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Since the Laplacian of a function tends to have a minimum where the function has a maximum, and *vice-versa*,

Positive Vorticity is associated with Low Pressure

that is, low values of the geopotential, and

Negative Vorticity is associated with High Pressure

We write the quasi-geostrophic *momentum equation* in component form

$$
\begin{aligned}
\frac{d_g u_g}{dt} - f_0 v_a - \beta y \, v_g &= 0 \\
\frac{d_g v_g}{dt} + f_0 u_a + \beta y \, u_g &= 0
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Exercise: Verify the derivation of the vorticity equation. Expand d/dt and proceed as indicated above.

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- **The advection of the absolute vorticity by the geostrophic wind**
- **The stretching or shrinking of fluid columns (divergence effect).**

The advection itself is the sum of two terms

$$-\mathbf{V_g}\cdot\nabla(\zeta_g + f) = -\mathbf{V_g}\cdot\nabla\zeta_g - \beta v_g$$

the advection of relative vorticity and the advection of planetary vorticity respectively.

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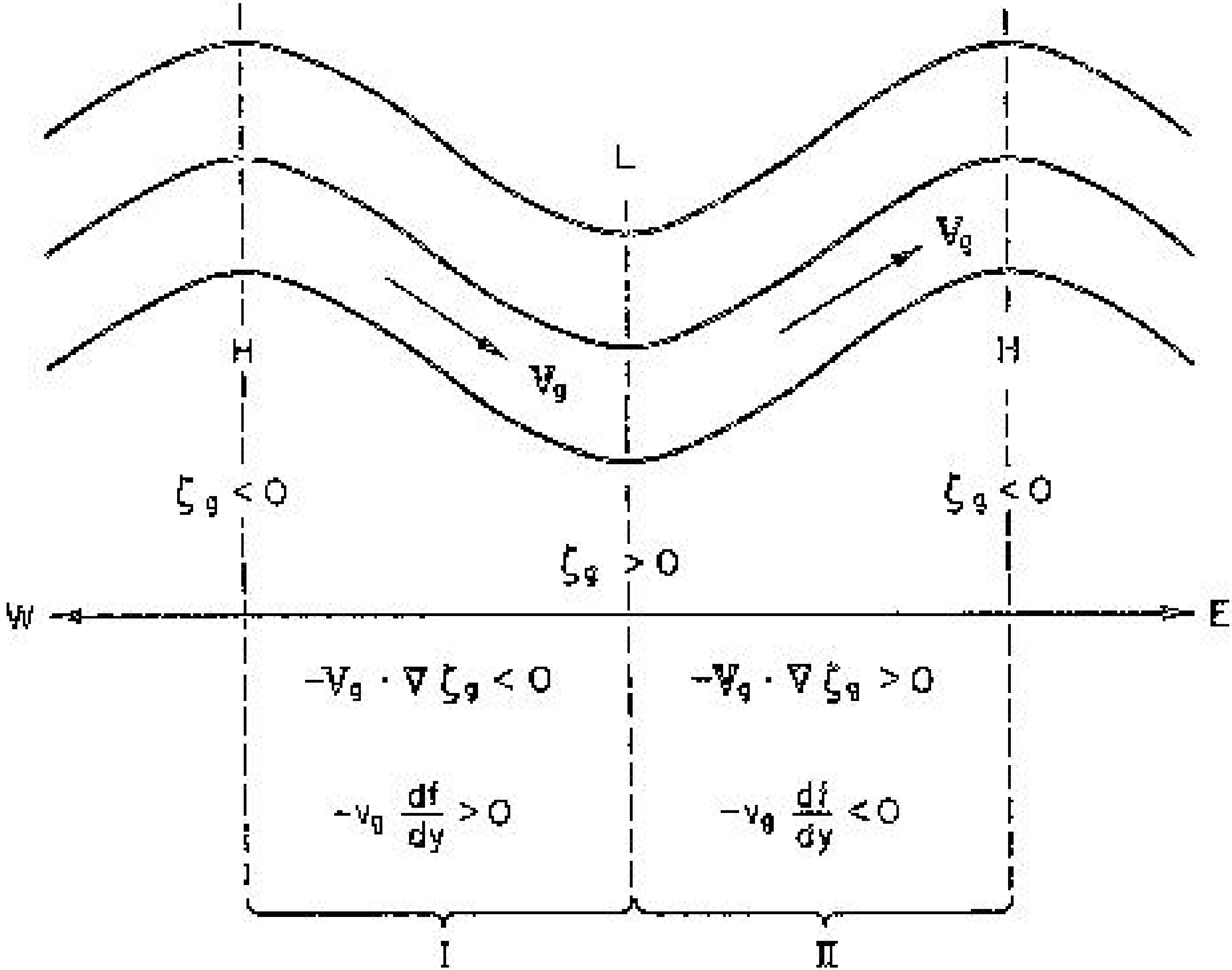
We can estimate their relative sizes:

$$\left|\frac{\mathbf{V_g}\cdot\nabla\zeta_g}{\beta v_g}\right| \sim \frac{Va}{L^2 f} = \frac{\mathbf{Ro}}{L/a} \sim 1$$

so the two terms are of comparable size.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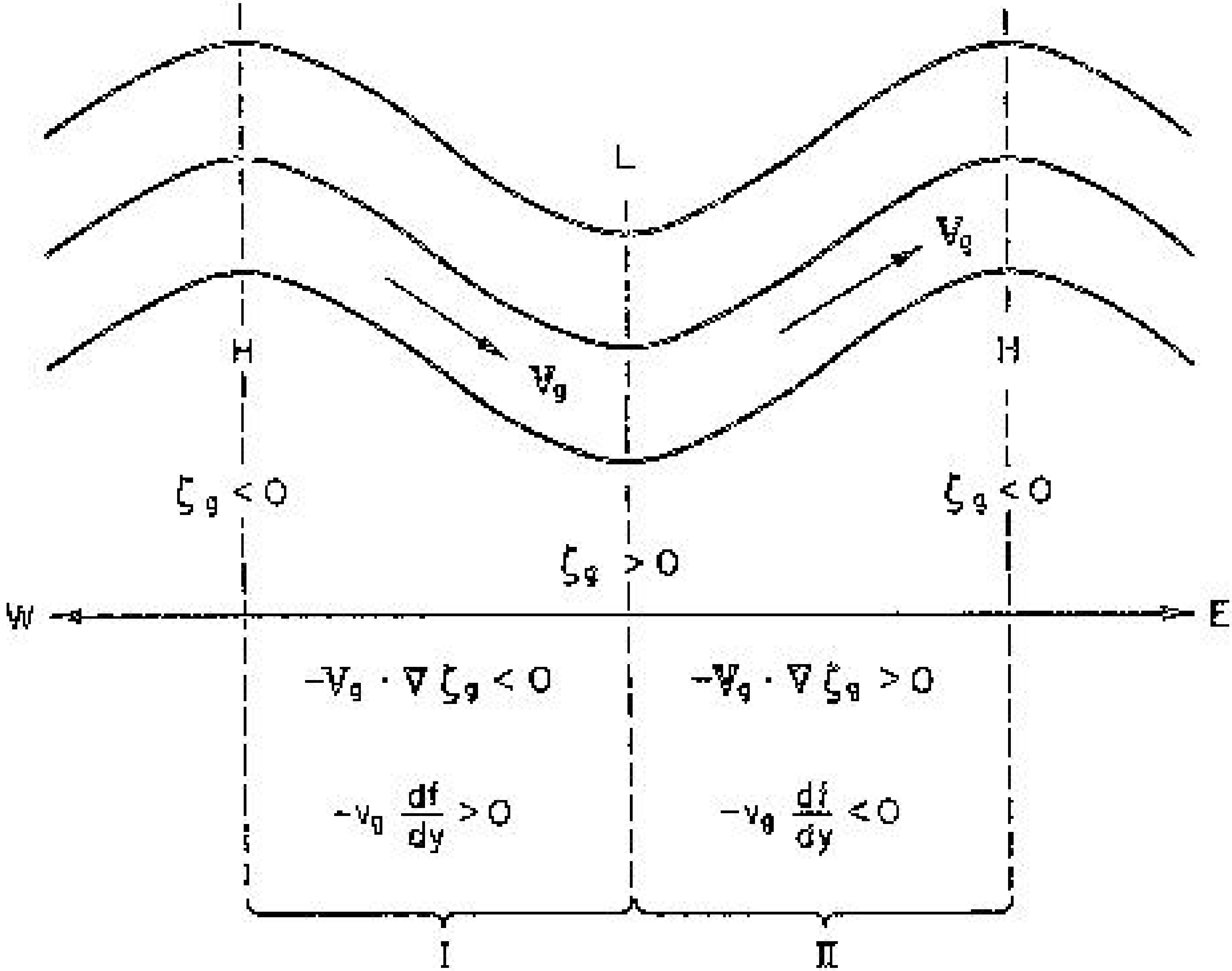
- **The advection of *relative* vorticity decreases ζ_g**
- **The advection of *planetary* vorticity increases ζ_g**



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On the other hand, the *planetary* vorticity advection tends to move the vorticity pattern *upstream* or westward, causing the wave to *regress*.

Since the two terms are of comparable magnitude, either may dominate, depending on the particular case.

Let us consider an *idealized streamfunction* on a midlatitude β-plane comprising a zonally averaged part and a wave disturbance

$$\Phi = \Phi_0 - f_0 \bar{u} y + f_0 A \sin kx \cos \ell y$$

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$$\begin{aligned} u_g &= -\frac{1}{f_0}\frac{\partial \Phi}{\partial y} = \bar{u} + u_g' = \bar{u} + \ell A \sin kx \sin \ell y \\ v_g &= +\frac{1}{f_0}\frac{\partial \Phi}{\partial x} = \quad + v_g' = \quad + kA \cos kx \cos \ell y \end{aligned}$$

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The geostrophic vorticity is

$$\zeta_g = \frac{1}{f_0}\nabla^2 \Phi = -(k^2 + \ell^2) A \sin kx \cos \ell y$$

It is easily shown that the advection of relative vorticity by the wave component vanishes,

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The total vorticity advection is

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Thus, as a <u>general rule</u>, short-wavelength synoptic-scale disturbances should move eastward in a westerly flow. Long planetary waves regress or remain stationary.

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Recalling that the vorticity and geopotential are related by $\zeta_g = (1/f_0)\nabla^2\Phi$ **and reversing the order of differentiation, we get**

$$\frac{1}{f_0}\nabla^2\Phi_t = -\mathbf{V_g}\cdot\nabla\left(\frac{1}{f_0}\nabla^2\Phi + f\right) + f_0\frac{\partial \omega}{\partial p}$$

Note: $\Phi_t \equiv \partial\Phi/\partial t$. **Holton uses** χ.

The thermodynamic equation is

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Let us multiply by f_0/σ and differentiate with respect to p:

$$\frac{\partial}{\partial p}\left(\frac{f_0}{\sigma}\frac{\partial \Phi_t}{\partial p}\right) = -\frac{\partial}{\partial p}\left[\frac{f_0}{\sigma}\mathbf{V_g} \cdot \nabla\left(\frac{\partial \Phi}{\partial p}\right)\right] - f_0\frac{\partial \omega}{\partial p} - f_0\frac{\partial}{\partial p}\left(\frac{\kappa \dot{Q}}{\sigma p}\right)$$

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It is simple to eliminate ω by addition of the thermodynamic and vorticity equations as expressed above. We then get

$$\underbrace{\left[\nabla^2 + \frac{\partial}{\partial p}\left(\frac{f_0^2}{\sigma}\frac{\partial}{\partial p}\right)\right]}_{A}\Phi_t = -\underbrace{f_0\mathbf{V_g}\cdot\nabla\left(\frac{1}{f_0}\nabla^2\Phi + f\right)}_{B}$$

$$+\underbrace{\frac{\partial}{\partial p}\left[\frac{f_0^2}{\sigma}\mathbf{V_g}\cdot\nabla\left(-\frac{\partial\Phi}{\partial p}\right)\right]}_{C}$$

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Term B: The advection of vorticity

Term C: The vertical shear of temperature advection.

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Thus, Term (B) makes Φ_t positive, so that a ridge tends to develop and, associated with this, the vorticity becomes negative.

Term (B) acts to *transport* the pattern of geopotential. However, since $\mathbf{V_g} \cdot \nabla\zeta = 0$ on the trough and ridge axes, *this term does not cause the wave to amplify or decay*.

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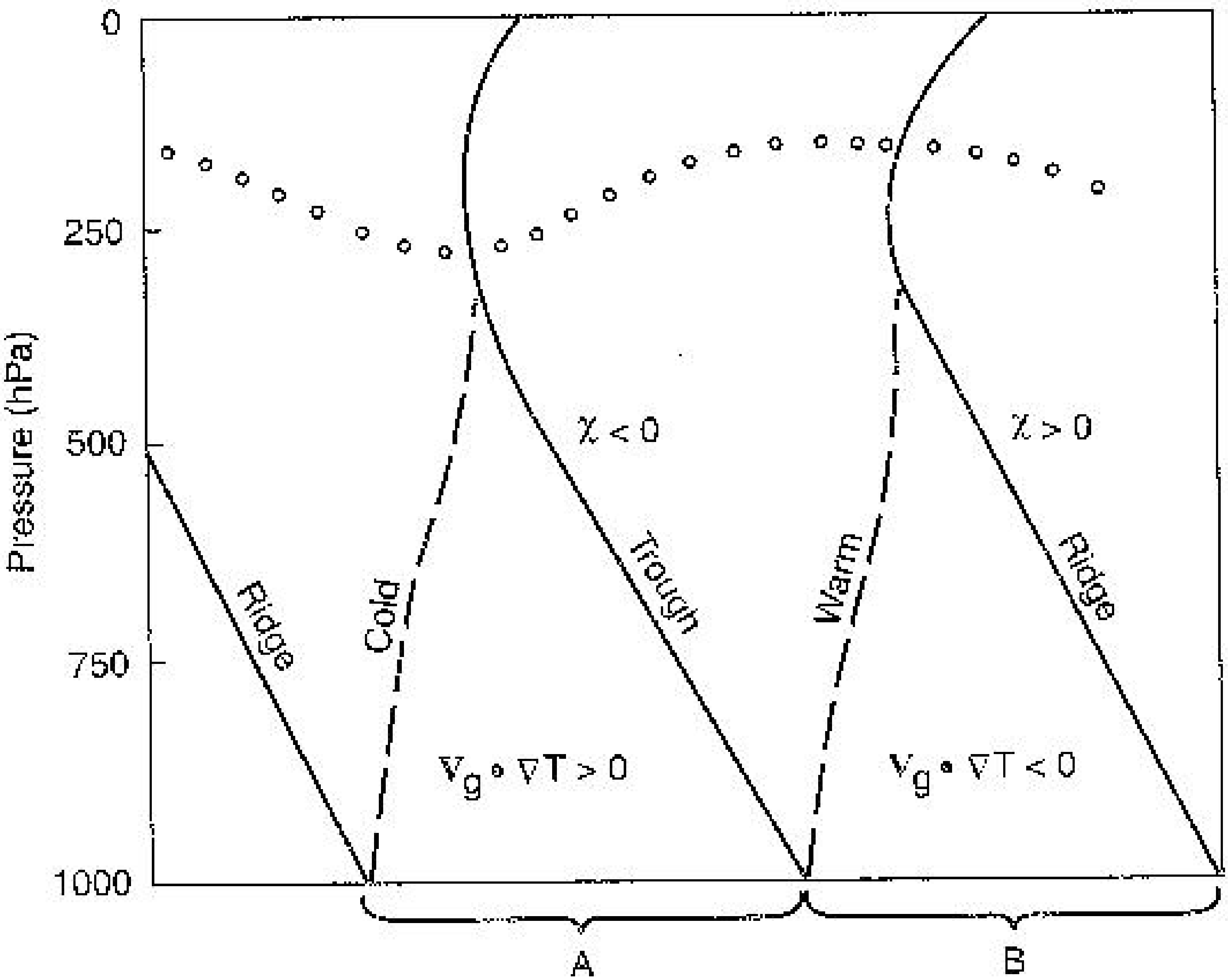
It is therefore related to *plus* the rate of change of temperature advection with respect to height. This is called the *differential temperature advection*.

The magnitude of the temperature (or thickness) advection tends to be largest in the lower troposphere, beneath the 500 hPa trough and ridge lines in a developing baroclinic wave.



0
250
500
750
1000
Pressure (hPa)
Ridge
Cold
Trough
Warm
Ridge
χ < 0
χ > 0
$V_g \cdot \nabla T > 0$
$V_g \cdot \nabla T < 0$
A
B

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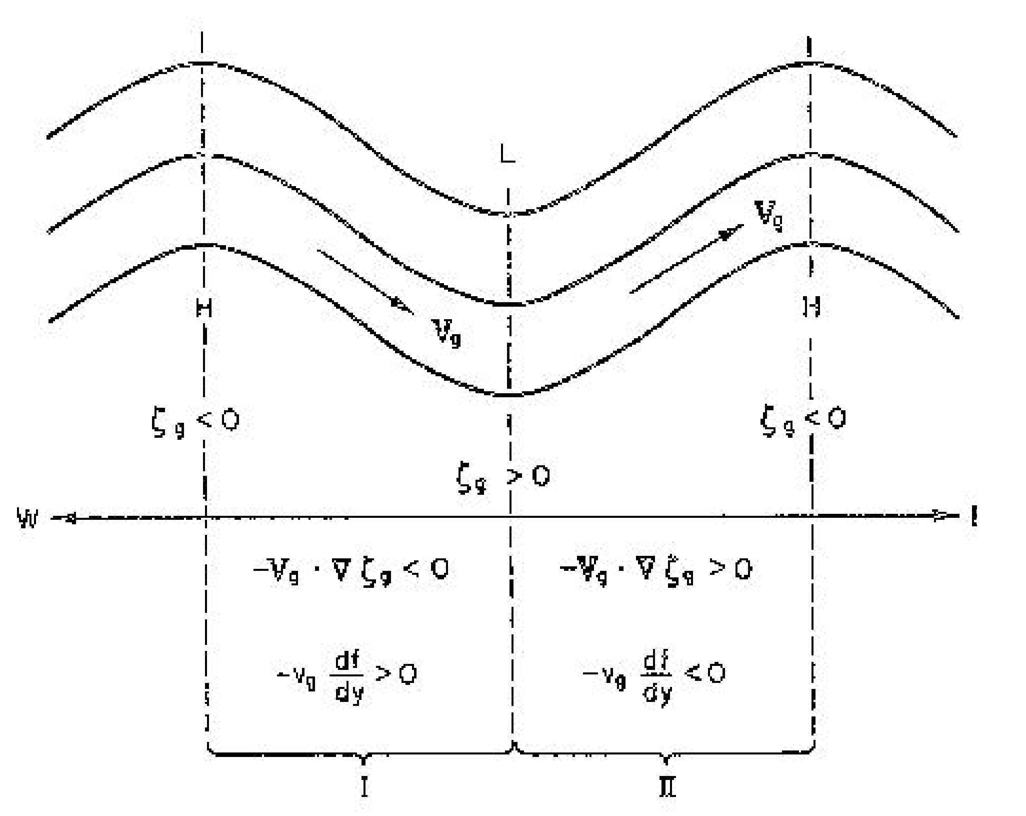
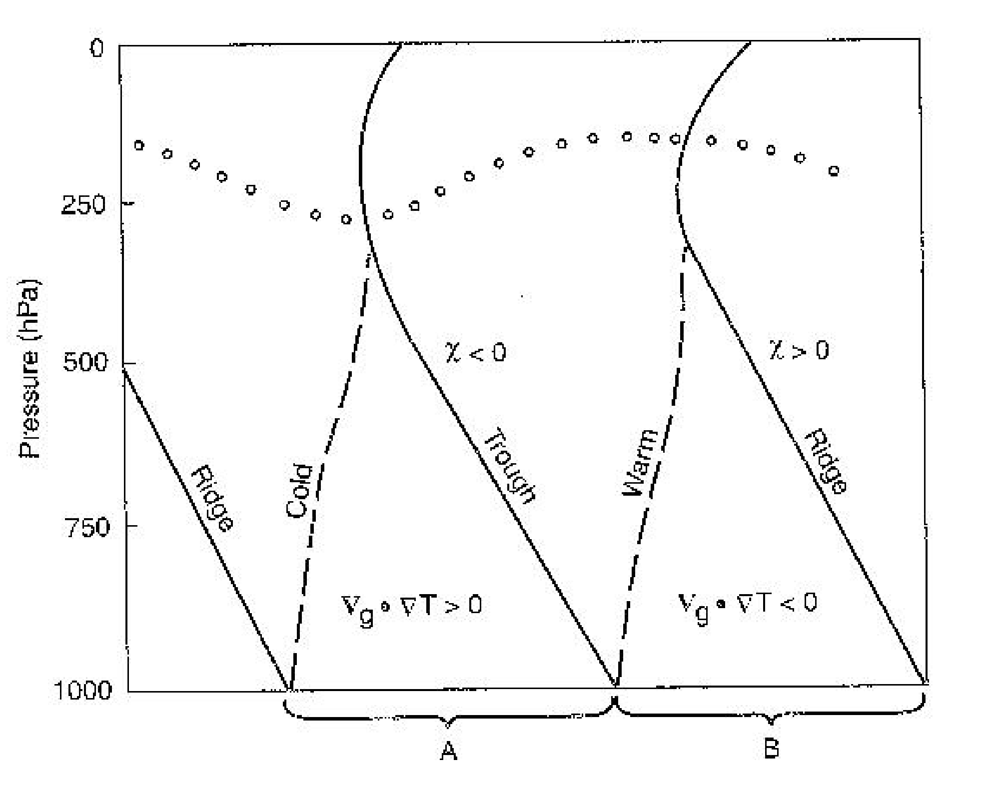
Thus in contrast to term (B), term (C) is dominant in the lower troposphere; but its effect is felt at higher levels.

In words, we may write the geopotential tendency equation:

$$\left[\begin{array}{c}\textbf{Falling}\\ \textbf{Pressure}\end{array}\right] \propto \left[\begin{array}{c}\textbf{Positive}\\ \textbf{Vorticity Advection}\end{array}\right] + \left[\begin{array}{c}\textbf{Differential}\\ \textbf{Temperature Advec'n}\end{array}\right]$$



Tendency due to vorticity advection



Tendence due to diff'l temperature advection

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We take the Laplacian of this and obtain

$$\nabla^2 \frac{\partial \Phi_t}{\partial p} = -\nabla^2\left[\mathbf{V_g} \cdot \nabla\left(\frac{\partial \Phi}{\partial p}\right)\right] - \sigma\nabla^2\omega$$

Recall that the vorticity equation may be written

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We now have two equations with *identical expressions* for the tendency Φ_t.

So we can subtract one from the other to obtain a *diagnostic equation* for the vertical velocity.

$$\underbrace{\left(\sigma\nabla^2 + f_0^2\frac{\partial^2}{\partial p^2}\right)}_{A}\omega = -\underbrace{f_0\frac{\partial}{\partial p}\left[-\mathbf{V_g}\cdot\nabla\left(\frac{1}{f_0}\nabla^2\Phi + f\right)\right]}_{B}$$

$$+\underbrace{\nabla^2\left[\mathbf{V_g}\cdot\nabla\left(-\frac{\partial\Phi}{\partial p}\right)\right]}_{C}$$

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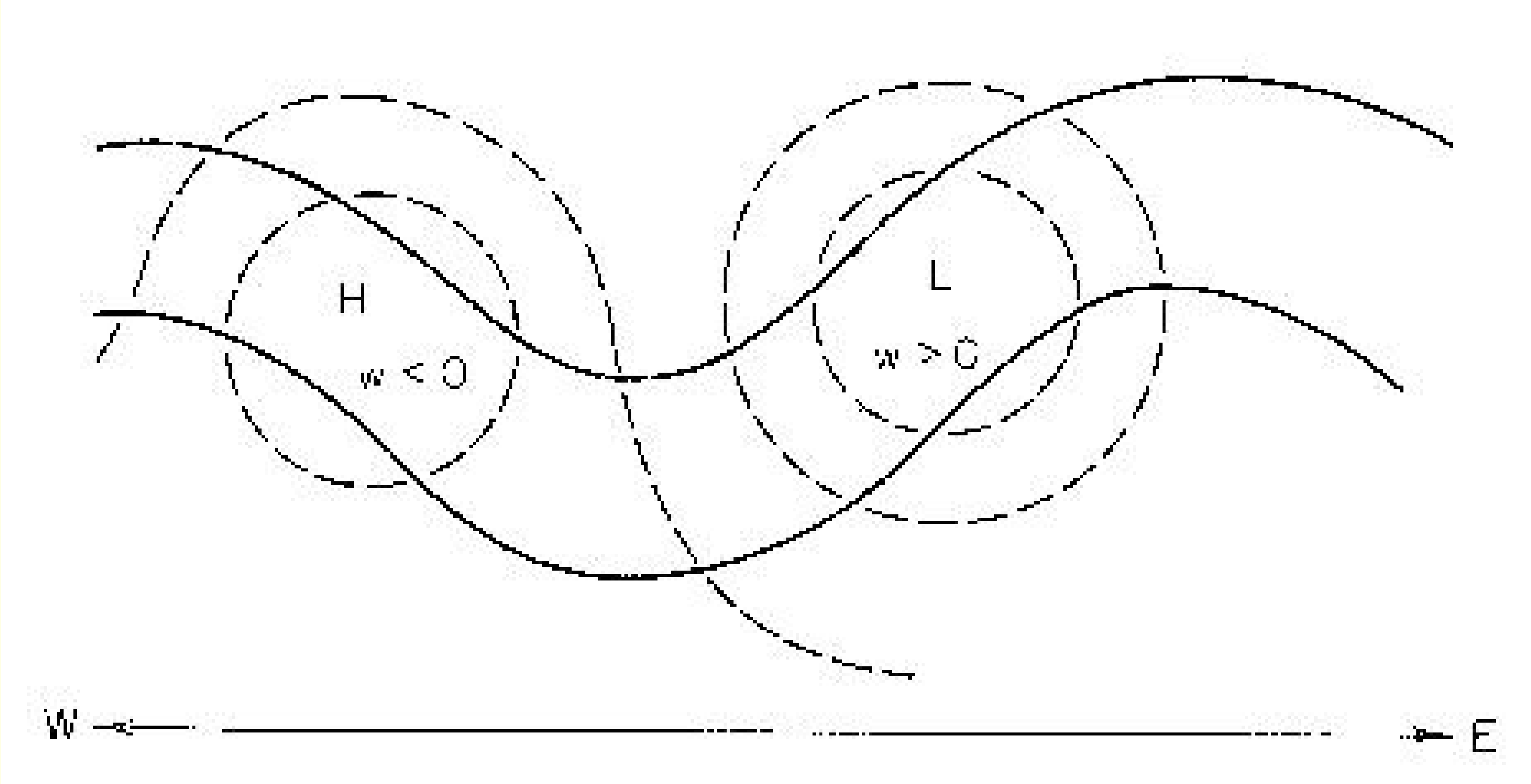
In words, we may write the omega equation as follows

$$\begin{bmatrix} \textbf{Rising} \\ \textbf{Motion} \end{bmatrix} \propto \begin{bmatrix} \textbf{Differential} \\ \textbf{Vorticity Advection} \end{bmatrix} + \begin{bmatrix} \textbf{Temperature} \\ \textbf{Advection} \end{bmatrix}$$



Idealized baroclinic wave. Solid: 500 hPa geopotential contours. Dashed: 1000 hPa contours. Regions of strong vertical motion due to differential vorticity advection are indicated.

Term (B) is

$$-f_0\frac{\partial}{\partial p}\left[-\mathbf{V_g}\cdot\nabla\left(\frac{1}{f_0}\nabla^2\Phi+f\right)\right]\propto\frac{\partial}{\partial z}\left[-\mathbf{V_g}\cdot\nabla\left(\zeta_g+f\right)\right]$$

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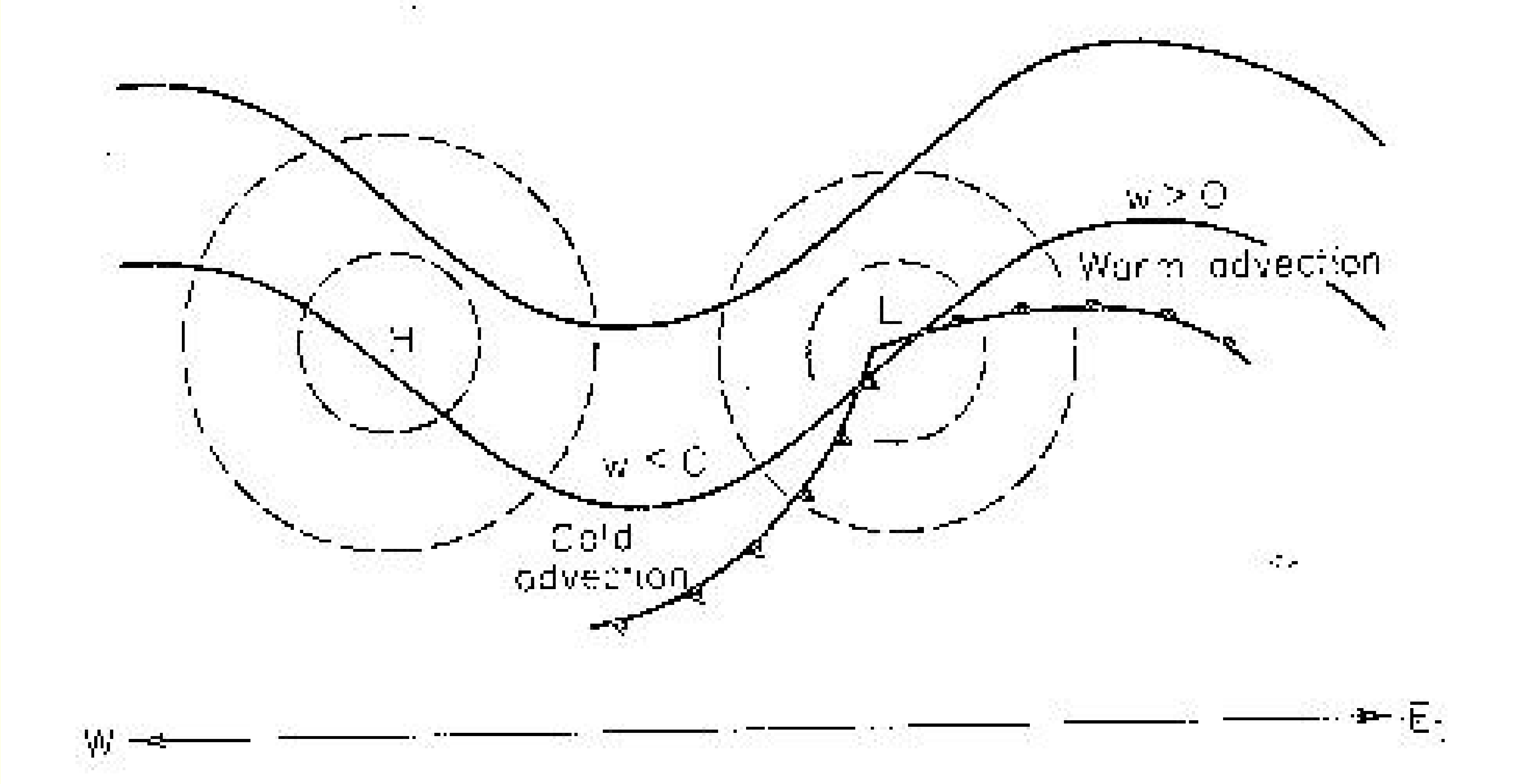
- **Correspondingly, $w < 0$ above the surface High Pressure**
- **We assume the scale is short enough that relative vorticity advection dominates planetary vorticity advection.**

Conclusion: Differential vorticity advection implies:
Rising motion above the surface low
Subsidence above the surface High.



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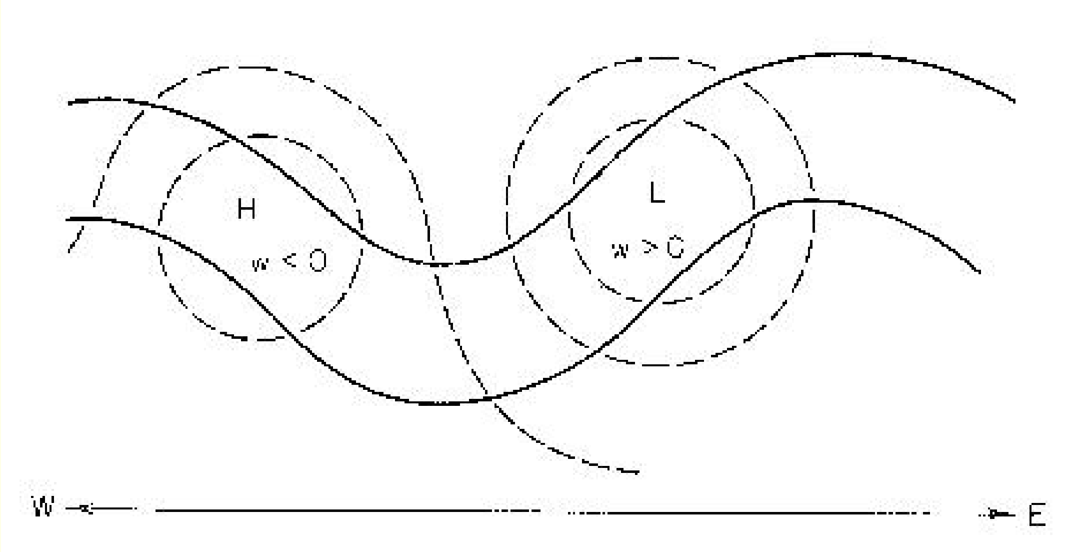
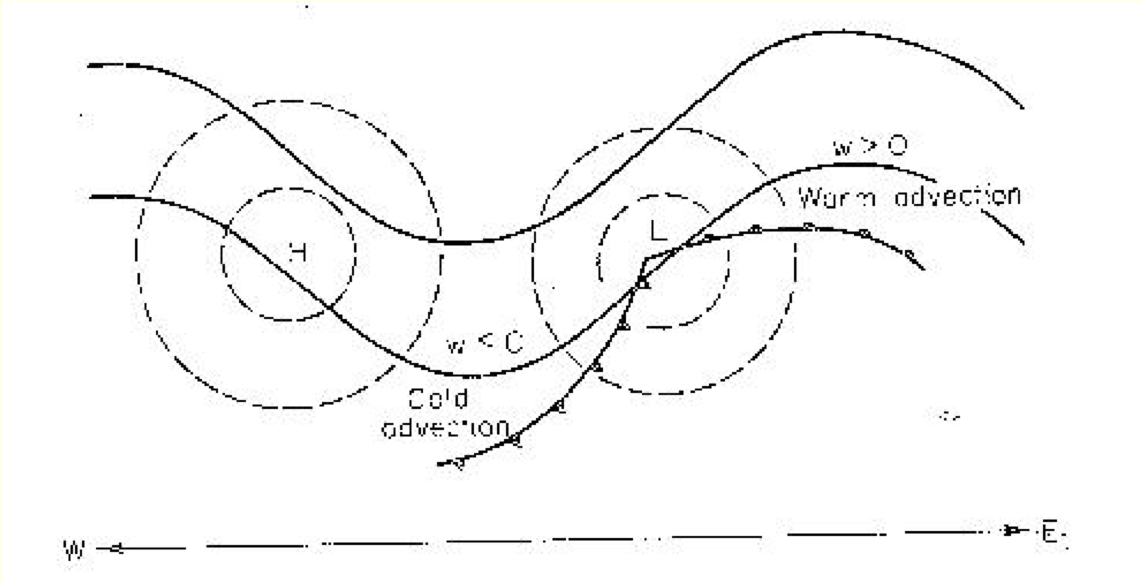
- Ahead of the surface Low there is warm advection
- Therefore, Term (C) is positive
- So, there is Rising Motion ahead of the Low centre.
- Behind the Low, the cold front is associated with cold advection
- Hence, there is subsidence at the 500 hPa trough



Vertical motion due to differential vorticity advection.



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The Tendence Equation allows us to predict the evolution of the mass field and the diagnostic relationships then yield all the other fields.

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Note the complimentarity between these two equations.

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- **How is the vertical velocity correlated with the geopotential field?**