M.Sc. in Meteorology UCD

Numerical Weather Prediction

Prof Peter Lynch

Meteorology & Climate Cehtre
School of Mathematical Sciences
University College Dublin

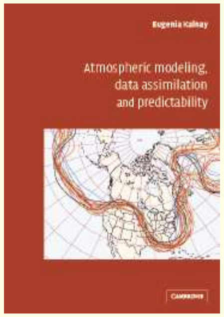
Second Semester, 2005–2006.

Text for the Course

The lectures will be based closely on the text

Atmospheric Modeling, Data Assimilation and Predictability
by
Eugenia Kalnay

published by Cambridge University Press (2002).



Introduction (Kalnay, Ch. 1)

- Numerical weather prediction provides the basic guidance for operational weather forecasting beyond the first few hours.
- Numerical forecasts are generated by running computer models of the atmosphere that can simulate the evolution of the atmosphere over the next few days.
- NWP is an *initial-value problem.* The initial conditions are provided by analysis of weather observations.
- The skill of NWP forecasts depends on accuracy of both *the computer model* and *the initial conditions.*

- Operational computer weather forecasts have been performed since about 1955.
- Since 1973, they have been global in extent.
- Over the years, the quality of the models and methods for using atmospheric observations has improved continuously, resulting in major forecast improvements.
- NCEP has the longest available record of the skill of numerical weather prediction.
- The "$S1$" score (Teweles and Wobus, 1954) measures the relative error in the horizontal gradient of the height of the 500 hPa pressure surface.
- A score of 70% or more corresponds to a useless forecast.
- A score of 20% or less corresponds to an essentially perfect forecast.

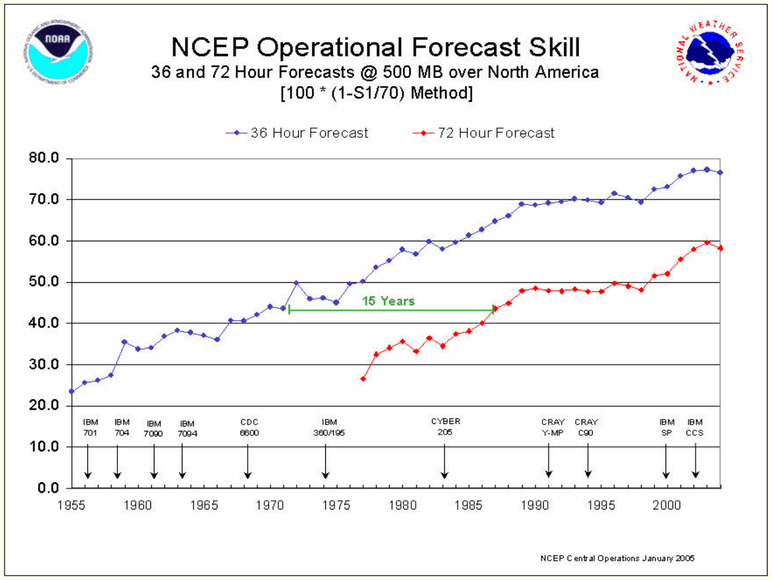
Definition of S1 Score

S1 Skill Score is the sum of the absolute horizontal gradients of the differences between the forecast and analysis values, normalised by the sum of the maximum absolute gradients of a forecast or analysis value.

It measures the model ability to forecast correctly the horizontal gradients of scalar variables such as MSLP. The full range of the S1 score is from 0 to 200, with a low score being better than a high score.

A perfect score of 0 occurs when the forecast and analysis gradients are the same, even though the values may be different.

Typically a score of about 70 represents the limit of usefulness and 20 is excellent.



Skill of the 36 hour (1955–2004) and 72 hour (1977–2004) 500 hPa forecasts produced at NCEP.

The accuracy of prediction is closely linked to the available computer power; the introduction of new machines is indicated in the figure.

Current 36-h 500-hPa forecasts over North America are close to what was considered essentially "perfect" 40 years ago.

The sea level pressure forecasts contain smaller-scale atmospheric structures, such as fronts, mesoscale convective systems that dominate summer precipitation, etc., and are still difficult to forecast in detail.

The 72-h forecasts of today are as accurate as the 36-h forecasts were 10–20 years ago.

Similarly, 5-day forecasts, which had no useful skill 15 years ago, are now moderately skilful.

The improvement in skill of numerical weather prediction over the last 50 years is due to four factors:

- Increased power of supercomputers, allowing much finer numerical resolution and fewer model approximations;
- Improved representation of small-scale physical processes (clouds, precipitation, turbulent transfers of heat, moisture, momentum, and radiation) within the models;
- increased availability of data, especially satellite and aircraft data over the oceans and the Southern Hemisphere.
- More accurate methods of data assimilation, which result in improved initial conditions for the models;

Major NWP research takes place in large national and international operational weather centres and in universities.

- European Center for Medium Range Weather Forecasts (ECMWF)
- National Centers for Environmental Prediction (NCEP)
- National Oceanic and Atmospheric Administration (NOAA)
- National Center for Atmospheric Research (NCAR)
- National Meteorological Services (NMSs):
  - UK, France, Germany, Scandinavian and other European countries
  - Canada, Japan, Australia, and others.
- International Research Projects
  - HIRLAM, COSMO, ALADIN, HARMONIE, etc.

In meteorology there has been a long tradition of sharing both data and research improvements.

All countries have benefited from this cooperation.

In this lecture, we give an overview of the major components and milestones in numerical forecasting. They will be discussed in detail in the following lectures.

Vilhelm Bjerknes (1862–1951)

Bjerknes' 1904 Manifesto

Objective:

To establish a science of meteorology

Acid test:

To predict future states of the atmosphere.

Necessary and sufficient conditions for the solution of the forecasting problem:

1. A knowledge of the initial state of the atmosphere
2. A knowledge of the physical laws which determine the evolution of the atmosphere.

Step (1) is Diagnostic. Step (2) is Prognostic.

Scientific Weather Forecasting in a Nut-Shell

- The atmosphere is a physical system
- Its behaviour is governed by the laws of physics
- These laws are expressed as mathematical equations
- Using observations, we determine the atmospheric state at a given initial time: “Today’s Weather”
- Using the equations, we calculate how this state changes over time: “Tomorrow’s Weather”

BUT:

- The equations are very complicated (non-linear) and a powerful computer is required to do the calculations
- The accuracy decreases as the range increases; there is an inherent limit of predictibility.

Lewis Fry Richardson, 1881–1953.


L. F. Richardson, 1931

During WWI, Richardson computed by hand the pressure change at a single point.

It took him two years !

His ‘forecast’ was a catastrophic failure:

$$\Delta p = 145 \text{ hPa in 6 hours}$$

His method was unimpeachable.

So, *what went wrong?*

Max Margules (1856–1920)

In 1904, Margules published a paper in the *Festschrift* marking the sixtieth birthday of his teacher Ludwig Boltzmann:

Über die Beziehung zwischen Barometerschwankungen und Kontinuitätsgleichung.

“On the Relationship between Barometric Variations and the Continuity Equation.”

- *Margules examined pressure changes predicted using the continuity equation.*
- *He found that, to obtain a realistic pressure tendency, the winds must be known to an unrealistic precision.*
- *He showed that synoptic forecasting by this means was doomed to failure.*

According to Fortak (2001), Margules was convinced that weather forecasting was:

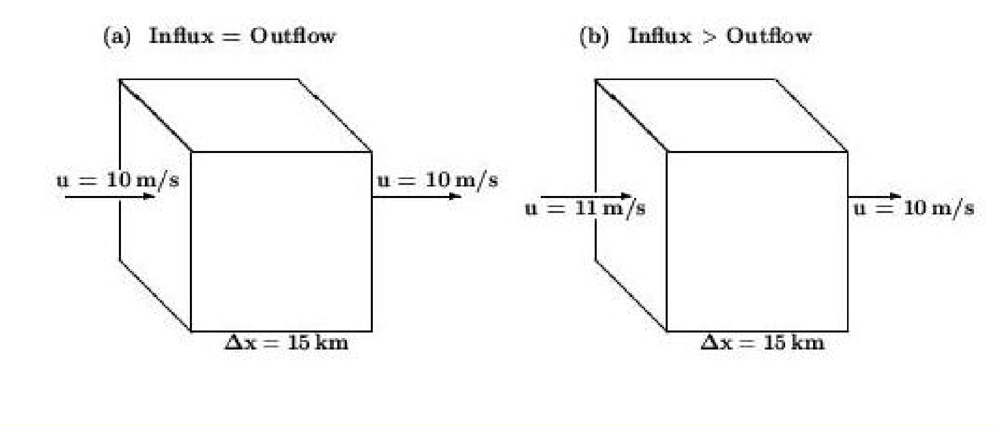
“immoral and damaging to the character of a meteorologist.”

Tendency *via* Continuity Equation

The environs of Dublin:

- A square of side ∼15km.
- Analogous to a cell of a finite difference model of the atmosphere.

A Box of Air over Dublin



Influx equals Outflow:
Pressure remains unchanged.

Influx exceeds Outflow:
Pressure will rise.

Pressure Tendency

Assume a westerly wind over Dublin

$$u > 0 \,, \qquad v = 0 \,.$$

Assume also that the surface pressure is initially 1000 hPa.

Using Conservation of Mass, a simple *back-of-the-envelope* calculation yields the following amazing result:

- If the speed on the western side *exceeds* that on the east by *just* 1 m/s, the pressure tendency is about 7 Pa/s.
- If this influx continues, the pressure will *double* in about 4 hours.

Conclusion:

We must apply the Continuity Equation with great care.

Felix Maria Exner (1876–1930)

A first attempt at calculating synoptic changes using physical principles was made by Felix Exner, working in Vienna.

Exner followed a radically different line from Bjerknes.

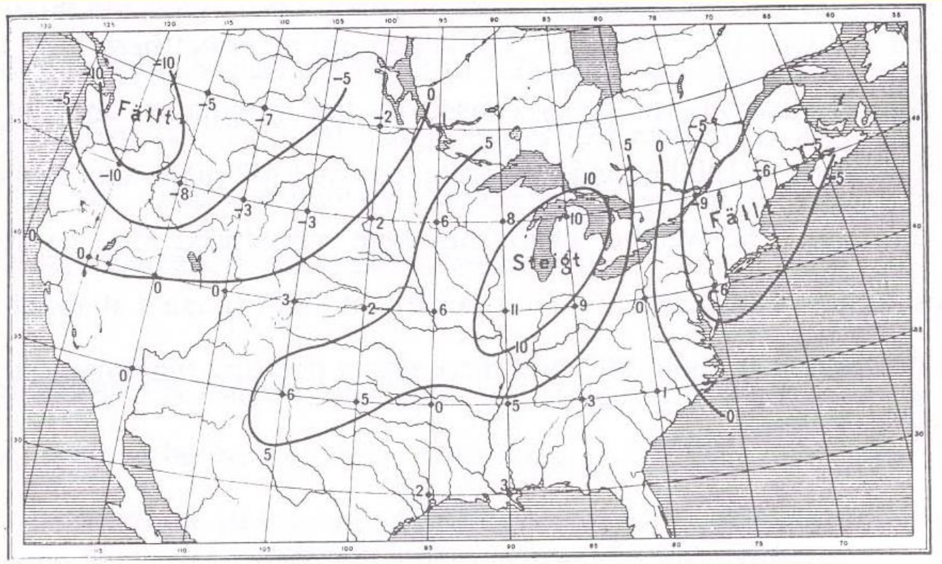
He did *not* make direct use of the continuity equation.

His method was based on a system reduced to the essentials.

Exner's Method

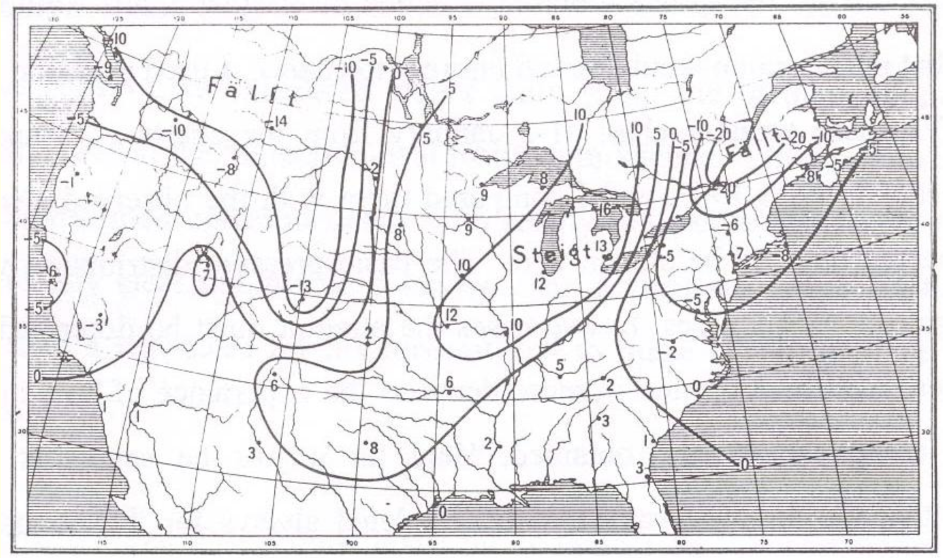
- ***Exner assumed geostrophic balance and thermal forcing constant in time.***
- ***He deduced a mean zonal wind from temperature observations.***
- ***He derived a prediction equation for advection of the pressure pattern with constant speed, modified by heating.***
- ***His method yielded a realistic forecast in the case illustrated in his paper.***

Exner's Forecast



Calculated Pressure Change
between 8pm and 12pm on 3 January, 1895
Hundredths of an inch. [*Steigt*=rises; Fällt=falls].

Verification



Observed Pressure Change
between 8pm and 12pm on 3 January, 1895
Hundredths of an inch. [*Steigt*=rises; Fällt=falls].

Richardson's Reaction

Exner's work deserves attention as a first attempt at systematic, scientific weather forecasting.

The only reference by Richardson to the method was a single sentence in his book *Weather Prediction by Numerical Process* (p. 43):

"F. M. Exner has published a prognostic method based on the source of air supply."

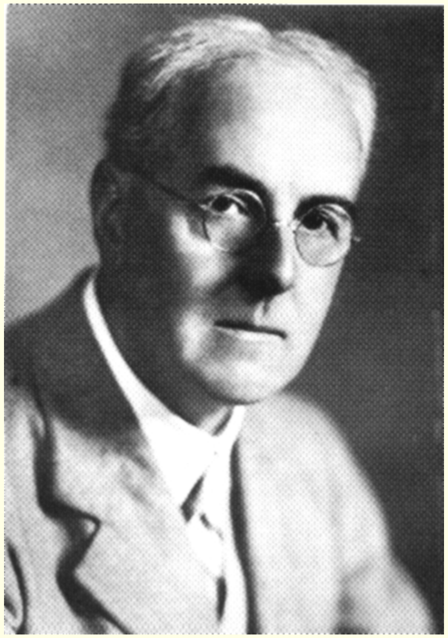
It would appear from this that Richardson was not particularly impressed by it!

⋆ ⋆ ⋆

It is noteworthy that

- Exner's forecast was unspectacular but reasonable.
- Richardson's forecast was spectacularly unreasonable.

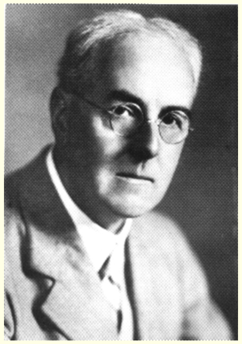
Lewis Fry Richardson, 1881–1953.



Bjerknes proposed graphical methods for the solution of the forecasting problem

Richardson was bolder — or perhaps more foolhardy — than Bjerknes.

He attempted a bulldozer approach, calculating changes from the full PDEs.



- Born, 11 October, 1881, Newcastle-upon-Tyne
- Family background: well-known quaker family
- 1900–1904: Kings College, Cambridge
- 1913–1916: Met. Office. Superintendent, Eskdalemuir Observatory
- Resigned from Met Office in May, 1916. Joined Friends' Ambulance Unit.
- 1919: Re-employed by Met. Office
- 1920: M.O. linked to the Air Ministry. LFR Resigned, on grounds of conscience
- 1922: *Weather Prediction by Numerical Process*
- 1926: Break with Meteorology. Worked on Psychometric Studies. Later on Mathematical causes of Warfare
- 1940: Resigned to pursue "peace studies"
- Died, September, 1953.

Richardson contributed to Meteorology, Numerical Analysis, Fractals, Psychology and Conflict Resolution.

Eskdalemuir Observatory in 1911

(where Richardson's dream began to take shape)

Chapters of Richardson's Book

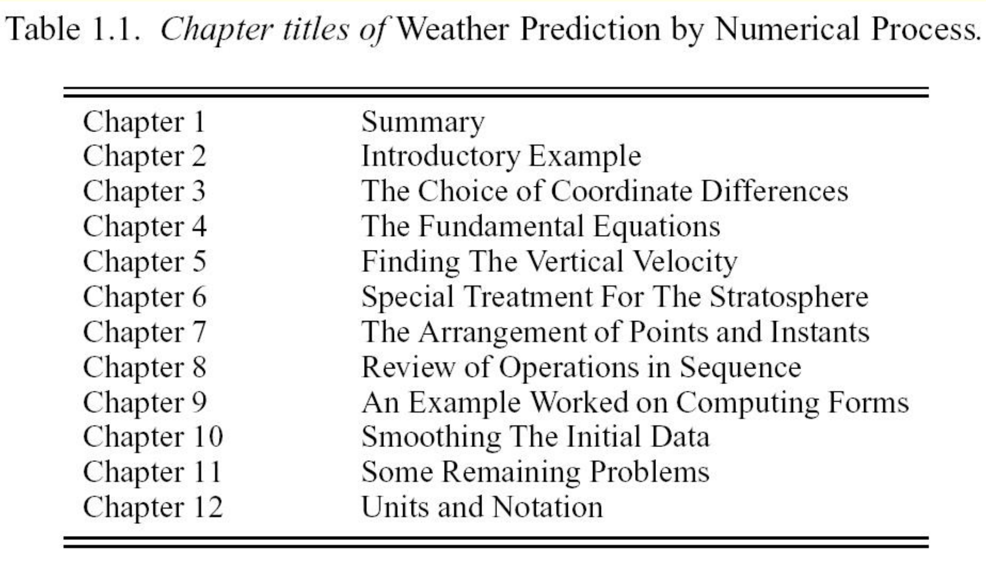


Table 1.1. *Chapter titles of* Weather Prediction by Numerical Process.

Chapter 1	Summary
Chapter 2	Introductory Example
Chapter 3	The Choice of Coordinate Differences
Chapter 4	The Fundamental Equations
Chapter 5	Finding The Vertical Velocity
Chapter 6	Special Treatment For The Stratosphere
Chapter 7	The Arrangement of Points and Instants
Chapter 8	Review of Operations in Sequence
Chapter 9	An Example Worked on Computing Forms
Chapter 10	Smoothing The Initial Data
Chapter 11	Some Remaining Problems
Chapter 12	Units and Notation

The Equations of the Atmosphere

GAS LAW (Boyle's Law and Charles' Law.)
Relates the pressure, temperature and density
CONTINUITY EQUATION
Conservation of mass; air neither created nor distroyed
WATER CONTINUITY EQUATION
Conservation of water (liquid, solid and gas)
EQUATIONS OF MOTION: Navier-Stokes Equations
Describe how the change of velocity is determined by the pressure gradient, Coriolis force and friction
THERMODYNAMIC EQUATION
Determines changes of temperature due to heating or cooling, compression or rarifaction, etc.

Seven equations; seven variables (u, v, w, ρ, p, T, q).

The Primitive Equations

$$\frac{du}{dt} - \left(f + \frac{u\tan\phi}{a}\right)v + \frac{1}{\rho}\frac{\partial p}{\partial x} + F_x = 0$$

$$\frac{dv}{dt} + \left(f + \frac{u\tan\phi}{a}\right)u + \frac{1}{\rho}\frac{\partial p}{\partial y} + F_y = 0$$

$$p = R\rho T$$

$$\frac{\partial p}{\partial y} + g\rho = 0$$

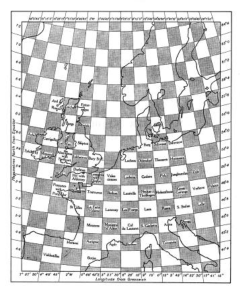
$$\frac{dT}{dt} + (\gamma - 1)T\nabla\cdot\mathbf{V} = \frac{Q}{c_p}$$

$$\frac{\partial\rho}{\partial t} + \nabla\cdot\rho\mathbf{V} = 0$$

$$\frac{\partial\rho_w}{\partial t} + \nabla\cdot\rho_w\mathbf{V} = [\mathrm{Sources} - \mathrm{Sinks}]$$

Seven equations; seven variables $(u, v, w, p, T, \rho, \rho_w)$.

The Finite Difference Scheme



The globe is divided into cells, like the checkers of a chess-board.
Spatial derivatives are replaced by finite differences:

$$\frac{df}{dx} \to \frac{f(x+\Delta x) - f(x-\Delta x)}{2\Delta x}.$$

Similarly for time derivatives:

$$\frac{dQ}{dt} \to \frac{Q^{n+1} - Q^{n-1}}{2\Delta t} = F^n$$

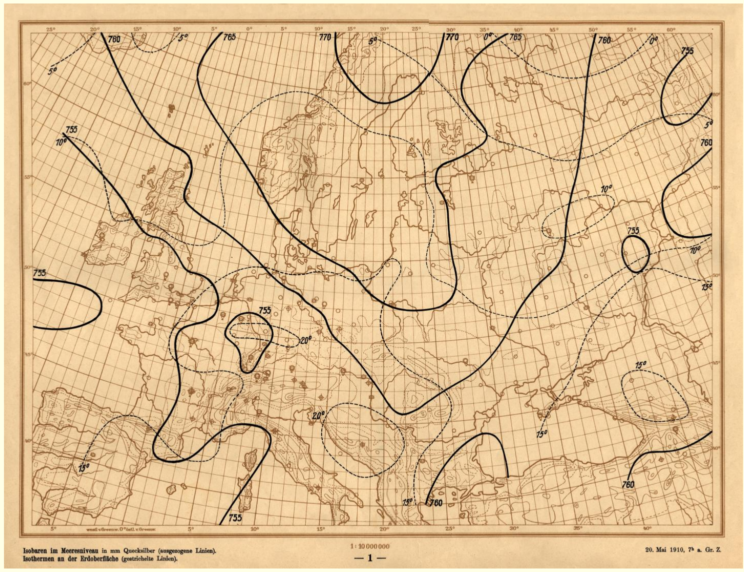
This can immediately be solved for Q^{n+1}:

$$Q^{n+1} = Q^{n-1} + 2\Delta t F^n.$$

By repeating the calculations for many time steps, we can get a forecast of any length.

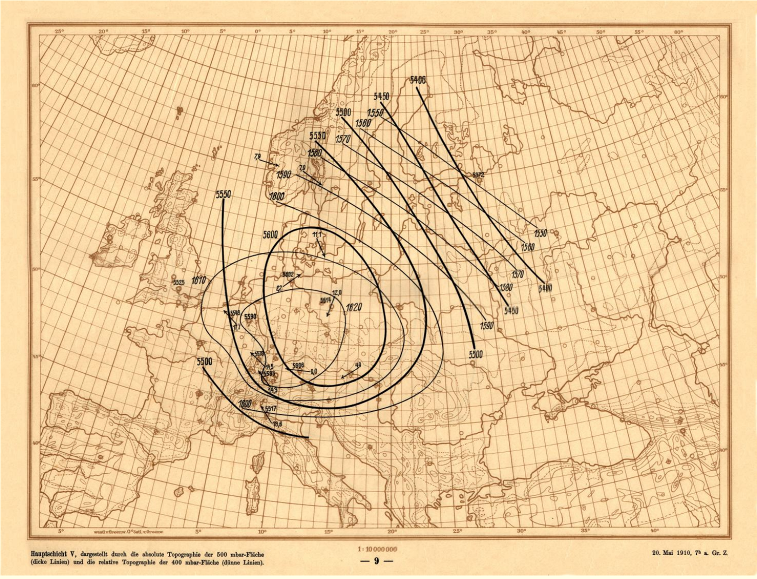
Richardson calculated only the initial rates of change.

The Leipzig Charts for 0700 UTC, May 20, 1910



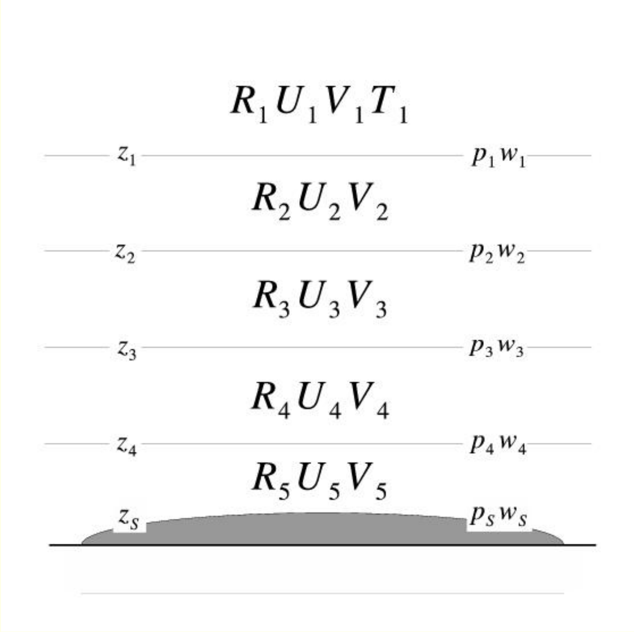
Bjerknes' sea level pressure analysis.

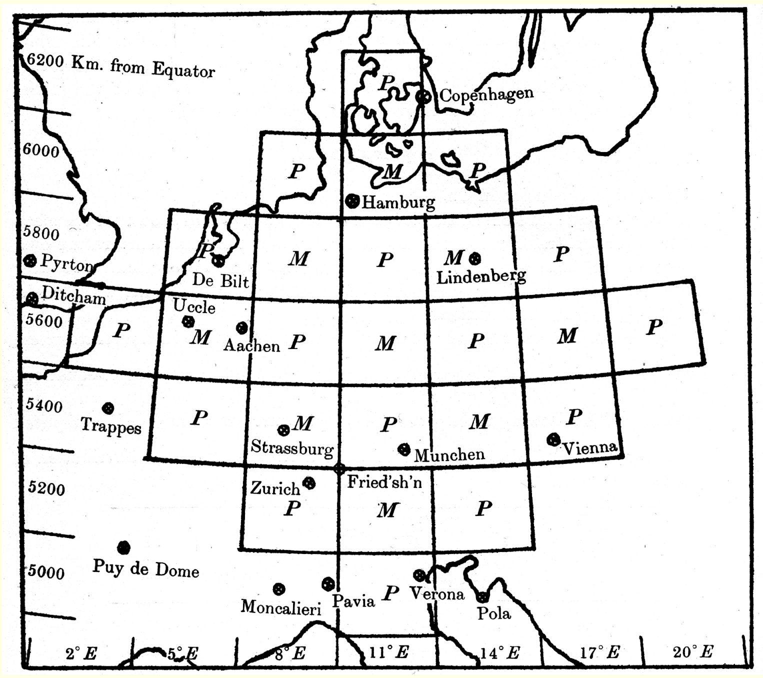
The Leipzig Charts for 0700 UTC, May 20, 1910



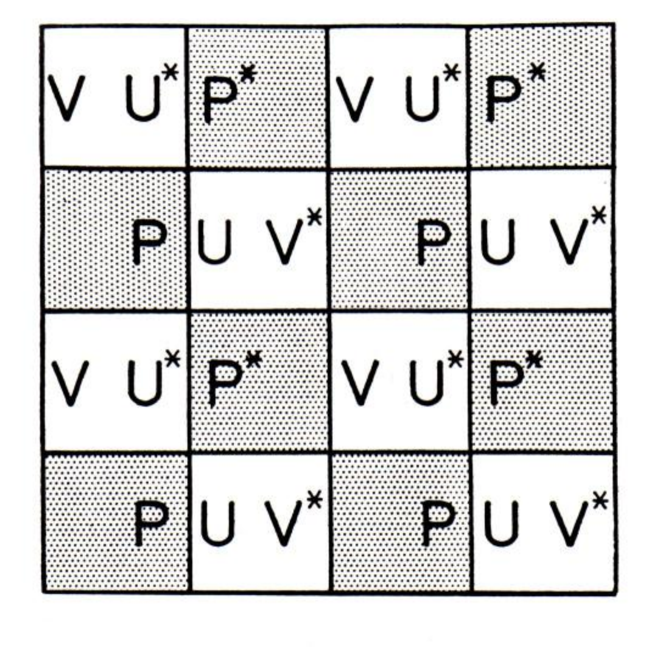
Bjerknes' 500 hPa height analysis.

Richardson's vertical stratification



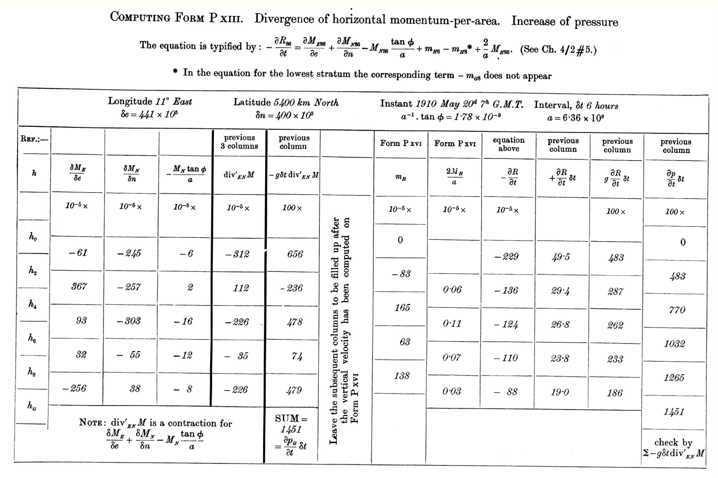


Grid used by Richardson for his forecast.



Richardson Grid (also called an Arakawa E-grid)

Richardson's *Spread-sheet*



Computing Form P XIII. Divergence of horizontal momentum-per-area. Increase of pressure

The equation is typified by: $-\frac{\partial R_{86}}{\partial t} = \frac{\partial M_{E86}}{\partial e} + \frac{\partial M_{N86}}{\partial n} - M_{N86}\frac{\tan\phi}{a} + m_{H6} - m_{H8}{}^{*} + \frac{2}{a}M_{H86}$. (See Ch. 4/2#5.)

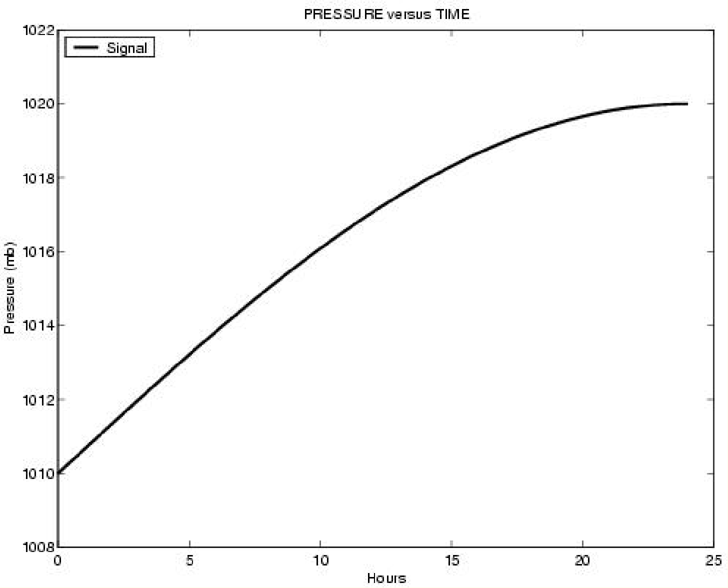
* In the equation for the lowest stratum the corresponding term $-m_{G8}$ does not appear

Longitude *11° East* $\delta e = 441 \times 10^5$ | Latitude *5400 km North* $\delta n = 400 \times 10^5$ | Instant *1910 May 20^d 7^h G.M.T.* $a^{-1}.\tan\phi = 1{\cdot}78 \times 10^{-9}$ | Interval, δt *6 hours* $a = 6{\cdot}36 \times 10^8$

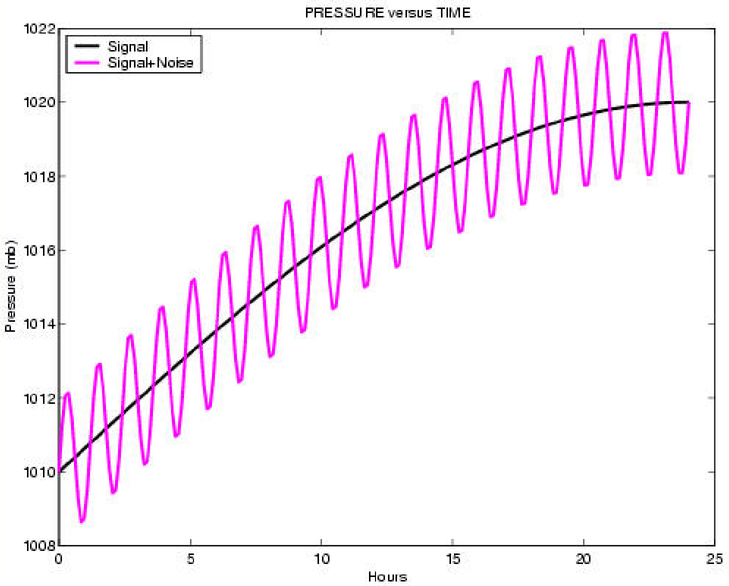
Ref.:—				previous 3 columns	previous column		Form P XVI	Form P XVI	equation above	previous column	previous column	previous column
h	$\frac{\delta M_E}{\delta e}$	$\frac{\delta M_N}{\delta n}$	$-\frac{M_N \tan\phi}{a}$	$\text{div}'_{EN}M$	$-g\delta t\,\text{div}'_{EN}M$		m_H	$\frac{2M_H}{a}$	$-\frac{\partial R}{\partial t}$	$+\frac{\partial R}{\partial t}\delta t$	$g\frac{\partial R}{\partial t}\delta t$	$\frac{\partial p}{\partial t}\delta t$
	$10^{-5}\times$	$10^{-5}\times$	$10^{-5}\times$	$10^{-5}\times$	$100\times$		$10^{-5}\times$	$10^{-6}\times$	$10^{-5}\times$		$100\times$	$100\times$
h_0							0					0
	−61	−245	−6	−312	656	Leave the subsequent columns to be filled up after the vertical velocity has been computed on Form P XVI			−229	49·5	483	
h_2							−83					483
	367	−257	2	112	−236			0·06	−136	29·4	287	
h_4							165					770
	93	−303	−16	−226	478			0·11	−124	26·8	262	
h_6							63					1032
	32	−55	−12	−35	74			0·07	−110	23·8	233	
h_8							138					1265
	−256	38	−8	−226	479			0·03	−88	19·0	186	
h_G												1451
	Note: $\text{div}'_{EN}M$ is a contraction for $\frac{\delta M_E}{\delta e} + \frac{\delta M_N}{\delta n} - M_N\frac{\tan\phi}{a}$				SUM = 1451 $= \frac{\partial p_G}{\partial t}\delta t$							check by $\Sigma - g\delta t\,\text{div}'_{EN}M$

Richardson's Computing Form $\mathrm{P}_{\mathrm{XIII}}$
The figure in the bottom right corner is the forecast change in surface pressure: 145 mb in six hours!

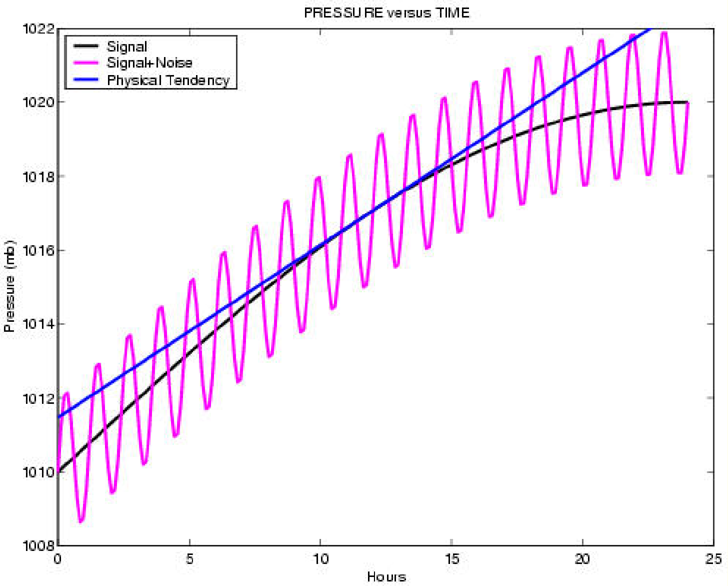
Smooth Evolution of Pressure



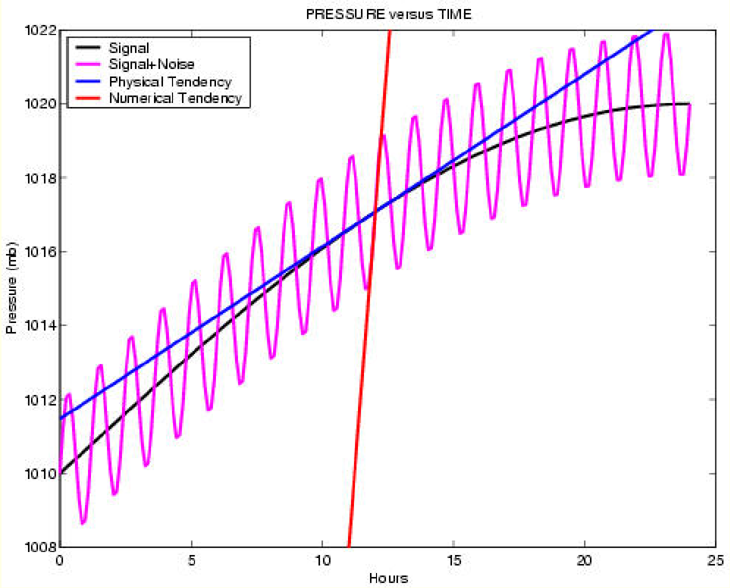
Noisy Evolution of Pressure

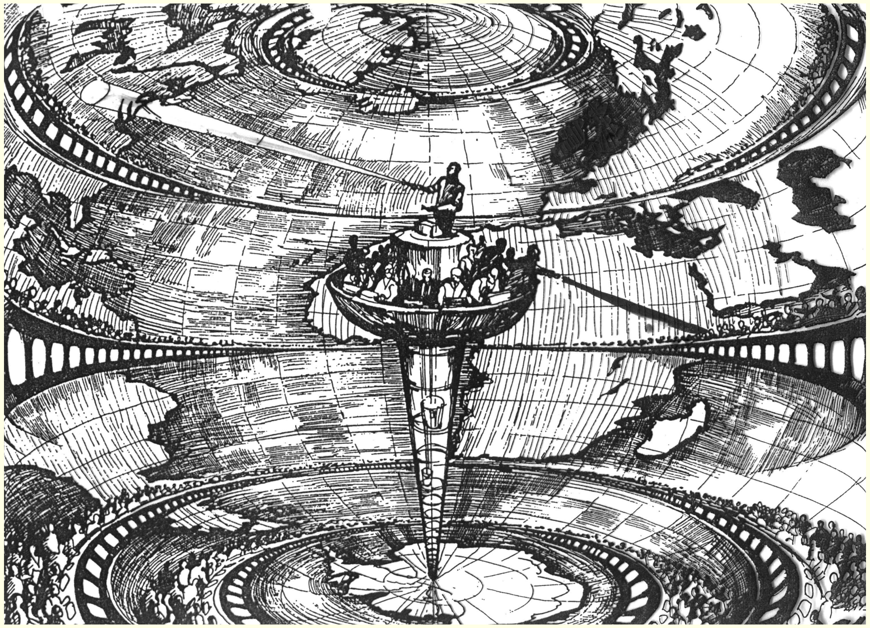


Tendency of a Smooth Signal



Tendency of a Noisy Signal





Richardson's Forecast Factory (A. Lannerback).

Dagens Nyheter, Stockholm. Reproduced from L. Bengtsson, *ECMWF*, 1984

64,000 Computers: The first Massively Parallel Processor

What went wrong?

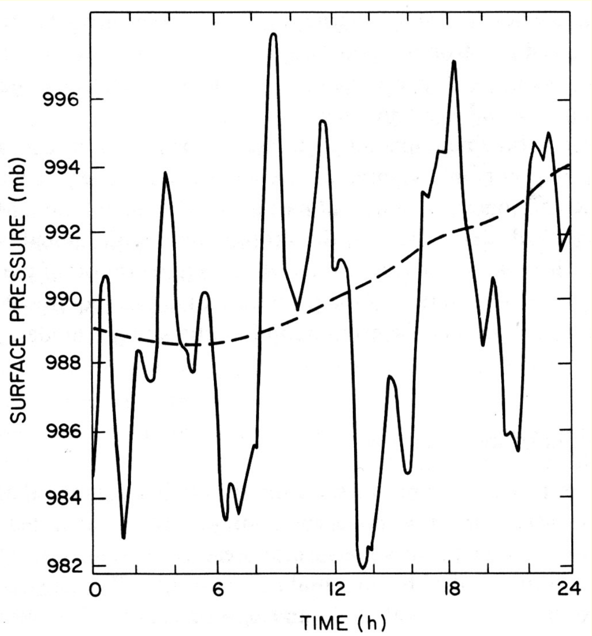
- Richardson extrapolated *instantaneous* pressure change, assuming it to remain constant over a long time period.
- This ignores the propensity of the atmosphere to respond rapidly to changes.
- An increase of pressure causes an immediate pressure gradient which acts to resist further change.
- The resulting gravity wave oscillations act in such a way as to restore balance.
- They result in pressure changes which may be large but which oscillate rapidly in time.

(Margules, 1893, was the first comprehensive study of gravity wave dynamics)

The ineluctable conclusion is that . . .

the instantaneous rate of change is not a reliable indicator of the long-term variation in pressure.

- ***To obtain an accurate prediction, the time step has to be short enough to allow the adjustment to take place.***
- ***Gravity-wave oscillations need not spoil the forecast.***
- ***They may be regarded as noise superimposed on the synoptic evolution.***
- ***They may also be effectively removed by an adjustment of the data, known as initialization.***



Evolution of surface pressure before and after NNMI.
(Williamson and Temperton, 1981)

Richardson on Smoothing

"The scheme of numerical forecasting has been developed so far that it is reasonable to expect that when the smoothing . . . has been arranged, it may give forecasts agreeing with the actual smoothed weather."

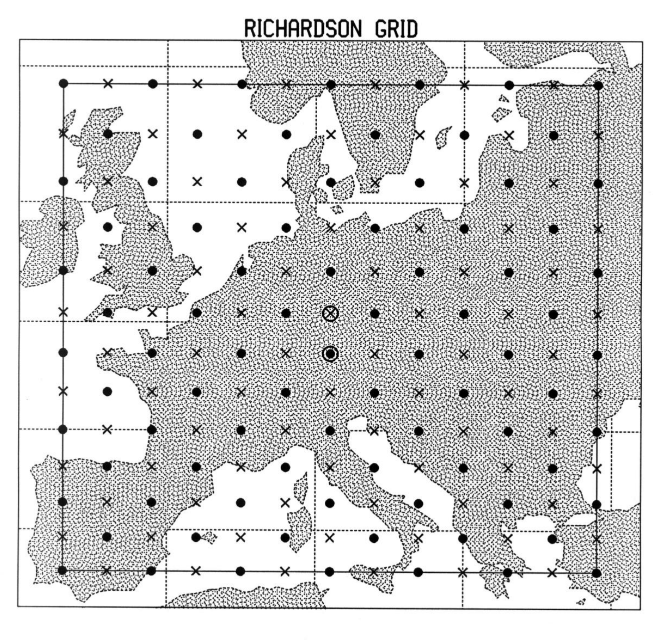
Richardson's Smoothing Methods

Richardson devoted a short chapter of his book to smoothing.

He outlined five smoothing methods:

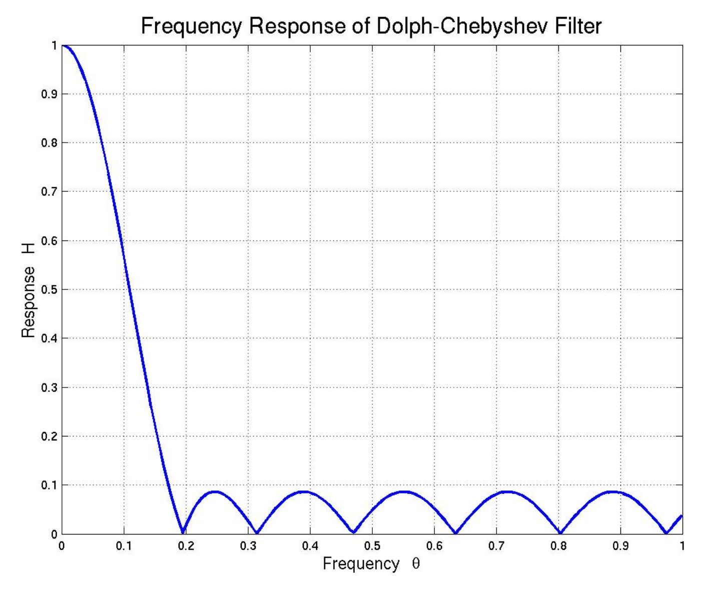
- **A. Space Means.**
- **B. Time Means.**
- **C. Potential Function.**
- **D. Stream Function.**
- **E. Smoothing during the Forecast.**

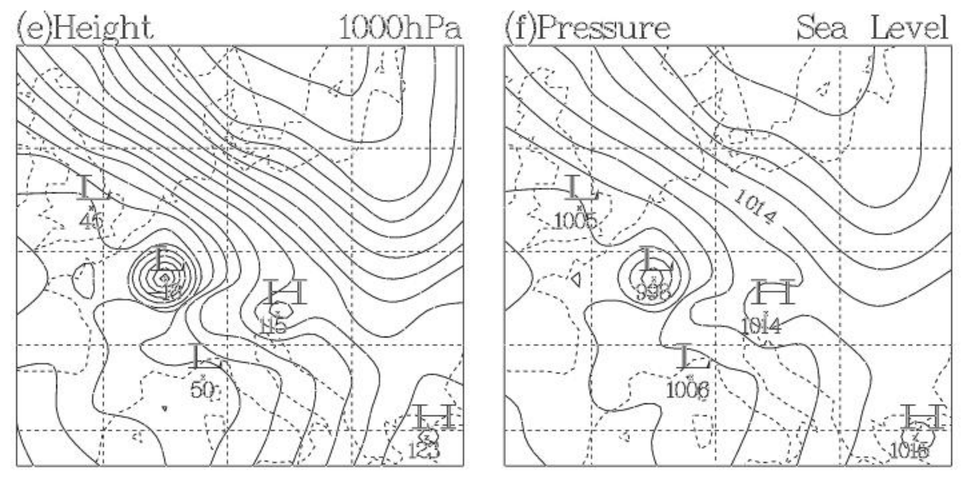
Richardson's *Method B* is a close cousin of Digital Filtering Initialization, which has some current popularity.



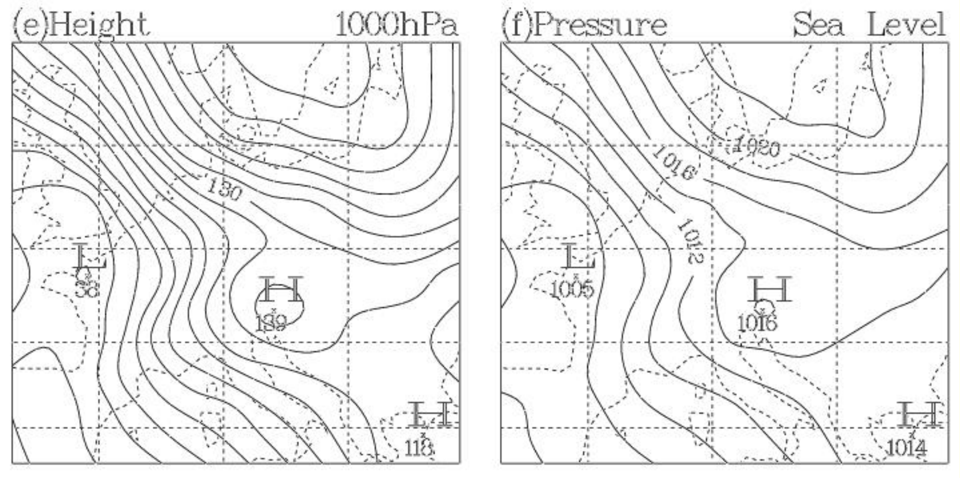
Grid for extending Richardson's forecast

Digital Filter Response



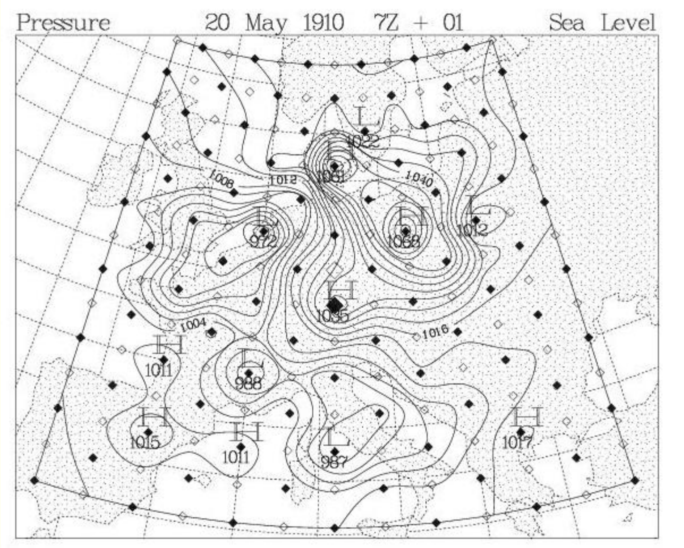


1000 hPa height and surface pressure before initialization



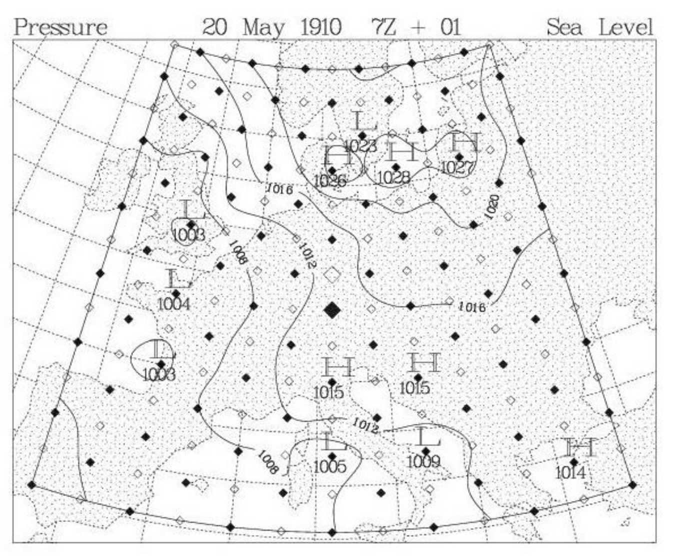
1000 hPa height and surface pressure after initialization

Forecast without Filtering



Short-range forecast of sea-level pressure, from *uninitialized data*. The contour interval is 4 hPa. Single forward time step of size $\Delta t = 3600\,\mathrm{s}$.

Forecast with Filtering



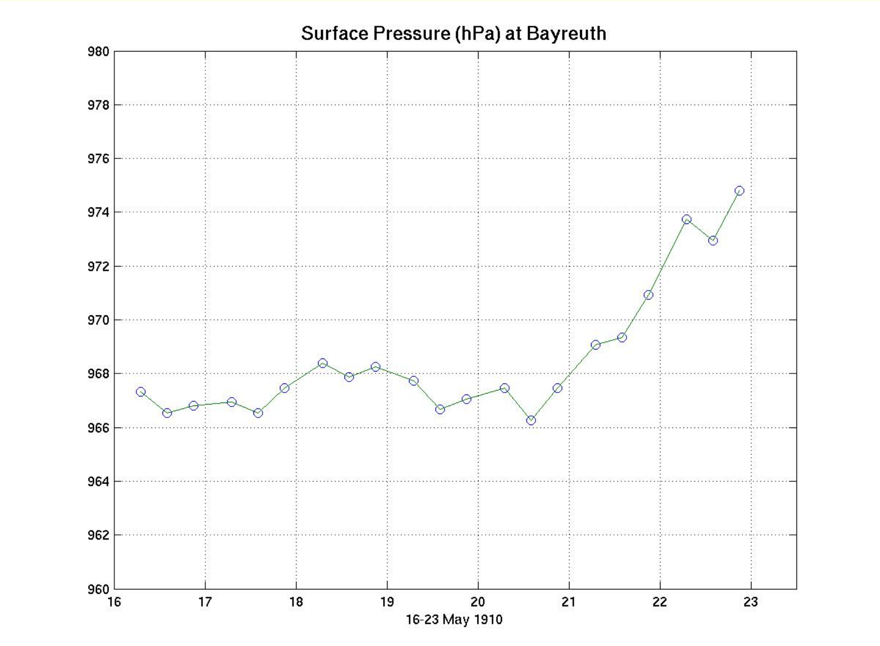
Short-range forecast of sea-level pressure, from *filtered data*. The contour interval is 4 hPa. Single forward time step of size $\Delta t = 3600\,\mathrm{s}$.

Table 1: Analysis of the pressure changes (hPa) across each layer, and the pressure change at the base of each layer.

		Total Divergence	Change in Pressure Thickness	Change in Base Pressure
Layer	Level	$\nabla \cdot \mathbf{U} + [\rho w]$	$\frac{\partial [p]}{\partial t}\Delta t$	$\frac{\partial p}{\partial t}\Delta t$
I		-0.0229	$+48.3$	
	1			$+48.3$
II		-0.0136	$+28.7$	
	2			$+77.1$
III		-0.0124	$+26.2$	
	3			$+103.2$
IV		-0.0110	$+23.3$	
	4			$+126.5$
V		-0.0088	$+18.6$	
	S			**$+145.1$**

Table 2: Analysis of the pressure changes (hPa) for the forecast from data after Digital Filtering.

		Total Divergence	Change in Pressure Thickness	Change in Base Pressure
Layer	Level	$\nabla \cdot \mathbf{U} + [\rho w]$	$\frac{\partial [p]}{\partial t}\Delta t$	$\frac{\partial p}{\partial t}\Delta t$
I		$+0.0001$	-0.2	
	1			-0.2
II		$+0.0011$	**-2.4**	
	2			-2.6
III		$+0.0002$	-0.4	
	3			-3.0
IV		$+0.0000$	-0.1	
	4			-3.1
V		-0.0010	**$+2.1$**	
	S			**-0.9**



Surface pressure at Bayreuth for a week in May, 1910

Crucial Advances, 1920–1950

- ***Dynamic Meteorology***
  - **Rossby Waves**
  - **Quasi-geostrophic Theory**
  - **Baroclinic Instability**
- ***Numerical Analysis***
  - **CFL Criterion**
- ***Atmopsheric Observations***
  - **Radiosonde**
- ***Electronic Computing***
  - **ENIAC**

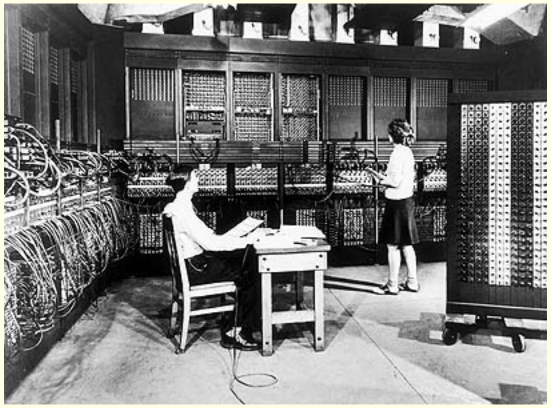
Electronic Computer Project

Von Neumann's idea (1946):

Weather forecasting was, *par excellence*, **a scientific problem suitable for solution using a large computer.**

Objective:
To predict the weather by simulating the dynamics of the atmosphere using a digital electronic computer.

The ENIAC



The ENIAC was the first multi-purpose programmable electronic digital computer.
It had:

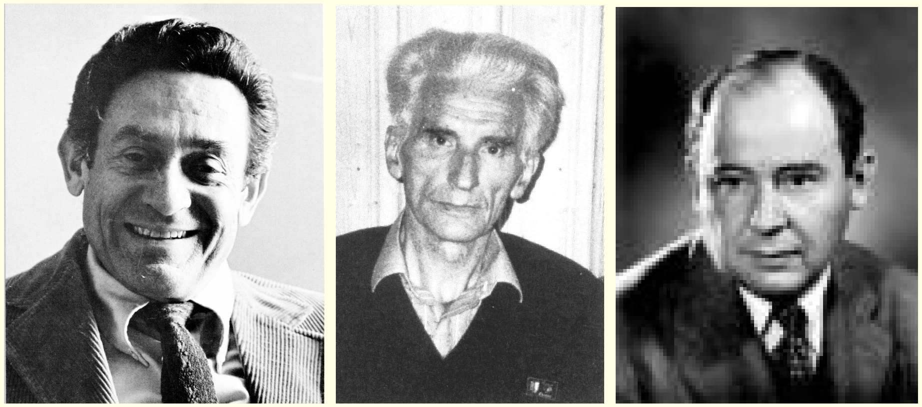
- **18,000 vacuum tubes**
- **70,000 resistors**
- **10,000 capacitors**
- **6,000 switches**
- **Power: 140 kWatts**

Evolution of the Meteorology Project:

- **Plan A: Integrate the Primitive Equations**
  Problems similar to Richardson's would arise
- **Plan B: Integrate baroclinic Q-G System**
  Too computationally demanding
- **Plan C: Solve barotropic vorticity equation**
  Very satisfactory initial results

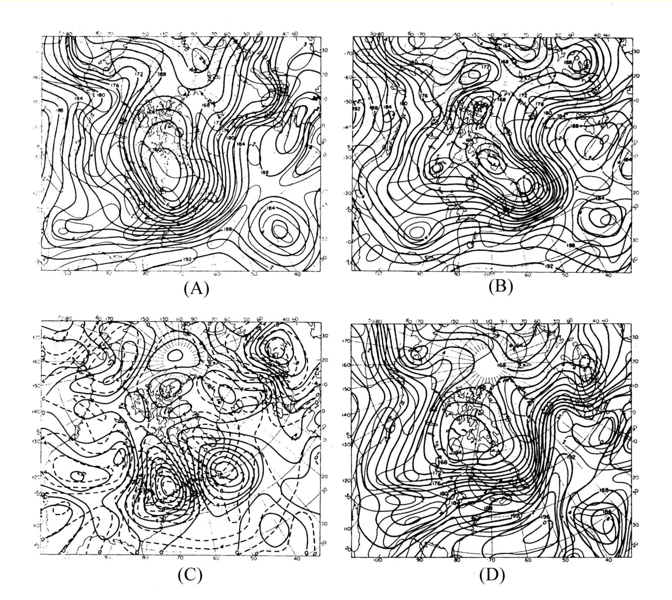
$$\frac{d}{dt}(\zeta + f) = 0$$

Charney, Fjørtoft, von Neumann



Charney, J.G., R. Fjørtoft and J. von Neumann, 1950:
Numerical integration of the barotropic vorticity equation. *Tellus*, 2, 237–254.

ENIAC: First Computer Forecast



Richardson's reaction

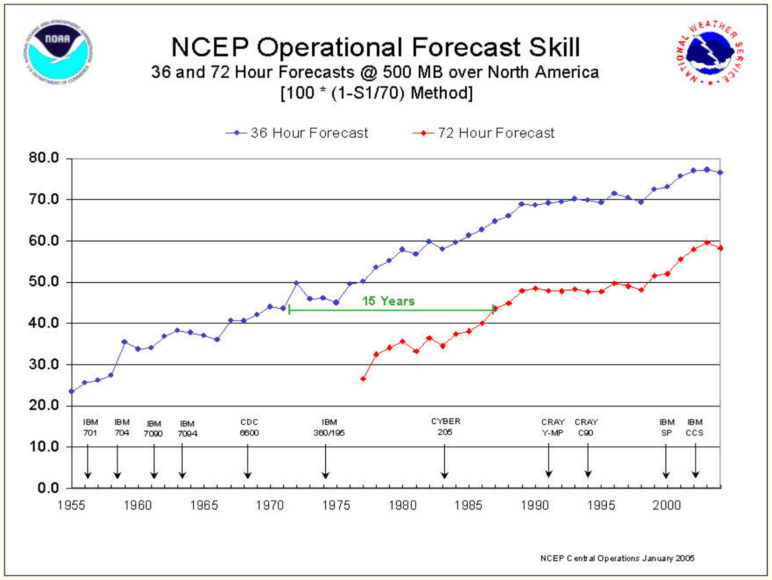
- ***"Allow me to congratulate you . . . on the remarkable progress which has been made.***
- ***"This is . . . an enormous scientific advance on the . . . result in Richardson (1922)."***

NWP Operations

The Joint Numerical Weather Prediction Unit was established on July 1, 1954:

- ***Air Weather Service of US Air Force***
- ***The US Weather Bureau***
- ***The Naval Weather Service.***

Operational numerical forecasting began in May, 1955, with a three-level quasi-geostrophic model.



Skill of the 36 hour (1955–2004) and 72 hour (1977–2004) 500 hPa forecasts produced at NCEP.

Move to Primitive Equations

In 1951, Jule Charney wrote:

The outlook for numerical forecasting would be indeed dismal if the quasi-geostrophic approximation represented the upper limit of attainable accuracy, for it is known that it applies only indifferently, if at all, to many of the small-scale but meteorologically significant motions.

All modern NWP centres have abandoned the QG equations for operational forecasting. (However, they are invaluable for theoretical studies).

Parameterization

Small-scale physical processes cannot be represented explicitly in computer models. They must be represented by bulk formulae. This is called parameterization of the subgrid-scale physics.

- **Condensation phenomena**
- **Solar radiation**
- **Long-wave radiation**
- **Orographic effects**
- **Land-atmosphere interactions**
- **Ocean-atmosphere interactions**
- **Turbulent transfer of momentum and heat.**

Data Assimilation

NWP is an initial-value problem.

The model integrates the equations forward in time, starting from the initial conditions.

In the early NWP experiments, hand interpolations of the observations to grid points were performed.

These fields of initial conditions were manually digitized.

The need for an automatic "objective analysis" quickly became apparent.

There is another important issue: the data available are not enough to initialize current models.

Modern primitive equations models have a number of degrees of freedom of the order of 10^7.

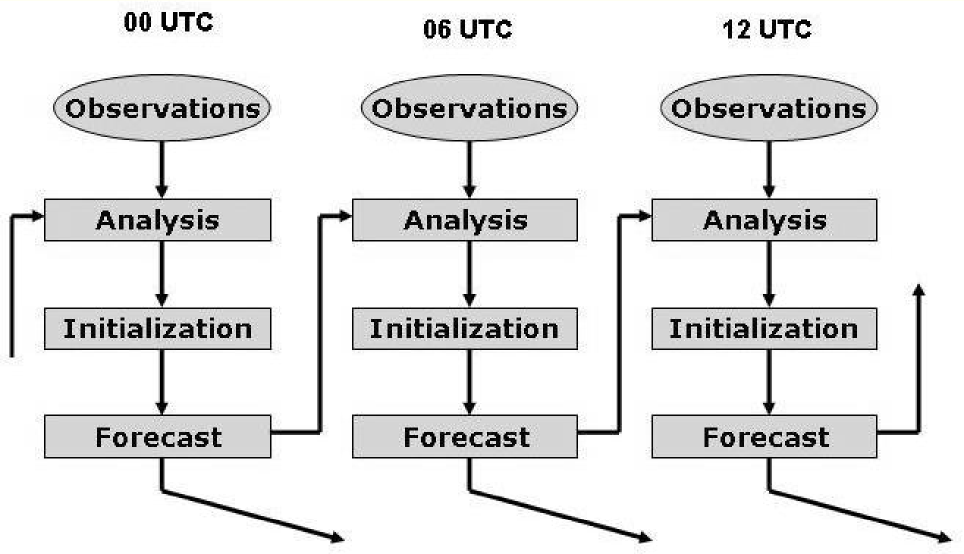
For a time window of ± 3 hours, there are typically 10 to 100 thousand observations of the atmosphere, two orders of magnitude less than the number of degrees of freedom of the model.

Moreover, their distribution in space and time is very nonuniform in space.

It is necessary to use additional information (denoted *background*, *first guess* or *prior information*).

A short-range forecast is used as the first guess in operational data assimilation systems.

Present-day operational systems typically use a 6-h cycle performed four times a day.



Typical 6-hour analysis cycle.

Operational NWP

For an introduction to
Operational NWP and the
evolution of forecast skill
read Kalnay, §1.5.

The Future

- Detailed short-range forecasts, using storm-scale models able to provide skilful predictions of severe weather;
- More sophisticated methods of data assimilation, capable of extracting the maximum possible information from observing systems, especially remote sensors such as satellites and radars;
- Development of adaptive observing systems, in which additional observations are placed where ensembles indicate that there is rapid error growth (low predictability);
- Improvement in the usefulness of medium-range forecasts, especially through the use of ensemble forecasting;
- Fully coupled atmospheric–hydrological systems, where the atmospheric model precipitation is down-scaled and used to extend the length of river flow prediction;

- More use of detailed atmosphere–ocean–land coupled models, in which long-lasting coupled anomalies such as SST and soil moisture anomalies lead to more skilful predictions of anomalies in weather patterns beyond the limit of weather predictability;
- More guidance to governments and the public on subjects such as air pollution, ultraviolet radiation and transport of contaminants, which affect health;
- An explosive growth of systems with emphasis on commercial applications of NWP, from guidance on the state of highways to air pollution, flood prediction, guidance to agriculture, construction, etc.