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Thus, the mean of a product has two components:

$$\begin{aligned}
\overline{w\theta} &= \overline{(\overline{w} + w')(\overline{\theta} + \theta')} \\
&= \overline{(\overline{w}\overline{\theta} + \overline{w}\theta' + w'\overline{\theta} + w'\theta')} \\
&= \overline{\overline{w}\overline{\theta}} + \overline{\overline{w}\theta'} + \overline{w'\overline{\theta}} + \overline{w'\theta'} \\
&= \overline{w}\overline{\theta} + \overline{w'\theta'}
\end{aligned}$$

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$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

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The total time derivative of u is

$$\begin{aligned} \frac{du}{dt} &= \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} + u\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) \\ &= \frac{\partial u}{\partial t} + \frac{\partial uu}{\partial x} + \frac{\partial uv}{\partial y} + \frac{\partial uw}{\partial z} \end{aligned}$$

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Writing sums of mean and eddy parts and averaging:

$$\overline{\frac{du}{dt}} = \frac{\partial \overline{u}}{\partial t} + \frac{\partial}{\partial x}\left(\overline{u}\,\overline{u} + \overline{u'u'}\right) + \frac{\partial}{\partial y}\left(\overline{u}\,\overline{v} + \overline{u'v'}\right) + \frac{\partial}{\partial z}\left(\overline{u}\,\overline{w} + \overline{u'w'}\right)$$

Repeat:

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$$\frac{\partial}{\partial x}\left(\overline{u}\,\overline{u}\right) + \frac{\partial}{\partial y}\left(\overline{u}\,\overline{v}\right) + \frac{\partial}{\partial z}\left(\overline{u}\,\overline{w}\right) = \overline{u}\frac{\partial \overline{u}}{\partial x} + \overline{v}\frac{\partial \overline{u}}{\partial y} + \overline{w}\frac{\partial \overline{u}}{\partial z}$$

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Now we can write the momentum equations as

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The thermodynamic equation may be expressed in a similar way:

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For example, $\overline{u'w'}$ is the vertical turbulent flux of zonal momentum, and $\overline{w'\theta'}$ is the vertical turbulent heat flux.

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In the free atmosphere, the terms in square brackets are sufficiently small that they can be neglected.

Within the boundary layer, the turbulent flux terms are comparable in magnitude to the remaining terms, and must be included in the analysis.

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Then the complete system of equations becomes

$$\frac{\overline{d}\overline{u}}{dt} - f\overline{v} + \frac{1}{\rho_0}\frac{\partial\overline{p}}{\partial x} + \frac{\partial}{\partial z}\left(\overline{w'u'}\right) = 0$$

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$$\frac{\partial\overline{p}}{\partial z} + g\rho_0 = 0$$

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We note that this system of five equations, for the variables $(\overline{u}, \overline{v}, \overline{w}, \overline{p}, \overline{\theta})$, also contains the variables (u', v', w', θ') in the turbulent fluxes.

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Next, we will consider a simple parameterization of the vertical momentum flux, $\partial(\overline{w'u'})/\partial z$.

Parameterization of Eddy Fluxes

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$$\frac{du}{dt} - fv + \frac{1}{\rho_0}\frac{\partial p}{\partial x} + \frac{\partial}{\partial z}\left(\overline{w'u'}\right) = 0$$

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In the free atmosphere, scale analysis shows that the inertial terms and turbulent flux terms are small compared to the Coriolis term and pressure gradient terms.

This leads us to the relation for geostrophic balance:

$$u \approx u_G \equiv -\frac{1}{f\rho_0}\frac{\partial p}{\partial y}, \qquad v \approx v_G \equiv +\frac{1}{f\rho_0}\frac{\partial p}{\partial x}$$

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Thus we get a three-way balance between the forces:

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-fv + \frac{1}{\rho_0}\frac{\partial p}{\partial x} + \frac{\partial}{\partial z}\left(\overline{w'u'}\right) &= 0 \\
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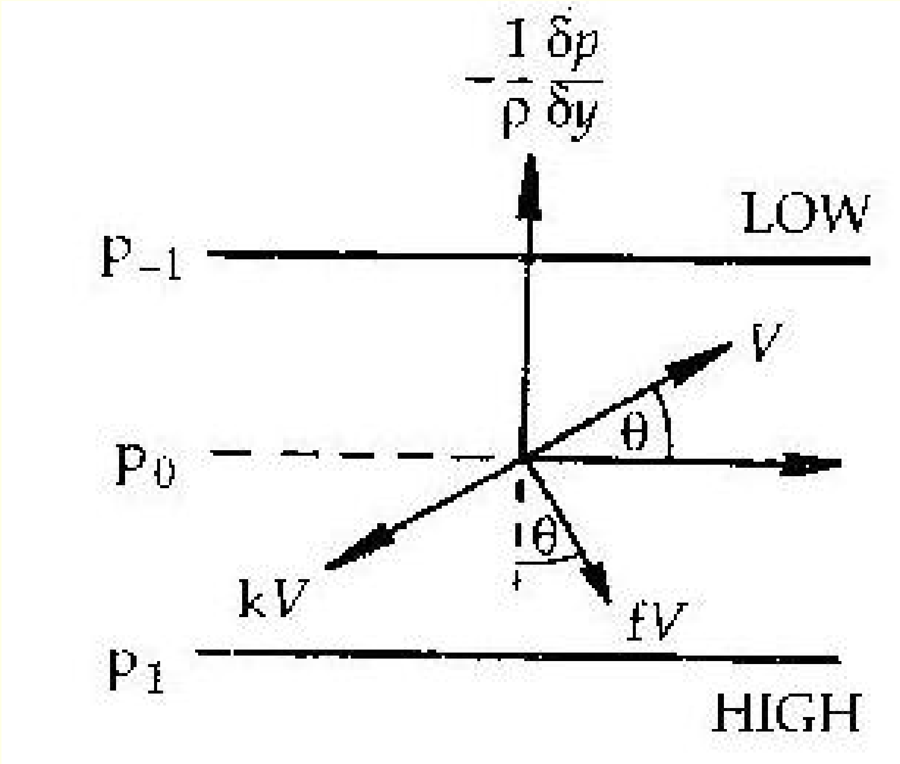

Using the definition of the geostrophic winds, we write the momentum equations as

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-f(v - v_G) + \frac{\partial}{\partial z}\left(\overline{w'u'}\right) &= 0 \\
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Thus, the flux of momentum is assumed to be proportional to the vertical gradient of the mean momentum. Then we can write

$$\overline{u'w'} = -K\left(\frac{\partial \overline{u}}{\partial z}\right)$$

where K is called the *eddy viscosity coefficient*.

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We will assume that K is constant. Thus, for example,

$$\frac{\partial}{\partial z}\left(\overline{w'u'}\right) = -\frac{\partial}{\partial z}\left(K\frac{\partial u}{\partial z}\right) = -K\frac{\partial^2 u}{\partial z^2}$$

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The momentum equations then become

$$-f(v - v_G) - K\frac{\partial^2 u}{\partial z^2} = 0$$

$$+f(u - u_G) - K\frac{\partial^2 v}{\partial z^2} = 0$$

In the following lecture, we will use these equations to model the *Ekman Layer*.

End of §5.3