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Weather Prediction by Numerical Process

> *Perhaps some day in the dim future it will be possible to advance the computations faster than the weather advances and at a cost less than the saving to mankind due to the information gained. But that is a dream. (WPNP, p.* vii*; Dover Edn., p.* xi*)*

Lewis Fry Richardson's extraordinary book *Weather Prediction by Numerical Process*, published in 1922, is a strikingly original scientific work, one of the most remarkable books on meteorology ever written. In this book—to which we will refer briefly as WPNP—Richardson constructed a systematic mathematical method for predicting the weather and demonstrated its application by carrying out a trial forecast. History has shown that his innovative ideas were fundamentally sound: the methodology proposed by him is essentially that used in practical weather forecasting today. However, the method devised by Richardson was utterly impractical at the time of its publication and the results of his trial forecast appeared to be little short of outlandish. As a result, his ideas were eclipsed for decades and his wonderful opus gathered dust and was all but forgotten.

1.1 The problem

Imagine you are standing by the ocean shore, watching the sea rise and fall as wave upon wave breaks on the rocks. At a given moment the water is rising at a rate of one metre per second—soon it will fall again. Is there an ebb or a flood tide? Suppose you use the observed rate of change and extrapolate it over the six hours that elapse between tidal extremes; you will obtain an extraordinary prediction: the water level should rise by some 20 km, twice the height of Mt Everest. This forecast is meaningless! The water level is governed by physical processes with a wide range of time-scales. The tidal variations, driven by lunar gravity, have a period of around twelve hours, linked to the Earth's rotation. But wind-driven

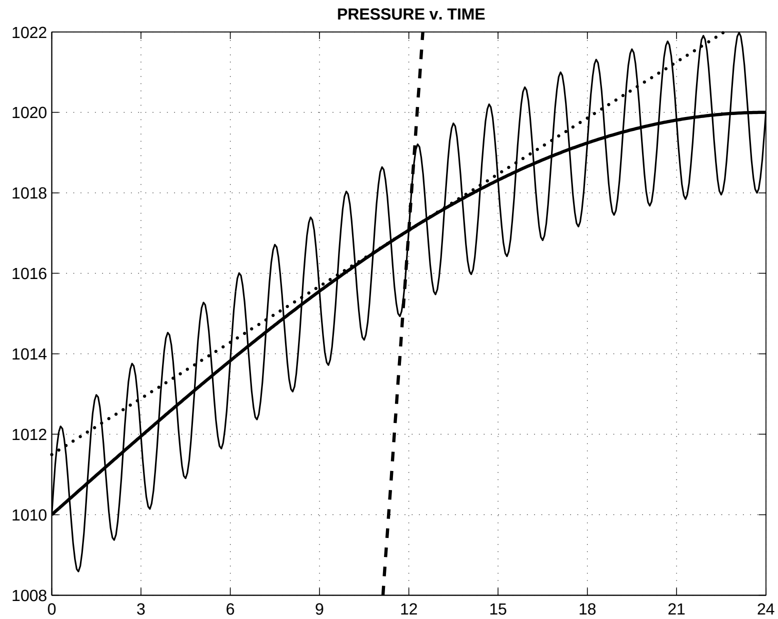

Fig. 1.1. Schematic illustration of pressure variation over a 24 hour period. The thick line is the mean, long-term variation, the thin line is the actual pressure, with high frequency noise. The dotted line shows the rate of change, at 12 hours, of the mean pressure and the dashed line shows the corresponding rate of change of the actual pressure (after Phillips, 1973).

waves and swell vary on a time-scale of seconds. The instantaneous change in level due to a wave is no guide to the long-term tidal variations: if the observed rise is extrapolated over a period much longer than the time-scale of the wave, the resulting forecast will be calamitous.

In 1922 Richardson presented such a forecast to the world. He calculated a change of atmospheric pressure, for a particular place and time, of 145 hPa in 6 hours. This was a totally unrealistic value, too large by two orders of magnitude. The prediction failed for reasons similar to those that destroy the hypothetical tidal forecast. The spectrum of motions in the atmosphere is analogous to that of the ocean: there are long-period variations dominated by the effects of the Earth's rotation—these are the meteorologically significant rotational modes—and short-period oscillations called gravity waves, having speeds comparable to that of sound. The interaction between the two types of variation is weak, just as is the interaction between wind-waves and tidal motions in the ocean; and, for many purposes, the gravity waves, which are normally of small amplitude, may be treated as irrelevant noise.

Although they have little effect on the long-term evolution of the flow, gravity waves may profoundly influence the way it changes on shorter time-scales. Fig. 1.1 (after Phillips, 1973) schematically depicts the pressure variation over a period of one day. The smooth curve represents the variation due to meteorological effects; its gentle slope (dotted line) indicates the long-term change. The rapidly varying curve represents the actual pressure changes when gravity waves are superimposed on the meteorological flow: the slope of the oscillating curve (dashed line) is precipitous and, if used to determine long-range variations, yields totally misleading results. What Richardson calculated was the instantaneous rate of change in pressure for an atmospheric state having gravity wave components of large amplitude. This tendency, $\partial p/\partial t \approx 0.7\,\mathrm{Pa\,s^{-1}}$, was a sizeable but not impossible value. Such variations are observed over short periods in intense, localized weather systems.[1] The problem arose when Richardson used the computed value in an attempt to deduce the long-term change. Multiplying the calculated tendency by a time step of six hours, he obtained the unacceptable value quoted above. The cause of the failure is this: *the instantaneous pressure tendency does not reflect the long-term change.*

This situation looks hopeless: how are we to make a forecast if the tendencies calculated using the basic equations of motion do not guide us? There are several possible ways out of the dilemma; their success depends crucially on the decoupling between the gravity waves and the motions of meteorological significance—we can distort the former without seriously corrupting the latter.

The most obvious approach is to construct a forecast by combining many time steps which are short enough to enable accurate simulation of the detailed high frequency variations depicted schematically in Fig. 1.1. The existence of these high frequency solutions leads to a stringent limitation on the size of the time step for accurate results; this limitation or *stability criterion* was discovered in a different context by Hans Lewy in Göttingen in the 1920s (see Reid, 1976), and was first published in Courant *et al.* (1928). Thus, although these oscillations are not of meteorological interest, their presence severely limits the range of applicability of the tendency calculated at the initial time. Small time steps are required to represent the rapid variations and ensure accuracy of the long-term solution. If such small steps are taken, the solution will contain gravity-wave oscillations about an essentially correct meteorological flow. One implication of this is that, if Richardson could have extended his calculations, taking a large number of small steps, his results would have been noisy but the mean values would have been meteorologically reasonable (Phillips, 1973). Of course, the attendant computational burden made this impossible for Richardson.

[1] For example, Loehrer & Johnson (1995) reported a surface pressure drop of $4\,\mathrm{hPa}$ in five minutes in a mesoscale convective system, or $\partial p/\partial t \approx -1.3\,\mathrm{Pa\,s^{-1}}$.

The second approach is to modify the governing equations in such a way that the gravity waves no longer occur as solutions. This process is known as filtering the equations. The approach is of great historical importance. The first successful computer forecasts (Charney, *et al.*, 1950) were made with the barotropic vorticity equation (see Ch. 10) which has low frequency but no high frequency solutions. Later, the quasi-geostrophic equations were used to construct more realistic filtered models and were used operationally for many years. An interesting account of the development of this system appeared in Phillips (1990). The quasi-geostrophic equations are still of great theoretical interest (Holton, 2004) but are no longer considered to be sufficiently accurate for numerical prediction.

The third approach is to adjust the initial data so as to reduce or eliminate the gravity wave components. The adjustments can be small in amplitude but large in effect. This process is called *initialization*, and it may be regarded as a form of smoothing. Richardson realized the requirement for smoothing the initial data and devoted a chapter of WPNP to this topic. We will examine several methods of initialization in this work, in particular in Chapter 8, and will show that the digital filtering initialization method yields realistic tendencies when applied to Richardson's data.

The absence of gravity waves from the initial data results in reasonable initial rates of change, but it does not automatically allow the use of large time steps. The existence of high frequency solutions of the governing equations imposes a severe restriction on the size of the time step allowable if reasonable results are to be obtained. The restriction can be circumvented by treating those terms of the equations that govern gravity waves in a numerically implicit manner; this distorts the structure of the gravity waves but not of the low frequency modes. In effect, implicit schemes slow down the faster waves thus removing the cause of numerical instability (see §5.2 below). Most modern forecasting models avoid the pitfall that trapped Richardson by means of initialization followed by semi-implicit integration.

1.2 Vilhelm Bjerknes and scientific forecasting

At the time of the First World War, weather forecasting was very imprecise and unreliable. Observations were scarce and irregular, especially for the upper air and over the oceans. The principles of theoretical physics played a relatively minor role in practical forecasting: the forecaster used crude techniques of extrapolation, knowledge of climatology and guesswork based on intuition; forecasting was more an art than a science. The observations of pressure and other variables were plotted in symbolic form on a weather map and lines were drawn through points with equal pressure to reveal the pattern of weather systems—depressions, anticyclones,

Fig. 1.2. A recent painting (from photographs) of the Norwegian scientist Vilhelm Bjerknes (1862–1951) standing on the quay in Bergen (©*Geophysical Institute, Bergen*. Artist: Rolf Groven).

troughs and ridges. The concept of *fronts*, surfaces of discontinuity between warm and cold airmasses, had yet to emerge. The forecaster used his experience, memory of similar patterns in the past and a menagerie of empirical rules to produce a forecast map. Particular attention was paid to the reported pressure changes or tendencies; to a great extent it was assumed that what had been happening up to now would continue for some time. The primary physical process attended to by the forecaster was *advection*, the transport of fluid characteristics and properties by the movement of the fluid itself.

The first explicit analysis of the weather prediction problem from a scientific viewpoint was undertaken at the beginning of the twentieth century when the Norwegian scientist Vilhelm Bjerknes set down a two-step plan for rational forecasting (Bjerknes, 1904):

> If it is true, as every scientist believes, that subsequent atmospheric states develop from the preceding ones according to physical law, then it is apparent that the necessary and sufficient conditions for the rational solution of forecasting problems are the following:

1. A sufficiently accurate knowledge of the state of the atmosphere at the initial time.
2. A sufficiently accurate knowledge of the laws according to which one state of the atmosphere develops from another.

Bjerknes used the medical terms *diagnostic* and *prognostic* for these two steps (Friedman, 1989). The diagnostic step requires adequate observational data to define the three-dimensional structure of the atmosphere at a particular time. There was a severe shortage of observations, particularly over the seas and for the upper air, but Bjerknes was optimistic:

> We can hope . . . that the time will soon come when either as a daily routine, or for certain designated days, a complete diagnosis of the state of the atmosphere will be available. The first condition for putting forecasting on a rational basis will then be satisfied.

In fact, such designated days, on which upper air observations were made throughout Europe, were organised around that time by the International Commission for Scientific Aeronautics.

The second or prognostic step was to be taken by assembling a set of equations, one for each dependent variable describing the atmosphere. Bjerknes listed seven basic variables: pressure, temperature, density, humidity and three components of velocity. He then identified seven independent equations: the three hydrodynamic equations of motion, the continuity equation, the equation of state and the equations expressing the two laws of thermodynamics. (As pointed out by Eliassen (1999), Bjerknes was in error in listing the second law of thermodynamics; he should instead have specified a continuity equation for water substance.) Bjerknes knew that an exact analytical integration was beyond our ability. His idea was to represent the initial state of the atmosphere by a number of charts giving the distribution of the variables at different levels. Graphical or mixed graphical and numerical methods, based on the fundamental equations, could then be applied to construct a new set of charts describing the state of the atmosphere, say, three hours later. This process could be repeated until the desired forecast length was reached. Bjerknes realized that the prognostic procedure could be conveniently separated into two stages, a purely hydrodynamic part and a purely thermodynamic part; the hydrodynamics would determine the movement of an airmass over the time interval and thermodynamic considerations could then be used to deduce changes in its state. He concluded:

> It may be possible some day, perhaps, to utilise a method of this kind as the basis for a daily practical weather service. But however that may be, the fundamental scientific study of atmospheric processes sooner or later has to follow a method based upon the laws of mechanics and physics.

Bjerknes' speculations are reminiscent of Richardson's 'dream' of practical scientific weather forecasting.

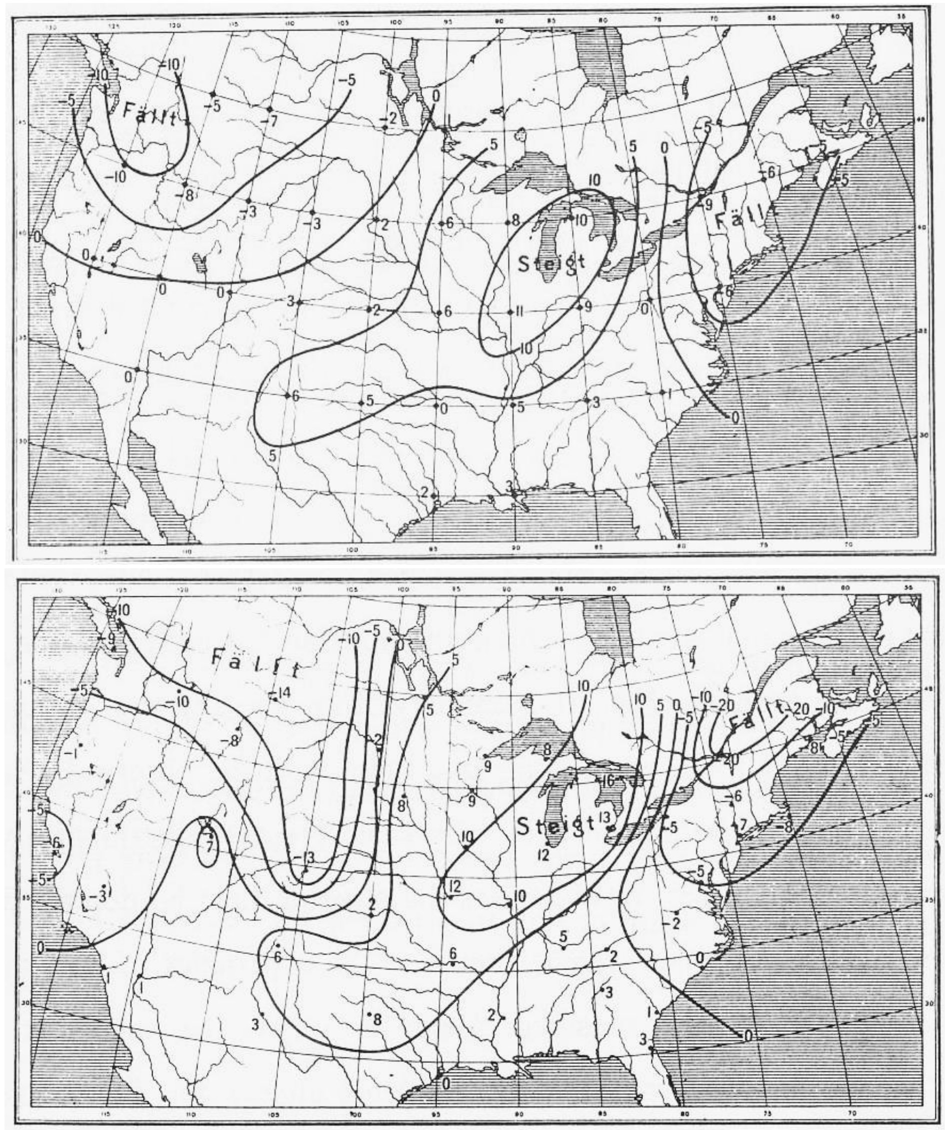

Fig. 1.3. Top: Exner's calculated pressure change between 8 p.m. and midnight, 3 January, 1895. Bottom: Observed pressure change for the same period [Units: Hundreths of an inch of mercury. *Steigt*=rises; *Fällt*=falls] (Exner, 1908).

A tentative first attempt at mathematically forecasting synoptic changes by the application of physical principles was made by Felix Exner, working in Vienna. His account (Exner, 1908) appeared only four years after Bjerknes' seminal paper. Exner makes no reference to Bjerknes' work, which was also published in *Meteo-*

rologische Zeitschrift. Though he may be presumed to have known about Bjerknes' ideas, Exner followed a radically different line: whereas Bjerknes proposed that the full system of hydrodynamic and thermodynamic equations be used, Exner's method was based on a system reduced to the essentials. He assumed that the atmospheric flow is geostrophically balanced and that the thermal forcing is constant in time. Using observed temperature values, he deduced a mean zonal wind. He then derived a prediction equation representing advection of the pressure pattern with constant westerly speed, modified by the effects of diabatic heating. It yielded a realistic forecast in the case illustrated in Exner's paper. Fig. 1.3 shows his calculated pressure change (top) and the observed change (bottom) over the four hour period between 8 p.m. and midnight on 3 January, 1895; there is reasonable agreement between the predicted and observed changes. However, the method could hardly be expected to be of general utility. Exner took pains to stress the limitations of his method, making no extravagant claims for it. But despite the very restricted applicability of the technique devised by him, the work is deserving of attention as a first attempt at systematic, scientific weather forecasting. Exner's numerical method was summarized in his textbook (Exner, 1917, §70). The only reference by Richardson to the method was a single sentence (WPNP, p. 43) 'F. M. Exner has published a prognostic method based on the source of air supply.' It would appear from this that Richardson was not particularly impressed by it!

In 1912 Bjerknes became the first Director of the new Geophysical Institute in Leipzig. In his inaugural lecture he returned to the theme of scientific forecasting. He observed that 'physics ranks among the so-called exact sciences, while one may be tempted to cite meteorology as an example of a radically inexact science.' He contrasted the methods of meteorology with those of astronomy, for which predictions of great accuracy are possible, and described the programme of work upon which he had already embarked: *to make meteorology into an exact physics of the atmosphere.* Considerable advances had been made in observational meteorology during the previous decade, so that now the diagnostic component of his two-step programme had become feasible.

> . . . now that complete observations from an extensive portion of the free air are being published in a regular series, a mighty problem looms before us and we can no longer disregard it. We must apply the equations of theoretical physics not to ideal cases only, but to the actual existing atmospheric conditions as they are revealed by modern observations. These equations contain the laws according to which subsequent atmospheric conditions develop from those that precede them. It is for us to discover a method of practically utilising the knowledge contained in the equations. From the conditions revealed by the observations we must learn to compute those that will follow. The problem of accurate pre-calculation that was solved for astronomy centuries ago must now be attacked in all ernest for meteorology (Bjerknes, 1914a).

Bjerknes expressed his conviction that the acid test of a science is its utility in forecasting: ‘There is after all but one problem worth attacking, *viz.*, the precalculation of future conditions.’ He recognised the complexity of the problem and realized that a rational forecasting procedure might require more time than the atmosphere itself takes to evolve, but concluded:

> If only the calculation shall agree with the facts, the scientific victory will be won. Meteorology would then have become an exact science, a true physics of the atmosphere. When that point is reached, *then* the practical results will soon develop.
>
> It may require many years to bore a tunnel through a mountain. Many a labourer may not live to see the cut finished. Nevertheless this will not prevent later comers from riding through the tunnel at express-train speed.

At Leipzig Bjerknes instigated the publication of a series of weather charts based on the data that were collected during the internationally-agreed intensive observation days and compiled and published by Hugo Hergesell in Strasbourg (these charts are discussed in detail in Chapter 6 below). One such publication (Bjerknes, 1914b), together with the ‘raw data’ in Hergesell (1913), was to provide Richardson with the initial conditions for his forecast.

Richardson first heard of Bjerknes’ plan for rational forecasting in 1913, when he took up employment with the Meteorological Office. In the Preface to WPNP he writes

> The extensive researches of V. Bjerknes and his School are pervaded by the idea of using the differential equations for all that they are worth. I read his volumes on *Statics* and *Kinematics* soon after beginning the present study, and they have exercised a considerable influence throughout it.

Richardson’s book opens with a discussion of then-current practice in the Met Office. He describes the use of an Index of Weather Maps, constructed by classifying old synoptic charts into categories. The Index (Gold, 1919) assisted the forecaster to find previous maps resembling the current one and therewith to deduce the likely development by studying the evolution of these earlier cases:

> The forecast is based on the supposition that what the atmosphere did then, it will do again now. There is no troublesome calculation, with its possibilities of theoretical or arithmetical error. The past history of the atmosphere is used, so to speak, as a full-scale working model of its present self (WPNP, p. *vii*; Dover Edn., p. *xi*).

Bjerknes had contrasted the precision of astronomical prediction with the ‘radically inexact’ methods of weather forecasting. Richardson returned to this theme in his Preface:

> —the *Nautical Almanac,* that marvel of accurate forecasting, is not based on the principle that astronomical history repeats itself in the aggregate. It would be safe to say that a particular disposition of stars, planets and satellites never occurs twice. Why then should we expect a present weather map to be exactly represented in a catalogue of past weather? . . . This alone is sufficient reason for presenting, in this book, a scheme of weather prediction which resembles the process by which the *Nautical Almanac* is produced, in so far as it is founded

upon the differential equations and not upon the partial recurrence of phenomena in their ensemble.

Richardson's forecasting scheme amounts to a precise and detailed implementation of the prognostic component of Bjerknes' programme. It is a highly intricate procedure: as Richardson observed, 'the scheme is complicated because the atmosphere is complicated.' It also involved an enormous volume of numerical computation and was quite impractical in the pre-computer era. But Richardson was undaunted, expressing his dream that 'some day in the dim future it will be possible to advance the computations faster than the weather advances'. Today, forecasts are prepared routinely on powerful computers running algorithms that are remarkably similar to Richardson's scheme — his dream has indeed come true.

Before discussing Richardson's forecast in more detail, we will digress briefly to consider his life and work from a more general viewpoint.

1.3 Outline of Richardson's life and work

Richardson's life and work are discussed in a comprehensive and readable biography (Ashford, 1985). The Royal Society Memoir of Gold (1954) provides a more succinct description and the *Collected Papers* of Richardson, edited by Drazin (LFR I) and Sutherland (LFR II), include a biographical essay by Hunt (1993); see also Hunt (1998). Brief introductions to Richardson's work in meteorology (by Henry Charnock), in numerical analysis (by Leslie Fox) and on fractals (by Philip Drazin) are also included in Volume 1 of the *Collected Papers*. The article by Chapman (1965) is worthy of attention and some fascinating historical background material may be found in the review by Platzman (1967). In a recent popular book on mathematics, Körner (1996) devotes two chapters (69 pages) to various aspects of Richardson's mathematical work. The National Cataloguing Unit for the Archives of Contemporary Scientists has produced a comprehensive catalogue of the papers and correspondence of Richardson, which were deposited by Oliver Ashford in Cambridge University Library (NCUACS, 1993). The following sketch of Richardson's life is based primarily on Ashford's book.

Lewis Fry Richardson was born in 1881, the youngest of seven children of David Richardson and Catherine Fry, both of whose families had been members of the Society of Friends for generations. He was educated at Bootham, the Quaker school in York, where he showed an early aptitude for mathematics, and at Durham College of Science in Newcastle. He entered King's College, Cambridge in 1900 and graduated with a First Class Honours in the Natural Science Tripos in 1903. In 1909 he married Dorothy Garnett. They had no offspring but adopted two sons and a daughter, Olaf (1916–1983), Stephen (b. 1920) and Elaine (b. 1927).

Over the ten years following his graduation, Richardson held several short re-

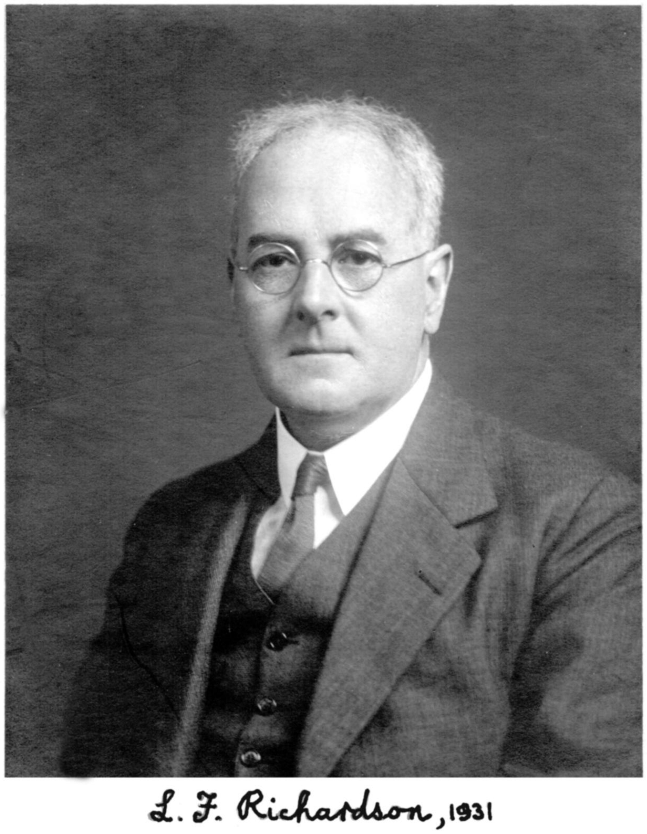

Fig. 1.4. Lewis Fry Richardson (1881–1953). Photograph by Walter Stoneman, 1931, when Richardson was aged 50 (copy of photograph courtesy of Oliver Ashford).

search posts (Appendix 2 contains a chronology of the milestones of his life and career). As a scientist with National Peat Industries, he investigated the optimum method of cutting drains to remove water from peat bogs. The problem was formulated in terms of Laplace’s equation on an irregularly-shaped domain. As this partial differential equation is not soluble by analytical means, except in special cases, he devised an approximate graphical method of solving it. More significantly, he then constructed a finite difference method for solving such systems and described this more powerful and flexible method in a comprehensive report (Richardson, 1910).

Around 1911, Richardson began to think about the application of his finite difference approach to the problem of forecasting the weather. He stated in the Preface of WPNP that the idea first came to him in the form of a fanciful idea about a forecast factory, to which we will return in the final chapter. Richardson began serious work on weather prediction in 1913 when he joined the Met Office and was appointed Superintendent of Eskdalemuir Observatory, at an isolated location in Dumfrieshire in the Southern Uplands of Scotland. In May 1916 he resigned from the Met Office in order to work with the Friends Ambulance Unit (FAU) in France. There he spent over two years as an ambulance driver, working in close proximity to the fighting and on occasions coming under heavy shell fire. He returned to England after the cessation of hostilities and was employed once again by the Met Office to work at Benson, between Reading and Oxford, with W. H. Dines. The conditions of his employment included *experiments with a view to forecasting by numerical process.* He also developed several ingenious instruments for making upper air observations. However, he was there only one year when the Office came under the authority of the Air Ministry, which also had responsibility for the Royal Air Force and, as a committed pacifist, he felt obliged to resign once more.

Richardson then obtained a post as a lecturer in mathematics and physics at Westminister Training College in London. His meteorological research now focussed primarily on atmospheric turbulence. Several of his publications during this period are still cited by scientists. In one of the most important — *The supply of energy from and to atmospheric eddies* (Richardson, 1920) — he derived a criterion for the onset of turbulence, introducing what is now known as the Richardson Number. In another, he investigated the separation of initially proximate tracers in a turbulent flow, and arrived empirically at his ‘four-thirds law’: the rate of diffusion is proportional to the separation raised to the power 4/3. This was later established more rigourously by Kolmogorov (1941) using dimensional analysis. Bachelor (1950) showed the consistency between Richardson’s four-thirds law and Kolmogorov’s similarity theory. A simple derivation of the four-thirds law using dimensional analysis is given by Körner (1996).

In 1926 Richardson was elected a Fellow of the Royal Society. Around that time he made a deliberate break with meteorological research. He was distressed that his turbulence research was being exploited for military purposes. Moreover, he had taken a degree in psychology and wanted to apply his mathematical knowledge in that field. Among his interests was the quantitative measurement of human sensation such as the perception of colour. He established for the first time a logarithmic relationship between the perceived loudness and the physical intensity of a stimulus. In 1929 he was appointed Principal of Paisley Technical College, near Glasgow, and he worked there until his retirement in 1940.

From about 1935 until his death in 1953, Richardson thrust himself energetically into *peace studies*, developing mathematical theories of human conflict and the causes of war. Once again he produced ideas and results of startling originality. He pioneered the application of quantitative methods in this extraordinarily difficult area. As with his work in numerical weather prediction, the value of his efforts was not immediately appreciated. He produced two books, *Arms and Insecurity* (1947), a mathematical theory of arms races, and *Statistics of Deadly Quarrels* (1950) in which he amassed data on all wars and conflicts between 1820 and 1949 in a systematic collection. His aim was to identify and understand the causes of war, with the ultimate humanitarian goal of preventing unnecessary waste of life. However, he was unsuccessful in finding a publisher for these books (the dates refer to the original microfilm editions). The books were eventually published posthumously in 1960, thanks to the efforts of Richardson's son Stephen. These studies continue to be a rich source of ideas. A recent review of Richardson's theories of war and peace has been written by Hess (1995).

Richardson's genius was to apply quantitative methods to problems that had traditionally been regarded as beyond 'mathematicization', and the continuing relevance and usefulness of his work confirms the value of his ideas. He generally worked in isolation, moving frequently from one subject to another. He lacked constructive collaboration with colleagues and, perhaps as a result, his work had great individuality but was also somewhat idiosyncratic. G. I. Taylor (1959) spoke of him as 'a very interesting and original character who seldom thought on the same lines as his contemporaries and often was not understood by them'. Just as for his work in meteorology, Richardson's mathematical studies of the causes of war were ahead of their time. In a letter to *Nature* (Richardson, 1951) he posed the question of whether an arms race must necessarily lead to warfare. Reviewing this work, his biographer (Ashford, 1985, p. 223) wrote 'Let us hope that before long history will show that an arms race can indeed end without fighting.' Just four years later the collapse of the Soviet Union brought the nuclear arms race to an abrupt end.[2]

Richardson's Quaker background and pacifist convictions profoundly influenced the course of his career. Late in his life, he wrote of the 'persistent influence of the Society of Friends, with its solemn emphasis on public and private duty'. Because of his pacifist principles, he resigned twice from the Met Office, first to face battlefield dangers in the Friends Ambulance Unit in France and again when the Office came under the Air Ministry. He destroyed some of his research results to prevent their use for military purposes (Brunt, 1954) and even ceased meteorological research for a time: he published no papers in meteorology between 1930 and 1948. He retired early on a meagre pension to devote all his energies to peace studies. His work was misunderstood by many but his conviction and vision gave him courage to persist in the face of the indifference and occasional ridicule of his contemporaries.

[2] Stommel (1985) noted that the only purchaser of the book *Arms and Insecurity*, which Richardson was offering for sale on microfilm in 1948, was the Soviet Embassy in London!

Richardson made important contributions in several fields, the most important being atmospheric diffusion, numerical analysis, quantitative psychology and the mathematical study of the causes of war. He is remembered by meteorologists through the Richardson Number, a fundamental quantity in turbulence theory, and for his extraordinary vision in formulating the process of numerical forecasting. The approximate methods that he developed for the solution of differential equations are extensively used in the numerical treatment of physical problems.

Richardson's pioneering work in studying the mathematical basis of human conflict has led to the establishment of a large number of university departments devoted to this area. In the course of his peace studies, he digressed to consider the lengths of geographical borders and coastlines, and discovered the scaling properties such that the length increases as the unit of measurement is reduced. This work inspired Benoit Mandelbrot's development of the theory of fractals (Mandelbrot, 1982). In a tribute to Richardson shortly after his death, his wife Dorothy recalled that one of his sayings was 'Our job in life is to make things better for those who follow us. What happens to ourselves afterwards is not our concern.' Richardson had the privilege to make contributions to human advancement in several areas. The lasting value of his work is a testimony of his wish to serve his fellow man.

1.4 The origin of *Weather Prediction by Numerical Process*

Richardson first applied his approximate method for the solution of differential equations to investigate the stresses in masonry dams (Richardson, 1910), a problem on which he had earlier worked with the statistician Karl Pearson. But the method was completely general and he realized that it had potential for use in a wide range of problems. The idea of numerical weather prediction appears to have germinated in his mind for several years. In a letter to Pearson dated 6 April 1907 he wrote in reference to the method that 'there should be applications to meteorology one would think' (Ashford, 1985, p. 25). This is the first inkling of his interest in the subject. In the Preface to WPNP he wrote that the investigation of numerical prediction

> grew out of a study of finite differences and first took shape in 1911 as the fantasy which is now relegated to Ch. 11/2. Serious attention to the problem was begun in 1913 at Eskdalemuir Observatory, with the permission and encouragement of Sir Napier Shaw, then Director of the Met Office, to whom I am greatly indebted for facilities, information and ideas.

The fantasy was that of a forecast factory, which we will discuss in detail in the final chapter. Richardson had had little or no previous experience of meteorology when he took up his position as Superintendent of the Observatory in what Gold (1954) described as 'the bleak and humid solitude of Eskdalemuir'. Perhaps it was this lack of formal training in the subject that enabled him to approach the prob-

Fig. 1.5. Eskdalemuir Observatory in 1911. Office and Computing Room, where Richardson's dream began to take shape (Photograph from MC-1911).

lem of weather forecasting from such a breathtakingly original and unconventional angle. His plan was to express the physical principles that govern the behaviour of the atmosphere as a system of mathematical equations and to solve this system using his approximate finite difference method. The basic equations had already been identified by Bjerknes (1904) but with the error noted above: the second law of thermodynamics was specified instead of conservation of water substance. The same error was repeated in Bjerknes' inaugural address at Leipzig (Bjerknes, 1914a). While this may seem a minor matter it proves that, while Bjerknes outlined a general philosophical approach, he did not attempt to formulate a detailed procedure, or algorithm, for applying his method. Indeed, he felt that such an approach was completely impractical. The complete system of fundamental equations was, for the first time, set down in a systematic way in Ch. 4 of WPNP. The equations had to be simplified, using the hydrostatic assumption, and transformed to render them amenable to approximate solution. Richardson also introduced a plethora of extra terms to account for various physical processes not considered by Bjerknes.

By the time of his resignation in 1916, Richardson had completed the formulation of his scheme and had set down the details in the first draft of his book, then called *Weather Prediction by Arithmetic Finite Differences.* But he was not concerned merely with theoretical rigour and wished to include a fully worked example to demonstrate how the method could be put to use. This example

> was worked out in France in the intervals of transporting wounded in 1916–1918. During the battle of Champagne in April 1917 the working copy was sent to the rear, where it became lost, to be re-discovered some months later under a heap of coal (WPNP, p. *ix*; Dover Edn., p. *xiii*).

One may easily imagine Richardson's distress at this loss and the great relief that the re-discovery must have brought him.[3] It is a source of wonder that in the appalling conditions prevailing at the front he had the buoyancy of spirit to carry out one of the most remarkable and prodigious feats of calculation ever accomplished.

Richardson assumed that the state of the atmosphere at any point could be specified by seven numbers: pressure, temperature, density, water content and velocity components eastward, northward and upward. He formulated a description of atmospheric phenomena in terms of seven differential equations. To solve them, Richardson divided the atmosphere into discrete columns of extent 3° east-west and 200 km north-south, giving $120 \times 100 = 12{,}000$ columns to cover the globe. Each of these columns was divided vertically into five cells. The values of the variables were given at the centre of each cell, and the differential equations were approximated by expressing them in finite difference form. The rates of change of the variables could then be calculated by arithmetical means. Richardson calculated the *initial changes in two columns* over central Europe, one for mass variables and one for winds. This was the extent of his 'forecast'.

How long did it take Richardson to make his forecast? It is generally believed that he took six weeks for the task but, given the volume of results presented on his 23 computing forms, it is difficult to understand how the work could have been expedited in so short a time. The question was discussed in Lynch (1993), which is reproduced in Appendix 4. The answer is contained in §11/2 of WPNP, but is expressed in a manner that has led to confusion. On page 219, under the heading 'The Speed and Organization of Computing', Richardson wrote

> It took me the best part of six weeks to draw up the computing forms and to work out the new distribution in two vertical columns for the first time. My office was a heap of hay in a cold rest billet. With practice the work of an average computer might go perhaps ten times faster. If the time-step were 3 hours, then 32 individuals could just compute two points so as to keep pace with the weather.

Could Richardson really have completed his task in six weeks? Given that 32 computers[4] working at ten times his speed would require 3 hours for the job, he himself must have taken some 960 hours — that is 40 days or 'the best part of six weeks' working flat-out at 24 hours a day! At a civilized 40-hour week the forecast would have extended over six months. It is more likely that Richardson

[3] In an obituary notice, Brunt (1954) stated that the manuscript was lost not once but twice.

[4] Richardson's 'computers' were made not of silicon but of flesh and blood.

spent perhaps ten hours per week at his chore and that it occupied him for about two years, the greater part of his stay in France.

In 1919 Richardson added an introductory example (WPNP, Ch 2) in which he integrated a system equivalent to the linearized shallow water equations, starting from idealised initial conditions defined by a simple analytic formula. This was done at Benson where he had 'the good fortune to be able to discuss the hypotheses with Mr W. H. Dines'. The chapter ends with an acknowledgement to Dines for having read and criticised it. It seems probable that the inclusion of this example was suggested by Dines, who might have been more sensitive than Richardson to the difficulties that readers of WPNP would likely experience. The book was thoroughly revised in 1920–21 and was finally published by Cambridge University Press in 1922 at a price of 30 shillings (£1.50), the print run being 750 copies.

Richardson's book was certainly not a commercial success. Akira Kasahara has told me that he bought a copy from Cambridge University Press in 1955, more than thirty years after publication. The book was re-issued in 1965 as a Dover paperback and the 3,000 copies, priced at $2, about the same as the original hard-back edition, were sold out within a decade. The Dover edition was identical to the original except for a six-page introduction by Sydney Chapman. Following its appearance, a retrospective appraisal of Richardson's work by George Platzman was published in the Bulletin of the American Meteorological Society (Platzman, 1967; 1968). This scholarly review has been of immense assistance in the preparation of the present work.

The initial response to WPNP was unremarkable and must have been disappointing to Richardson. The book was widely reviewed with generally favourable comments—Ashford (1985) includes a good coverage of reactions—but the impracticality of the method and the abysmal failure of the solitary sample forecast inevitably attracted adverse criticism. Napier Shaw, reviewing the book for *Nature*, wrote that Richardson 'presents to us a *magnum opus* on weather prediction'. However, in regard to the forecast, he observed that the wildest guess at the pressure change would not have been wider of the mark. More importantly for our purposes, he questioned Richardson's conclusion that wind observations were the real cause of the error, and also his dismissal of the geostrophic wind. Edgar W. Woolard, a meteorologist with the U.S. Weather Bureau, wrote

> The book is an admirable study of an eminently important problem … a first attempt in this extraordinarily difficult and complex field … it indicates a line of attack on the problem, and invites further study with a view to improvement and extension. … It is sincerely to be hoped that the author will continue his excellent work along these lines, and that other investigators will be attracted to the field which he has opened up. The results cannot fail to be of direct practical importance as well as of immense scientific value.

However, other investigators were not attracted to the field, perhaps because the forecast failure acted as a deterrent, perhaps because the book was so difficult to read, with its encyclopædic but distracting range of topics. Alexander McAdie, Professor of Meteorology at Harvard, wrote 'It can have but a limited number of readers and will probably be quickly placed on a library shelf and allowed to rest undisturbed by most of those who purchase a copy' (McAdie, 1923). Indeed, this is essentially what happened to the book.

A most perceptive review by F. J. W. Whipple of the Met Office came closest to understanding Richardson's unrealistic forecast, postulating that rapidly-travelling waves contributed to its failure:

> The trouble that he meets is that quite small discrepancies in the estimate of the strengths of the winds may lead to comparatively large errors in the computed changes of pressure. It is very doubtful whether sufficiently accurate results will ever be arrived at by the straightforward application of the principle of conservation of matter. In nature any excess of air in one place originates waves which are propagated with the velocity of sound, and therefore much faster than ordinary meteorological phenomena.
>
> One of the difficulties in the mathematical analysis of pressure changes on the Earth is that the great rapidity of these adjustments by the elasticity of the air has to be allowed for. The difficulty does not crop up explicitly in Mr Richardson's work, but it may contribute to the failure of his method when he comes to close quarters with a numerical problem.

The hydrostatic approximation used by Richardson eliminates vertically propagating sound waves, but gravity waves and also horizontally propagating sound waves (Lamb waves) are present as solutions of his equations. These do indeed travel 'much faster than ordinary meteorological phenomena'. Nowhere in his book does Richardson allude to this fact. Whipple appears to have had a far clearer understanding of the causes of Richardson's forecast catastrophe that did Richardson himself. The consideration of these causes is a central theme of the present work.

A humourist has observed that publishing a book of verse is like dropping a feather down the Grand Canyon and awaiting the echo. Richardson's work was not taken seriously and his book failed to have any significant impact on the practice of meteorology during the decades following its publication. But the echo finally arrived and continues to resound around the world to this day: Richardson's brilliant and prescient ideas are now universally recognised among meteorologists and his work is the foundation upon which modern forecasting is built.

1.5 Outline of the contents of WPNP

We will examine Richardson's numerical forecast in considerable detail in the chapters that follow. For now, it is useful to present a broad outline — a synoptic view — of his book. The chapter titles are given in Table 1.1. Ch. 1 is a summary of

Table 1.1. *Chapter titles of* Weather Prediction by Numerical Process.

Chapter 1	Summary
Chapter 2	Introductory Example
Chapter 3	The Choice of Coordinate Differences
Chapter 4	The Fundamental Equations
Chapter 5	Finding The Vertical Velocity
Chapter 6	Special Treatment For The Stratosphere
Chapter 7	The Arrangement of Points and Instants
Chapter 8	Review of Operations in Sequence
Chapter 9	An Example Worked on Computing Forms
Chapter 10	Smoothing The Initial Data
Chapter 11	Some Remaining Problems
Chapter 12	Units and Notation

the contents of the book. Richardson's plan is to apply his finite difference method to the problem of weather forecasting. He had previously used both graphical and numerical methods for solving differential equations and had come to favour the latter:

> whereas Prof. Bjerknes mostly employs graphs, I have thought it better to procede by way of numerical tables. The reason for this is that a previous comparison of the two methods, in dealing with differential equations, had convinced me that the arithmetical procedure is the more exact and the more powerful in coping with otherwise awkward equations. (WPNP, p. *viii*; Dover Edn., p. *xii*)

The fundamental idea is that the numerical values of atmospheric pressures, velocities, etc., are tabulated at certain latitudes, longitudes and heights so as to give a general description of the state of the atmosphere at an instant. The physical laws determine how these quantities change with time. The laws are used to formulate an arithmetical procedure which, when applied to the numerical tables, yields the corresponding values after a brief interval of time, Δt. The process can be repeated so as to yield the state of the atmosphere after $2\Delta t$, $3\Delta t$, and so on, until the desired forecast length is reached.

In Ch. 2 the method of numerical integration is illustrated by application to a simple linear 'shallow-water' model. The step-by-step description of Richardson's method and calculations in this chapter is clear and explicit and is a splendid introduction to the process of numerical weather prediction. In contrast, the remainder of the book is heavy going, containing so much extraneous material that the central ideas are often obscured.

Ch. 3 describes the choice of coordinates and the discrete grid to be used. The choice is guided by (1) the scale of variation of atmospheric variables, (2) the errors due to replacing infinitesimal by finite differences, (3) the accuracy that is necessary to satisfy public requirements, (4) the cost, which increases with the number of points in space and time that have to be dealt with' (WPNP, p. 16). Richardson considered the distribution of observing stations in the British Isles, which were separated, on average, by a distance of 130 km. Over the oceans, observations were 'scarce and irregular'. He concluded that a grid with 128 equally spaced meridians and 200 km in latitude would be a reasonable choice. In the vertical he chose five layers, or *conventional strata*, separated by horizontal surfaces at 2.0, 4.2, 7.2 and 11.8 km, corresponding approximately to the mean heights of the 800, 600, 400 and 200 hPa surfaces. The alternative of using isobaric coordinates was considered but dismissed. The time interval chosen by Richardson was six hours, but this corresponds to $2\Delta t$ for the leapfrog method of integration; in modern terms, we have $\Delta t = 3$ h. The cells of the horizontal grid were coloured alternately red and white, like the checkers of a chess-board. The grid was illustrated on the frontispiece of WPNP, reproduced in Fig. 1.6.[5]

The next three chapters, comprising half the book, are devoted to assembling a system of equations suitable for Richardson's purposes. In Ch. 4

> the fundamental equations are collected from various sources, set in order and completed where necessary. Those for the atmosphere are then integrated with respect to height so as to make them apply to the mean values of the pressure, density, velocity, etc., in the several conventional strata.

As hydrostatic balance is assumed, there is no prognostic equation for the vertical velocity. Ch. 5 is devoted to the derivation of a diagnostic equation for this quantity. Platzman (1967) wrote that Richardson's vertical velocity equation 'is the principal, substantive contribution of the book to dynamic meteorology.' Ch. 6 considers the special measures which must be taken for the uppermost layer, the stratosphere, a region later described as 'a happy hunting-ground for meteorological theorists' (Richardson and Munday, 1926).

Ch. 7 gives details of the finite difference scheme, explaining the rationale for the choice of a staggered grid. Richardson considers several possible time-stepping techniques, including a fully implicit scheme, but opts for the simple leapfrog or 'step-over' method. Here can also be found a discussion of variable grid resolution and the special treatment of the polar caps. In Ch. 8 the forecasting 'algorithm' is presented in detail. It is carefully constructed so as to be, in Richardson's words, *lattice reproducing*; that is, where a quantity is known at a particular time and place, the algorithm enables its value at a later time to be calculated at the same place. The description of the method is sufficiently detailed and precise to enable a computer program based on it to be written, so that Richardson's results can be replicated (without the toil of two year's manual calculation).

[5] Richardson used 120 meridians, giving a 3° east-west distance, for his actual forecast, later realizing that 128 meridians (or 2.8125°) would more conveniently facilitate sub-division near the poles.

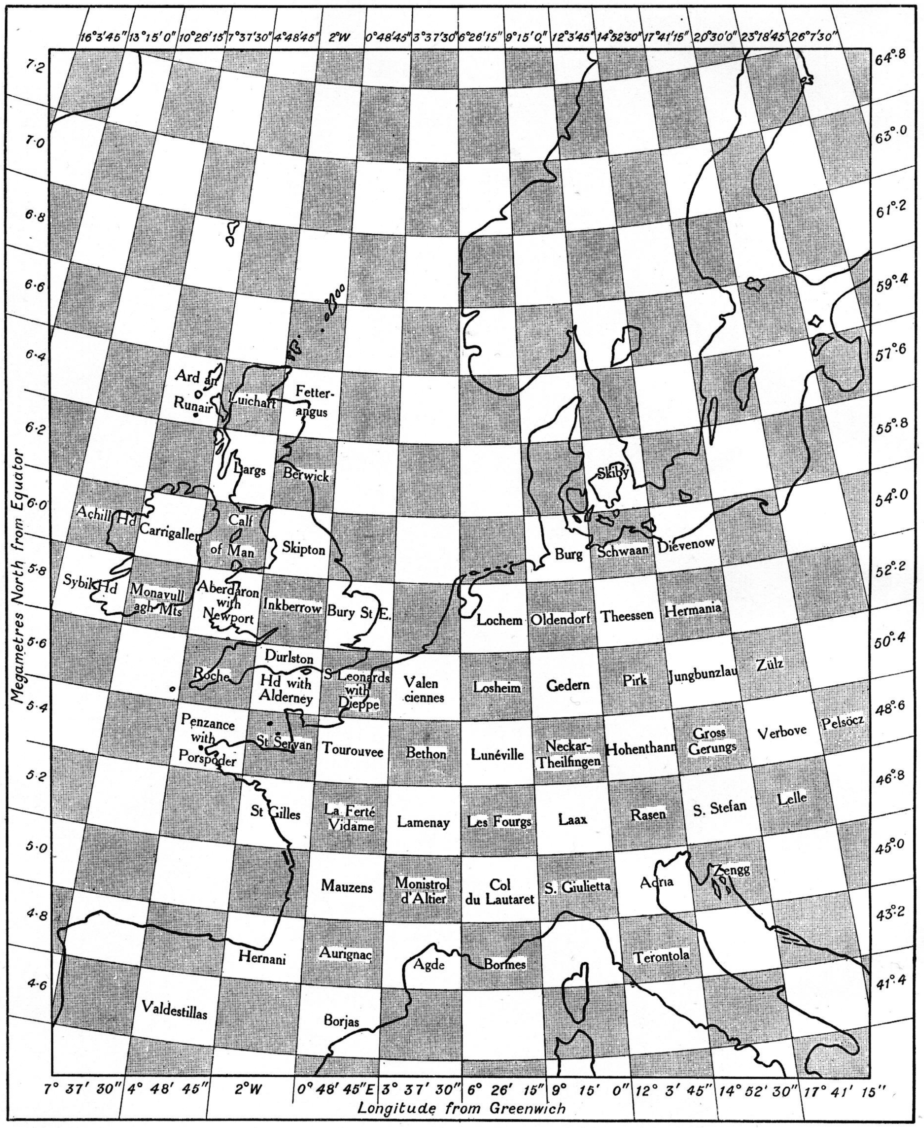

Fig. 1.6. Richardson's idealized computational grid (Frontispiece of WPNP).

Ch. 9 describes the celebrated trial forecast and its unfortunate results. The preparation of the initial data is outlined—the data are tabulated on page 185 of WPNP. The calculations themselves are presented on a set of 23 Computer Forms. These were completed manually: 'multiplications were mostly worked by a 25 centim slide rule' (WPNP, p. 186). The calculated changes in the primary variables over a six hour period are compiled on page 211. It is characteristic of Richardson's whimsical sense of humour that, on the heading of this page, the word "prediction" is enclosed in quotes; the results certainly cannot be taken literally. Richardson explains the chief result thus:

> The rate of rise of surface pressure, $\partial p_G/\partial t$, is found on Form $\mathrm{P_{XIII}}$ as 145 millibars in 6 hours, whereas observations show that the barometer was nearly steady. This glaring error is examined in detail below in Ch. 9/3, and is traced to errors in the representation of the initial winds.

(Here, p_G is the surface pressure). Richardson described his forecast as 'a fairly correct deduction from a somewhat unnatural initial distribution' (WPNP, p. 211). We will consider this surprising claim in detail in the ensuing chapters.

The following chapter is given short shrift by Richardson in his summary: 'In Ch. 10 the smoothing of observations is discussed.' The brevity of this resumé should not be taken to reflect the status of the chapter. In its three pages, Richardson discusses five alternative smoothing techniques. Such methods are crucial for the success of modern computer forecasting models. In a sense, Ch. 10 contains the key to solving the difficulties with Richardson's forecast. He certainly appreciated its importance for he stated, at the beginning of the following chapter,

> The scheme of numerical forecasting has developed so far that it is reasonable to expect that when the smoothing of Ch. 10 has been arranged, it may give forecasts agreeing with the actual smoothed weather.

This chapter considers 'Some Remaining Problems' relating to observations and to eddy diffusion, and also contains the oft-quoted passage depicting the forecast factory.

Finally, Ch. 12 deals with units and notation and contains a full list of symbols, giving their meanings in English and in Ido, a then-popular international language. Richardson had considered such a vast panoply of physical processes that the Roman and Greek alphabets were inadequate. His array includes several Coptic letters and a few specially-constructed symbols, such as a little leaf indicating evaporation from vegetation. As a tribute to Richardson's internationalism, the present book contains a similar table, giving the modern equivalents of Richardson's archaic notation, with meanings in English and Esperanto (see Appendix 1).

The emphasis laid by Richardson on different topics may be gauged from a page count of WPNP. Roughly half the book is devoted to discussions of a vast range of physical processes, some having a negligible effect on the forecast. The approximate *budget* in Table 1.2 is based on an examination of the contents of WPNP and on the earlier analyses of Platzman (1967) and Hollingsworth (1994). Due to the imprecision of the attribution process, the figures should be interpreted only in a qualitative sense.

The 23 computing forms on which the results of the forecast were presented, were designed and arranged in accordance with the systematic algorithmic proce-

Table 1.2. *Page-count of* Weather Prediction by Numerical Process.

Dynamics	Momentum Equations	11		
	Vertical Velocity	10		
	The Stratosphere	24		
	Total Dynamics		45	
Numerics	Finite Differences	12		
	Numerical Algorithm	25		
	Total Numerics		37	
	Dynamics+Numerics			82
Physics	Clouds and Water	12		
	Energy and Entropy	8		
	Radiation	19		
	Turbulence	36		
	Surface, Soil, Sea	23		
	Total Physics			98
Miscellaneous	Summary	3		
	Initial Data	7		
	Analysis of Results	5		
	Smoothing	3		
	Forecast Factory	1		
	Computing Forms	23		
	Notation and Index	14		
	Total Miscellaneous			56
Total Pages				236

dure that Richardson had devised for calculating the solution of the equations. The completed forms appear on pages 188–210 of WPNP so that the arithmetical work can be followed in great detail. Richardson arranged, at his own expense, for sets of blank forms to be printed to assist intrepid disciples to carry out experimental forecasts with whatever observational data were available. It is not known if these forms, which cost two shillings per set, were ever put to their intended use.[6]

The headings of the computing forms (see Table 1.3) indicate the scope of the computations. 'The forms are divided into two groups marked P and M according as the point on the map to which they refer is one where pressure P or momenta M are tabulated' (WPNP, p. 186). This arrangement of the computations is quite analogous to a modern spread-sheet program such as Excel, where the data are entered and the program calculates results according to prescribed rules. The first three forms contain input data and physical parameters. The forms may be classified as follows (Platzman, 1967):

[6] I am grateful to Oliver Ashford for providing me with a set of blank forms; they remain to be completed.

Table 1.3. *Headings of the 23 computing forms designed and used by Richardson. Copies were available separately from his book as* Forms whereon to write the numerical calculations described in Weather Prediction by Numerical Process by Lewis F Richardson. *Cambridge University Press, 1922. Price two shillings.*

Computing Form	Title
P_{I}	Pressure, Temperature, Density, Water and Continuous Cloud
P_{II}	Gas constant. Thermal capacities. Entropy derivatives
P_{III}	Stability, Turbulence, Heterogeneity, Detatched Cloud
P_{IV}	For Solar Radiation in the grouped ranges of wave-lengths known as *BANDS*
P_{V}	For Solar Radiation in the grouped ranges of wave-lengths known as *REMAINDER*
P_{VI}	For Radiation due to atmospheric and terrestrial temperature
P_{VII}	Evaopration at the interface
P_{VIII}	Fluxes of Heat at the interface
P_{IX}	For Temperature of Radiating Surface. Part I, Numerator of Ch. 8/2/15#20
P_{X}	For Temperature of Radiating Surface. Part II Denominator of Ch. 8/2/15#20
P_{XI}	Diffusion produced by eddies. See Ch. 4/8. Ch. 8/2/13
P_{XII}	Summary of gains of entropyand of water, both per mass of atmosphere during δt
P_{XIII}	Divergence of horizontal momentum-per-area. Increase of pressure
P_{XIV}	Stratosphere. Vertical Velocity by Ch. 6/6#21 Temperature Change by Ch. 6/7/3#8
P_{XV}	For Vertical Velocity in general, by equation Ch. 8/2/23#1. Preliminary
P_{XVI}	For Vertical Velocity. Conclusion
P_{XVII}	For the transport of water and its increase in a fixed element of volume
P_{XVIII}	For water in soil $\frac{\partial w}{\partial t} = \cdots$, which is equation Ch. 4/10/2#5
P_{XIX}	For Temperature in soil. The equation is Ch. 4/10/2, namely $\frac{\partial \theta}{\partial t} = \cdots$
M_{I}	For Stresses due to Eddy Viscosity
M_{II}	Stratosphere. Horizontal velocities and special terms in dynamical equations
M_{III}	For the Dynamical Equation for the Eastward Component
M_{IV}	For the Dynamical Equation for the Northward Component

- *Hydrodynamic calculations (11 forms)*
  - Input data and physical parameters: $\mathrm{P_{I}}$–$\mathrm{P_{III}}$
  - Mass tendency and pressure tendency: $\mathrm{P_{XIII}}$
  - Vertical velocity: $\mathrm{P_{XIV}}$–$\mathrm{P_{XVI}}$
  - Momentum tendency: $\mathrm{M_{I}}$–$\mathrm{M_{IV}}$
- *Thermodynamic and hydrologic calculations (12 forms)*
  - Radiation: $\mathrm{P_{IV}}$–$\mathrm{P_{VI}}$
  - Ground surface and subsurface: $\mathrm{P_{VII}}$–$\mathrm{P_{X}}, \mathrm{P_{XVIII}}, \mathrm{P_{XIX}}$
  - Free air: $\mathrm{P_{XI}}, \mathrm{P_{XII}}, \mathrm{P_{XVII}}$

The hydrodynamic calculations are by far the more important. In repeating the forecast we will omit the thermodynamic and hydrological calculations, which prove to have only a minor effect on the computed tendencies. The results on Form $\mathrm{P_{XIII}}$ are of particular interest and include the calculated surface pressure change of 145 hPa/6 h (the observed change in pressure over the period was less than one hPa).

Throughout his career, Richardson continued to consider the possibility of a second edition of WPNP. He maintained a file in which he kept material for this purpose and added to it from time to time, the last entry being in 1951. Platzman (1967) stressed the importance of this *Revision File* and discussed several items in it. The file contained an unbound copy of WPNP, on the sheets of which Richardson added numerous annotations. Interleaved among the printed pages were manuscript notes and correspondence relating to the book. In 1936, C. L. Godske, an assistant of Bjerknes, visited Richardson in Paisley to discuss the possibility of continuing his work using more modern observational data. Richardson gave him access to the Revision File and, after the visit, wrote to Cambridge University Press suggesting Godske as a suitable author if a second edition should be called for at a later time (Ashford, 1985, p. 157). After Richardson's death, the Revision File passed to Oliver Ashford who in 1967 deposited it in the archives of the Royal Meteorological Society. The file was misplaced, along with other Richardson papers, when the Society moved its head-quarters from London to Bracknell in 1971. Ashford expressed a hope that 'perhaps it too will turn up some day "under a heap of coal".' The file serendipitously re-appeared around 2000 and Ashford wrote in a letter to *Weather* that 'there is still something of a mystery' about where the file had been (Ashford, 2001). The file has now been transferred to the National Meteorological Archive of the Met Office in Exeter. We will refer repeatedly in the sequel to this peripatetic file.

1.6 Preview of remaining chapters

The fundamental equations of motion are introduced in Chapter 2. The prognostic equations, which follow from the physical conservation laws, are presented and a number of diagnostic relationships necessary to complete the system are derived. In the case of small amplitude horizontal flow the equations assume a particularly simple form, reducing to the linear shallow water equations or Laplace tidal equations. These are discussed in Chapter 3, and an analysis of their normal mode solutions is presented. The numerical integration of the linear shallow water equations is dealt with in Chapter 4. Richardson devoted a chapter of his book to this barotropic case, with the aim of verifying that his finite difference method could yield results of acceptable accuracy. We consider his use of geostrophic initial winds and show how the noise in his forecast may be filtered out.

The transformation of the full system of differential equations into algebraic form is undertaken in Chapter 5. This is done by the method of finite differences in which continuous variables are represented by their values at a discrete set of grid-points in space and time, and derivatives are approximated by differences between the values at adjacent points. The vertical stratification of the atmosphere is considered: the continuous variation is averaged out by integration through each of five layers and the equations for the mean values in each layer are derived. A complete system of equations suitable for numerical solution is thus obtained. A detailed step-by-step description of Richardson's solution procedure is given in this chapter.

The preparation of the initial conditions is described in Chapter 6. The sources of the initial data are discussed, and the transformations required to produce the needed initial values are outlined. There is also a brief description of the instruments used in 1910 in the making of these observations. In Chapter 7 the initial tendencies produced by the numerical model are presented. They are in excellent agreement with the values that Richardson obtained. The reasons for the small discrepancies are explained. The results are unrealistic: the reasons for this are analysed and we begin to consider ways around the difficulties.

The process of initialization is discussed in Chapter 8. We review early attempts to define a balanced state for the initial data. The ideas of normal mode initialization, filtered equations and the slow manifold are introduced by consideration of a particularly simple mechanical system, an elastic pendulum or 'swinging spring'. These concepts are examined in greater detail in the remaining sections of the chapter. Finally, the digital filter initialization technique, which is later applied to Richardson's forecast, is presented.

In Chapter 9 we discuss the initialization of Richardson's forecast. Richardson's discussion on smoothing the initial data is re-examined. When appropriate smoothing is applied to the initial data, using a simple digital filter, the initial tendency of surface pressure is reduced from the unrealistic $145\,\mathrm{hPa}/6\,\mathrm{h}$ to a reasonable value of less than $1\,\mathrm{hPa}/6\,\mathrm{h}$. The forecast is shown to be in good agreement with the observed pressure change. The rates of change of temperature and wind are also realistic. To extend the forecast, smoothing in space is found to be necessary. The results of a 24 hour forecast with such smoothing are presented.

Chapter 10 considers the development of NWP in the 1950s, when high-speed electronic computers first came into use. The first demonstration that computer forecasting might be practically feasible was carried out by the Princeton Group (Charney, *et al.*, 1950). These pioneers were strongly impressed by Richardson's work as presented in his book. With the benefit of advances in understanding of atmospheric dynamics made since Richardson's time, they were able to devise means of avoiding the problems that had ruined his forecast. The ENIAC integrations are described in detail. There follows a description of the development of primitive equation modelling. The chapter concludes with a discussion of general circulation models and climate modelling.

The state of numerical weather prediction today is summarized in Chapter 11. The global observational system is reviewed, and methods of objectively analysing the data are described. The exponential growth in computational power is illustrated by considering the sequence of computers at the Met Office. To present the state of the art of NWP, the operations of the European Centre for Medium-Range Weather Forecasts (ECMWF) are reviewed. There follows a brief outline of current meso-scale modelling. The implications of chaos theory for atmospheric predictability are considered, and probabilistic forecasting using ensemble prediction systems is described.

In Chapter 12 we review Richardson's understanding of the causes of the failure of his forecast. His wonderful fantasy about a forecast factory is then re-visited. A parallel between this fantasy and modern massively parallel computers is drawn. Finally, we arrive at the conclusion that modern weather prediction systems provide a spectacular realization of Richardson's dream of practical numerical weather forecasting.