useful for those modes that are very fast but of little meteorological importance.

We will next consider schemes in which the gravity wave terms are implicit while the remaining terms are explicit.

These semi-implicit schemes are of crucial importance in modern operational NWP.

Introduction to the SI Method

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Formally, we separate the terms into two groups.
Thus, the equation

$$\frac{\partial u}{\partial t} = F(u) = F_1(u) + F_2(u)$$

is discretised by something like

$$\left(\frac{U^{n+1} - U^{n-1}}{2\Delta t}\right) = F_1(U^n) + F_2\left(\frac{U^{n-1} + U^{n+1}}{2}\right)$$

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Schemes of this sort are pivotal in modern NWP models, due to their excellent stability properties.

Semi-implicit Scheme for SWE

We consider the Shallow Water Equations:

$$\left.\begin{aligned}
&\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\underline{\frac{\partial \Phi}{\partial x}} + fv \\
&\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\underline{\frac{\partial \Phi}{\partial y}} - fu \\
&\frac{\partial \Phi}{\partial t} + u\frac{\partial \Phi}{\partial x} + v\frac{\partial \Phi}{\partial y} = -\underline{\bar{\Phi}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)} - \left(\Phi - \bar{\Phi}\right)\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)
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The phase speed of the inertia-gravity wave is given by

$$c = \bar{U} \pm \sqrt{\bar{\Phi} + \frac{f^2}{k^2}} \approx \bar{U} \pm 300\,\mathbf{m/s}$$

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The Courant number $\mu = c\Delta t/\Delta x$ is dominated by the speed of external inertia-gravity waves, $c = c_{IGW}$.

An *explicit scheme* thus requires a time step an order of magnitude smaller than that required for advection.

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The shallow water equations may be written

$$\left.\begin{aligned}\frac{\partial u}{\partial t} &= -\underline{\frac{\partial \Phi}{\partial x}} + R_u \\ \frac{\partial v}{\partial t} &= -\underline{\frac{\partial \Phi}{\partial y}} + R_v \\ \frac{\partial \Phi}{\partial t} &= -\underline{\bar{\Phi}\delta} + R_\Phi\end{aligned}\right\}$$

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We discretise implicitly the terms that result in gravity waves, and explicitly the remaining terms.

$$\left(\frac{u^{n+1}-u^{n-1}}{2\Delta t}\right) = -\left(\frac{\Phi_x^{n+1}+\Phi_x^{n-1}}{2}\right) + R_u^n$$

$$\left(\frac{v^{n+1}-v^{n-1}}{2\Delta t}\right) = -\left(\frac{\Phi_y^{n+1}+\Phi_y^{n-1}}{2}\right) + R_v^n$$

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Now solve for u^{n+1} and v^{n+1}:

$$u^{n+1} = -\Delta t\,\Phi_x^{n+1} + S_u$$
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$$\begin{aligned} u^{n+1} &= -\Delta t\, \Phi_x^{n+1} + S_u \\ v^{n+1} &= -\Delta t\, \Phi_y^{n+1} + S_v \end{aligned}$$

Then, the divergence at time $(n+1)\Delta t$ is

$$\begin{aligned} \delta^{n+1} &= (u_x^{n+1} + v_y^{n+1}) \\ &= -\Delta t(\Phi_{xx}^{n+1} + \Phi_{yy}^{n+1}) + R_\delta \\ &= -\Delta t \nabla^2 \Phi^{n+1} + R_\delta \end{aligned}$$

We substitute this in the continuity equation to get

$$\left(\frac{\Phi^{n+1} - \Phi^{n-1}}{2\Delta t}\right) = \frac{1}{2}\bar{\Phi}\Delta t\,\nabla^2\Phi^{n+1} + S_\Phi$$

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This can be written as a Helmholtz Equation for Φ^{n+1}:

$$\left[\nabla^2 - \left(\frac{1}{\bar{\Phi}\Delta t^2}\right)\right]\Phi^{n+1} = F_\Phi$$

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$$\begin{aligned} u^{n+1} &= -\Delta t\,\Phi_x^{n+1} + S_u \\ v^{n+1} &= -\Delta t\,\Phi_y^{n+1} + S_v \end{aligned}$$

All the variables are now known at time $(n+1)\Delta t$ **and the next time-step can be computed.**

Spatial Discretisation

We introduce compact notation for finite differences and averages:

$$\left.\begin{array}{ll} \delta_x f = \dfrac{f_{j+1/2} - f_{j-1/2}}{\Delta x} & \textbf{Space difference} \\ \overline{f}^x = \frac{1}{2}(f_{j+1/2} + f_{j-1/2}) & \textbf{Space average} \end{array}\right\}$$

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With this notation, assuming uniform resolution, we have

$$\left.\begin{array}{l} \delta_{2x} f = \delta_x \overline{f}^x = \dfrac{f_{i+1} - f_{i-1}}{2\Delta x} \\[2ex] \overline{f}^{2x} = \frac{1}{2}(f_{i+1} + f_{i-1}) \end{array}\right\}$$

and, for time,

$$\overline{f}^{2t} = \frac{1}{2}(f^{n+1} + f^{n-1})$$

Using this compact finite difference notation, we can write the leapfrog semi-implicit SWE as

$$\left.\begin{aligned}
\delta_{2t}u + u\,\delta_{2x}u + v\,\delta_{2y}u &= -\delta_{2x}\overline{\Phi}^{2t} + fv \\
\delta_{2t}v + u\,\delta_{2x}v + v\,\delta_{2y}v &= -\delta_{2y}\overline{\Phi}^{2t} - fu \\
\delta_{2t}\Phi + u\,\delta_{2x}\Phi + v\,\delta_{2y}\Phi &= -\bar{\Phi}\overline{(\delta_{2x}u + \delta_{2y}v)}^{2t} - \left(\Phi - \bar{\Phi}\right)(\delta_{2x}u + \delta_{2y}v)
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We can rearrange the FDEs as

$$\left.\begin{aligned}
\frac{u^{n+1} - u^{n-1}}{2\Delta t} &= -\delta_{2x}\left(\frac{\Phi^{n+1} + \Phi^{n-1}}{2}\right) + R_u \\
\frac{v^{n+1} - v^{n-1}}{2\Delta t} &= -\delta_{2y}\left(\frac{\Phi^{n+1} + \Phi^{n-1}}{2}\right) + R_v \\
\frac{\Phi^{n+1} - \Phi^{n-1}}{2\Delta t} &= -\bar{\Phi}\left[\delta_{2x}\left(\frac{u^{n+1} + u^{n-1}}{2}\right) + \delta_{2y}\left(\frac{v^{n+1} + v^{n-1}}{2}\right)\right] + R_\Phi
\end{aligned}\right\}$$

The terms R_u, R_v and R_Φ are the remaining terms, evaluated at the central time $n\Delta t$:

$$\begin{aligned} R_u &= -u\delta_{2x}u - v\delta_{2y}u + fv \\ R_v &= -u\delta_{2x}v - v\delta_{2y}v - fu \\ R_\Phi &= -u\delta_{2x}\Phi - v\delta_{2y}\Phi - \left(\Phi - \bar{\Phi}\right)(\delta_{2x}u + \delta_{2y}v) \end{aligned}$$

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We can solve the first two equations for u^{n+1} and v^{n+1}:

$$\left.\begin{aligned} u^{n+1} &= -\Delta t\delta_{2x}\Phi^{n+1} + S_u \\ v^{n+1} &= -\Delta t\delta_{2y}\Phi^{n+1} + S_v \end{aligned}\right\}$$

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where S_u and S_v can be computed from known quantities.

Eliminating u^{n+1} and v^{n+1} from the third equation, we obtain an elliptic equation for Φ^{n+1}:

$$\left(\delta_{2x}^2 + \delta_{2y}^2 - \frac{1}{\bar{\Phi}\Delta t^2}\right)\Phi^{n+1} = F_{i,j}^n$$

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The right-hand side of this Helmholtz equation depends only on values at $t = n\Delta t$ or $(n-1)\Delta t$, so that it is known.

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Solving this elliptic equation provides Φ^{n+1}.

Once this is known, it can be plugged back into the first two equations. Thus (u^{n+1}, v^{n+1}) can be obtained.

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The elliptic operator on the left-hand side of the Helmholtz equation is a finite difference equivalent to $(\nabla^2 - \lambda^2)$,

$$\left(\delta_{2x}^2 + \delta_{2y}^2 - \frac{1}{\bar{\Phi}\Delta t^2}\right)\Phi = \frac{\Phi_{i+2,j} + \Phi_{i-2,j} + \Phi_{i,j+2} + \Phi_{i,j-2} - \left(4 + \frac{1}{\mu^2}\right)\Phi_{i,j}}{4\Delta^2}$$

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We assume for simplicity that $\Delta x = \Delta y = \Delta$.

Here, $\mu^2 = \bar\Phi\Delta t^2/\Delta^2$ is the Courant number (squared) for gravity waves.

Since $\mu^2 = \bar{\Phi}\Delta t^2/\Delta^2 >> 1$, the semi-implicit scheme distorts the gravity wave solution, slowing the gravity wave down until they satisfy the CFL criterion.

This is an acceptable distortion since we are interested in the slower "weather-like" processes.

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In a nut-shell:

- **The fast gravity-waves are slowed down by the semi-implicit scheme. Thus, their behaviour is distorted.**
- **The slower, meteorologically significant Rossby-Haurwitz waves are represented accurately, as long as the Courant Number for these waves is not too large.**

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It treats the terms generating sound waves (anelastic terms, i.e., three-dimensional divergence), and the terms generating gravity waves (pressure gradient and horizontal divergence) semi-implicitly,

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This model, called the Mesoscale Compressible Community (MCC) model, is a *universal model*, designed to tackle accurately atmospheric problems from the planetary scale through mesoscale, convective and smaller.

Google for MCC model

See documentation of LM model

See documentation of ECMWF model

Conclusion of §3.2.5