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But the thermal wind relationship is

$$f_0\frac{\partial\mathbf{V_g}}{\partial p} = \mathbf{k}\times\nabla\frac{\partial\Phi}{\partial p}$$

This is just the p-derivative of $f_0\mathbf{V_g} = \mathbf{k}\times\nabla\Phi$.

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$$\mathbf{RHS} = -f_0\mathbf{V_g}\cdot\nabla\left[\frac{1}{f_0}\nabla^2\Phi + f + \frac{\partial}{\partial p}\left(\frac{f_0}{\sigma}\frac{\partial\Phi}{\partial p}\right)\right] = -f_0\mathbf{V_g}\cdot\nabla q$$

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The quantity in square brackets is called the *quasi-geostrophic potential vorticity*

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The left side of the tendency equation may be written

$$\mathbf{LHS} = f_0 \frac{\partial}{\partial t} \left[\frac{1}{f_0} \nabla^2 \Phi + \frac{\partial}{\partial p} \left(\frac{f_0}{\sigma} \frac{\partial \Phi}{\partial p} \right) \right] = f_0 \frac{\partial q}{\partial t}$$

since f does not vary with time.

The tendency equation may now be written in a conservative form called the ***quasi-geostrophic potential vorticity equation*** **or QGPV equation:**

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Note that q is completely determined once the three-dimensional distribution of geopotential Φ is given.

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This was the equation

$$\frac{d}{dt}(\zeta + f) = 0$$

for the conservation of *absolute vorticity*.

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An idealized geopotential field is given at time $t = 0$ by

$$\Phi = \Phi_0 - f_0 \bar{u} y + A \sin(kx - mp)$$

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(c) Compute the variations in the temperature field due to the wave.

(d) Compute the vorticity advection and temperature advection.

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(f) Using the omega equation, describe the pattern of vertical velocity associated with the wave disturbance.

The ENIAC Integrations

(ENIAC: Electronic Numerical Integrator and Computer)

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3. **The first step towards influencing the weather by rational human intervention will have been made.**

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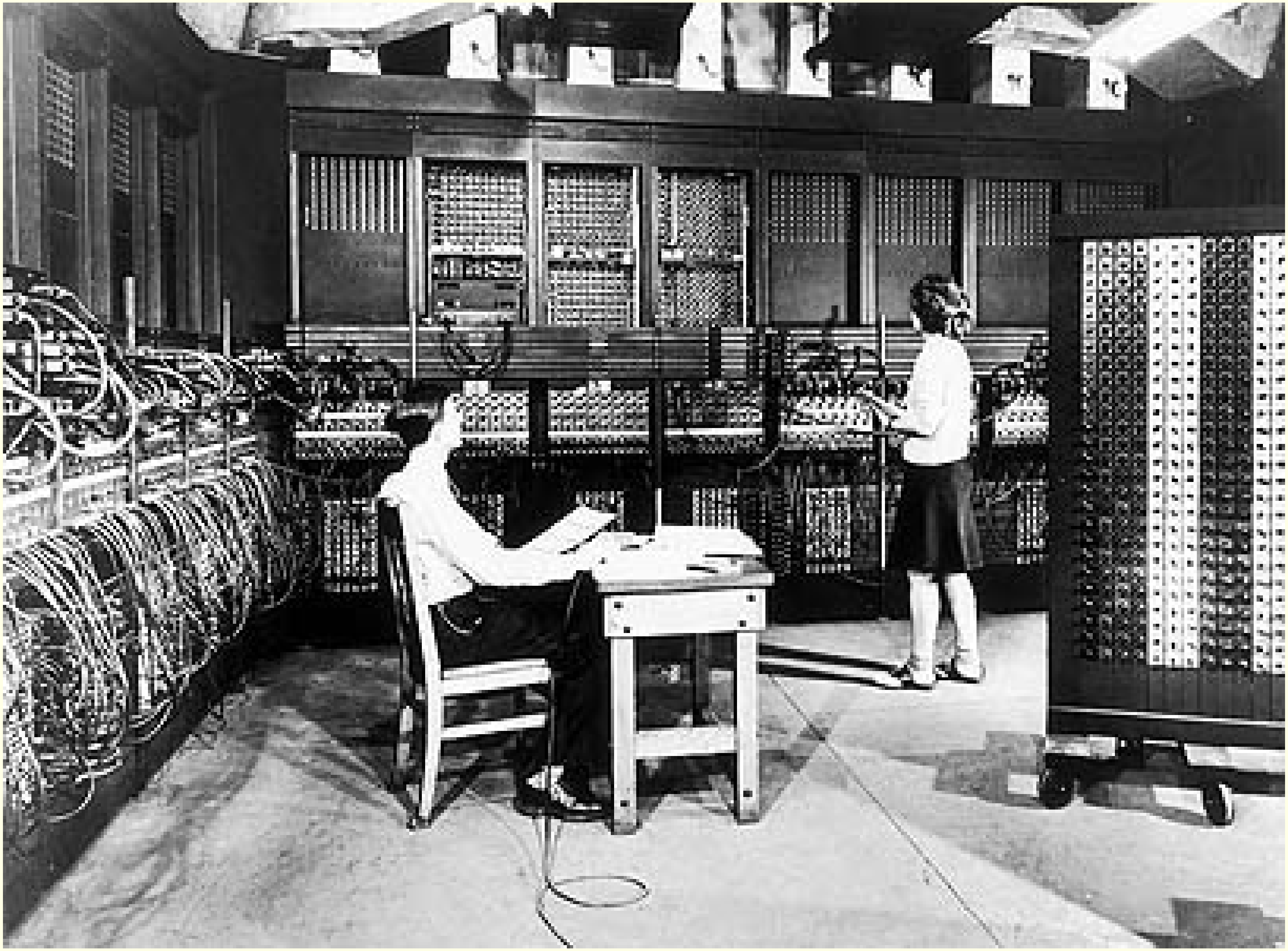
The ENIAC



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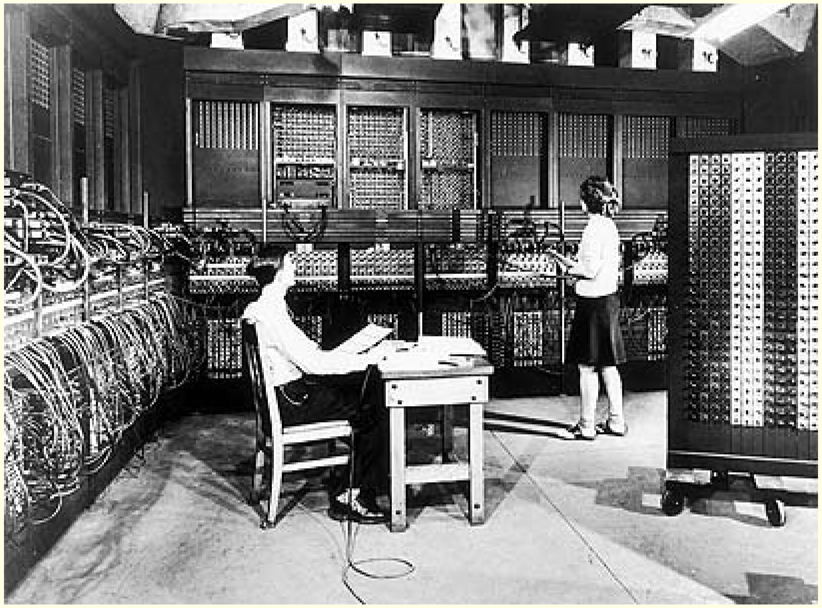

The ENIAC (Electronic Numerical Integrator and Computer) was the *first multi-purpose programmable electronic digital computer*.

It had:

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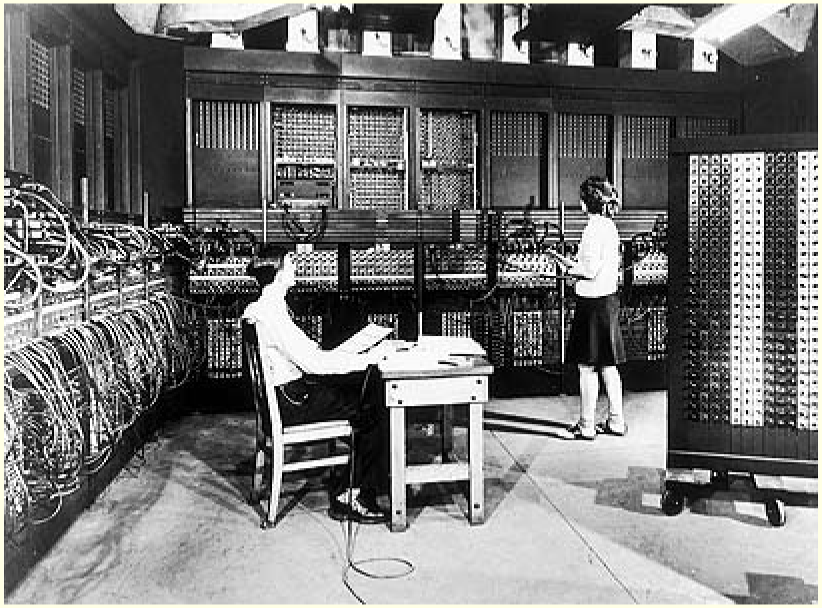

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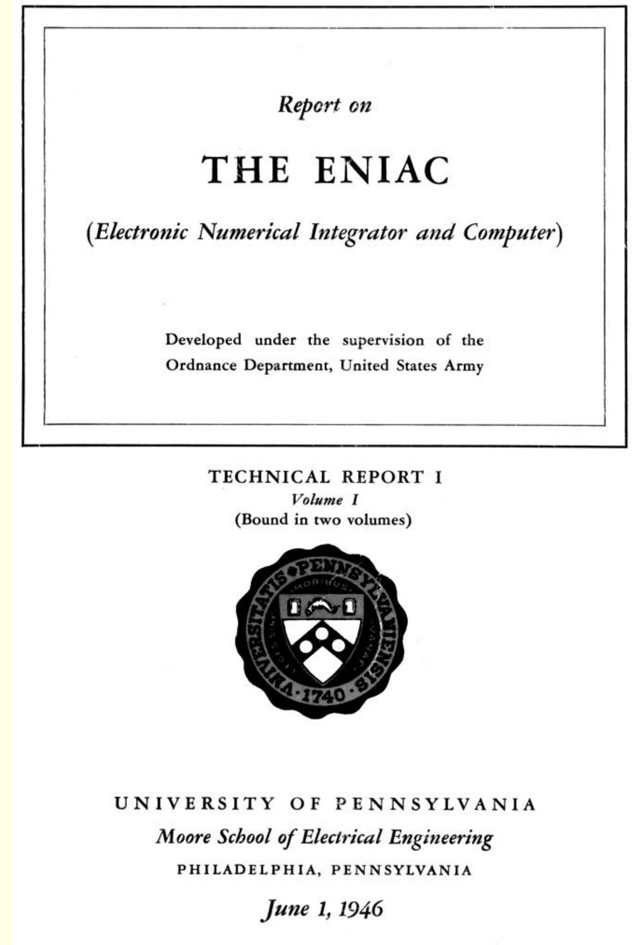
The ENIAC: Technical Details.

ENIAC was a decimal machine. No high-level language. Assembly language. Fixed-point arithmetic: $-1 < x < +1$**.**

10 registers, that is,
Ten words of high-speed memory.
Function Tables:
624 6-digit words of "ROM", set on ten-pole rotary switches.
"Peripheral Memory":
Punch-cards.
Speed: FP multiply: 2ms (say, 500 Flops).
Access to Function Tables: 1ms.
Access to Punch-card equipment: You can imagine!


Report on

THE ENIAC

(*Electronic Numerical Integrator and Computer*)

Developed under the supervision of the
Ordnance Department, United States Army

TECHNICAL REPORT I
Volume I
(Bound in two volumes)

UNIVERSITY OF PENNSYLVANIA
Moore School of Electrical Engineering
PHILADELPHIA, PENNSYLVANIA

June 1, 1946

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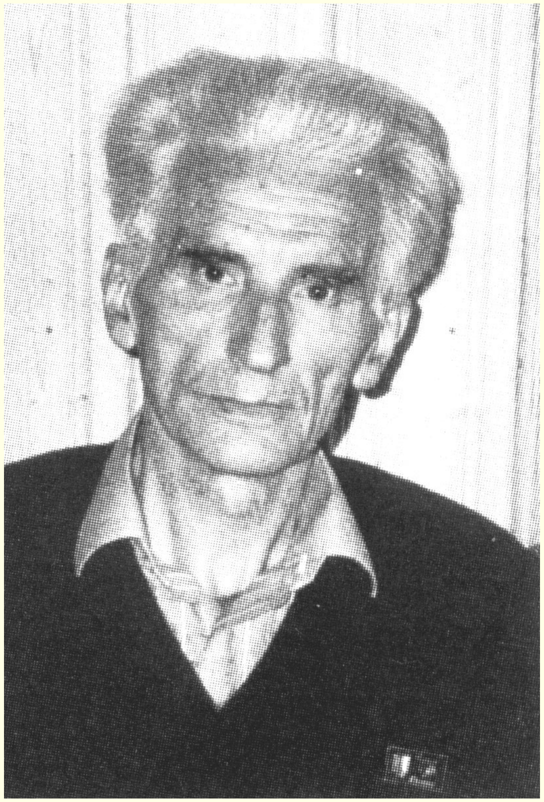
- **Plan C: Solve barotropic vorticity equation**
  Very satisfactory initial results

Charney, Fjørtoft, von Neumann

Charney, et al., *Tellus*, 1950.

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In more detail:

$$\frac{\partial}{\partial t}[\nabla^2\psi - F\psi] + \left\{\frac{\partial\psi}{\partial x}\frac{\partial\nabla^2\psi}{\partial y} - \frac{\partial\psi}{\partial y}\frac{\partial\nabla^2\psi}{\partial x}\right\} + \beta\frac{\partial\psi}{\partial x} = 0$$

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- **Gridstep : Δx = 750 km (approximately)**
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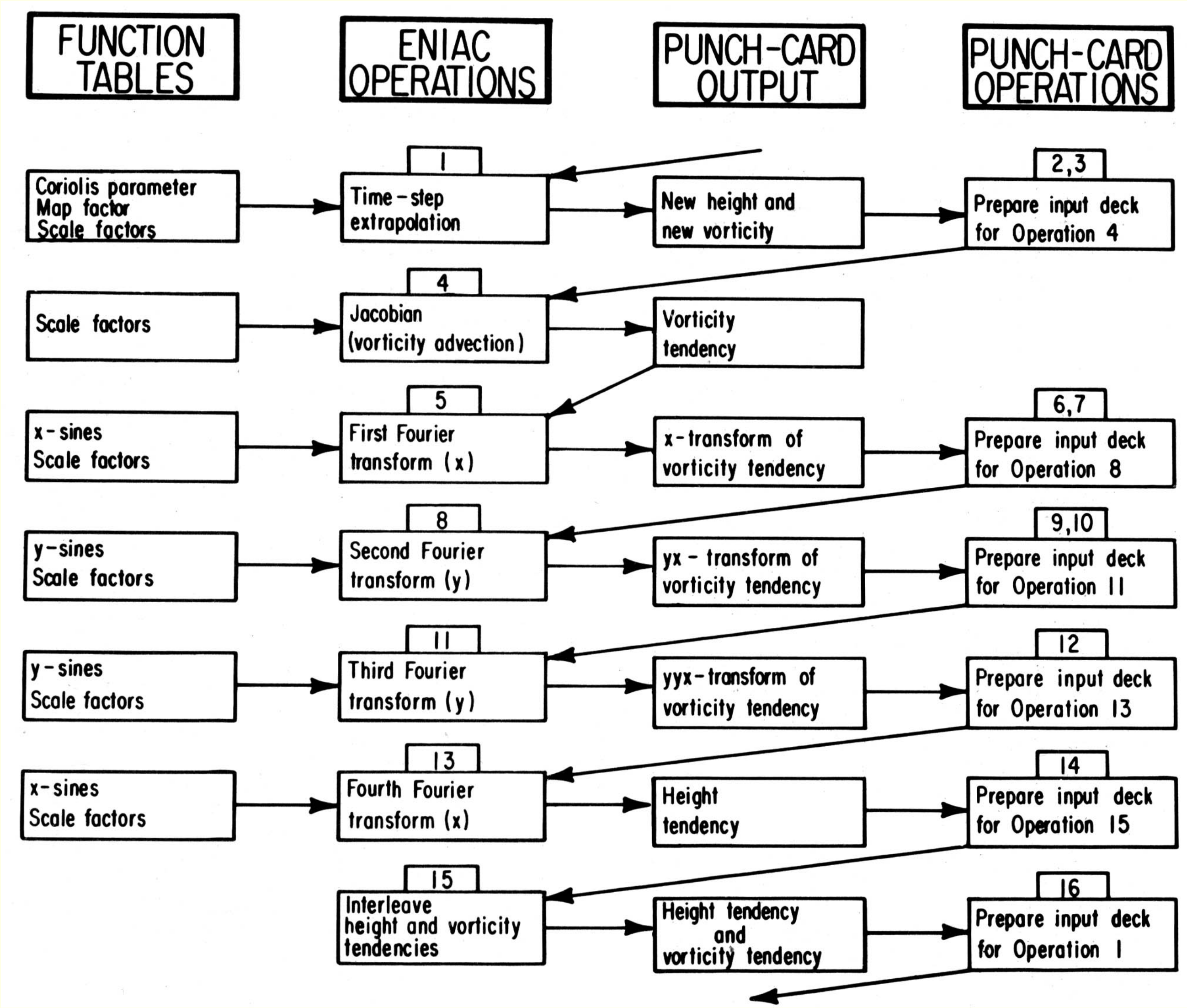
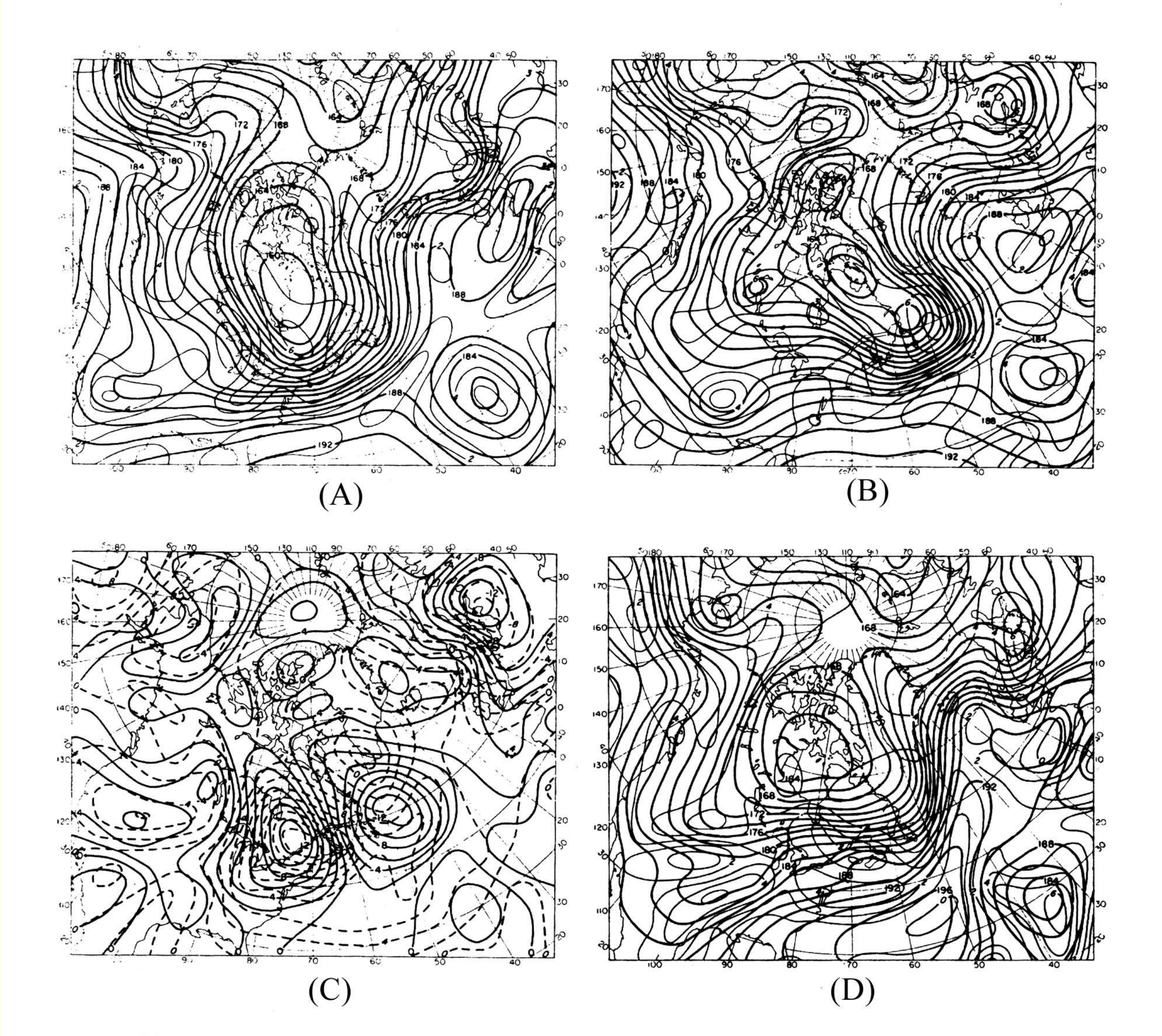
Forecast involved punching about 25,000 cards. Most of the elapsed time was spent handling these.

ENIAC Algorithm



ENIAC: First Computer Forecast



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- **"This is . . . an enormous scientific advance on the single, and quite wrong, result in which . . . [Richardson (1922)] ended."**