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(A) Growth by Condensation

We saw from Kelvin's Equation that, if the supersaturation is large enough to activate a droplet, the droplet will continue to grow. We will now consider the rate at which such a droplet grows by condensation.

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Then the rate of increase in the mass M of the droplet is given by

$$\frac{dM}{dt} = 4\pi R^2 D \frac{d\rho_v}{dR}$$

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Since, under steady-state conditions, dM/dt is independent of R, the above equation can be integrated as follows

$$\frac{dM}{dt} \int_{R=r}^{R=\infty} \frac{dR}{R^2} = 4\pi D \int_{\rho_v(r)}^{\rho_v(\infty)} d\rho_v$$

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This gives

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Substituting $M = \frac{4}{3}\pi r^3 \rho_\ell$, where ρ_ℓ is the density of liquid water, into this last expression, we obtain

$$r\frac{dr}{dt} = \frac{D}{\rho_\ell}[\rho_v(\infty) - \rho_v(r)]$$

Finally, using the ideal gas equation for water vapour,

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If e **is not too different from** e_s**, then**

$$\frac{e(\infty) - e(r)}{e(\infty)} \approx \frac{e(\infty) - e_s}{e_s} = \left(\frac{e(\infty)}{e_s} - 1\right) = S$$

where S **is the supersaturation of the ambient air (expressed as a fraction rather than a percentage).**

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Hence, we get

$$r\frac{dr}{dt} = G_\ell S$$

where $G_\ell = D\rho_v(\infty)/\rho_\ell$.

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$$r\,dr = G_\ell S\,dt$$

which can be integrated immediately to give

$$r = \sqrt{2G_\ell S t} \qquad \textbf{so that} \qquad r \propto t^{1/2}$$

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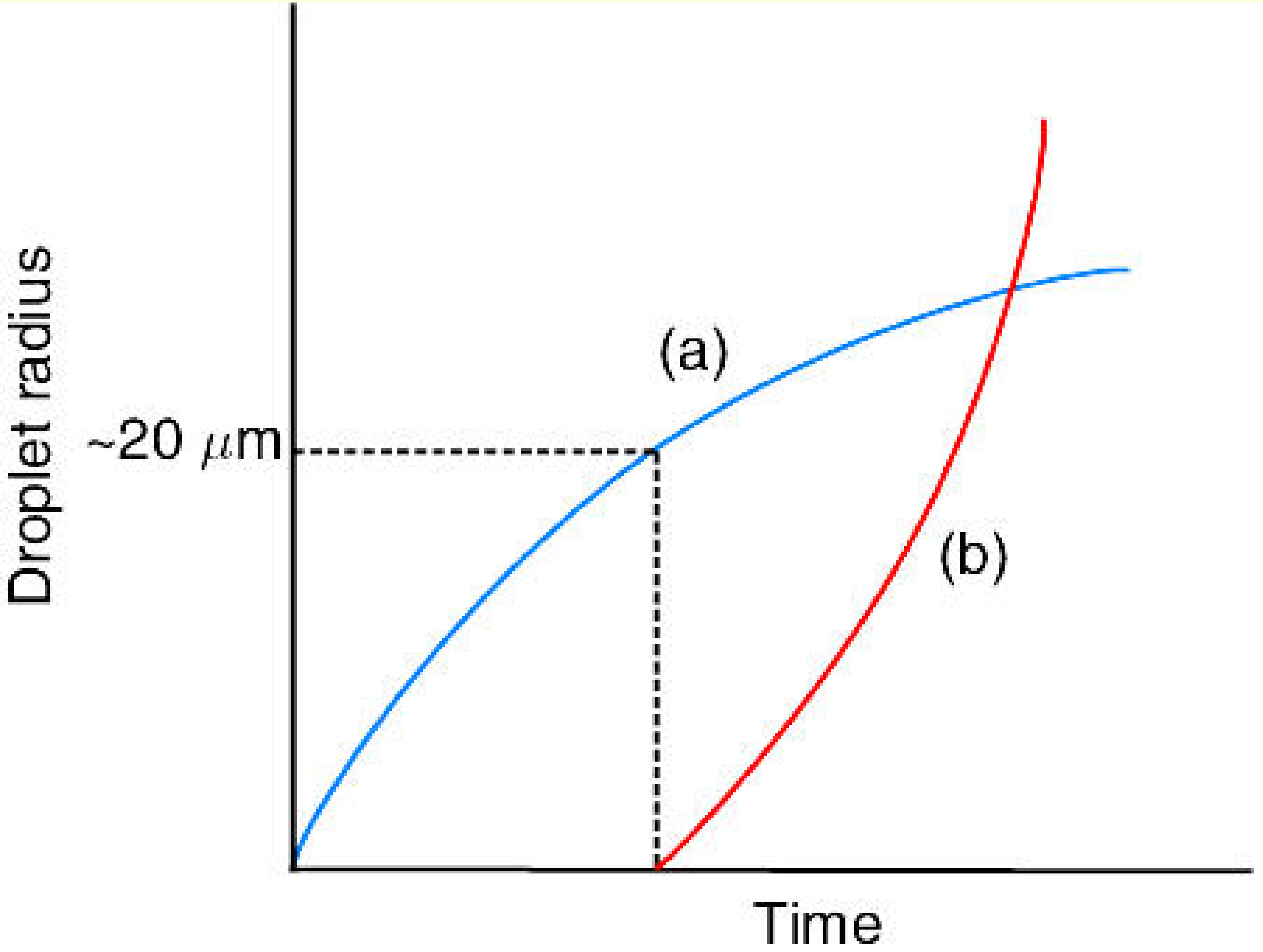
Thus, droplets growing by condensation initially increase in radius very rapidly but their rate of growth diminishes with time (see following figure).



Schematic curves of droplet growth (a) by condensation from the vapour phase (blue curve) and (b) by collection of droplets (red curve).

Since the rate of growth of a droplet by condensation is inversely proportional to its radius, the *smaller activated droplets grow faster than the larger droplets*.

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Consequently, in this simplified model, the sizes of the droplets in the cloud become *increasingly uniform with time* (that is, the droplets approach a *monodispersed distribution*).

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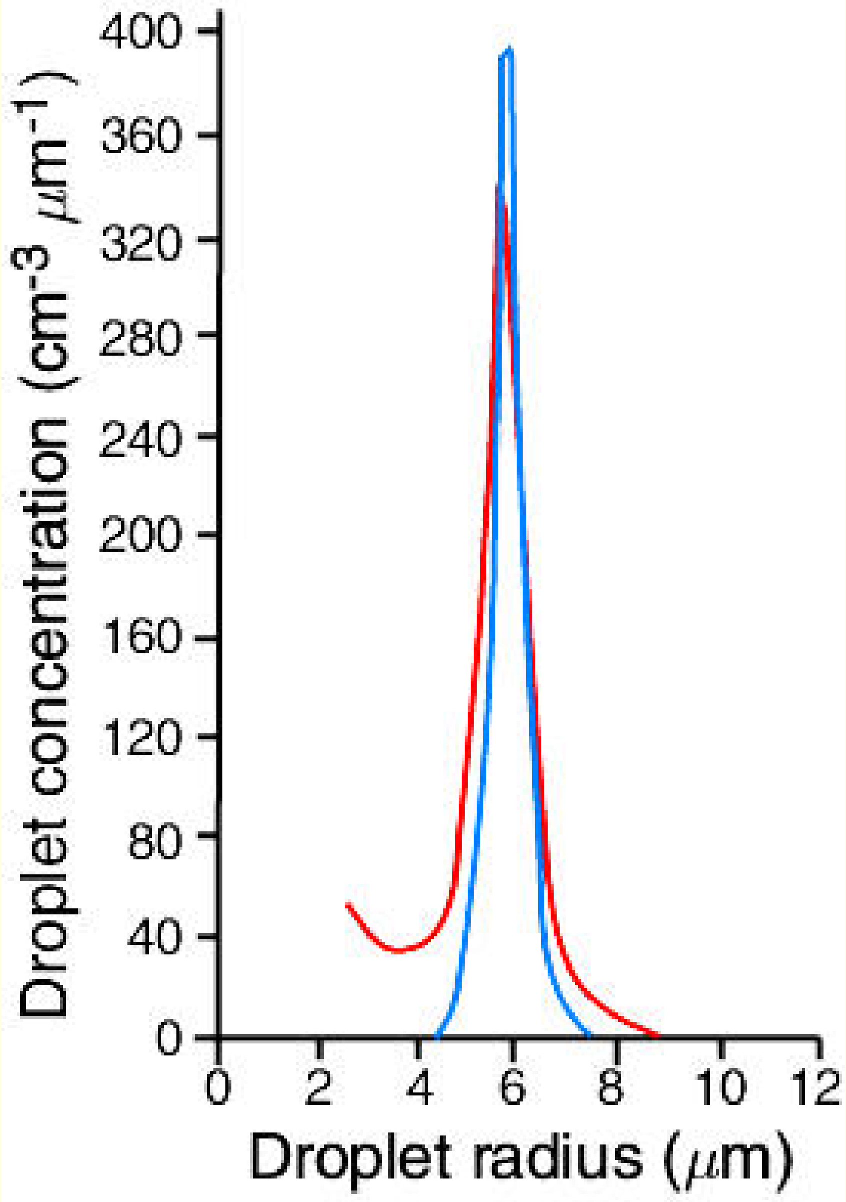
Comparisons of cloud droplet size distributions measured a few hundred meters above the bases of non-precipitating warm cumulus clouds with droplet size distributions computed assuming growth by condensation for about 5 min show good agreement (figure follows).



Cloud droplet size distribution measured 244 m above the base of a warm cumulus cloud (red) and the corresponding computed droplet size distribution assuming growth by condensation only (blue).

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For a cloud droplet $10\,\mu$m in radius to grow to a raindrop 1 mm in radius, an increase in volume of one millionfold is required! However, only about one droplet in a million (about 1 liter^{-1}) in a cloud has to grow by this amount for the cloud to rain.

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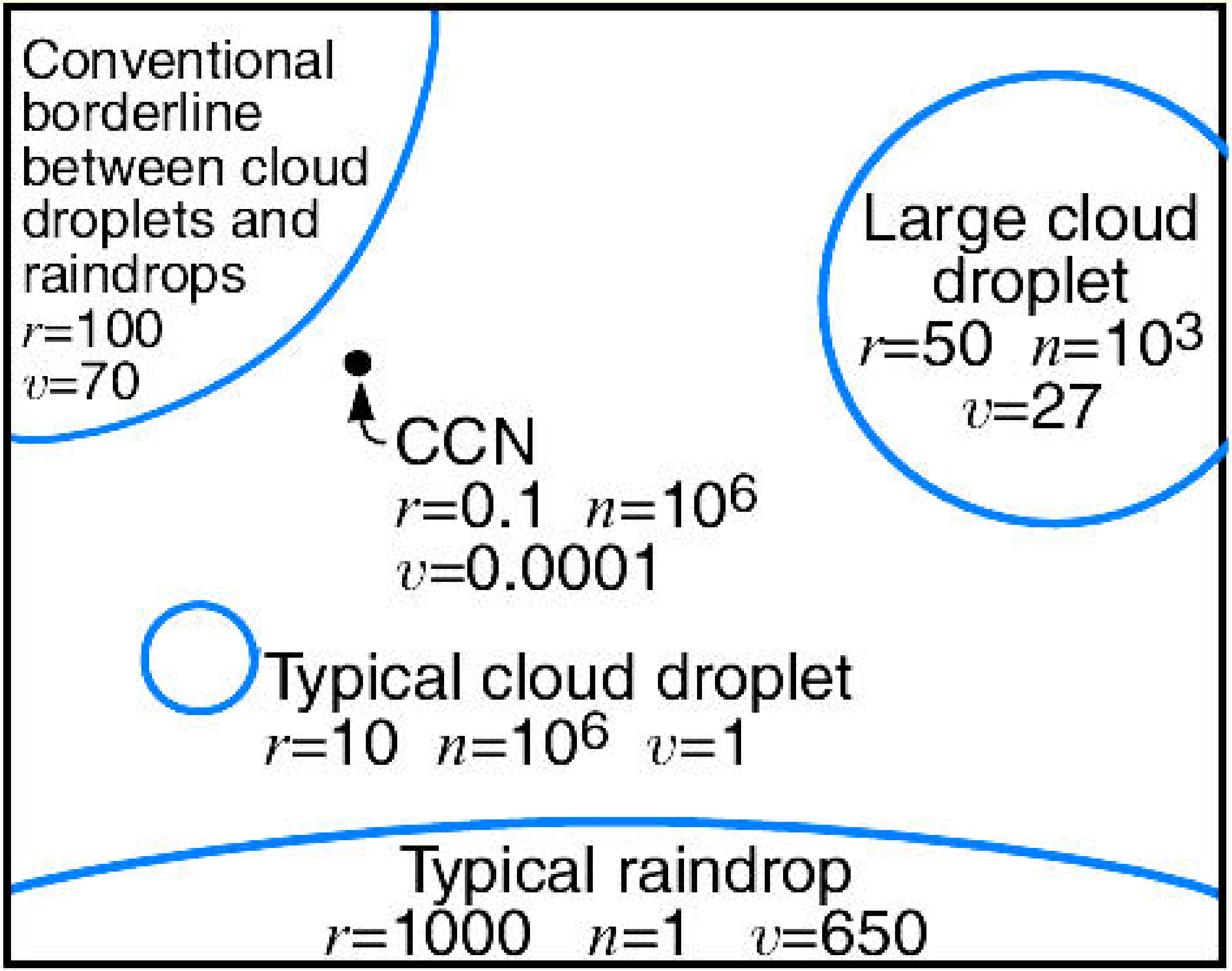
The enormous increases in size required to transform cloud droplets into raindrops is illustrated by the next diagram.



Relative sizes of cloud droplets and raindrops; r is the radius in micrometers, n the number per liter of air, and v the terminal fall speed in centimeters per second.

(B) Growth by Collection

In warm clouds the growth of some droplets from the relatively small sizes achieved by condensation to the sizes of raindrops is achieved by the *collision* and *coalescence* of droplets.

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Since the *terminal fall speed* increases with the size of the droplet, larger droplets have a higher than average terminal fall speed.

Thus, they will collide with smaller droplets lying in their paths.

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In addition, the air exerts a drag force F_{drag} on the body, which acts upwards.

The body will attain a steady terminal fall speed when these three forces are in balance:

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From the above equations, it follows that

$$v = \frac{2}{9}\frac{g(\rho' - \rho)r^2}{\eta}$$

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This is because as a drop increases in size it becomes *increasingly non-spherical* and has an increasing wake. This gives rise to a drag force that is much greater than that given above.

Collision and Coalescence

Consider a single drop of radius r_1 that is overtaking a smaller droplet of radius r_2.

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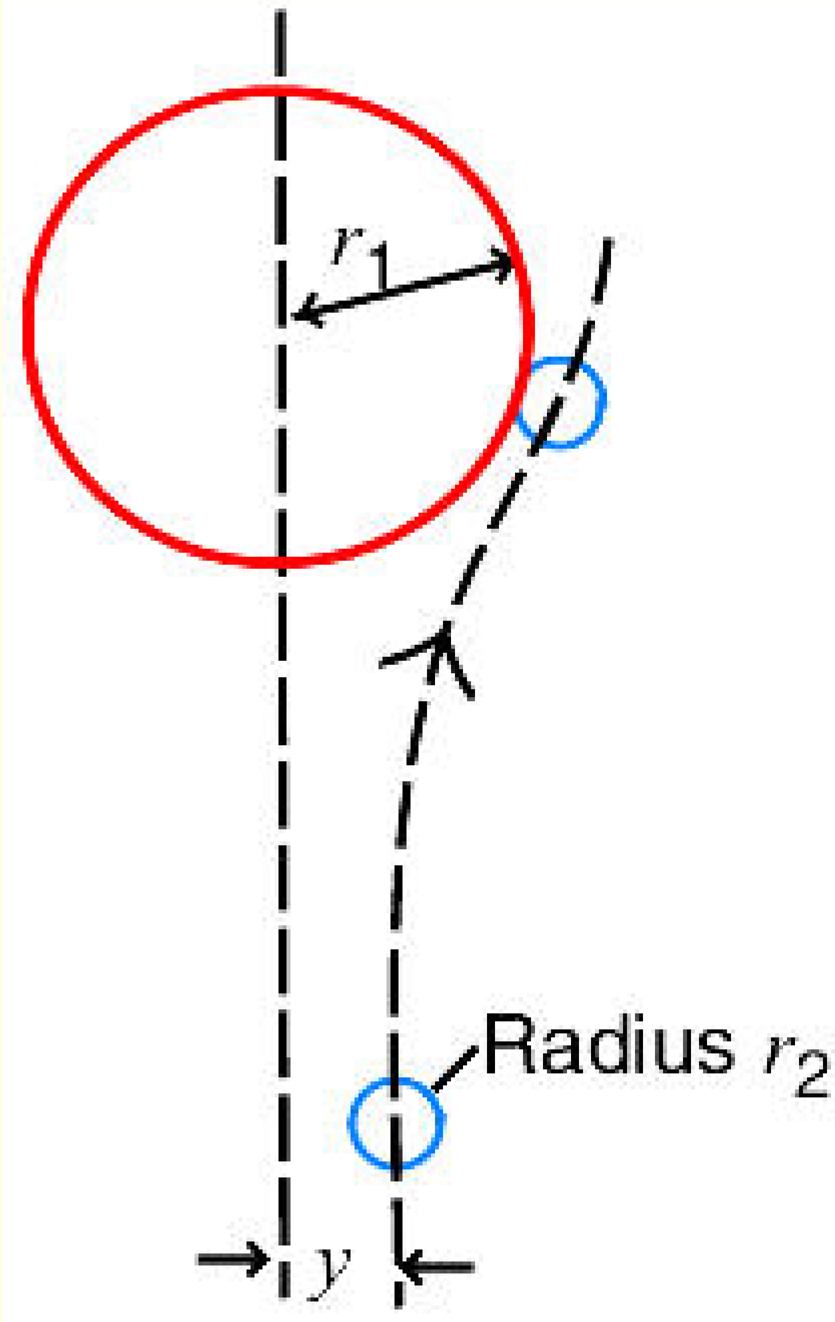
The *collision efficiency* E of a droplet of radius r_2 with a drop of radius r_1 is defined as

$$E = \frac{y^2}{(r_1 + r_2)^2}$$

where y is the distance from the central line for which the droplet just makes a grazing collision with the large drop (see Figure).



Relative motion of a small droplet (blue) with respect to a collector drop (red). y **is the maximum impact parameter for a droplet (radius** r_2**) with a collector drop (radius** r_1**).**

The next issue is whether or not a droplet is captured (i.e., does *coalescence* occur?) when it collides with a larger drop. Droplets can bounce off one another or off a plane surface of water, as illustrated below.

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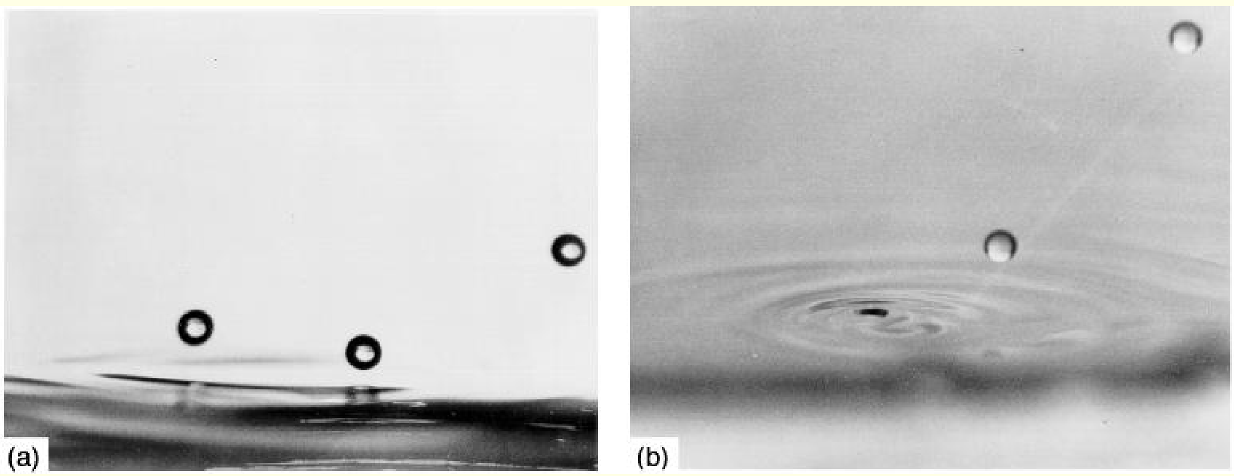

Left: A stream of water droplets, about 100 μm in diameter, rebounding from a plane surface of water. Right: When the angle between the stream of droplets and the surface of the water is increased beyond a critical value, the droplets coalesce with the water.

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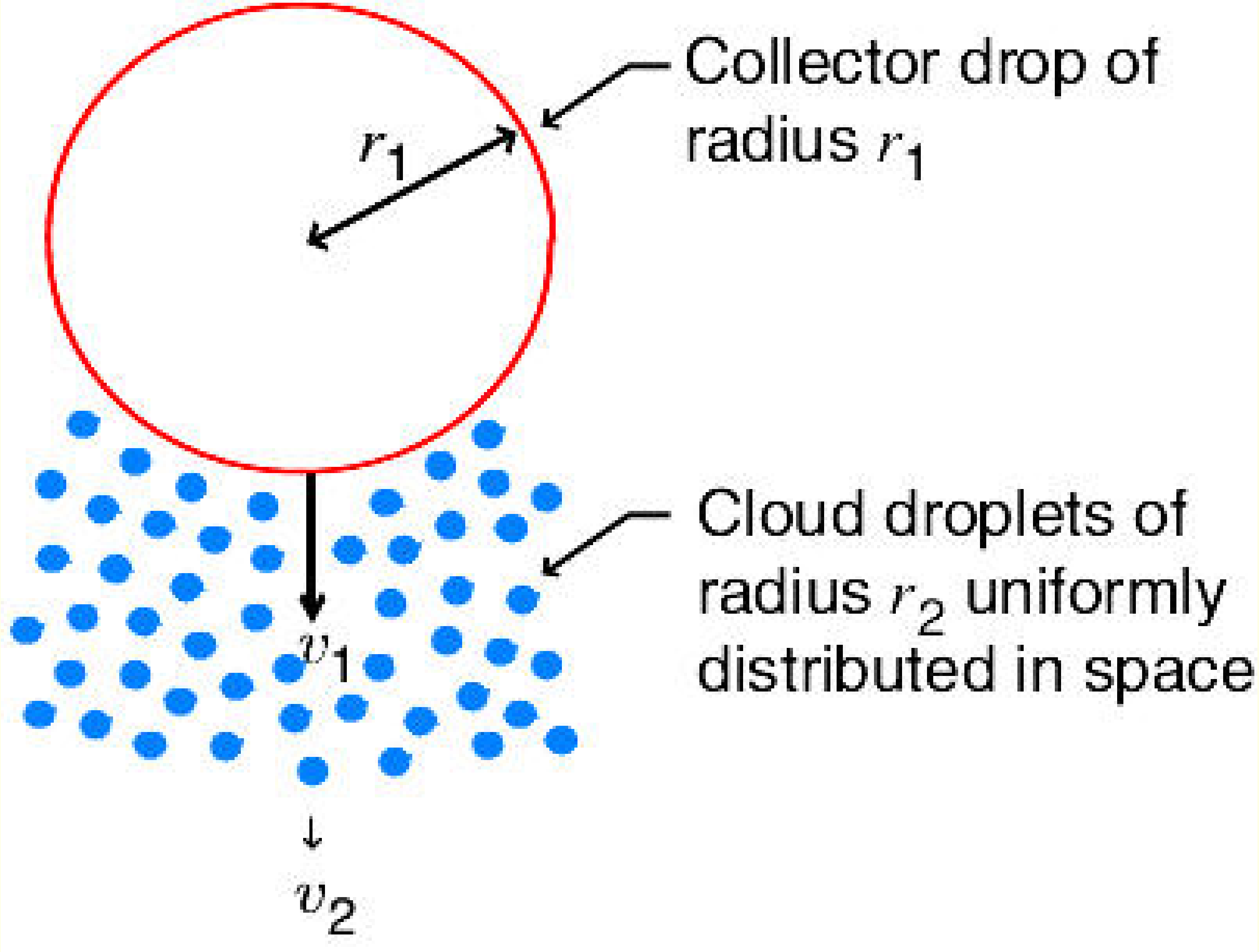
This so-called *continuous collection model* is illustrated in the following diagram.



Schematic to illustrate the continuous collection model for the growth of a cloud drop by collisions and coalescence.

The *rate of increase in mass* M of the large drop due to collisions is given by

$$\frac{dM}{dt} = \pi r_1^2 (v_1 - v_2) w_\ell E_c$$

where w_ℓ is the LWC (in $\mathrm{kg\,m^{-3}}$) of the cloud droplets of radius r_2.

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Substituting $M = \frac{4}{3}\pi r_1^3 \rho_\ell$ here, where ρ_ℓ is the density of liquid water, we obtain

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If $v_1 \gg v_2$ and we assume that the coalescence effciency is unity, then

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The *rate of increase in mass* M of the large drop due to collisions is given by

$$\frac{dM}{dt} = \pi r_1^2 (v_1 - v_2) w_\ell E_c$$

where w_ℓ is the LWC (in $\mathrm{kg\,m^{-3}}$) of the cloud droplets of radius r_2.

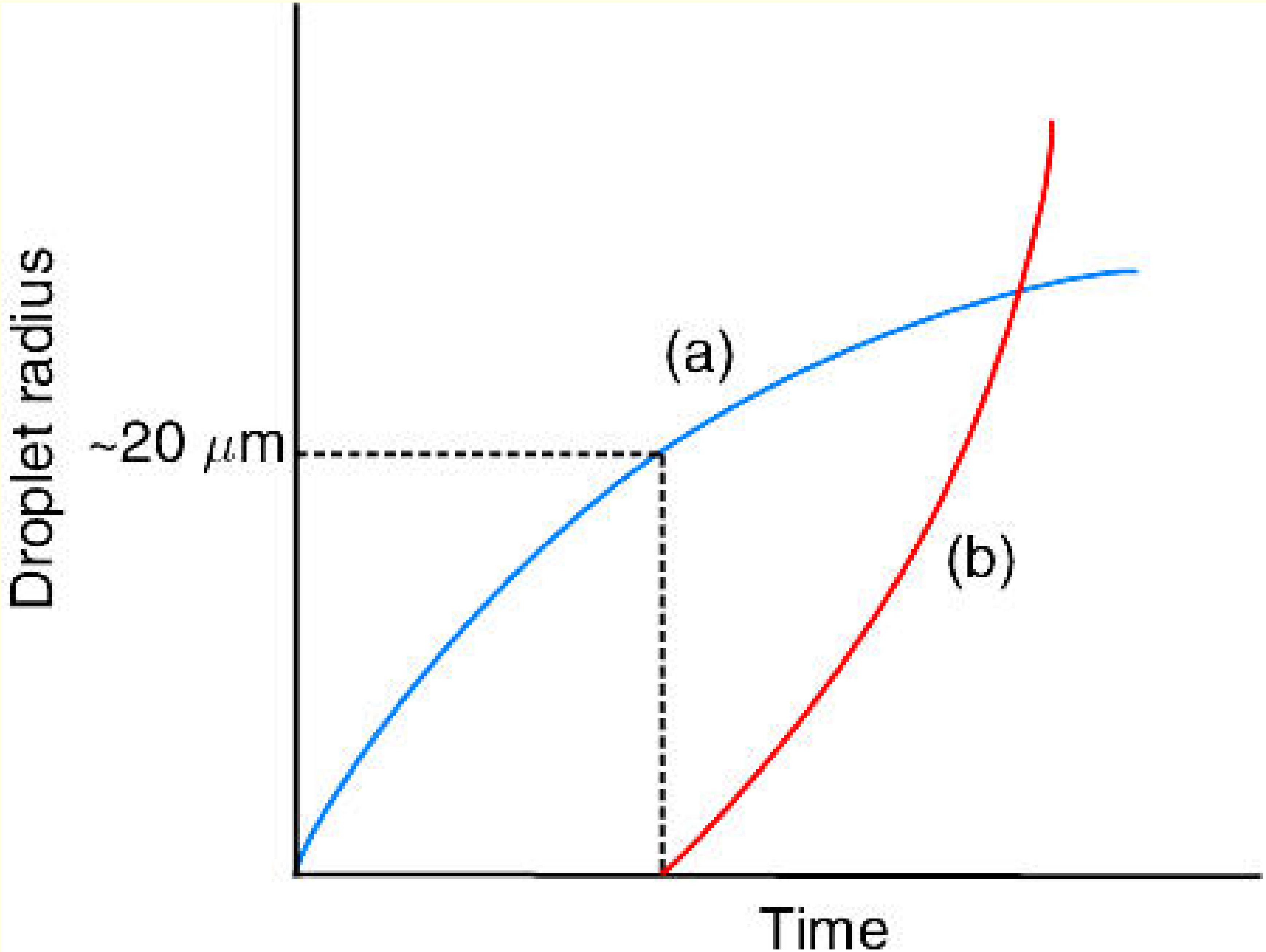
Substituting $M = \frac{4}{3}\pi r_1^3 \rho_\ell$ here, where ρ_ℓ is the density of liquid water, we obtain

$$\frac{dr_1}{dt} = \frac{(v_1 - v_2) w_\ell E_c}{4\rho_\ell}$$

If $v_1 \gg v_2$ and we assume that the coalescence effciency is unity, then

$$\frac{dr_1}{dt} = \frac{v_1 w_\ell E}{4\rho_\ell}$$

Since v_1 and E both increase as r_1 increases, it follows that dr_1/dt increases with increasing r_1; that is, the growth of a drop by collection is an *accelerating process*.



Schematic curves of droplet growth (a) by condensation from the vapour phase (blue curve) and (b) by collection of droplets (red curve).

This *“accelerating”* behavior is illustrated by the red curve in the figure above, which indicates negligible growth by collection until the collector drop has reached a radius of about 20 μm.

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Eventually, as the drop grows, v_1 becomes greater than the updraft velocity w and the drop begins to fall through the updraft and will eventually pass through the cloud base and *may reach the ground as a raindrop*.

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Clearly, deep clouds with strong updrafts should produce rain quicker than shallower clouds with weak updrafts.

Computer simulation

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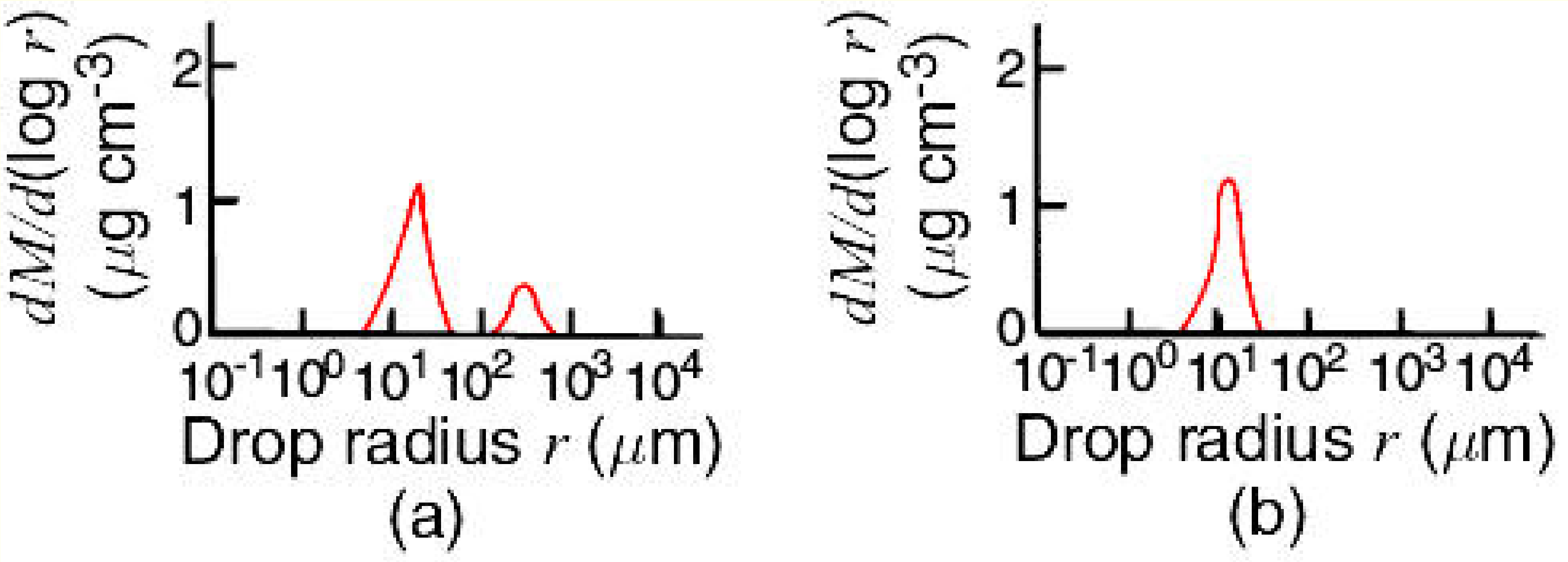

Numerical predictions of the mass spectrum of drops in
(a) a warm marine cumulus cloud, and
(b) a warm continental cumulus cloud
after about one hour of growth.

The Figure above illustrates the effects of these differences in cloud microstructures on the development of larger drops.

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These model results support the observation that a *marine cumulus cloud is more likely to rain* than a continental cumulus cloud with similar updraft velocity, LWC and depth.

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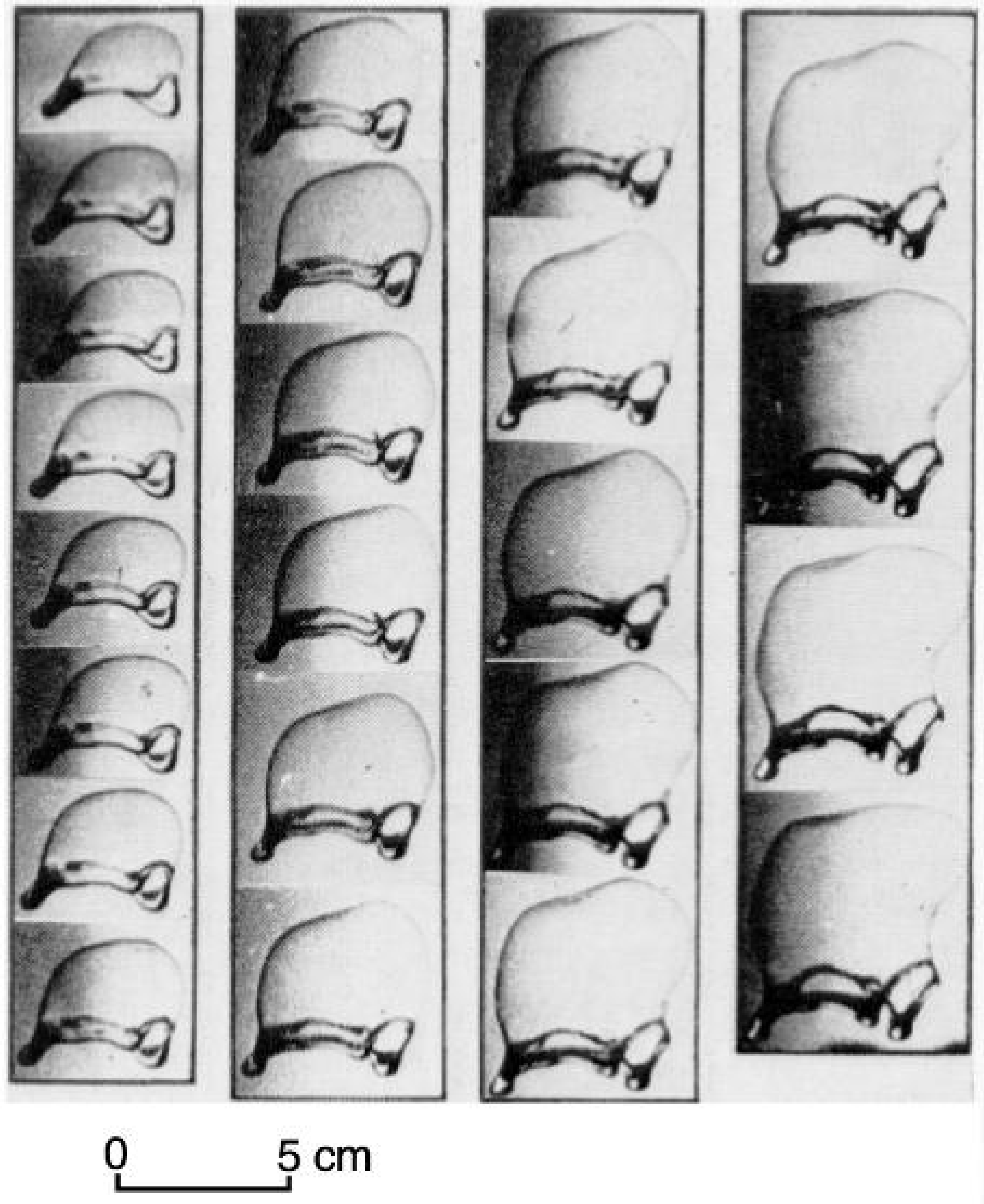
If the initial radius of the drop exceeds about 2.5 mm, the shape becomes a large inverted bag, with a toroidal ring of water around its lower rim.

Laboratory and theoretical studies indicate that when the bag bursts, it produces a fine spray of droplets and the toroidal ring breaks up into a number of large drops (see Figure to follow).



Sequence of high-speed photographs showing how a large drop in free fall forms a parachute-like shape with a toroidal ring of water around its lower rim. Time interval between photographs = 1 ms.

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Zipf's Law: In the English language, the probability of encountering the nth most common word is given roughly by $P(n) = 0.1/n$ for n up to 1000 or so. The law breaks down for less frequent words, since the harmonic series diverges.