M.Sc. in Computational Science

Fundamentals of Atmospheric Modelling

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Lecture 3

The Equations of Motion

The Thin Atmosphere

The atmosphere is *infinitely thick but very thin!*
90% of its mass lies within $10\,\text{km}$ of the earth's surface.

The lowest layer of the atmosphere, where temperature decreases with height, is called the *troposphere*. It about $10\,\text{km}$ in depth.

The vertical extent of the large scale motion systems is very much smaller than their horizontal scales. Characteristic horizontal and vertical scales for *synoptic* systems are

$$L = 10^6\,\text{m} = 1000\,\text{km} \qquad H = 10^4\,\text{m} = 10\,\text{km}.$$

This geometrical situation is intimately linked to the existence of <u>hydrostatic balance</u>.

A typical grid-box of a numerical model might have dimensions $10\,\text{km} \times 10\,\text{km} \times 100\,\text{m}$, which has an <u>aspect ratio</u> of one hundred to one, comparable to that of a credit card!

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Digression

Did you know?

The ratio of the length to the breadth of a credit card is equal to the ratio of the *Golden Section*:

$$\left(\begin{matrix}\text{Aspect}\\\text{Ratio}\end{matrix}\right) = \left[\frac{\text{Length}}{\text{Breadth}}\right] = \frac{1+\sqrt{5}}{2} \approx 1.618\,.$$

It is allegedly the most aesthetically pleasing rectangular shape, and is found in numerous classical works of art.

This ratio is ubiquitous throughout nature. It is closely associated with the Fibonacci sequence of numbers

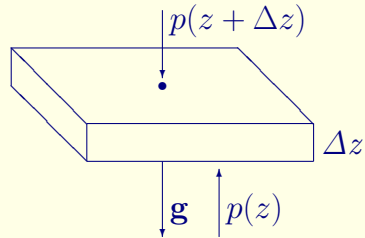$$\{1, 1, 2, 3, 5, 8, 13, 21, 34, \dots\}\,,$$

where each term is the sum of the preceding two terms.

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Hydrostatic Balance

For a fluid at rest, the pressure at a point depends on the weight of fluid vertically above that point.

The pressure difference between two points on the same vertical line depends only on the weight of fluid between them.

$$\text{Force Upward on Box}: \qquad + [p(z) \cdot \Delta x \Delta y]$$
$$\text{Force Downward on Box}: \qquad - [p(z + \Delta z) \cdot \Delta x \Delta y + mg]$$

For equilibrium, the net force must be zero:
$$\frac{p(z + \Delta z) - p(z)}{\Delta z} \cdot \Delta x \Delta y \Delta z + mg = 0 \,.$$
This may be written
$$\frac{\partial p}{\partial z} \cdot \mathcal{V} + mg = 0 \,,$$
or, dividing through by the volume,
$$\frac{\partial p}{\partial z} + \rho g = 0 \,.$$

This is the **Hydrostatic balance equation**. It implies an exact balance between the <u>vertical pressure gradient</u> and <u>gravity</u>.

For an atmosphere at rest, hydrostatic balance holds exactly.

Exercise: Vertical Pressure Gradient

Suppose the atmosphere is in a state of hydrostatic balance. Calculate approximately the pressure drop over a vertical distance of $100\,\text{m}$, assuming the density is constant at $\rho = 1.2\,\text{kg m}^{-3}$ and $g = 9.8\,\text{m s}^{-2}$.

$$\star \qquad \star \qquad \star$$

The hydrostatic equation gives
$$\frac{\Delta p}{\Delta z} + \rho g = 0 \,.$$
Substituting the numerical values gives
$$\Delta p = -\Delta z \rho g = -100\,\text{m} \times 1.2\,\text{kg m}^{-3} \times 9.8\,\text{m s}^{-2}$$
(negative, since pressure decreases upwards). Evaluating this gives
$$|\Delta p| = 1176\,\text{kg m}^{-1}\text{s}^{-2} = 1176\,\text{Pa} = 11.76\,\text{hPa} \,.$$

The <u>hectoPascal</u>, numerically equal to the <u>millibar</u>, is the pressure unit most commonly used in practice.

Note that the assumption of constant density is unrealistic over large vertical distances. We will relax this assumption presently.

Hydrostatic Approximation

The **hydrostatic approximation** consists of assuming that balance between the vertical pressure gradient and gravity holds <u>even when the fluid is in motion</u>. For the large scale motions of the atmosphere and ocean, hydrostatic balance holds to a high degree of accuracy.

Until recently, most numerical models used to predict atmospheric flow were hydrostatic. For these models, the vertical velocity is a *diagnostic variable*, deduced from the other dependent variables at each point in time.

Non-hydrostatic models are now growing in popularity, particularly where spatial grids of a few kilometres are used. For these models, the vertical velocity is a *prognostic variable*, predicted in the same way as the other dependent variables.

The Equation of State

We are familiar from elementary physics with Boyle's Law and Charles' Law of gases. They are special cases of the *Equation of State* for a perfect gas:

$$pV = nR^*T$$

where $R^* = 8314 \, \mathrm{J \, K^{-1} \, kmol^{-1}}$ is the universal gas constant and n is the number of kilomoles of gas (a *kmole* is the molecular weight in kg).

The mean molecular weight of air is $\mu \approx 29$. Thus, $m = \mu n$. Dividing by the volume, we get the *equation of state*

$$p = R\rho T$$

where $R = R^*/\mu = 287 \, \mathrm{J \, K^{-1} \, kmol^{-1}}$ is the gas constant for dry air.

Table 1: **Main Constitutents of the Atmosphere**

Gas		Percentage	Mol. Wt.
Nitrogen	N_2	80%	28
Oxygen	O_2	20%	32
Air		100%	29

The atmosphere is composed primarily of nitrogen (80%) and oxygen (20%), so the mean molecular weight of air is about **29**.

Other constituents, such as carbon dioxide and methane, are vitally important for <u>radiative balance</u>, but their concentrations are quite small.

Water occurs in all three phases, and is enormously important. However, we will be concentrating on the large-scale dynamics of the atmosphere and will largely ignore water, as it introduces great complexity.

Vertical Variation of Pressure

Let's consider an *isothermal* atmosphere at rest. Let the constant temperature be T_0. The hydrostatic equation and the equation of state are

$$\frac{\partial p}{\partial z} + g\rho = 0 \,, \qquad p = R\rho T_0 \,.$$

Combining these we have

$$\frac{\partial p}{\partial z} = -g\frac{p}{RT_0} \,, \qquad \text{so} \qquad \frac{dp}{p} = -\frac{g \, dz}{RT_0} = -\frac{dz}{H} \,,$$

where we define the <u>scale-height</u> by $H = RT_0/g$.

We integrate over the range $p_0 = p(0)$ to $p = p(z)$ to get

$$\log\left(\frac{p}{p_0}\right) = -\frac{z}{H}$$

or

$$p(z) = p_0 \exp(-z/H) \,.$$

Thus, *pressure decreases exponentially with height*.

Since $\rho = p/RT_0$, *density also decreases exponentially* with height.

The scale height is easily computed. Suppose $T_0 = 265 \, \mathrm{K}$. Then

$$H = \frac{RT_0}{g} = \frac{287 \times 265}{9.8066} = 7755 \, \mathrm{m} = 7.755 \, \mathrm{km} \,,$$

so the scale height is about $8 \, \mathrm{km}$. Pressure decreases by a factor of $1/e$ over this height.

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Exercise

Relax the assumption of an *isothermal atmosphere*: Assume that temperature decreases linearly with height

$$T = T_0 + \gamma z$$

where the <u>lapse rate</u>, $\gamma = \partial T/\partial z < 0$, is constant.

Calculate the dependence of p on height.

Exercise: Constant Lapse-rate

Answer: Assume that temperature decreases linearly with height

$$T = T_0 + \gamma z$$

where the lapse rate $\gamma = \partial T/\partial z < 0$ is constant.

Combining the hydrostatic equation and the equation of state as before, we get

$$\frac{dp}{p} = -\frac{g\,dz}{RT} = -\frac{g}{RT_0}\frac{dz}{1 + \gamma z/T_0}.$$

Integrating this yields

$$\log\left(\frac{p}{p_0}\right) = -\frac{T_0}{\gamma H}\log\left(1 + \frac{\gamma z}{T_0}\right) = \log\left(1 + \frac{\gamma z}{T_0}\right)^{-T_0/\gamma H}$$

so that

$$p = p_0\left(1 + \frac{\gamma z}{T_0}\right)^{-T_0/\gamma H}$$

[**More Work:** Show that this reduces to the previous result when $\gamma \to 0$. Use $\lim_{n\to\infty}(1 + x/n)^{-n} = \exp(-x)$.]

Exercise: Mass of Air Column

Consider a vertical column of air, of cross-section A and infinite height. Suppose the pressure at the bottom of the column is p_0.
What is the mass M of the column?

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If we knew the density, we could calculate the mass using

$$M = \iiint_{\mathcal{V}} \rho\,d\mathcal{V} = A\int_0^\infty \rho\,dz.$$

However, there is a simpler way: the **force** of the column on the surface below it is given by two quantities, which must be equal:

$$p_0 \times A = M \times g, \qquad \text{so that} \qquad M = p_0 A/g.$$

For a unit cross-section, $A = 1\,\mathrm{m}^2$, the total column mass is $M = p_0/g \approx 10^4\,\mathrm{kg}$ or ten tonnes!

More Work: Recall the TV screen. The force was $12\,\mathrm{kN}$ for an area $A = 0.12\,\mathrm{m}^2$. Show that this result is consistent with the above.

Equivalent Incompressible Fluid

What is the depth of a layer of *incompressible* fluid having the same bottom pressure as the atmosphere?

The mass of a vertical column of fluid of ***constant density*** ρ_0 and depth h is

$$M = A\int_0^h \rho_0\,dz = A\rho_0 h.$$

The force exerted by the column on the surface below is Mg. Since pressure is force-per-unit-area, the pressure is

$$p_0 = g\rho_0 h$$

so, for given pressure p_0, the depth is

$$h = \frac{p_0}{g\rho_0}.$$

But the scale height of the atmosphere is

$$H = \frac{RT_0}{g} = \frac{p_0}{g\rho_0},$$

so we see that **the depth h equals the scale height H.**

The Equations of Motion

Forces Acting on a Parcel of Air

(1) Recall that the pressure force <u>per unit mass</u> is

$$\mathbf{F}_p = -\frac{1}{\rho}\nabla p\,.$$

It points towards low pressure.

(2) The force due to gravity acts vertically downward, towards the centre of the earth. Per unit mass, it is:

$$\mathbf{g}^* = -g\mathbf{k}\,.$$

The star on g* will be explained below.

(3) The force of friction acts in a direction opposite to the velocity of the flow. We could model it as

$$\mathbf{F}_f = -\nu\nabla^2\mathbf{V}, \qquad \text{or} \qquad \mathbf{F}_f = -\kappa\mathbf{V}\,.$$

The friction coefficient κ will depend on position and, perhaps, on velocity.

Equations in an Inertial Frame

Independent Variables: Space and time , r and t.
Dependent Variables: $\mathbf{V} = (u, v, w)$, p, ρ and T.

The basic equations of motion ($\mathbf{a} = \mathbf{F}/m$) are:

$$\frac{d\mathbf{V}}{dt} = -\frac{1}{\rho}\nabla p + \mathbf{g}^\star + \mathbf{F}_f \qquad (1)$$

where the total, material or Lagrangian derivative is

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \mathbf{V}\cdot\nabla$$

which measures the change with time of a variable moving along with the fluid flow.

The continuity equation, representing the conservation of mass, may be written in Lagrangian form:

$$\frac{d\rho}{dt} + \rho\nabla\cdot\mathbf{V} = 0\,.$$

If the fluid is *incompressible*, it is especially simple:

$$\nabla\cdot\mathbf{V} = 0 \qquad (2)$$

A Complete System

If we assume incompressible, inviscid flow, equations (1) and (2) comprise a system of four equations for the four variables $(u, v, w; p)$:

$$\frac{d\mathbf{V}}{dt} = -\frac{1}{\rho}\nabla p + \mathbf{g}^\star$$
$$\nabla\cdot\mathbf{V} = 0$$

Written out in *(local) cartesian coordinates*, they are

$$\left(\frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}\right)u = -\frac{1}{\rho}\frac{\partial p}{\partial x}$$
$$\left(\frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}\right)v = -\frac{1}{\rho}\frac{\partial p}{\partial y}$$
$$\left(\frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}\right)w = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g$$
$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0\,.$$

Rotating Coordinate Frames

Theorem: Consider a vector $\mathbf{A}$ fixed in a frame which is rotating with constant angular velocity Ω. Then the rate of change of $\mathbf{A}$ is

$$\frac{d\mathbf{A}}{dt} = \mathbf{\Omega}\times\mathbf{A}\,.$$

(see, e.g., Synge and Griffith, pg. 278.)

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The vector rotates through an angle $\Omega\Delta t$ in time Δt.

The projection of $\mathbf{A}$ on the Ω-axis does not change

The projection of $\mathbf{A}$ in the X-Y-plane is $A\sin\theta$. It does not change in magnitude, but its direction changes (see Figure). We have
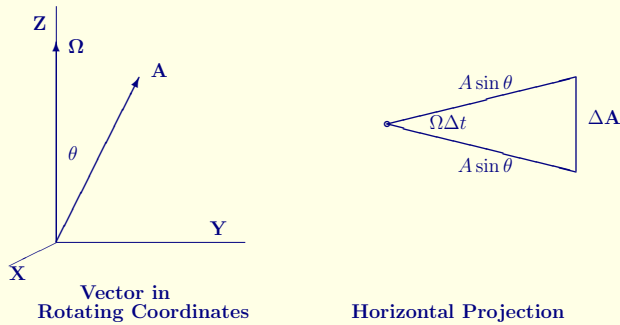
$$\Delta\mathbf{A} = (\Omega A\sin\theta\,\hat{\mathbf{n}})\cdot\Delta t$$

where $\hat{\mathbf{n}}$ is a unit vector perpendicular to both $\mathbf{\Omega}$ and $\mathbf{A}$.
Thus

$$\frac{d\mathbf{A}}{dt} = \mathbf{\Omega}\times\mathbf{A}\,.$$

z Ω

A

θ

Y

X

Vector in
Rotating Coordinates

$A\sin\theta$

$\Omega\Delta t$ ΔA

$A\sin\theta$

Horizontal Projection

Consider a vector
$$\mathbf{A} = A_1\mathbf{i} + A_2\mathbf{j} + A_3\mathbf{k}\,.$$
The rate of change of **A** in the rotating frame is
$$\left(\frac{d\mathbf{A}}{dt}\right)_{\mathrm{R}} = \frac{dA_1}{dt}\mathbf{i} + \frac{dA_2}{dt}\mathbf{j} + \frac{dA_3}{dt}\mathbf{k}\,.$$
The rate of change of **A** in the inertial frame is
$$\left(\frac{d\mathbf{A}}{dt}\right)_{\mathrm{I}} = \left(\frac{dA_1}{dt}\mathbf{i} + \frac{dA_2}{dt}\mathbf{j} + \frac{dA_3}{dt}\mathbf{k}\right) + \left(A_1\frac{d\mathbf{i}}{dt} + A_2\frac{d\mathbf{j}}{dt} + A_3\frac{d\mathbf{k}}{dt}\right)\,.$$
The changes in the unit vectors are (by the above theorem)
$$\frac{d\mathbf{i}}{dt} = \boldsymbol{\Omega} \times \mathbf{i}\,; \quad \frac{d\mathbf{j}}{dt} = \boldsymbol{\Omega} \times \mathbf{j}\,; \quad \frac{d\mathbf{k}}{dt} = \boldsymbol{\Omega} \times \mathbf{k}\,.$$
Therefore,
$$\left(\frac{d\mathbf{A}}{dt}\right)_{\mathrm{I}} = \left(\frac{dA_1}{dt}\mathbf{i} + \frac{dA_2}{dt}\mathbf{j} + \frac{dA_3}{dt}\mathbf{k}\right) + \boldsymbol{\Omega} \times (A_1\mathbf{i} + A_2\mathbf{j} + A_3\mathbf{k})\,.$$

Thus, the relationship between the relative and absolute rates of change of **A** is:
$$\left(\frac{d\mathbf{A}}{dt}\right)_{\mathrm{I}} = \left(\frac{d\mathbf{A}}{dt}\right)_{\mathrm{R}} + \boldsymbol{\Omega} \times \mathbf{A}\,. \qquad (*)$$

$\star \qquad \star \qquad \star$

Now let **A** be the position vector **r**. Since $(d\mathbf{r}/dt)_{\mathrm{I}} = \mathbf{V}_{\mathrm{I}}$ and $(d\mathbf{r}/dt)_{\mathrm{R}} = \mathbf{V}_{\mathrm{R}}$, we get:
$$\mathbf{V}_{\mathrm{I}} = \mathbf{V}_{\mathrm{R}} + \boldsymbol{\Omega} \times \mathbf{r}\,,$$
which relates the relative velocity to that in the inertial frame:
$$\begin{bmatrix}\textbf{Inertial} \\ \textbf{Velocity}\end{bmatrix} = \begin{bmatrix}\textbf{Relative} \\ \textbf{Velocity}\end{bmatrix} + \begin{bmatrix}\textbf{Velocity} \\ \textbf{of Frame}\end{bmatrix}$$

$\star \qquad \star \qquad \star$

Exercise

If $\mathbf{A} = \mathbf{r}_0$ is a point *fixed* in the rotating frame, the velocity in the absolute frame is
$$\mathbf{V} = \boldsymbol{\Omega} \times \mathbf{r_0}\,.$$

Find the absolute velocity of a point on the earth's surface (i) at the Equator, and (ii) at $60°$ North.

$\star \qquad \star \qquad \star$

The rotation of the earth (assuming solar day $\approx$ siderial day) is:
$$\boldsymbol{\Omega} = 1\,\text{rev. per day} = \frac{2\pi}{24 \times 60 \times 60}\,\text{rad/sec} = 7.29 \times 10^{-5}\,\text{s}^{-1}$$
The radius of the earth is
$$a = \frac{2 \times 10^7}{\pi}\,\text{m} \approx 6.37 \times 10^6\,\text{m}\,.$$
(i) At the equator, $\phi = 0°$ and $\theta = 90°$, so that $\sin\theta = 1$ and
$$\boldsymbol{\Omega} \times \mathbf{r_0} = \Omega a = (7.29 \times 10^{-5}\,\text{s}^{-1}) \times (6.37 \times 10^6\,\text{m}) = 4.64 \times 10^2\,\text{m s}^{-1} \sim 1000\,\text{m.p.h.}$$
(ii) At $\phi = 60°$, we have $\theta = 30°$, so that $\sin\theta = 0.5$, and the value of the velocity due to the earth's rotation is half that at the equator.

Relative Acceleration

Recall from (*) above that

$$\left(\frac{d\mathbf{A}}{dt}\right)_{\mathrm{I}} = \left(\frac{d\mathbf{A}}{dt}\right)_{\mathrm{R}} + \mathbf{\Omega} \times \mathbf{A}\,.$$

Now let A be the absolute velocity $\mathbf{V}_{\mathrm{I}} = \mathbf{V}_{\mathrm{R}} + \mathbf{\Omega} \times \mathbf{r}$:

$$\begin{aligned}
\left(\frac{d\mathbf{V}_{\mathrm{I}}}{dt}\right)_{\mathrm{I}} &= \left(\frac{d\mathbf{V}_{\mathrm{R}}}{dt}\right)_{\mathrm{R}} + \left(\frac{d\mathbf{\Omega} \times \mathbf{r}}{dt}\right)_{\mathrm{R}} + \mathbf{\Omega} \times \mathbf{V}_{\mathrm{R}} + \mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{r}) \\
&= \left(\frac{d\mathbf{V}_{\mathrm{R}}}{dt}\right)_{\mathrm{R}} + 2\mathbf{\Omega} \times \mathbf{V}_{\mathrm{R}} + \mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{r})\,.
\end{aligned}$$

The term $\mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{r})$ is called the <u>centrifugal acceleration</u>. Since it depends only on position, it can be combined with the gravitational acceleration to give an **apparent gravitational attraction**

$$\mathbf{g} = \mathbf{g}^{\star} - \mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{r})\,.$$

This is a small adjustment to the true gravitational acceleration.

Exercise: Centrifugal Acceleration

Calculate the magnitude of the centrifugal acceleration at the Equator and compare it to the magnitude of the gravitational acceleration.

$$\star \qquad \star \qquad \star$$

First note that

$$\mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{r}) = -\Omega^2 \mathbf{R}\,,$$

where R is the projection of r on the equatorial plane. Thus, near the earth's surface, the magnitude of the centrifugal acceleration is

$$\Omega^2 a = (7.29 \times 10^{-5}\,\mathrm{s}^{-1})^2 \times (6.37 \times 10^6\,\mathrm{m}) = 3.18 \times 10^{-2}\,\mathrm{m\,s}^{-2}$$

Now, comparing with true gravity, the percentage correction is

$$\frac{\Omega^2 a}{g} \times 100 \approx 0.3\%\,.$$

The centrifugal acceleration is responsible for the **flattened form of the earth**, which assumes an <u>oblate spheroidal</u> shape.

Do you lose weight when you travel to the Tropics? If so, how much?

The Coriolis Acceleration

The term $2\mathbf{\Omega} \times \mathbf{V}$ is called the Coriolis acceleration. It varies linearly with the speed V and is perpendicular to the velocity $\mathbf{V}$

The Coriolis acceleration is of primary importance for the dynamics of the atmosphere. It is a dominant factor in all large-scale weather systems.

Since $2\mathbf{\Omega} \times \mathbf{V}$ has no component in the direction of motion, it does no work.

However, once the air is moving, it is subject to the deflecting effect of this term. This is why the atmospheric flow is *predominantly rotational* in character.

Exercises

(1) Calculate the deflection of a golf ball travelling for 10 seconds at 10 m/s. Assume a latitude of $60°$N, and make reasonable assumptions to simplify the problem.

(2) Suppose the pressure at Cork is $1014\,\mathrm{hPa}$ and at Sligo is $1008\,\mathrm{hPa}$. Take the distance between Cork and Sligo to be $330\,\mathrm{km}$. Assume the isobars are east-west, and assume that Cork and Sligo are on the same meridian.
Calculate the acceleration due to the pressure gradient (assume $\rho = 1.2\,\mathrm{kg\,m}^{-3}$).
What wind speed would give a Coriolis acceleration of the same magnitude (take $2\Omega \sin\phi = 10^{-4}\,\mathrm{s}^{-1}$)?

[For further problems, see Holton, Chapter 1.]

Alternative Treatment of Rotation

Let (X, Y, Z) be the coordinates in a **fixed frame**, and (x, y, z) be those in a rotating frame. Suppose the rotating frame spins about the Z-axis with angular velocity Ω. The coordinates are related by

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} \cos \Omega t & -\sin \Omega t & 0 \\ \sin \Omega t & \cos \Omega t & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}.$$

If we differentiate with respect to time, we get

$$\begin{pmatrix} \dot{X} \\ \dot{Y} \\ \dot{Z} \end{pmatrix} = \begin{pmatrix} \cos \Omega t & -\sin \Omega t & 0 \\ \sin \Omega t & \cos \Omega t & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \dot{x} - \Omega y \\ \dot{y} + \Omega x \\ \dot{z} \end{pmatrix}.$$

If we differentiate once again, we get

$$\begin{pmatrix} \ddot{X} \\ \ddot{Y} \\ \ddot{Z} \end{pmatrix} = \begin{pmatrix} \cos \Omega t & -\sin \Omega t & 0 \\ \sin \Omega t & \cos \Omega t & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \ddot{x} - 2\Omega \dot{y} - \Omega^2 x \\ \ddot{y} + 2\Omega \dot{x} - \Omega^2 y \\ \ddot{z} \end{pmatrix}.$$

The equations of motion in non-rotating coordinates are

$$\ddot{X} = F_X, \qquad \ddot{Y} = F_Y, \qquad \ddot{Z} = F_Z.$$

Substituting from the matrix equation, we get

$$(\ddot{x} - 2\Omega \dot{y} - \Omega^2 x) \cos \Omega t - (\ddot{y} + 2\Omega \dot{x} - \Omega^2 y) \sin \Omega t = F_X$$
$$(\ddot{x} - 2\Omega \dot{y} - \Omega^2 x) \sin \Omega t + (\ddot{y} + 2\Omega \dot{x} - \Omega^2 y) \cos \Omega t = F_Y$$
$$\ddot{z} = F_Z$$

Solving for the terms with $\ddot{x}$ and $\ddot{y}$, we get

$$\ddot{x} - 2\Omega \dot{y} - \Omega^2 x = \cos \Omega t F_X + \sin \Omega t F_Y \equiv F_x$$
$$\ddot{y} + 2\Omega \dot{x} - \Omega^2 y = \cos \Omega t F_Y - \sin \Omega t F_X \equiv F_y$$
$$\ddot{z} = F_Z \equiv F_z$$

Thus, the rotation introduces <u>additional terms</u>:
The terms $-2\Omega \dot{y}$ and $2\Omega \dot{x}$ are the **Coriolis acceleration.**
The terms $\Omega^2 x$ and $\Omega^2 y$ are the **centrifugal acceleration.**