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From Richardson to early numerical weather prediction

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The development of computer models for numerical simulation of the atmosphere and oceans is one of the great scientific triumphs of the past fifty years. These models have added enormously to our understanding of the complex processes in the atmosphere and oceans. The consequences for humankind of ongoing climate change will be far-reaching. Earth system models are the best means we have of predicting the future of our climate.

The basic ideas of numerical forecasting and climate modeling were developed about a century ago, long before the first electronic computer was constructed. However, advances on several fronts were necessary before numerical prediction could be put into practice. A fuller understanding of atmospheric dynamics allowed the development of simplified systems of equations; regular observations of the free atmosphere provided the initial conditions; stable finite difference schemes were developed; and powerful electronic computers provided a practical means of carrying out the calculations required to predict the changes in the weather.

In this chapter, we trace the history of computer forecasting from Richardson's prodigious manual computation, through the ENIAC (Electronic Numerical Integrator and Computer) integrations to the early days of operational numerical weather prediction and climate modeling. The useful range of deterministic prediction is increasing by about one day each decade. We set the scene for the story of the remarkable progress in weather forecasting and in climate modeling over the past fifty years, which will be treated in subsequent chapters.

2.1 Pioneers of scientific forecasting

The fundamental idea of computing changes in the weather by numerical means was formulated around the turn of the twentieth century, before electronic computers were available. The great American meteorologist Cleveland Abbe recognized that meteorology is essentially the application of hydrodynamics and thermodynamics

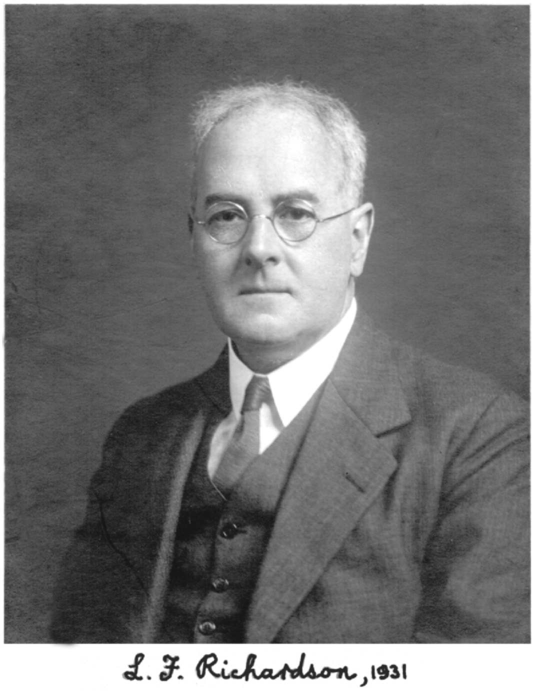

Figure 2.1 Lewis Fry Richardson (1881–1953), signed and dated 1931, when Richardson was aged 50 (photograph by Walter Stoneman; copy courtesy of Oliver Ashford).

to the atmosphere (Abbe, 1901), and he identified the system of mathematical equations that govern the evolution of the atmosphere. Abbe's work was reviewed recently by Willis and Hooke (2006). The Norwegian scientist Vilhelm Bjerknes undertook a more explicit analysis of the weather prediction problem from a scientific perspective (Bjerknes, 1904). His stated goal was to make meteorology an exact science, a true physics of the atmosphere.

Later, Lewis Fry Richardson (Figure 2.1) attempted a direct solution of the equations of motion, and presented the results in his book *Weather Prediction by Numerical Process* (Richardson, 1922). The book opened with a discussion of then-current practice. Richardson described the use of an index of weather maps, constructed by classifying old synoptic charts into categories. The index assisted the forecaster to find previous maps resembling the current one and thus deduce the likely development by studying the evolution of these earlier cases. This "analog approach" was at the heart of operational forecasting until the modern era, and forecast skill remained rather static. Indeed, Sverre Petterssen (2001) described the advances prior to the computer era as occurring in "homeopathic doses".

Table 2.1 *Six-hour changes in pressure (units: hPa/6 h). [LFR: Richardson; MOD: Model; DFI: Filtered.]*

Level (km)	LFR	MOD	DFI
1 (11.8)	48.3	48.5	−0.2
2 (7.2)	77.0	76.7	−2.6
3 (4.2)	103.2	102.1	−3.0
4 (2.0)	126.5	124.5	−3.1
Surface	**145.1**	**145.4**	**−0.9**

The full story of Richardson's work, the reason for his catastrophic results, and a complete reconstruction of the forecast are described in a recent book (Lynch, 2006). Richardson calculated a change in surface pressure over a six-hour period of 145 hPa, a totally unrealistic value. His extrapolation over six hours exacerbated the problem, but was not the root cause of it. Lynch showed that, when the analyzed data are balanced through the process of initialization, a realistic value of pressure change is obtained. In Table 2.1 we show the six-hour changes in pressure at each level of the numerical model he used. The column marked LFR (Lewis Fry Richardson) has the values obtained by Richardson (1922). The column marked MOD (model) has the values reconstructed using a computer model based directly on Richardson's method: they are very close to Richardson's values. The column marked DFI (digital filter initialization) is for a forecast from data initialized using a digital filter: the initial tendency of surface pressure is reduced from the unrealistic 145 hPa/6 h to a reasonable value of less than 1 hPa/6 h (bottom row, Table 2.1). These results indicate clearly that Richardson's unrealistic prediction was due to imbalance in the initial data that he used. Complete details of the forecast reconstruction may be found in Lynch (2006).

Richardson's forecasting scheme involved a phenomenal volume of numerical computation and was quite impractical in the pre-computer era. But he was undaunted, speculating that

> "some day in the dim future it will be possible to advance the computations faster than the weather advances".

The work of Max Margules, published almost twenty years before Richardson's forecast, pointed to serious problems with Richardson's methodology. Margules examined the relationship between the continuity equation (which expresses conservation of mass) and changes in surface pressure (Margules, 1904). He considered the possibility of predicting pressure changes by direct use of the mass conservation principle. He showed that, due to strong cancellation between terms, the calculation is very error-prone, and may give ridiculous results. Therefore, it is not possible, using the continuity equation alone, to derive a reliable estimate of synoptic-scale changes in pressure. Margules concluded that any attempt to forecast the weather was *immoral and damaging to the character of a meteorologist* (Fortak, 2001).

To make his forecast of the change in pressure, Richardson used the continuity equation, employing precisely the method that Margules had shown to be seriously problematical. The resulting prediction of pressure change was completely unrealistic. The question of what influence, if any, Margules' results had on Richardson's approach to forecasting was considered by Lynch (2003). At a later stage, Richardson did come to a realization that his original method was unfeasible. In a note contained in the Revision File, inserted in the manuscript version of his book, he wrote,

"Perhaps the most important change to be made in the second edition is that the <u>equation of continuity of mass must be eliminated"</u>.

(Richardson's underlining)

He went on to speculate that the vertical component of vorticity or rotation rate of the fluid might be a suitable prognostic variable. This was indeed a visionary adumbration of the use of the vorticity equation for the first successful numerical integration in 1950.

Of course, we now know that Margules was unduly pessimistic. The continuity equation is an essential component of primitive equation models which are used in the majority of current computer weather prediction systems. Primitive equation models use the exact equations of motion except for the hydrostatic approximation: they support gravity wave solutions and, when changes in pressure are computed using the continuity equation, large tendencies can arise if the atmospheric conditions are far from balance. However, spuriously large tendencies are avoided in practice by an adjustment of the initial data to reduce gravity wave components to realistic amplitudes, by the process of initialization, as indicated in Table 2.1.

2.2 Pre-computer forecasting

Weather forecasts are now so reliable, accurate, and readily available that it is easy to forget how things were only a few decades ago. Before the computer era, forecasting was imprecise and undependable. Analysis of the global atmospheric state was severely hampered by lack of observations, and the principles of theoretical physics played little role in practical forecasting. Although much of the underlying physics was known, its application to the prediction of atmospheric conditions was impractical. Observations were sparse and irregular, especially for the upper air and over the oceans.

Forecasting was a haphazard process, very imprecise and unreliable. The forecaster used crude techniques of extrapolation, knowledge of local climatology, and guesswork based on intuition; forecasting was more an art than a science. The observations of pressure and other variables were plotted in symbolic form on a weather map and lines were drawn through points with equal pressure to reveal the pattern of weather systems – depressions, anticyclones, troughs, and ridges. The forecaster used experience, memory, and a variety of empirical rules to produce a forecast map. The primary physical process attended to by the forecaster was *advection*, the transport of fluid characteristics and properties by the movement of the fluid itself. But the crucial quality of advection is that it is *nonlinear*; the human forecaster may extrapolate trends using an assumption of constant wind, but is quite incapable of intuiting the subtleties of complex advective processes.

The technique of "weather typing" was used with limited success. This was the method underlying the index of weather maps, mentioned by Richardson. Current meteorological conditions were compared with the historical record. If a close match was found, it was assumed that the evolution of the flow for the following days would be similar to that observed on the previous occasion. However, the atmosphere shows little tendency to repeat itself. Richardson was not optimistic about this method. He wrote in his Preface that

> "The forecast is based on the supposition that what the atmosphere did then, it will do again now . . . The past history of the atmosphere is used, so to speak, as a full-scale working model of its present self".
>
> *(Richardson, 1922)*

Bjerknes had contrasted the precision of astronomical prediction with the "radically inexact" methods of weather forecasting. Richardson returned to this theme, pointing out that the *Nautical Almanac*,

> "that marvel of accurate forecasting"

is not based on the principle that astronomical history repeats itself. Given the complexity of the atmosphere, why should we expect a present weather map to be exactly represented in a catalogue of past weather?

In Europe meteorology was studied in many universities, and researchers applied physical principles to atmospheric problems. Bjerknes had dreamed of mathematical forecasting but, finding it impractical, had marshaled his team in Bergen to develop more feasible methods. They developed mechanistic models of extra-tropical weather systems and described the life-cycles of mid-latitude depressions and their associated warm and cold fronts. Although the models of polar fronts and the life-cycles of extra-tropical depressions are conceptual in nature, they are founded on sound scientific principles. Frontal and air-mass theory gradually came to have a profound influence on operational practice on both sides of the Atlantic. Bjerknes visited America to promote the new ideas and many of his Bergen students played major roles in the advancement of American meteorology, both in government agencies and universities.

2.3 Key developments, 1920–1950

Richardson was several decades ahead of his time in what he attempted to do. At the time of the First World War, computational weather forecasting was impractical for at least four reasons.

First, observations of the three-dimensional structure of the atmosphere were available only on a very occasional basis, with inadequate coverage and never in real time. The registering balloons had to be recovered and the recordings analyzed to recover the data, a process that took days or even weeks. Second, the numerical algorithms for solving the atmospheric equations were subject to instabilities that were not understood. Thus, the numerical solution might bear little or no resemblance to the solution of the continuous equations. Third, the balanced nature of atmospheric flow was inadequately understood, and the imbalances arising from observational and analysis errors confounded Richardson's forecast. Fourth, the massive volume of computation required to advance the numerical solution could not be done, even by a huge team of human computers. Indeed, Richardson's estimate of 64,000 computers, the number of people needed to do the calculations for a useful forecast in real time, was a serious under-estimate. It has been reckoned that one million people would have been required for the task (Lynch, 1993). Thus, what Richardson devised was a "method without a means".

A number of key developments in the ensuing decades set the scene for progress. Developments in the theory of meteorology provided crucial understanding of atmospheric dynamics, in particular the balance of the atmosphere and the means of eliminating spurious high-frequency gravity waves. Advances in numerical analysis led to the design of algorithms which were stable and faithfully replicated the true solution. Timely observations of the atmosphere in three dimensions were becoming available following the invention of the radiosonde, which provided measurements of pressure, temperature, humidity, and winds through a vertical column of the atmosphere. Finally, the development of digital computers provided a way of attacking the enormous computational task involved in weather forecasting.

In addition to the technical developments, the socio-political framework of the mid century provided a crucial impetus. Progress in meteorology has often followed from natural or human-made catastrophes. The Second World War was a spectacular example. Military operations on land, air, and sea all depend heavily on accurate weather forecasts. The role of weather in Operation Overlord – the D-day invasion of Normandy – was recounted by Pettersen (2001). In the United States an intensive training program for meteorologists was organized under the inspiration of Carl-Gustav Rossby. As a result, the professional meteorological community grew by a factor of fifteen during the war, from 400 before to 6000 afterwards. Many of the new entrants were highly skilled in mathematics and physics and wished to develop rigorous methods of forecasting that were based on scientific principles.

2.4 The ENIAC integrations

The first general-purpose electronic computer, ENIAC (Electronic Numerical Integrator and Computer) was commissioned by the U.S. Army for use in calculating the dynamics of projectiles. The principal designers of ENIAC were John Mauchly and Presper Eckert. It is noteworthy that Mauchley's interest in computers arose from his desire to forecast the weather by calculation. The computer was originally called the Electronic Numerical Integrator. A U.S. Army colonel suggested adding the words "and Computer" to give the catchy acronym ENIAC (McCartney, 1999). The ENIAC, which had been completed in 1945, was the first general-purpose electronic digital computer ever built. It was a gigantic machine, with 18,000 thermionic valves, filling a large room and consuming 140 kW of power. Input and output were by means of punch-cards. McCartney (1999) provides an absorbing account of the origins, design, development, and destiny of ENIAC.

John von Neumann recognized weather forecasting, a problem of both great practical significance and intrinsic scientific interest, as ideal for an automatic computer. He was in close contact with Rossby, who was the person best placed to understand the challenges that would have to be addressed to achieve success in this venture. Von Neumann established a Meteorology Project at the Institute for Advanced Study in Princeton and recruited Jule Charney to lead it. Arrangements were made to compute a solution of a simple equation, the barotropic vorticity equation (BVE), on the only computer available, the ENIAC. Barotropic models treat the atmosphere as a single layer, averaging out variations in the vertical. The resulting numerical predictions were truly ground-breaking. Four 24-hour forecasts were made, and the results clearly indicated that the large-scale features of the mid-tropospheric flow could be forecast numerically with a reasonable resemblance to reality.

The ENIAC forecasts were described in a seminal paper by Jule Charney, Ragnar Fjørtoft, and John von Neumann (Charney, *et al*., 1950, referenced below as CFvN). The story of this work was recounted by George Platzman in his Victor P. Starr Memorial Lecture (Platzman, 1979). The atmosphere was treated as a single layer, represented by conditions at the 500 hPa level, modeled by the BVE. This equation, expressing the conservation of absolute vorticity following the flow, gives the rate of change of the Laplacian of the height of the 500 hPa surface in terms of the

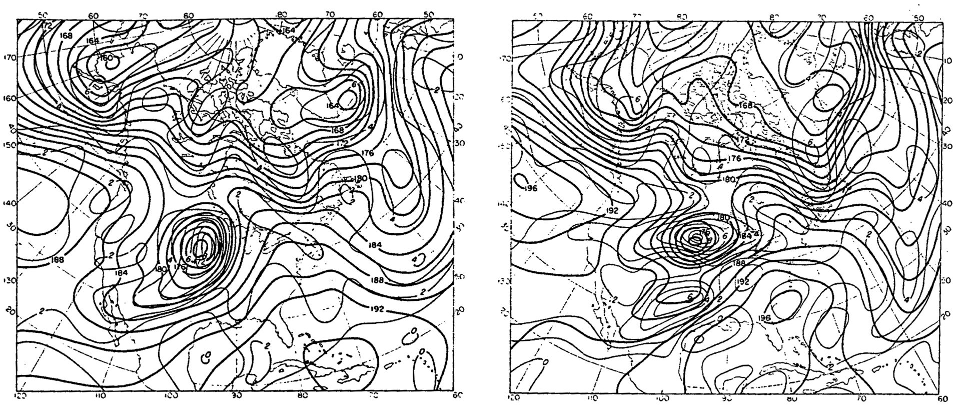

Figure 2.2 The ENIAC forecast starting at 0300 UTC, January 5, 1949. Left panel: analysis of 500 hPa height (thick lines) and absolute vorticity (thin lines). Right panel: forecast height and vorticity (from Charney, *et al*., 1950). Height units are hundreds of feet, contour interval is 200 ft. Vorticity units and contour interval are $10^{-5}s^{-1}$.

advection. The tendency of the height field is obtained by solving a Poisson equation with homogeneous boundary conditions. The height field may then be advanced to the next time level. With a one-hour timestep, this cycle is repeated 24 times for a one-day forecast.

The initial data for the forecasts were prepared manually from standard operational 500 hPa analysis charts of the U.S. Weather Bureau, discretized to a grid of 19 by 16 points, with grid interval of 736 km. Centered spatial finite differences and a leapfrog time-scheme were used. The boundary values of height were held constant throughout each 24-hour integration. The forecast starting at 0300 UTC, January 5, 1949 is shown in Figure 2.2 (from CFvN). The left panel is the analysis of 500 hPa height and absolute vorticity. The forecast height and vorticity are shown in the right panel. The feature of primary interest was an intense depression over the United States. This deepened, moving NE to the 90°W meridian in 24 hours. A discussion of this forecast, which underestimated the development of the depression, may be found in CFvN and in Lynch (2008).

The success of the ENIAC forecasts had an electrifying effect on the world meteorological community. Several baroclinic (multi-level) models were developed in the following years. They were all based on the filtered or quasi-geostrophic system of equations, an approximate system derived using geostrophic balance between the pressure and winds. Later, models using the more accurate primitive equations were introduced. Charney had anticipated this as a necessary development, and indeed André Robert later identified it as the key development in numerical weather prediction (see Lin *et al*. 1997).

The Princeton team studied the severe storm of Thanksgiving Day, 1950 using two-and three-level quasi-geostrophic models. After some tuning, they found that the cyclogenesis could be reasonably well simulated. Thus, it appeared that the central problem of operational forecasting had been cracked. However, it transpired that the success of the Thanksgiving forecast had been something of a fluke: early multi-level models were consistently worse than the simple barotropic equation; and it was the single-level model that was used when regular operations commenced in 1958. A fuller discussion can be found in Lynch (2006). The trials and triumphs of the Joint Numerical Weather Prediction Unit, which will appear again in Chapter 3, and the establishment of operational computer forecasting are described comprehensively in a recent book (Harper, 2008).

2.5 Advancing computer technology

Advances in computer technology over the past half-century have been spectacular. The increase in computing power is encapsulated in an empirical rule called Moore's Law, which implies that computing speed doubles about every 18 months. Thus, a modern micro-processor has far greater power than the ENIAC had. Recently, Lynch & Lynch (2008) decided to repeat the ENIAC integrations using a programmable cell-phone, which was called the Portable Hand-Operated Numerical Integrator and Computer, or PHONIAC. This technology has great potential for generation and display of operational weather forecast products.

The oft-cited paper in *Tellus* (CFvN) gives a complete account of the computational algorithm and discusses four forecast cases. Lynch (2008) presented the results of repeating the ENIAC forecasts using a MATLAB program `eniac.m`, run on a laptop computer (a Sony Vaio, model VGN-TX2XP). The main loop of the 24-hour forecast ran in about 30 ms. Given that the original ENIAC integrations each took about one day, this time ratio – about three million to one – indicates the dramatic increase in computing power over the past half-century. The program `eniac.m` was converted from MATLAB to a Java application, `phoniac.jar`, for implementation on a cell-phone. The program was tested on a PC using emulators for three different mobile phones. A basic graphics routine was also written in Java. When working correctly, the program was downloaded onto a Nokia 6300 cell-phone for execution.

Charney *et al*. (1950) provided a full description of the solution algorithm for the BVE. The programs `eniac.m` and `phoniac.jar` were constructed following the original algorithm precisely, including the specification of the boundary conditions and the Fourier transform solution method for the Poisson equation. Hence, given initial data identical to that used in CFvN, the recreated forecasts should be

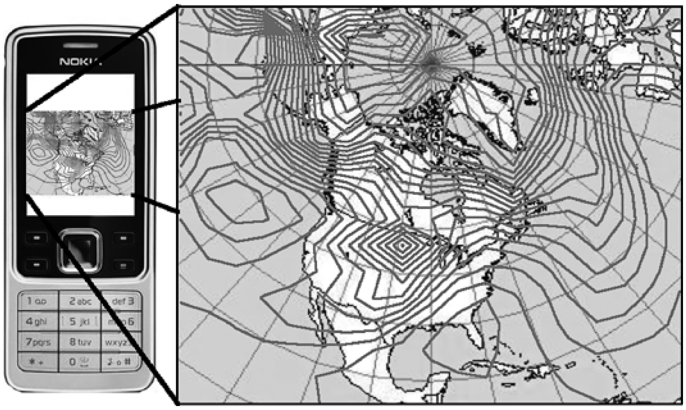

Figure 2.3 The Nokia 6300, dubbed PHONIAC (left) and the forecast for 0300 UTC, January 6, 1949 (right) made with the program `phoniac.jar`. The contour interval is 50 m (from Lynch and Lynch, 2008).

identical to those made in 1950. Of course, the re-analyzed fields are not identical to those originally used, and the verification analyses are also different. Nevertheless, the original and new results are very similar.

The initial fields for the four ENIAC forecasts were valid for dates in January and February, 1949. A retrospective global analysis of the atmosphere, covering more than fifty years, has been undertaken by the National Centers for Environmental Prediction (NCEP) and the National Center for Atmospheric Research (NCAR) (Kistler *et al*., 2001). This re-analysis extends back to 1948, including the period chosen for the ENIAC integrations. The re-analyzed data are available on a 2.5° by 2.5° grid. The GRIB fields of the 500 hPa analyses were downloaded from the NCEP/NCAR re-analysis website and interpolated to the ENIAC grid.

In Figure 2.3 we show PHONIAC and the forecast for 0300 UTC, January 6, 1949 made with the program `phoniac.jar`. The main features of the forecast (right panel) are in broad agreement with the originals (right panel, Figure 2.2). In CFvN it is noted that the computation time for a 24-hour forecast was about 24 hours, that is, the team could just keep pace with the weather provided the ENIAC did not fail. This time included off-line operations: reading, punching, and interfiling of punch cards. PHONIAC executed the main loop of the 24-hour forecast in less than one second. The main steps in the solution algorithm are presented in Lynch (2008, Appendix B). For the benefit of students, the MATLAB and Java codes are available on a website (http://maths.ucd.ie/~plynch/eniac/). Maps of the four original and recreated forecasts are also available there, along with miscellaneous supplementary material relating to the ENIAC integrations.

2.6 Climate modeling

We can trace the beginnings of climate modeling back to 1956, when Norman Phillips carried out the first long-range simulation of the general circulation of the atmosphere. He used a two-level quasi-geostrophic model on a beta-plane channel, ignoring the effects of sphericity except for variations of the Coriolis force with latitude. The computation used a spatial grid of 16 × 17 points, and the simulation was for a period of about one month. Starting from a zonal flow with small random perturbations, a wave disturbance with a wavelength of 6000 km developed. It had the characteristic westward tilt with the height of a developing baroclinic wave, and moved eastward at about 20 m s^{-1}. Figure 2.4 shows the configuration of the flow after twenty days' simulation. Phillips examined the energy exchanges of the developing wave and found good qualitative agreement with observations of baroclinic systems in the atmosphere. He also examined the mean meridional flow, the average circulation in a vertical cross-section along a meridian, and found circulations corresponding to the Hadley, Ferrel, and Polar cells, large-scale

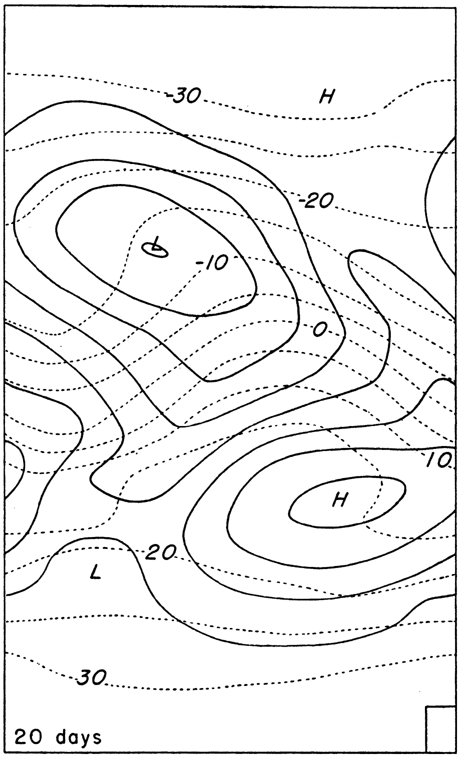

Figure 2.4 Configuration of the flow after 20-days' simulation with a simple, two-level filtered model. Solid lines: 1000 hPa heights at 200 foot intervals. Dashed lines: 500 hPa temperatures at 5 °C intervals (Phillips, 1956).

circulations with ascending motion at one latitude coupled to descending motion at another through meridional flow:

> "We see the appearance of a definite three-celled circulation, with an indirect [or reverse] cell in middle latitudes and two somewhat weaker cells to the north and south. This is a characteristic feature of . . . unstable baroclinic waves".
>
> *(Phillips, 1956, p. 144)*

Von Neumann was hugely impressed by Phillips' work, and arranged a conference at Princeton University in October 1955, *Application of Numerical Integration Techniques to the Problem of the General Circulation*, to consider its implications. The work had a galvanizing effect on the meteorological community. Within ten years, there were several major research groups modeling the general circulation of the atmosphere, the leading ones being at the Geophysical Fluid Dynamics Laboratory, the National Center for Atmospheric Research, the University of California, Los Angeles, the Lawrence Livermore National Laboratory, and the United Kingdom Meteorological Office These research efforts will appear again in Chapters 3 and 9.

The development of comprehensive models of the atmosphere is undoubtedly one of the finest achievements of meteorology in the twentieth century. Following Phillips' seminal work, several general circulation models (GCMs) were developed, including various physical processes such as solar heating, terrestrial radiation, convection, and small-scale turbulence. Advanced models are under continuing refinement and extension, and are increasing in sophistication and comprehensiveness. They simulate not only the atmosphere and oceans but also a wide range of geophysical, chemical, and biological processes and feedbacks. The models, now called *Earth system models*, are applied to the eminently practical problem of weather prediction and also to the study of climate variability and humankind's impact on it. There is no doubt that the study of climate change and its impacts is of enormous importance for our future. Global climate models are the best means we have of anticipating likely changes.

2.7 Uncertainty and probability

The chaotic nature of the atmospheric flow, most clearly elucidated by Lorenz (1963), imposes a limit on predictability, as unavoidable errors in the initial state grow rapidly and render the forecast useless after some days. The most successful means of confronting this obstacle is to run a collection, or ensemble, of forecasts, each starting from a slightly different initial state, and to use the combined outputs to deduce probabilistic information about future changes in the atmosphere. Probability forecasts for a wide range of weather events are generated for use in the operational centers. These have become the key guidance for medium-range prediction.

Computer prediction models are now the primary input for operational forecasters and are vital for a wide range of applications. Perhaps the most important application is to provide timely warning of weather extremes. The ensemble approach provides valuable quantitative guidance on the probability and likely severity of extreme events. The warnings which result from computer guidance enable great saving of life and property.

Transportation, energy consumption, construction, tourism, and agriculture are all sensitive to weather conditions. There are expectations from all these sectors of increasing accuracy and detail of forecasts, as decisions with heavy financial implications must continually be made. Numerical weather prediction models are used to generate special guidance for the marine community. Trajectories for modeling pollution drift, for nuclear fallout, and smoke from forest fires are easily derived. Aviation benefits significantly from computer guidance, which provides warnings of hazards. Automatic generation of terminal aerodrome forecasts enables servicing of a large number of airports from a central forecasting facility.

Interaction between the atmosphere and ocean becomes a dominant factor at longer forecast ranges. For seasonal forecasting, coupled atmosphere–ocean models are essential. Once again, the ensemble approach is an effective means of addressing uncertainty in the predictions. Good progress has been made in seasonal forecasting for the tropics. Considerable effort is being made to produce useful long-range forecasts for temperate regions, but many challenges remain. The ensemble approach has also become a central aspect of climate change prediction.

2.8 Dreams fulfilled

Developments in atmospheric dynamics, instrumentation, observing practice, and digital computing have made the dreams of Abbe, Bjerknes, and Richardson an everyday reality. Numerical weather prediction models are now at the center of operational forecasting. It is no exaggeration to describe the advances made over the past half-century as revolutionary. Progress in weather forecasting and in climate modeling has been dramatic. We can now predict the weather for several days in advance with a high degree of confidence, and the useful range of deterministic prediction is increasing by about one day each decade. Using Earth system models, we are gaining great insight into the factors causing changes in our climate, and their likely timing and severity.

Meteorology is now firmly established as a quantitative science, and its value and validity are demonstrated daily by the acid test of any science, its ability to predict the future. The development of comprehensive models of the atmosphere is undoubtedly one of the finest achievements of meteorology in the twentieth century. The story of how the models have developed over the past fifty years, and the current state of numerical weather prediction and climate modeling, is told in the following chapters of this volume.

References

Abbe, C. (1901). The physical basis of long-range weather forecasts. *Monthly Weather Review*, 29, 551–61.

Bjerknes, V. (1904). Das Problem der Wettervorhersage, betrachtet vom Standpunkte der Mechanik und der Physik. *Meteorologische Zeitschrift*, 21, 1–7.

Charney, J. G., Fjörtoft, R., and von Neumann, J. (1950). Numerical integration of the barotropic vorticity equation. *Tellus*, 2, 237–254.

Fortak, H. (2001). Felix Maria Exner und die österreichische Schule der Meteorologie. Pp. 354–386, in Hammerl, Christa, Wolfgang Lenhardt, Reinhold Steinacker, and Peter Steinhauser, 2001: *Die Zentralanstalt für Meteorologie und Geodynamik 1851–2001. 150 Jahre Meteorologie und Geophysik in Österreich*. Leykam Buchverlags GmbH, Graz. ISBN 3–7011–7437–7.

Harper, K. J. (2008). *Weather by the Numbers: the Genesis of Modern Meteorology*. MIT Press, 308pp.

Kistler, R., Kalnay, E., Collins, W., Saha, S., White, G., Woollen, J., Chelliah, M., Ebisuzaki, W., Kanamitsu, M., Kousky, V., van den Dool, H., Jenne, R., and Fiorino, M. (2001). The NCEP-NCAR 50-year reanalysis: Monthly means CD-ROM and documentation. *Bulletin of the American Meteorological Society*, 82, 247–267.

Lin, C. A., Laprise, R., and Richie, H. ed. (1997). Numerical Methods in Atmospheric and Oceanic Modelling. The André J. Robert Memorial Volume. Canadian Meteorological and Oceanographic Society.

Lorenz, E. N. (1963) Deterministic nonperiodic flow. *Journal of the Atmospheric Sciences*, 20, 130–141.

Lynch, P. (1993). Richardson's forecast factory: the $64,000 question. *Meteorological Magazine*, 122, 69–70.

Lynch, P. (2003). Margules' tendency equation and Richardson's forecast. *Weather*, 58, 186–193.

Lynch, P. (2006). *The Emergence of Numerical Weather Prediction: Richardson's Dream*. Cambridge University Press, 279 pp. (ISBN: 0–521–85729–5).

Lynch, P. (2008). The ENIAC forecasts: a recreation. *Bulletin of the American Meteorological Society*, 89, 45–55.

Lynch, P. and Lynch, O. (2008). Forecasts by PHONIAC. *Weather*, 63, 324–326.

McCartney, S. (1999). *ENIAC: The Triumphs and Tragedies of the World's First Computer*. Berkley Books, New York, 262pp.

Margules, M., (1904). Über die Beziehung zwischen Barometerschwankungen und Kontinuitätsgleichung [On the relationship between barometric variations and the continuity equation]. *Ludwig Boltzmann Festschrift*. Leipzig, Barth, J. A., 930 pp.

Petterssen, S. (2001). *Weathering the Storm. Sverre Petterssen, the D-Day Forecast, and the Rise of Modern Meteorology*. Fleming, J. R. ed. American Meteorological Society.

Phillips, N. A. (1956). The general circulation of the atmosphere : A numerical experiment. *Quarterly Journal of the Royal Meteorological Society*, 82, 123–164.

Platzman, G. W. (1979). The ENIAC computations of 1950 – Gateway to numerical weather prediction. *Bulletin of the American Meteorological Society*, 60, 302–312.
Richardson, L. F. (1922). *Weather Prediction by Numerical Process*. Cambridge University Press. 2nd Edn. with Foreward by Peter Lynch (2007).
Willis, E. P. and Hooke, W. H. (2006). Cleveland Abbe and American meteorology, 1871–1901. *Bulletin of the American Meteorological Society*, 87, 315–26.